Letter

Exponentially faster preparation of quantum dimers via driven-dissipative stabilization

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We propose a rapid, high-fidelity, and noise-resistant scheme to generate many-body entanglement between multiple qubits stabilized by dissipation into a 1D bath. Using a carefully designed time-dependent drive, our scheme achieves a provably exponential speedup over state-of-the-art dissipative stabilization schemes in 1D baths, which require a timescale that diverges as the target fidelity approaches unity and scales exponentially with the number of qubits. To prepare quantum dimer pairs, our scheme only requires local 2-qubit control Hamiltonians, with a protocol time that is independent of system size. This provides a scalable and robust protocol for generating a large number of entangled dimer pairs on-demand, serving as a fundamental resource for many quantum metrology and quantum information processing tasks.

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Introduction. Entangled quantum states are essential for quantum computation [1] and metrology [2], which demand their high fidelity generation in a way that is resilient to noise and dissipation. Dissipation, once seen as detrimental, is now explored as a resource for entanglement generation [3]. However, despite a plethora of theoretical proposals and experimental realizations for generating entangled states with cavity quantum electrodynamics (QED) systems [3–7], ion traps [8–10], Rydberg atoms [11–14], color centers [15–18], circuit QED [19,20], and optical lattices and spin chains [21–25], limitations persist in either the speed of state generation, entanglement fidelity or the aforementioned robustness to noise and dissipation. For instance, the dissipative entanglement generation schemes based on Ref. [3] rely on perturbative expansions in the system's driving strengths, which fundamentally limits the speed of entanglement generation.

It was also shown in [26,27] that when multiple locally driven system qubits are coupled to a chiral 1D bath (which could either be a waveguide or a spin chain), one can obtain many-body entangled states stabilized by the dissipation into the 1D bath. In this theoretical scheme, no perturbative expansions in the system's driving strengths are required, which circumvents the aforementioned speed limit. An atomic implementation of this scheme on cold quantum gases was proposed in [21], and experimentally implemented recently on superconducting qubits [28].

However, as we will demonstrate in this manuscript, time-independent many-body entanglement generation schemes using engineered dissipation as proposed in [21,26,27] require a timescale that diverges as the target fidelity approaches unity, leading to an inevitable tradeoff between fidelity and speed. Furthermore, for existing steady state schemes including [29], the protocol time scales exponentially with the number of qubits. This presents a severe limitation for scaling up to many qubits, especially in the presence of noise. We propose a new scalable protocol based on carefully designed time-dependent driving to generate many-body entanglement in 1D systems in a fast, high-fidelity, and noise-robust manner.

An important application of our scheme is in preparing a large number of quantum dimer pairs on demand, which are valuable resource states for various quantum technologies such as quantum metrology [29,30] and quantum information processing. Our scheme achieves a high-fidelity preparation using only local 2-qubit control Hamiltonians, rendering it feasible to current experimental capabilities. Crucially, our protocol time is independent of the number of qubits, thereby exponentially faster than the previously proposed schemes [21,26,27,29]. We perform a systematic study of robustness of our scheme against various sources of noise and decoherence. We show that in the presence of any amount of spontaneous decay outside of the 1D bath, previous time-independent schemes eventually fail for a sufficiently large number of qubits due to the exponentially long timescales required. On the contrary, our scheme is robust against such losses for any number of qubits.

Many-body entangled dark states of 1D systems. In waveguide QED, one often considers the case where there are N qubits coupled to a 1D bath [27,31]. The 1D bath serves first

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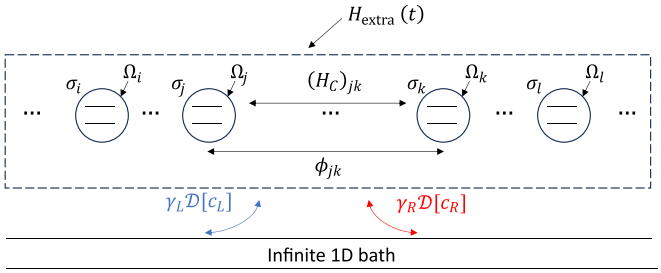

