Letter

Hidden quantum criticality and entanglement in quench dynamics

Sanku Paul,[1,2,3] Paraj Titum,[4,3,5] and Mohammad Maghrebi[2,3]

[1]Department of Physics and Complex Systems, S.N. Bose National Centre for Basic Sciences, Kolkata 700106, India
[2]Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA
[3]Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106-4030, USA
[4]Johns Hopkins University Applied Physics Laboratory, Laurel, Maryland 20723, USA
[5]Joint Quantum Institute, NIST/University of Maryland, College Park, Maryland 20742, USA

(Received 8 March 2022; accepted 11 June 2024; published 3 July 2024)

Entanglement exhibits universal behavior near the ground-state critical point where correlations are long ranged and the thermodynamic entropy is vanishing. On the other hand, a quantum quench imparts extensive energy and results in a build up of entropy, hence no critical behavior is expected at long times. In this work, we present a new paradigm in the quench dynamics of integrable spin chains which exhibit a ground-state order-disorder phase transition at a critical line. Specifically, we consider a quench along the critical line which displays a volume-law behavior of the entropy and exponentially decaying correlations; however, we show that quantum criticality is hidden in higher-order correlations and becomes manifest via measures such as the mutual information and logarithmic negativity. Furthermore, we showcase the scale invariance of the Rényi mutual information between disjoint regions as further evidence for genuine critical behavior. We attribute the emergent quantum criticality to the soft mode not getting excited in spite of the quench. Moreover, the results presented here are universal to models whose low-energy or long-wavelength dynamics are well described by a free-fermionic field theory. Our results are amenable to an experimental realization on different quantum simulator platforms, particularly the Rydberg simulators.

DOI: 10.1103/PhysRevResearch.6.L032003

Introduction. Entanglement characterizes nonclassical correlations in a quantum state and has a wide variety of applications in quantum computing, networking, and metrology. Furthermore, entanglement provides a powerful diagnostic for quantum phase transitions [1–4]. For a pure state, it can be quantified by the von Neumann entropy S_A of a given subsystem A. For a one-dimensional (1D) spin chain, we generically have [5]

$$S_A = a|A| + b\ln|A| + \text{const}, \tag{1}$$

with $|A|$ the subsystem size and a, b constants independent of $|A|$. Highly excited states (or finite-temperature states) typically obey a volume law with $a \neq 0$, reflecting the thermodynamic entropy of the state. In contrast, the ground state of gapped Hamiltonians exhibit an area law where S_A is a constant independent of system size (i.e., a, $b = 0$). In both cases, only short-range correlations are present in the state. On the other hand, a leading logarithmic term emerges at a quantum critical point in the ground state—as well as quantum scars [6–8]—with b a universal coefficient (while $a = 0$). For a conformal field theory (CFT), this coefficient is $b = c/3$ with c the central charge [9]. The logarithmic scaling of entanglement entropy is typically a powerful indicator of criticality which is more conventionally diagnosed with power-law correlations [10]. Conversely, the absence of universal logarithms means no critical behavior. For instance, thermal states do not exhibit logarithmic corrections [11], consistent with the fact that there are no 1D phase transitions at finite temperature [12].

How does this paradigm change for nonequilibrium states? Here, we consider the stationary states of an isolated system upon a sudden quench. Generic quantum systems are widely believed to thermalize [13–15], hence the volume law emerges while universal logarithms do not [16–19]. In contrast, integrable systems evade thermalization and approach a stationary state at long times [20–24]. However, even in this case, the long-time stationary states typically exhibit exponentially decaying correlations [25,26] and extensive energy/entropy [27–32]. A rather special exception is coupled harmonic oscillators where subleading logarithms appear in the dynamics [33,34], due to their zero modes harboring an arbitrary large entropy [34,35]. Current-carrying steady states could also lead to subleading logarithms [36–38]. However, generic settings of quench dynamics do not exhibit criticality at late times, akin to thermal states, hence $a \neq 0$ while $b = 0$.

