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Letter

Shepherding and herdability in complex multiagent systems

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We study the shepherding control problem where a group of "herders" need to orchestrate their collective behavior in order to steer the dynamics of a group of "target" agents towards a desired goal. We relax the assumptions, often made in the existing literature, of targets showing cohesive collective behavior in the absence of the herders, and herders owning global sensing capabilities. We find scaling laws linking the number of targets with the minimum number of herders needed to shepherd them, and we unveil the existence of a critical threshold of the density of the targets, below which the number of herders needed for success significantly increases. We explain the existence of such a threshold in terms of the percolation of a suitably defined herdability graph and support our numerical evidence by analyzing a partial differential equation describing the herders dynamics in a simplified one-dimensional setting. Extensive numerical experiments validate our methodology.

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In various physical contexts, a group of agents (the *herders*) must cooperate and self-organize in order to orchestrate the emergence of some desired collective behavior in another group (the *targets*), which would naturally act differently. Examples include shepherd dogs herding sheep towards a location [1], predators such as dolphins corralling prey [2,3], and robotic systems managing environmental pollutants [4] or guiding agents to safety [5–9]. This is known as the "shepherding" control problem in the control theoretic and robotics literature, where typically artificial herders need to drive targets to a specified area [8]. Previous research has primarily been focused on one herder with multiple targets [7,10,11], with less emphasis on the case of multiple herders [5,6,12].

Notably, when herders are outnumbered, solutions often assume targets exhibit cohesive behavior, such as sheep flocking together [1,5,10,13,14], allowing herders to leverage this for efficient problem solving [5,10,11]. Relaxing this assumption makes the problem much more cumbersome to solve theoretically, as recently noted in [10]. Moreover, the assumption that targets are cohesive is also unrealistic in many applications such as environmental cleanup via multirobot systems [4] or the confinement of microbial populations [15] where target agents (pollutant particles or bacteria) do not necessarily fulfill this hypothesis.

There is also a second strong and, most importantly, unrealistic assumption that most of the current solutions often adopt: that the herders possess unlimited sensing capabilities, i.e., that they all know the positions of all other herders and all targets in the region of interest [1,12].

Moreover, as noted independently in the recent literature, e.g., [11,12], most existing approaches adopt centralized or distributed strategies that do not exploit a crucial feature of complex systems. Specifically, effective shepherding control should not be preengineered but should emerge from herders following simple local rules in their interactions with targets and each other, leading to collective behavior suited for the shepherding task. A striking example is that described in [16], where a phenomenological model is used to describe the emergent collective behavior that two or more humans show when asked to solve the shepherding problem in a virtual reality setting (e.g., starting oscillating around the targets to contain them).

In this Letter, we simultaneously remove both of the assumptions mentioned earlier. Our investigation aims to determine if, and under what "herdability" conditions, multiple cooperating herders—operating solely based on local information—can effectively shepherd a group of target agents towards a desired state. To this aim, we propose and analyze a minimal model, general enough to comprise all the crucial features of the problem in the presence of limited sensing capabilities of the herders and the absence of any inherent collective behavior among the targets.

Our approach is fundamentally different from typical studies on active-passive particle systems (see, e.g., [17–19]), where the emergence of spontaneous behavior similar to shepherding is also observed. It is also distinct from previous research on bipartite systems for understanding prey-predator

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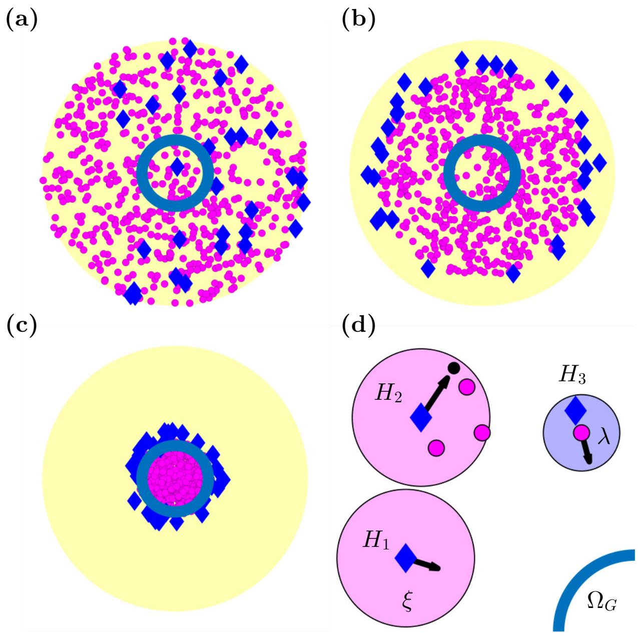