FIG. 1. Schematic for the setup described by Eqs. (1) and (2). N qubits are coupled to a waveguide as per [26] or to a 1D spin chain with a synthetic gauge field as per [27]. Here, σ_j is the lowering operator for the jth system qubit that is driven with an external local drive Ω_j. ϕ_{jk} describes the phase picked up by the bath excitation as it travels between the jth and kth qubit along the infinite 1D bath, which affects the bath-mediated chiral interaction between the jth and kth qubit. H_{extra} is the extra external field in our scheme which we will introduce later. All the qubits decay collectively into the 1D bath through collective jump operators c_L (left-going modes) and c_R (right-going modes) with decay rates γ_L and γ_R, respectively.

as a decay channel for the system qubit excitations, and second to mediate long-distance coherent interactions between the system qubits. With reference to Fig 1, under the Born-Markov and rotating wave approximations, by tracing out the 1D bath, we obtain the following Hamiltonian (setting $\hbar = 1$) for the N system qubits:

$$H_S = -\sum_{i=1}^{N} \delta_i \sigma_i^\dagger \sigma_i + H_{\text{drive}}(t) + \sum_{j<k} (H_C)_{jk}, \tag{1}$$

where $(H_C)_{jk} = \frac{i}{2}(\gamma_R e^{-i\phi_{jk}} - \gamma_L e^{i\phi_{jk}})\sigma_j^\dagger \sigma_k + \text{H.c.}$ describes the coherent interaction mediated by the 1D bath between the jth and kth system qubits, $H_{\text{drive}}(t) = \sum_{i=1}^{N} (\Omega_i(t)/2)\sigma_i + \text{H.c.}$ describes the local driving on the qubits with Rabi frequency $\Omega_i(t)$, and δ_i describes the detuning between the ith qubit and the carrier frequency of the 1D bath. The dissipation of the system into the 1D bath is described by a master equation for the N system qubits [26,27]

$$\dot{\rho} = -i[H_S, \rho] + \gamma_L \mathcal{D}[c_L]\rho + +\gamma_R \mathcal{D}[c_R]\rho. \tag{2}$$

Here, $\mathcal{D}[c_{L(R)}]\rho = c_{L(R)}\rho c_{L(R)}^\dagger - \{c_{L(R)}^\dagger c_{L(R)}, \rho\}/2$ describes the leftward (rightward) dissipation of the system qubits into the bath, where $c_L = \sum_{j=1}^{N} e^{i\phi_j}\sigma_j$, $c_R = \sum_{j=1}^{N} e^{-i\phi_j}\sigma_j$ are the collective jump operators. The system is chiral if $\gamma_L \neq \gamma_R$, physically manifesting as an asymmetric emission into the bath. While entanglement generation schemes which operate in the transient regime for these 1D systems have been proposed [32,33], a higher fidelity that is also stabilized by the dissipation into the bath can be attained in the steady state [26,29]. In particular, it was shown [26] that when $\phi_{jk} \bmod 2\pi = 0$, together with certain conditions on δ_i (or in the chiral case $\gamma_L \neq \gamma_R$) with homogeneous time-independent driving $\Omega_i(t) = \Omega$, it is possible to obtain the following multipartite entangled dark steady state for even N :

$$\rho_{ss} = |\Phi\rangle\langle\Phi|, \quad \text{where } |\Phi\rangle = \prod_{q=1}^{N_m} |M_q\rangle \tag{3a}$$

$$|M_q\rangle = a^{(0)}|g\rangle^{\otimes M_q} + \sum_{j_1<j_2} a^{(1)}_{j_1,j_2}|S\rangle_{j_1 j_2}|g\rangle^{\otimes M_q-2} + \cdots + \sum a^{(M_q/2)}_{j_1,\ldots j_{M_q}}|S\rangle_{j_1 j_2}\ldots|S\rangle_{j_{M_q-1}j_{M_q}}. \tag{3b}$$