In this work, we present a new paradigm for entanglement and criticality in the long-time stationary state of quench dynamics. We consider the anisotropic XY chain as a paradigmatic integrable model and show that a quench along the critical line leads to a volume law plus logarithmic corrections. While the latter indicate criticality, characteristic correlation functions decay exponentially. However, we show that a form of quantum criticality is hidden in higher-order correlation functions, captured via quantum information measures such as mutual information and logarithmic negativity. We attribute this critical behavior to the absence of an excitation of the soft mode at criticality. This behavior is expected in any free-fermion model whose long-wavelength dynamics are governed by a free-fermionic field theory highlighting its inherent universality.

Model. We consider the quench dynamics in the anisotropic XY model given by the Hamiltonian

$$H(h,\gamma) = -\sum_j \frac{1+\gamma}{2}\sigma_j^x\sigma_{j+1}^x + \frac{1-\gamma}{2}\sigma_j^y\sigma_{j+1}^y + h\sigma_j^z, \quad (2)$$

where σ^α ($\alpha = x, y, z$) are the Pauli operators. Here, h is the transverse field, and γ defines an anisotropy parameter. The ground-state phase diagram of this model is shown in Fig. 1 (top): an order-disorder phase transition occurs at $h_c = 1$ for any value of γ. Quench dynamics of the XY model has been studied extensively [39–46]. A general sudden quench can be parametrized as a sudden change in the parameters of the Hamiltonian defined in Eq. (2): $(h_0, \gamma_0) \to (h, \gamma)$; the initial state is the ground state of $H(h_0, \gamma_0)$ and evolves under $H(h, \gamma)$. Without loss of generality, we fix $\gamma = 1$ in the post-quench Hamiltonian.

Here, we investigate the critical properties of the stationary state at late times. Indeed, recent works have shown that quantum phase transitions leave their fingerprints on quench dynamics [47–51]. However, a quantum quench imparts extensive energy and entropy and generically leads to exponentially decaying correlations. Hence, genuine critical behavior (e.g., the divergence of correlation length, scale invariance, etc.) is lacking. Contrary to this picture, we show that, depending on the initial state, the long-time stationary state could in fact exhibit critical behavior. To highlight the role of the initial state, we consider two different initial states. (i) Noncritical, $(h_0, \gamma_0) = (2, 1)$ corresponding to a disordered state; and (ii) critical, $(h_0, \gamma_0) = (1, -1)$. The two quench protocols are schematically represented in the top panel of Fig. 1. We study the stationary state near the (post quench) critical point $h = h_c$ (dotted line in Fig. 1). Left (right) columns in Fig. 1 correspond to the first (second) protocol, respectively.

Let us start by discussing longitudinal correlation functions. While in a critical ground state ($\gamma > 0$), they fall off algebraically, $\rho_l^{xx} \equiv \langle\sigma_j^x\sigma_{j+l}^x\rangle \propto l^{-1/4}$; away from criticality they decay exponentially, $\rho_l^{xx} \sim \exp(-l/\xi_x)$, with $\xi_x \sim 1/|h - h_c|$ the correlation length. In contrast, quench dynamics always leads to a disordered stationary state ($\langle\sigma^x\rangle = 0$) and a finite correlation length (as predicted from CFT [26]), although the latter features a kink at the critical point; see Figs. 1(a) and 1(b). A finite correlation length seems to indicate a lack of criticality; however, the critical behavior is hidden in higher-order correlations that are captured via information-theoretic measures.