FIG. 1. Representative snapshots of the system configuration (with herders represented by blue diamonds and targets by magenta circles) at (a) the initial time $t = 0$ with the agents uniformly distributed in Ω_0 (yellow shaded disk), (b) at an intermediate time during shepherding control when herders surround all targets, and (c) when the task is successfully achieved with all the targets in Ω_G (dark blue circle). (d) Schematic of the herders' and targets' sensing (magenta shaded disks) and repulsion (blue shaded disk) regions of radius ξ and λ, respectively. The solid black arrows represent the direction of motion of the herders when moving in the absence of nearby targets ($\mathbf{H}_1$), selecting the target to chase $\mathbf{T}^*$ with the largest distance from to goal ($\mathbf{H}_2$), or when the herder pushes a selected target towards the goal region ($\mathbf{H}_3$).

and foraging behaviors [20]. In particular, in the problem we study, herders actively make decisions on what targets to select and maneuver incorporating a feedback mechanism based on their proximity to themselves and the goal region. This unique integration of feedback control theory into physics-inspired models marks our study as distinct in the field of complex systems control. Our goal is to engineer the collective behavior of a complex multiagent system (the herders) in order for another group of agents (the targets) to perform a desired task and solve a distributed control problem, an aspect that has been rarely considered in the vast literature on control and controllability of complex systems (see, e.g., [21] and references therein).

We consider the shepherding problem in $\mathbb{R}^2$ [see Fig. 1(a)], where N herders must corral M targets to a goal region Ω_G. We assume that both the herders and the targets are initially randomly and uniformly distributed in a circle Ω_0 of radius R, and that the Ω_G region is a circle of radius $r^* < R$, both Ω_0 and Ω_G being centered around the origin. Let $\mathbf{H} \in \mathbb{R}^{2N}$ be the vector of the herders' positions $\mathbf{H} = [\mathbf{H}_1, \mathbf{H}_2, \dots, \mathbf{H}_N]$ with $\mathbf{H}_i \in \mathbb{R}^2$ being the Cartesian coordinates of the ith herder, $i = 1, \dots, N$, and $\mathbf{T} \in \mathbb{R}^{2M}$ the vector of the targets' positions $\mathbf{T} = [\mathbf{T}_1, \mathbf{T}_2, \dots, \mathbf{T}_M]$, with $\mathbf{T}_a \in \mathbb{R}^2$ being the Cartesian coordinates of target a, $a = 1, \dots, M$.

We assume the targets do not exhibit any type of cohesive collective behavior with their dynamics being described by the following overdamped Langevin equation

$$\dot{\mathbf{T}}_a = \sqrt{2D}\boldsymbol{\mathcal{N}} + \beta \sum_{i \in \mathcal{N}_a} (\lambda - |\mathbf{d}_{ia}|)\hat{\mathbf{d}}_{ia}, \tag{1}$$

where, analogously to what is typically considered in the literature on soft matter, e.g., [18,19,22], $\boldsymbol{\mathcal{N}}$ is white Gaussian noise, β and D are positive constants, $\mathbf{d}_{ia} = \mathbf{H}_i - \mathbf{T}_a$ is the vector of the difference between the position of herder i and target a, $\lambda > 0$ is the radius of the region where targets are repelled by nearby herders, and $\mathcal{N}_a$ represents the set of indices of all the herders, if any, whose positions are such that $|\mathbf{d}_{ia}| \leqslant \lambda$. Note that $\beta\lambda \equiv v_T$ is the maximum escaping speed of a target due to the presence of one nearby herder and that we assume $\beta\lambda^2 \gg D$ so that the harmonic repulsive action eventually exerted by the herders onto the targets dominates over their own Brownian dynamics

We model the dynamics of the herders as made of two mutually exclusive terms, one capturing their own dynamics and the other their interaction with the targets (see, e.g., [23]). Specifically, we set

$$\dot{\mathbf{H}}_i = (1 - \eta_i)\mathbf{F}_i(\mathbf{H}_i, \mathbf{r}^*) + \eta_i \mathbf{I}_i(\mathbf{T}, \mathbf{H}, \xi), \tag{2}$$

where $\eta_i = \eta_i(\mathbf{T}, \mathbf{H}, \xi)$ is an indicator function activating when herder i has at least one target to chase in its sensing region of radius ξ, $\mathbf{F}_i(\mathbf{H}_i, r^*)$ describes the herder's own dynamics when it is not chasing any targets, while $\mathbf{I}_i(\mathbf{T}, \mathbf{H}, \xi)$ is a feedback term capturing the herder's reaction to the presence of targets in its sensing region.