We define $|S\rangle_{ij} = (|e\rangle_i|g\rangle_j - |g\rangle_i|e\rangle_j)/\sqrt{2}$ as a singlet state (or a dimer pair) between qubits i and j. $|\Phi\rangle$ is a product of N_m adjacent multimers $|M_q\rangle$, and each $|M_q\rangle$ is an entangled state over M_q qubits as defined in Eq. (3b), where M_q is an even integer. Note that the summation in the last line of Eq. (3b) runs over all different pairings of qubits $\{(j_1, j_2), \ldots (j_{M_q-1}, j_{M_q})\}$ with $j_k < j_{k+1}$. It can also be shown that $a^{(i)} \propto |\Omega|^{-M_q/2+i}$ [26]. In the above equation, of particular interest is the $N_m = 1$, $M_q = N$ case, since that corresponds to the maximal genuine entanglement (across all bipartite cuts of qubits). We also consider $|\Omega| \to \infty$, since it is the most relevant for metrology [29,30]. Hence, we shall focus on obtaining the state

$$|\Phi\rangle \propto \sum |S\rangle_{i_1 i_2}|S\rangle_{i_3 i_4}\ldots|S\rangle_{i_{N-1}i_N}, \tag{4}$$

where the summation in Eq. (4) runs over different pairings of qubits $\{(i_1, i_2), (i_3, i_4), \ldots (i_{N-1}, i_N)\}$ where $i_j < i_{j+1}$. By a suitable detuning pattern, it is also possible to obtain the special case where there is only one term in the sum, such that the system forms dimerized pairs of qubits in the steady state. However, we will now show that such schemes require a prohibitively long time to generate high-fidelity, many-body entanglement.

Divergent timescale of preparing entangled dark states. As mentioned in [26], the timescale required to form one dimer pair from $N = 2$ qubits diverges as the target fidelity approaches one. This can also be seen by analyzing the Liouvillian gap [34,35] (see the Supplemental Material [36]), but is analytically challenging for large N. By using a recently developed general framework for analyzing quantum speed limits in dissipative state preparation [37], we derive a lower bound on the time T required to generate the state in Eq. (3) for any system size N (see the Supplemental Material [36] for a derivation),

$$T \geqslant T_{\text{QSL}} \propto \prod_{q=1}^{N_m} |\Omega|^{M_q/2} = |\Omega|^{N/2} \sim \left(\frac{1}{1-F}\right)^{N/4}. \tag{5}$$

The preparation time diverges as $|\Omega| \to \infty$, or equivalently as the fidelity F to the target state in Eq. (4) approaches unity. Crucially, for any fixed target fidelity F, the preparation time scales exponentially with the number of qubits N.

In the presence of any spontaneous decay rate Γ_f outside of the 1D bath, the time-independent scheme would fail when the preparation time required exceeds $\sim 1/\Gamma_f$. From Eq. (5), we can estimate that the time-independent scheme fails for $N \gtrsim \log(\Gamma/\Gamma_f)$, where $\Gamma = \gamma_L + \gamma_R$ is the total decay rate into the 1D bath. This can be interpreted as a fundamental trade off between fidelity and speed, and highlights a severe limitation to the scalability of such schemes. We now propose an exponentially faster scheme that circumvents all

these problems while retaining the robustness from dissipative stabilization.

Exponentially faster scheme for many-body entanglement generation. Our scheme deviates from the previously proposed time-independent schemes in two important aspects. First, instead of a time-independent homogeneous drive $\Omega_j = \Omega$, we consider $\Omega_j = \Omega(t)$ such that $\Omega(0) = 0$ and $\Omega(t)$ is any nondecreasing real-valued function of t. Second, all the detunings δ_j are zero, even at zero chirality. In this case, with $\phi_{jk} \bmod 2\pi = 0$, in the master equation, Eq. (2), we have $\gamma_L \mathcal{D}[c_L] + \gamma_R \mathcal{D}[c_R] = \Gamma \mathcal{D}[c]\rho$ where $c = \sum_{j=1}^{N} \sigma_j$, $\Gamma = \gamma_L + \gamma_R$, and $(H_C)_{jk} = (i\Delta\gamma/2)(\sigma_j^\dagger \sigma_k - \sigma_j \sigma_k^\dagger)$ where $\Delta\gamma = \gamma_R - \gamma_L$. We define the total coherent interaction term as $H(t) \equiv H_C + H_{\text{drive}}(t)$, where $H_C = \sum_{j<k} (H_C)_{jk}$.