Next, we turn to the von Neumann entropy of a connected block of spins A $(= A_1 \cup A_2)$ of size $2L$; see Figs. 1(c) and 1(d) [52]. Regardless of the quench protocol, the entropy of the stationary state obeys a volume law in agreement with CFT [27]. A logarithmic term, if any, would appear to the subleading order in Eq. (1). To this end, we consider the mutual information between two (sub)systems A_1 and A_2, $I_{A_1:A_2} = S_{A_1} + S_{A_2} - S_{A_1\cup A_2}$, which measures the total amount of correlations. Figures 1(e) and 1(f) show the mutual information between the two adjacent regions. Both quench protocols are sensitive to the critical point, but exhibit very different trends in the stationary state. For the first protocol [Fig. 1(e)], the mutual information is bounded but exhibits a dip at the

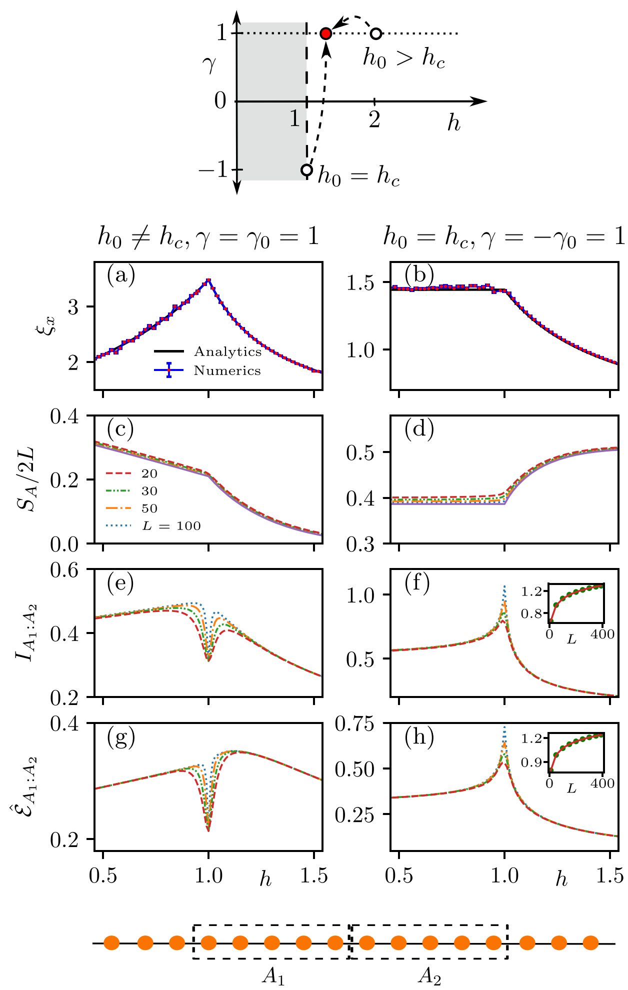

FIG. 1. (Top) Ground-state phase diagram of the XY model as a function of h the transverse field and γ the anisotropy factor; $h = h_c \equiv 1$ defines the critical line. The arrows schematically represent two quench protocols starting from a noncritical ($h_0 > h_c$) or a critical ($h_0 = h_c$) initial state. The following quantities are plotted in the long-time stationary state as a function of h along the horizontal dotted line: (a), (b) longitudinal correlation length (the solid line is the analytical result from [26].); (c), (d) von Neumann entropy density $S_A/2L$ of a region of size $2L$ (the solid line is the analytical result from [27]); (e), (f) mutual information $I_{A_1:A_2}$; and (g), (h) upper bound on log-negativity $\hat{\mathcal{E}}_{A_1:A_2}$ for two adjacent regions of size L; see the schematics. Different curves in panels (c)–(h) correspond to different system sizes; see panel (c). Solid (purple) lines in (c) and (d) are the analytical prediction in the limit $L \to \infty$. Both $I_{A_1:A_2}$ and $\hat{\mathcal{E}}_{A_1:A_2}$ exhibit strong dependence on subsystem size near the critical point ($h = h_c$), but exhibit contrasting behaviors (a sharp dip vs a peak) for $h_0 \neq h_c$ and $h = h_c$; the data in the insets (f) and (h) are consistent with $I_{A_1:A_2} \sim \frac{1}{6}\ln L$ and $\hat{\mathcal{E}}_{A_1:A_2} \sim \frac{1}{8}\ln L$.