Without loss of generality, we choose $\mathbf{F}_i(\mathbf{H}_i, r^*)$ so that the herders, in the absence of nearby targets, converge towards the origin if outside the goal region of radius r^*; namely we set

$$\mathbf{F}_i(\mathbf{H}_i, r^*) = \begin{cases} -v_H \widehat{\mathbf{H}}_i, & \text{if } |\mathbf{H}_i| \geqslant r^*, \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

As typically done in the control theoretic and robotics literature, e.g., [12], we assume that at each time step, herder i selects a target within its sensing region, say $\mathbf{T}_i^* = \mathbf{T}_i(\mathbf{H}, \mathbf{T}, \xi)$, to corral and chase. Then, we choose

$$\mathbf{I}_i(\mathbf{T}, \mathbf{H}, \xi) = -\{\alpha[\mathbf{H}_i - (\mathbf{T}_i^* + \delta\widehat{\mathbf{T}}_i^*)]\}_{v_H}, \tag{4}$$

where $\delta = \lambda/2$ is the distance at which the herder places itself behind the chosen target to corral it towards the goal region, α is a positive dimensional constant, and $\{\cdot\}_{v_H}$ is a saturation operator that limits the herders' maximum speed to v_H when chasing a target.

The target to chase $\mathbf{T}_i^*$ is selected by herder i as the target with the largest distance from the origin among those, if any, within the sensing radius of herder i. Furthermore, if herder i detects other herders $\mathbf{H}_j$ in its sensing region (i.e., such that $|\mathbf{H}_j - \mathbf{H}_i| \leqslant \xi$), it only considers those targets $\mathbf{T}_a$ for which $|\mathbf{T}_a - \mathbf{H}_i| \leqslant |\mathbf{T}_a - \mathbf{H}_j|$. Through this simple local rule, nearby herders effectively cooperate so as to decide which target to chase without needing any global information on the positions of other herders and targets.

We assume that the herders' velocity $v_H > v_T$ as typically done in the control literature [8] and to prevent the formation

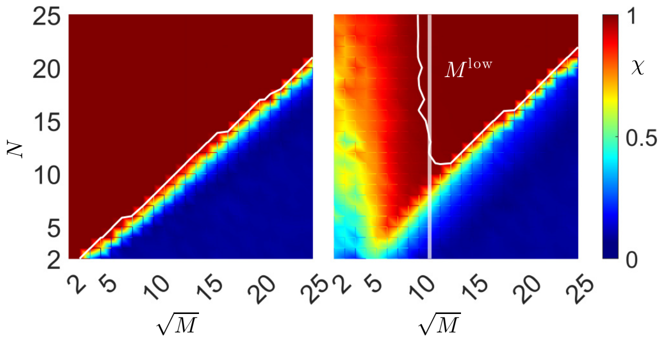

FIG. 2. Values of the fraction χ of successfully herd targets obtained for different values of M and N when $R = 50$. Results are averaged over 50 simulations; the increments of N and $\sqrt{M}$ have values $\Delta N = 1$, $\Delta\sqrt{M} = 1$. The level curve for $\chi^* = 0.99$ is depicted by the white curve. The left panel shows the case of infinite sensing ($\xi = \infty$) where $N^* \propto \sqrt{M}$ while the right panel the case of limited sensing ($\xi < \infty$) where we recover $N^* \propto \sqrt{M}$ only above a critical threshold $M > M^{low}$ (white vertical line).