Our scheme begins by choosing a target state $|\Phi\rangle$ of the form in Eq. (4), where in the summation, we have the freedom to choose which different pairings of qubits to sum over. Let $\theta(\Omega(t))$ be a function where $\theta(\Omega(t) = 0) = 0$, $\theta(\Omega(t) = \infty) = \pi/2$. For example, $\theta(\Omega(t))$ could be

$$\theta(\Omega(t)) = \frac{\pi}{2}(1 - e^{-k\Omega(t)/\Gamma}), \; k > 0, \tag{6}$$

though many other examples exist. The main idea is that both the initial state $|g\rangle^{\otimes N}$ and the target state $|\Phi\rangle$ at $\Omega(t) \to \infty$ are instantaneous steady states, which means that if we can generate the unitary evolution $U(\theta(\Omega))|g\rangle^{\otimes N} = \cos(\theta(\Omega))|g\rangle^{\otimes N} - i\sin(\theta(\Omega))|\Phi\rangle$, then $\Omega \to \infty$ gives us $U(\theta(\Omega))|g\rangle^{\otimes N} = |\Phi\rangle$. In practice, we do not require $\Omega(t) \to \infty$, since at large $\Omega(t)$ such that $\theta(\Omega) = \pi/2 - \epsilon$, $U(\theta)$ already prepares a state $|\psi(\theta)\rangle \equiv U(\theta)|g \ldots g\rangle$ with a fidelity of $F = |\langle\psi(\theta)|\Phi\rangle|^2 = \cos^2(\epsilon) \approx 1 - \epsilon^2$ to $|\Phi\rangle$. Hence, by a judicious choice of $\theta(\Omega(t))$ and $\Omega(t)$, we can achieve a state $|\psi(\theta)\rangle$ that has very high fidelity to $|\Phi\rangle$ at times much shorter than the dissipation timescale Γ^{-1}. Using Eq. (6) as an example, for $k\Omega(t)/\Gamma \approx 4$, we have $F \approx 0.999$. After preparing $|\psi(\theta)\rangle$ at a short time t_f, we keep $\Omega(t > t_f)$ constant. This causes the state to relax toward the steady state close to $|\Phi\rangle$. Thus, our scheme works with a high fidelity even for a finite Ω, rendering its practicality. In short, our scheme moves along a trajectory within the decoherence-free subspace spanned by $|g\rangle^{\otimes N}$ and $|\Phi\rangle$ and is thus dissipation stabilized.

To construct $U(\theta)$, we first define $X \equiv |g^{\otimes N}\rangle\langle\Phi| + |\Phi\rangle\langle g^{\otimes N}|$ and then see that $U(\theta) = \exp(-i\int_0^t (\partial_{t'}\theta) X dt')$ which means that the desired $U(\theta)$ can be generated by the Hamiltonian $H_u(t) = (\partial_t\theta)X$. Thus, we simply need to add an extra time-dependent control field $H_{\text{extra}}(t) \approx H_u(t)$ to our system Hamiltonian $H(t)$. This extra time-dependent control field would only need to be switched on from $t = 0$ to $t = t_f$ for some finite t_f to generate $U(\theta)$, after which the time dependence can be switched off and $\Omega(t)$ held constant. One might be concerned about spurious effects from the coherent interactions mediated by the 1D bath. While this can be entirely mitigated in $H_{\text{extra}}(t)$, we find that it is unnecessary. The validity of the approximation $H_{\text{extra}}(t) \approx H_u(t)$ is discussed in detail in the Supplemental Material [36], but here we note the following two points. First, the approximation is better for a smaller $\Delta\gamma$, with the best case being zero chirality ($\Delta\gamma = 0$). This is actually an advantage when compared to [26] which requires $\Delta\gamma \neq 0$ when all the detunings δ_i are zero. Second, by choosing $\partial_t\theta$ to be as large as possible, we can perform the transformation $|g\rangle^{\otimes N} \to |\psi(\theta)\rangle \approx |\Phi\rangle$ in this decoherence-free subspace arbitrarily quickly, which also improves the approximation $H_{\text{extra}}(t) \approx H_u(t)$.