critical point, which becomes sharper as $L \to \infty$. Interestingly, $I_{A_1:A_2} \equiv I(L, h)$ displays a finite-size scaling near $h = h_c$:

$$I(L, h) = I(\infty, h) - \mathcal{F}[(h-1)L], \tag{3}$$

with $\mathcal{F}(x)$ a scaling function [52]. In contrast, for the second protocol, the mutual information diverges as $I \sim \frac{1}{6} \ln L$ at the critical point (just like the critical ground state); see the inset in Fig. 1(f). Translated to the entanglement entropy, this means that a critical-to-critical quench results in both volume and logarithmic terms in Eq. (1). Such logarithmic scaling is indicative of criticality [53–56], as we further argue below. Away from the critical point and in the limit $L \to \infty$, we find $I \sim -\frac{1}{6} \ln |h - h_c|$ [52]. Mimicking the ground state, we define a "mutual information correlation length" $\xi_{\rm MI}$ as $I \sim \frac{1}{6} \ln \xi_{\rm MI}$ [57]. We then conclude that $\xi_{\rm MI} \sim 1/|h - h_c|$ diverges in the stationary state, although $\xi_x \sim \mathcal{O}(1)$. Indeed, mutual information does not overlook any hidden correlations which could be invisible to two-point correlations, a property that could be useful for quantum data hiding [11,58–60].

Next, we address the (classical vs quantum) nature of correlations. To this end, we consider an entanglement monotone known as the logarithmic (log)negativity defined as $\mathcal{E}_{A_1:A_2} = \ln {\rm Tr}|\rho_A^{T_2}|$ where $A = A_1 \cup A_2$, and T_2 represents partial transposition with respect to A_2 [61,62]. For technical reasons, we calculate a relatively tight upper bound $\hat{\mathcal{E}}$ ($\geqslant \mathcal{E}$) on log negativity [52,63,64]. Figures 1(g) and 1(h) show that $\hat{\mathcal{E}}$ behaves similarly to mutual information. In particular, $\hat{\mathcal{E}} \sim \frac{1}{8} \ln L$ grows logarithmically with L in the critical-to-critical quench; see the inset in Fig. 1(h). We conclude that the correlations captured by the mutual information are indeed quantum in nature.

Fermionic picture. To find insight into our results, we consider an exact mapping to free fermions via the Jordan-Wigner transformation $c_j = (\prod_{m<j} \sigma_m^z)(\sigma_j^x - i\sigma_j^y)/2$ [52]. The resulting fermionic Hamiltonian is (restoring γ)

$$H = \sum_k H_k = -\sum_k (\cos k - h)\hat{\tau}_k^z + \gamma \sin k\, \hat{\tau}_k^y, \tag{4}$$

where $\hat{\tau}_k^z = c_k^\dagger c_k - c_{-k} c_{-k}^\dagger$ and $\hat{\tau}_k^y = i(c_k c_{-k} + c_k^\dagger c_{-k}^\dagger)$ are akin to Pauli operators acting in an even parity sector [52]; it is similar for the prequench Hamiltonian with $\gamma, h \to \gamma_0, h_0$. We are interested in the soft mode ($k \to 0$) with vanishing energy near criticality, $h \to h_c$. It captures the long wavelength properties of the system. The corresponding Hamiltonian in this limit is given by $H_{k\to 0} = -(1-h)\hat{\tau}_k^z$ away from criticality ($h \neq h_c$), while $H_{k\to 0} = -\gamma k\, \hat{\tau}_k^y$ at the critical point ($h = h_c$). Now, under a critical-to-critical quench, the low-energy eigenbasis of the quench Hamiltonian is unchanged from the prequench Hamiltonian. Therefore, the soft mode does not get excited, resulting in quantum-critical behavior. The absence of soft mode excitation further reflects on the long-distance fermionic correlations in the stationary state. The latter is observed in a correlator $g_l^{\rm st} \equiv i\langle a_j^x a_{j+l}^y \rangle$, where $a^x = c_j + c_j^\dagger$ and $a^y = i(c_j^\dagger - c_j)$ are Majorana fermions [52], and is expressed as

$$g_l^{\rm st} = \frac{2 + 4l}{-3\pi + 4\pi l(l+1)} \xrightarrow[l\to\infty]{} \frac{1}{\pi l}. \tag{5}$$