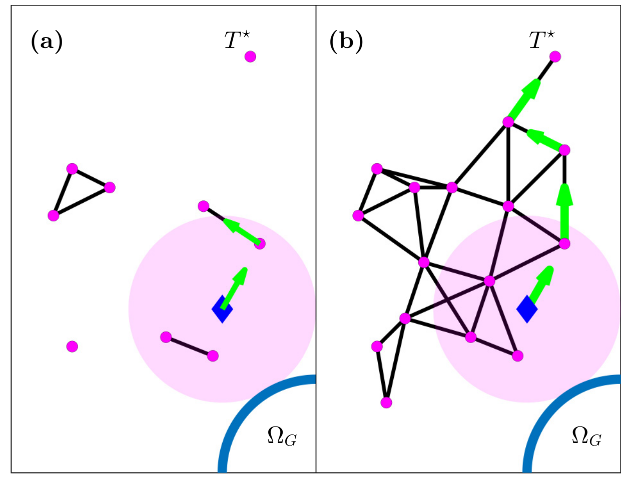

FIG. 3. Two representative configurations of targets and the structure of the corresponding herdability graph $\mathcal{G}$ (whose edges are depicted as solid black lines) (a) below and (b) above the critical percolation threshold $\widehat{M^{low}}$. Green arrows show possible paths the herder could potentially navigate to reach the furthermost targets, denoted as $\mathbf{T}^{\star}$, showing that when the graph is too sparse [panel (a)] more distant targets can be lost.

of stable chasing patterns that can be observed for $v_T \lesssim v_H$ (similar to those reported in [24–26]) that can hinder the achievement of the control goal. In addition, the radius of the repulsion zone λ is chosen smaller than that of the sensing area ξ.

Next, we study the *herdability* of a group of M targets by a group of N herders [27]. Specifically, we define a group of M target agents as "herdable" by N herders if the latter can successfully guide at least a certain fraction $\chi > \chi^*$ of the former towards Ω_G within a finite time [see Supplemental Material (SM) [28] for further details]. The threshold fraction χ^* is set based on standard values in control theory, typically $\chi^* \in \{0.9, 0.95, 0.99\}$ [29]. Given the dynamics of the agents, we will then look for the *minimal* number of herders, denoted as $N^*(M)$, required to achieve herdability of M targets.

For the sake of comparison, we start by considering herders with infinite sensing capabilities, setting $\xi = \infty$. As shown in Fig. 2(a), for a broad range of target group sizes, the required number of herders, $N^*(M)$, exhibits a quadratic relationship with the number of targets. Conversely, in scenarios with finite sensing [Fig. 2(b)], the scaling $N^*(M) \propto \sqrt{M}$ is observed, but only when the number of targets, M, exceeds a certain critical threshold, M^{low}. Below this threshold, the task notably demands more herders, indicating, counterintuitively, that fewer targets do not necessarily ease the control task with herders' limited sensing abilities.

In general, the minimum number of herders, $N^*(M)$, needed to shepherd M targets depends on two things, namely the herders' ability to (i) collectively sense all targets, which are random independent walkers, and (ii) to counterbalance the diffusion of the M targets with the transport flow they induce.

From a simple dimensional argument, as the M targets are distributed in a *two*-dimensional circular domain while the N herders tend to arrange themselves on its *one*-dimensional boundary [see Fig. 1(b) and SM videos [28]], condition (ii) is satisfied for $N^*(M) \propto \sqrt{M}$ [as observed in Fig. 2(a)] while condition (i) is trivially satisfied when the herders possess infinite sensing ($\xi = \infty$).

However, with finite sensing $\xi < \infty$, meeting condition (i) becomes increasingly more cumbersome as targets' density decreases (e.g., $M < M^{low}$). In this case, targets can become too sparse, hindering herders from efficiently scouting the area based on local information alone. Consequently, a larger number of herders, N^* is required to ensure all targets, particularly those farthest from the goal Ω_G, are observed. This requirement deviates from the quadratic scaling observed with infinite sensing. For $M > M^{low}$, the higher density of targets enables herders to effectively navigate and explore the area of interest moving from target to target, even without sensing each target at every time instant, thus aligning with the scaling law observed in the infinite sensing scenario.

To explain the critical threshold M^{low}, we analyze how herders, relying on local information, can satisfy the condition of sensing and corralling also distant targets from Ω_G.

To this aim, we define the herdability graph $\mathcal{G}$ as the random geometric graph [30] where nodes represent targets, and an edge exists between two targets, say $\mathbf{T}_a$ and $\mathbf{T}_b$, if their distance is within the sensing radius of the herders, i.e., if $|\mathbf{T}_a - \mathbf{T}_b| \leqslant \xi$.

Then, a path in $\mathcal{G}$ from one target, T_a, to another generic target, say T_c, indicates the potential for a herder to transition from sensing T_a to sensing T_c. Therefore, we propose to estimate the critical threshold M^{low} by calculating the percolation threshold of the graph $\mathcal{G}$, denoted as $\widehat{M^{low}}(R, \xi)$; in particular, we compute $\widehat{M^{low}}(R, \xi)$ in the worst case at $t = 0$ when targets are randomly and uniformly distributed within a circle of radius R (see Sec. II of the SM [28] for further details).