We stress that while this protocol looks similar to the idea of counterdiabatic driving in decoherence-free subspaces [38–40] due to the presence of an additional time-dependent control Hamiltonian, it is different in many ways. Unlike counterdiabatic driving, the state $|\psi(\theta)\rangle$ does not need to be an instantaneous eigenstate of $H(t)$. In fact, moving along the adiabatic trajectory in the Hilbert space as proposed in [26] requires $\Delta\gamma \neq 0$, whereas our scheme allows for $\Delta\gamma = 0$. Thus, our scheme is fundamentally different from the various shortcut-to-adiabaticity schemes [41]. In our computation of the extra driving field $H_{\text{extra}}(t)$, unlike the various counterdiabatic driving schemes, we do not require all the instantaneous eigenstates of $H(t)$. This is highly advantageous in many situations where an exact diagonalization of $H(t)$ is difficult, such as for large N. More details about the differences between our proposed scheme and counterdiabatic driving can be found in the Supplemental Material [36].

In our scheme, the key part is implementing the X operator, which can be experimentally difficult for certain target states $|\Phi\rangle$ due to the many-body interactions required to generate X. An example for $N = 6$ qubits is shown in the Supplemental Material [36]. However, when $|\Phi\rangle$ describes the state of $N/2$ dimerized pairs, applying the above formalism gives us $U(\theta) = U_{i_1 i_2}(\theta) U_{i_3 i_4}(\theta) \ldots U_{i_{N-1} i_N}(\theta)$ where $U_{i_k i_{k+1}}(\theta) = \cos(\theta)\mathbb{1} - i\sin(\theta) X_{i_k i_{k+1}}$, and $X_{i_k i_{k+1}} = |gg\rangle\langle S|_{i_k i_{k+1}} + |S\rangle\langle gg|_{i_k i_{k+1}}$ is a two-body interaction term between qubits i_k and i_{k+1}. $U(\theta)$ can then be generated by the Hamiltonian $H_u(t) = (\partial_t\theta)X$ where $X = X_{i_1 i_2} + X_{i_2 i_3} + \cdots + X_{i_{N-1} i_N}$. Finally, we have $H_{\text{extra}}(t) \approx H_u(t)$, which means that it suffices for the engineered control Hamiltonian to be 2-qubit interactions. Explicitly, for geometrically local dimer pairs, we have $H_{\text{extra}}(t) \approx \sum_{k\,\text{odd}} V_{k,k+1}$ where $V_{k,k+1} = (\partial_t\theta)(\frac{1}{2}(\sigma_k^x - \sigma_{k+1}^x) + \frac{1}{2}(\sigma_k^x \sigma_{k+1}^z - \sigma_k^z \sigma_{k+1}^x))$. Since the control Hamiltonian is local and can be applied in parallel, our protocol time is independent of N, which is exponentially faster than state-of-the-art time-independent schemes [26,29] while still benefiting from dissipative stabilization.

Figure 2 shows the results of numerical experiments comparing our scheme against previous proposals. We also benchmark our scheme against an adiabatic scheme. As can be seen, at short timescales $\Gamma t \ll 1$, our scheme achieves concurrence $\approx$1 for the case where $\Delta\gamma = 0$ and concurrence $\approx$0.97 for the case where $\Delta\gamma/\Gamma = 1$. On the other hand, the adiabatic scheme fails at timescales $\Gamma t \ll 1$ as the driving strength is modulated too quickly, violating the adiabatic condition for open quantum systems [42]. This is corroborated by a sharp drop in purity between $0 < t < t_f$. After $t > t_f$ where the driving strengths become fixed, the adiabatic and the time-independent schemes become very similar. Our scheme is scalable and can be used to generate many dimer pairs simultaneously.