This, while compared with the critical ground state, $g_l^{\rm gs} = -2/(\pi + 2\pi l) \to -1/(\pi l)$ as $l \to \infty$, indicates that the asymptotic behavior of $g_l^{\rm st}$ is identical to that of the ground state up to a sign. This provides further evidence of quantum criticality in a critical-to-critical quench. The sign in $g_l^{\rm st}$ results from a sign change in $H_{k\to 0}$ during the quench protocol. In contrast, in a noncritical-to-critical quench, the Hamiltonian operator changes from $\hat{\tau}^z$ to $\hat{\tau}^y$. Since these operators are mutually unbiased [65], the soft mode heats up to infinite temperature, leading to the exponential decay of $g_l^{\rm st}$. This accounts for the lack of genuine critical behavior in the latter quench scenario.

In addition to the fermionic correlation function, mutual information is also independent of the ultraviolet cutoff [66] and captures long-range correlations [67]. Let us first consider the mutual information $I_{A_1:A_2}^f$ corresponding to the fermionic lattice model. Given that the model is Gaussian and exhibits the same long-range correlations as the critical ground state [Eq. (5)], we conclude that $I_{A_1:A_2}^f \sim \frac{1}{6} \ln L$. We remark that highly-excited states of free fermions can be constructed where mutual information scales logarithmically [42,68]. Here, we have shown that such behavior emerges naturally in a critical-to-critical quench. For the spin model, note that $I_{A_1:A_2}^f = I_{A_1:A_2}$ for adjacent regions, since the corresponding spin operators can be written in terms of fermions in the same region [4]. It follows that $I_{A_1:A_2} \sim \frac{1}{6} \ln L$ for the critical-to-critical quench, consistent with our numerics. Our results are consistent with CFT calculations where it was shown that the entanglement of the initial state survives in dynamics [69]; however, this conclusion does not hold in a quench to a noncritical point even if the initial state is critical and highly entangled [see Figs. 1(f) and 1(h)].

Next, we note that a sharp dip emerges in mutual information and log negativity in the noncritical-to-critical quench [Figs. 1(e) and 1(g)]. This behavior is due to the discontinuity in the behavior of the soft mode: away from a critical point $h \neq h_c$, the soft mode remains unexcited, while quenching to a critical point $h = h_c$ it heats up to infinite temperature.

Hidden criticality. The critical nature of the model, while manifest in terms of fermions, becomes hidden when examined through longitudinal spin correlations. This surprising feature is attributed to the Jordan-Wigner string operator. On the other hand, the connected transverse spin correlations are directly determined from fermionic correlations; in the critical-to-critical quench, they scale as $\langle \sigma_{j+l}^z \sigma_j^z \rangle_c = -g_{l+1}^{\rm st} g_{1-l}^{\rm st} \sim 1/\pi^2 l^2$ as $l \to \infty$, identical to the critical ground state. In spite of this, we argue that the universal logarithm in the information measures cannot be attributed just to the algebraic decay of transverse correlations. First, if long-range correlations only involve σ^z operators, they will be of a classical nature, but this would be incompatible with the logarithmic divergence of log negativity. Furthermore, two-point correlations alone cannot violate the area law for the mutual information if they decay faster than $1/l$ [52]. In contrast with transverse correlations, the fermionic two-point