Figure 3 presents two schematic examples illustrating target configurations below and above the estimated threshold $\widehat{M^{low}}$ along with their respective herdability graph structures

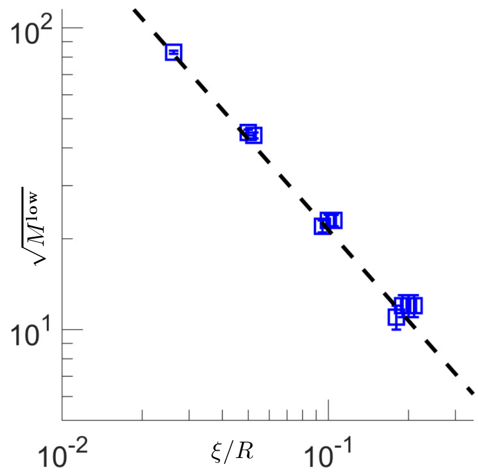

FIG. 4. Scaling of the critical threshold $M^{\rm low}$ as a function of ξ/R. The numerically observed values of $M^{\rm low}$ (scatter dots), evaluated by direct inspection, are compared with the theoretical estimate $\widehat{M^{\rm low}}$ (dashed line) for different values of ξ and R (see Table 1 in the SM [28] for the values of ξ and R selected). Error bars represent the maximum precision of the computation given the step size $\Delta\sqrt{M} = 1$ used in the simulations. Results for $\chi^* \in \{0.90, 0.95\}$ are reported in Sec. I of the SM [28] confirming the observed scaling. For the same ξ/R value, scatter points were shifted on the x axis to increase visibility.

$\mathcal{G}$. These examples clarify how $\widehat{M^{\rm low}}$ can serve as an approximation for the critical threshold $M^{\rm low}$. Given the known scaling of the percolation threshold of a two-dimensional random geometric graph as R^2/ξ^2 [31], we anticipate $M^{\rm low} \sim R^2/\xi^2$. This expectation aligns with our numerical findings shown in Fig. 4 when $\chi^* = 0.99$, confirming that our theoretical approach effectively captures the observed trend. For more details and additional validation for different χ^* values and noise levels in the targets dynamics, we refer the reader to the SM [28].

Our study's central finding is that effective herdability hinges on sufficient connectivity of the herdability graph $\mathcal{G}$. To analytically substantiate this, we examine a simpler one-dimensional scenario and derive a partial differential equation (PDE) characterizing the spatiotemporal dynamics of the herders' density, denoted as ρ^H. A pivotal aspect of our analysis involves translating the decision-making process herders use to select the target to chase, $\mathbf{T}_i^*$, into a continuum framework. We propose this can be done by expressing the target selection rule used by the herders as a weighted average approximated by

$$\mathbf{T}_i^* = \lim_{\gamma\to\infty} \frac{\sum_{a\in\mathcal{N}_i} e^{\gamma|\mathbf{T}_a|}\,\mathbf{T}_a}{\sum_{a\in\mathcal{N}_i} e^{\gamma|\mathbf{T}_a|}}, \tag{5}$$

with $\mathcal{N}_i$ being the set of target indices such that $\mathbf{d}_{ia} \leqslant \xi$.

Then, recasting (5) in a continuum framework, ignoring the herders' own dynamics, $\mathbf{F}_i$, allowing different herders to select the same target, and setting $\delta = 0$ in (4) for the sake of simplicity, the dynamics of the herders' density $\rho^H(x,t)$ can be captured heuristically by the following PDE,