Robustness analysis. We consider the robustness of our scheme to two types of noise which arise from imperfect control. Let $\xi_1(t)$ and $\xi_2(t)$ be two independent Gaussian white noise random variables with zero mean and unit variance. A stochastic fluctuation in $H_{\text{drive}}(t)$ can be modeled by making the replacement $H_{\text{drive}}(t) \to (1 + \eta_1\xi_1(t))H_{\text{drive}}(t)$. Similarly,

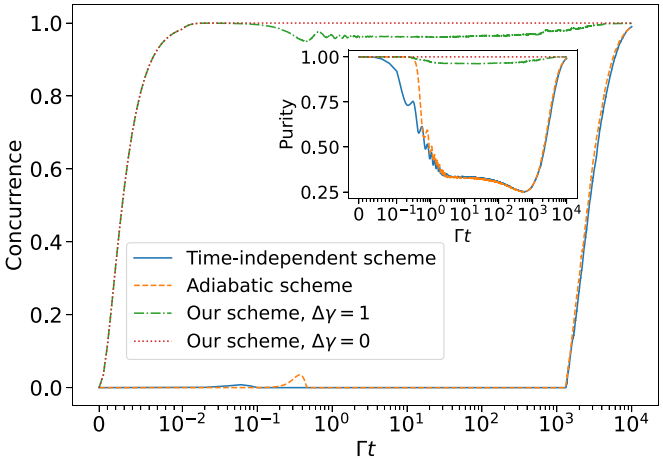

FIG. 2. The case of $N = 8$ qubits forming $N/2 = 4$ geometrically local dimer pairs $\{(1, 2), (3, 4), (5, 6), (7, 8)\}$. Since all dimer pairs are treated equally, we plot the concurrence [43] and the purity (in the inset) between the qubits (1,2) for various entanglement generation schemes mentioned in the main text. For our scheme, we use Eq. (6) with $k = 10$. For both our scheme and the adiabatic scheme, we use the linear ramp function $\Omega(t)/\Gamma = m\Gamma t\Theta(t_f - t) + m\Gamma t_f\Theta(t - t_f)$ saturating at $t = t_f$, where $\Theta(t)$ is the Heaviside step function with $\Theta(0) = 1/2$, and with $m = 25$, $t_f = \Gamma^{-1}$, whereas for the time-independent scheme, we have $\Omega/\Gamma = 25$. For both the adiabatic scheme and the time-independent scheme, we have $\Delta\gamma/\Gamma = 1$, and also the appropriate detuning conditions as proposed in [26]. The adiabatic schemes and the time-independent schemes are very similar after $t > t_f$ because the driving strengths $\Omega(t)$ become fixed after $t > t_f$. Clearly, only our scheme succeeds at short timescales $\Gamma t \ll 1$.

a stochastic fluctuation in $H_{\text{extra}}(t)$ can be modeled by making the replacement $H_{\text{extra}}(t) \to (1 + \eta_2\xi_2(t))H_{\text{extra}}(t)$. Following [41,44], we average over the white noise random variables using Novikov's theorem for white noise [45] to obtain the following modified master equation: $\dot{\rho} = -i[H_{\text{drive}}(t) + H_{\text{extra}}(t), \rho] + \Gamma\mathcal{D}[c]\rho + \eta_1^2\mathcal{D}[H_{\text{drive}}(t)]\rho + \eta_2^2\mathcal{D}[H_{\text{extra}}(t)]\rho$. Using $\theta(\Omega(t))$ from Eq. (6) with $k = 10$, and $\Omega(t) = mt$, $m > 0$, we numerically study the effect of η_i separately in Fig. 3 for the $\Delta\gamma = 0$ case. Our scheme is robust against noise in H_{drive} regardless of how fast $\Omega(t)$ is increased. The reason is that our scheme works as long as $\Omega(t)/\Gamma \gg 1$ at large t, such that the fluctuations $\Omega(t)/\Gamma$ are insignificant. On the other hand, when dealing with noise in $H_{\text{extra}}(t)$, there is a tradeoff between the amount of noise present η_2 and the maximum m allowed such that the concurrence remains high, which can be explained by the adiabatic theorem for open quantum systems [42].

Another common source of noise is spontaneous decay outside of the 1D bath. As discussed earlier, the time-independent schemes fail completely for $N \gtrsim \log(\Gamma/\Gamma_f)$ where Γ_f is the spontaneous decay rate, due to the exponentially long timescales needed. In contrast, our scheme is able to generate quantum dimers with high concurrence for any N on the relevant system timescale Γ^{-1}, as long as $\Gamma_f/\Gamma \ll 1$, which is achievable in current experiments (see the Supplemental Material [36] for more details).