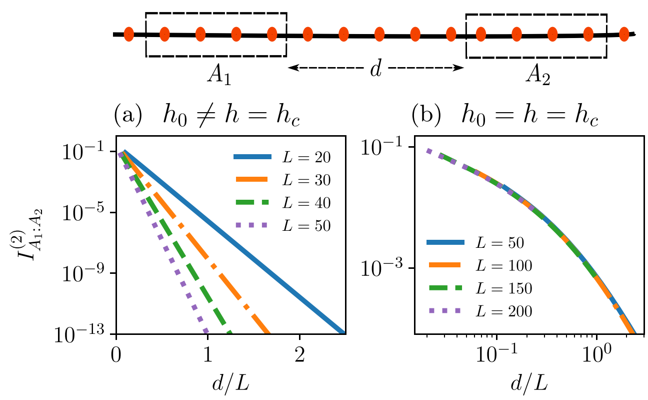

FIG. 2. Scaling behavior of disjoint Rényi information $I^{(2)}_{A_1:A_2}$ in the stationary state of quench dynamics. Schematics represent two disjoint regions A_1 and A_2 each of length L separated by a distance d. Disjoint Rényi information I_2 is plotted for (a) noncritical-to-critical quench, and (b) critical-to-critical quench. The latter exhibits scaling invariance, indicative of criticality. In each case, h_0, γ_0, γ are the same as Fig. 1.

correlations in Eq. (5) decay as $1/l$, which, in the spin language, are given in terms of a string operator:

$$\left\langle \sigma^x_j \sigma^x_{j+l} \prod_{j<m<j+l} \sigma^z_m \right\rangle \sim -\frac{1}{\pi l}. \tag{6}$$

This explicitly shows that criticality is hidden in higher-order spin correlations.

Scaling invariance of disjoint Rényi mutual information. A direct signature of criticality is scale invariance due to the divergence of the correlation length. While correlation functions decay exponentially, information-theoretic measures are a suitable candidate to display such scale invariance. We inspect the mutual information between two disjoint regions of size L separated by a distance d. For technical reasons, we consider the Rényi mutual information $I^{(\alpha)}_{A_1:A_2}$ defined analogously from the Rényi entropy, $R^{(\alpha)}_A = \frac{1}{1-\alpha} \ln \mathrm{Tr}_A \rho^\alpha_A$ and take $\alpha = 2$ for simplicity. Critical behavior, if any, dictates

$$I^{(2)}_{A_1:A_2} = \mathcal{I}(d/L), \tag{7}$$

with $\mathcal{I}$ a scaling function which only depends on the ratio d/L, independent of any intrinsic scales. In Fig. 2, we show that the noncritical-to-critical quench shows no such scaling, but the critical-to-critical quench is manifestly scale invariant. This provides further evidence that the latter quench leads to genuinely quantum-critical behavior. However, the scale invariance noted is markedly distinct from that of a critical ground state [52].

Universality. Quantum criticality emerges due to the nonexcitation of the soft mode as we argued in our discussion following Eq. (4). This reasoning applies not only to the nearest-neighbor Ising model, but also to any free-fermion model where the soft mode exhibits similar behavior (an example is given by beyond-nearest-neighbor Ising models that can be mapped to free fermions [52]). Such models are described at long wavelengths by a free-fermionic field theory given by [70]

$$H = \int dx \left[\frac{c}{2} (\Psi^\dagger \partial_x \Psi^\dagger - \Psi \partial_x \Psi) + \Delta \Psi^\dagger \Psi \right]. \tag{8}$$

Here, the parameter Δ denotes the distance from the critical point; for the Ising model considered so far, $\Delta \sim h - 1$. A similar construction to Eq. (4) shows that the critical-to-critical quench does not excite the zero mode (here defined by $\Psi_{k\to 0}$). Our conclusions even apply to long-range variants of such models (e.g., long-range Kitaev model [52]). Universality is at the heart of equilibrium phase transitions—our work features an example of universal critical behavior in quench dynamics and far from equilibrium. A final remark is in order: critical quench dynamics studied in the Luttinger liquid [71,72] also result in post-quench power-law correlations indicative of criticality. However, the models considered here are distinguished by their Ising symmetry and ground-state symmetry-breaking phase transition.