$$\rho_t^H + \left[-\frac{dV}{dx}\rho^H\right]_x = 0, \tag{6}$$

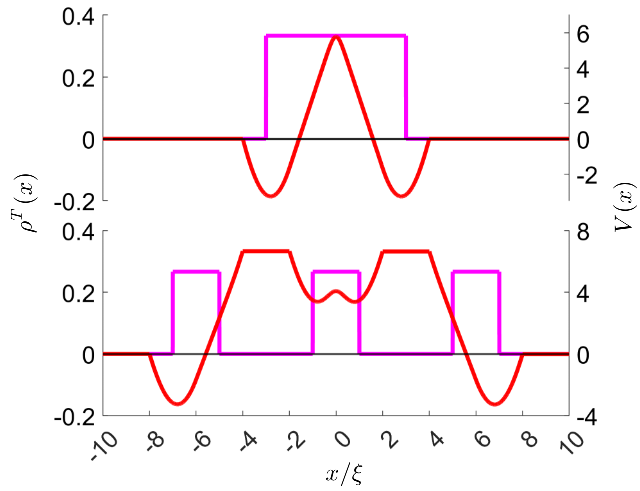

FIG. 5. Stationary distributions of the targets, ρ^T (magenta lines), over a one-dimensional domain together with the corresponding potential V (red line) computed by (7) showing global stability when no regions exist where $\rho^T|_{x\in\Delta} = 0$ with $|\Delta| > \xi$ (top panel) and local stability otherwise (bottom panel).

where

$$-\frac{dV}{dx} = -\frac{1}{\mathcal{M}}\int_{\mathbf{B}_\xi(x)} e^{\gamma|y|}\rho^T(y)(x-y)dy, \tag{7}$$

with ρ^T being the density of the targets supposed to be stationary when the herders are sufficiently faster than the targets, and $\mathbf{B}_\xi(x)$ denotes a ball of radius ξ centered in x, and $\mathcal{M}$ is a normalization factor.

Using (6), we find, as shown in Fig. 5, that a globally stable equilibrium configuration, in which herders completely encircle all targets, is attainable only under the condition that no regions exist where $\rho^T|_{x\in\Delta} = 0$ with $|\Delta| > \xi$, a situation corresponding to the herdability graph $\mathcal{G}$ being disconnected. This further substantiates our finding that the targets need to maintain a sufficient level of connectivity within the herdability graph $\mathcal{G}$ in order to enable the herders to collectively detect and guide even the most distant ones.

In summary, the relevance of our findings is twofold. Firstly, we solved the shepherding control problem in a setting with non cohesive stochastic targets and with herders with limited sensing capabilities. Secondly, we introduced and analyzed the *herdability* of a group of targets by a group of herders providing conditions for the herders to corral and chase all targets. This is achieved solely by means of local information and knowledge of the control goal enabling the herders to select the next target to chase according to a decentralized feedback control mechanism. We found that the minimal number of herders N^* required to successfully shepherd a group of M targets scales as $N^* \propto \sqrt{M}$ only above a critical threshold $M^{\rm low}$ in the case of limited sensing. We showed that an estimate of such critical threshold can be effectively obtained in terms of the percolation of a suitably defined *herdability graph*. Finally, we provided a strategy to translate to a continuum framework the target selection strategy used by the herders and, consequently, derived an appropriate PDE describing the evolution of the herders' density in a one-dimensional representative setting. The stability analysis of the asymptotic herders' distributions confirmed our claim that in order for the task to be successful the herdability graph must be adequately connected.

Ongoing and future work will be devoted to further explore the continuum formulation of the shepherding control problem. Indeed, we anticipate that such a formulation will allow to investigate herdability for a variety of targets dynamics [32], leveraging on results from the literature on the physics of nonreciprocal interactions [25] and on the control of PDEs, e.g., [33,34]. For example, the collective dynamics of the targets could be described as a wave equation where perturbations can effectively propagate in space, e.g., [35], and then solutions to the boundary-control problem of the wave equation described in [33] could be adapted to find novel herder configurations to solve the shepherding control problem.

All the numerical simulations were carried out in MATLAB using a forward Euler scheme for the herders, and a Euler-Maruyama scheme for the targets, with time steps $\Delta t = 0.03$, and total duration t depending on the settling time required for stationarity (see Fig. S8 of the Supplemental Material [28]). The parameters of the targets' dynamics were chosen as $D = 1$, $\beta = 3$, $\lambda = 2.5$. The parameters of the herders' dynamics were chosen as $v_H = 8$, $\alpha = 5$, $\gamma = 5$. The radius r^* of the goal region Ω_G was chosen as $R/5$. The source code to run the simulations can be downloaded from [36].

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[27] This concept is related to but not entirely equivalent to the definition of herdability of complex systems that was given independently in Ref. [37].

[28] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevResearch.6.L032012 for the scaling analysis of the critical threshold in the number of targets M^{low} for different values of the success threshold; the effects of different noise levels on M^{low}; the calculation of the estimate of M^{low} via the percolation threshold of the herdability graph; the effects of periodic boundary conditions; Supplemental tables, figures, and videos.

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