Discussion. We present a scheme for rapid, high fidelity generation of many-body entanglement for qubits coupled to a 1D bath, which is also robust to noise. Our scheme is exponentially faster than previously proposed time-independent schemes in [21,26,27,29], and does not require chirality or specific detuning patterns on the qubits, which makes it more convenient for experimental implementation. Our scheme avoids the usual drawbacks of dissipative state preparation in open systems such as the use of time-dependent dissipators or potentially unphysical dynamics [39]. Remarkably, to generate geometrically local dimer pairs, we only require 2-qubit control Hamiltonians $H_{\text{extra}}(t)$, which can be experimentally

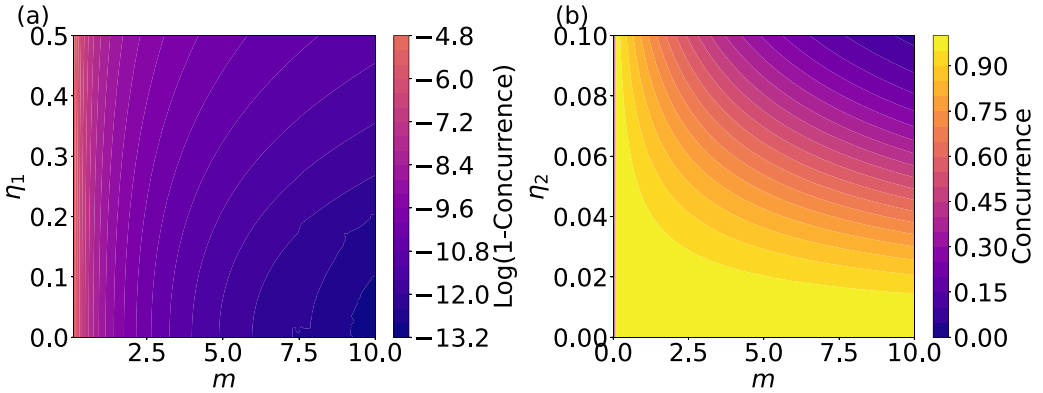

FIG. 3. Analysis of the robustness of our scheme against noise. Here, we use Eq. (6) for $\theta(\Omega(t))$ with $k = 10$ and $\Omega(t) = mt$, and we consider the case where $\Delta\gamma = 0$ in generating four dimerized pairs from $N = 8$ qubits. Since the concurrences of all the dimers are the same, we use the concurrence of a dimer pair at the steady state to characterize the entanglement generated. In (a), since the values of concurrence C of the final state obtained are all close to one, we plot $\log(1 - C)$ against η_1 and m while assuming $\eta_2 = 0$, and in (b), we plot the concurrence of the final state obtained as a function of η_2 and m but with assuming $\eta_1 = 0$. From (a), since the values of the concurrence C are all close to one, we see that our scheme is relatively insensitive to fluctuations in the driving strength $\Omega(t)$ regardless of how fast we increase the driving, though there is still some trade off. From (b), we see that there is a trade off between the amount of noise allowed and the rate m at which we can increase the driving strength Ω.

implemented in superconducting qubits [46–49]. Nonlocal interactions between the dimers are suppressed by destructive interference.

Furthermore, recent experiments using superconducting qubits work with free space spontaneous emission and dephasing decay rates of $\Gamma_f/2\pi \approx 15$ kHz and $K_\phi/2\pi \approx 100$ KHz [50]. Considering a typical decay rate of a single qubit into a waveguide $\Gamma/2\pi \approx 15$MHz, from Fig. 2, it is clear that our scheme is faster than the superconducting qubit decoherence times. Since the time-independent scheme has been recently demonstrated experimentally with superconducting qubits [28], it is a promising platform to realize our exponentially faster protocol. As potential future work, it is worth exploring the possibility of approximating the many-body interaction terms in our general scheme using local driving terms, following the formalism developed in [51,52] for counterdiabatic driving.

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