Experimental realization. The quench experiments studied here may be experimentally realized in a variety of quantum simulator platforms. Particularly, we envision 1D arrays of Rydberg atoms trapped using optical tweezers as the ideal platform to study these Hamiltonians [73]. These systems have long coherence times, tunable interactions, and have been used to implement a variety of spin models as well as universal quantum computing [74]. To investigate the hidden criticality in experiment, a challenging aspect is the preparation of the critical state which may be possible using variational algorithms [75]. The measurement of entanglement, such as the Rényi entropy of a quantum state, can be performed using the statistical correlations in randomized measurements [76,77]. Finally, we note that while the results in this paper have focused on the stationary state of the quench dynamics, we expect the essential features of the hidden criticality, especially its scaling behavior, to be manifest in the intermediate time dynamics.

Conclusion and outlook. We have studied the critical behavior in the long-time stationary state of an integrable spin chain upon a sudden quench. We have shown that, for critical-to-critical quenches, the stationary state exhibits quantum critical behavior which cannot be detected through the local order parameter and is instead hidden in higher-order correlations, which we identify through information-theoretic measures. Our findings open up a new frontier for investigating quantum criticality in quench dynamics beyond the ground-state order-disorder phase transitions. Our conclusions immediately apply to free fermion models whose long-wavelength dynamics is described by free-fermionic field theory. Furthermore, it is natural to expect that these results hold at intermediate times even for interacting (i.e., nonintegrable) spin models [50]. This interesting direction will be investigated in future work. Finally, exploring connections with data hiding in quantum information is worthwhile [58,59].

Acknowledgments. We thank M. A. Rajabpour, A. Polkovnikov, M. Rigol, and H. Shapourian for insightful discussions. M. F. Maghrebi acknowledges support from the NSF under Grant No. DMR-1912799. S. Paul is supported by the start-up funding from Michigan State University and DST India. P. Titum acknowledges funding from the U.S. Department of Energy (DOE), Office of Science, Office of Advanced Scientific Computing Research (ASCR) Quantum Computing Application Teams program, under fieldwork Proposal No. ERKJ347. This research was supported in part by the National Science Foundation under Grant No. NSF PHY-1748958.

[1] T. J. Osborne and M. A. Nielsen, Entanglement in a simple quantum phase transition, Phys. Rev. A **66**, 032110 (2002).
[2] A. Osterloh, L. Amico, G. Falci, and R. Fazio, Scaling of entanglement close to a quantum phase transition, Nature (London) **416**, 608 (2002).
[3] G. Vidal, J. I. Latorre, E. Rico, and A. Kitaev, Entanglement in quantum critical phenomena, Phys. Rev. Lett. **90**, 227902 (2003).
[4] J. I. Latorre, E. Rico, and G. Vidal, Ground state entanglement in quantum spin chains, Quantum Info. Comput. **4**, 48 (2004).
[5] P. Calabrese and J. Cardy, Entanglement entropy and quantum field theory, J. Stat. Mech. (2004) P06002.
[6] S. Moudgalya, N. Regnault, and B. A. Bernevig, Entanglement of exact excited states of Affleck-Kennedy-Lieb-Tasaki models: Exact results, many-body scars, and violation of the strong eigenstate thermalization hypothesis, Phys. Rev. B **98**, 235156 (2018).
[7] M. Schecter and T. Iadecola, Weak ergodicity breaking and quantum many-body scars in spin-1 XY magnets, Phys. Rev. Lett. **123**, 147201 (2019).
[8] Z. Yao, L. Pan, S. Liu, and H. Zhai, Quantum many-body scars and quantum criticality, Phys. Rev. B **105**, 125123 (2022).
[9] P. Calabrese and J. Cardy, Entanglement entropy and conformal field theory, J. Phys. A: Math. Theor. **42**, 504005 (2009).
[10] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena* (Oxford University Press, United Kingdom, 2021), Vol. 171.
[11] M. M. Wolf, F. Verstraete, M. B. Hastings, and J. I. Cirac, Area laws in quantum systems: Mutual information and correlations, Phys. Rev. Lett. **100**, 070502 (2008).
[12] L. D. Landau and E. M. Lifshitz, *Course of Theoretical Physics* (Elsevier, United Kingdom, 2013).
[13] L. D'Alessio, Y. Kafri, A. Polkovnikov, and M. Rigol, From quantum chaos and eigenstate thermalization to statistical mechanics and thermodynamics, Adv. Phys. **65**, 239 (2016).
[14] C. Gogolin and J. Eisert, Equilibration, thermalisation, and the emergence of statistical mechanics in closed quantum systems, Rep. Prog. Phys. **79**, 056001 (2016).
[15] M. Srednicki, The approach to thermal equilibrium in quantized chaotic systems, J. Phys. A **32**, 1163 (1999).
[16] M. Žnidarič, T. Prosen, and I. Pižorn, Complexity of thermal states in quantum spin chains, Phys. Rev. A **78**, 022103 (2008).
[17] A. M. Lóuchli and C. Kollath, Spreading of correlations and entanglement after a quench in the one-dimensional Bose–Hubbard model, J. Stat. Mech. (2008) P05018.
[18] J. Schachenmayer, B. P. Lanyon, C. F. Roos, and A. J. Daley, Entanglement growth in quench dynamics with variable range interactions, Phys. Rev. X **3**, 031015 (2013).
[19] H. Kim and D. A. Huse, Ballistic spreading of entanglement in a diffusive nonintegrable system, Phys. Rev. Lett. **111**, 127205 (2013).
[20] A. C. Cassidy, C. W. Clark, and M. Rigol, Generalized thermalization in an integrable lattice system, Phys. Rev. Lett. **106**, 140405 (2011).
[21] J.-S. Caux and R. M. Konik, Constructing the generalized Gibbs ensemble after a quantum quench, Phys. Rev. Lett. **109**, 175301 (2012).
[22] M. Fagotti and F. H. L. Essler, Reduced density matrix after a quantum quench, Phys. Rev. B **87**, 245107 (2013).
[23] J. Sirker, N. P. Konstantinidis, F. Andraschko, and N. Sedlmayr, Locality and thermalization in closed quantum systems, Phys. Rev. A **89**, 042104 (2014).
[24] L. Vidmar and M. Rigol, Generalized Gibbs ensemble in integrable lattice models, J. Stat. Mech. (2016) 064007.
[25] P. Calabrese, F. H. L. Essler, and M. Fagotti, Quantum quench in the transverse field Ising chain: I. Time evolution of order parameter correlators, J. Stat. Mech. (2012) P07016.
[26] P. Calabrese, F. H. L. Essler, and M. Fagotti, Quantum quenches in the transverse field Ising chain: II. Stationary state properties, J. Stat. Mech. (2012) P07022.
[27] P. Calabrese and J. Cardy, Evolution of entanglement entropy in one-dimensional systems, J. Stat. Mech. (2005) P04010.
[28] P. Calabrese and J. Cardy, Entanglement and correlation functions following a local quench: A conformal field theory approach, J. Stat. Mech. (2007) P10004.
[29] G. De Chiara, S. Montangero, P. Calabrese, and R. Fazio, Entanglement entropy dynamics of Heisenberg chains, J. Stat. Mech. (2006) P03001.
[30] V. Alba and P. Calabrese, Entanglement and thermodynamics after a quantum quench in integrable systems, Proc. Natl. Acad. Sci. USA **114**, 7947 (2017).
[31] V. Alba and P. Calabrese, Quantum information dynamics in multipartite integrable systems, Europhys. Lett. **126**, 60001 (2019).
[32] L. Vidmar, L. Hackl, E. Bianchi, and M. Rigol, Volume law and quantum criticality in the entanglement entropy of excited eigenstates of the quantum Ising model, Phys. Rev. Lett. **121**, 220602 (2018).
[33] M. G. Nezhadhaghighi and M. A. Rajabpour, Entanglement dynamics in short- and long-range harmonic oscillators, Phys. Rev. B **90**, 205438 (2014).
[34] J. S. Cotler, M. P. Hertzberg, M. Mezei, and M. T. Mueller, Entanglement growth after a global quench in free scalar field theory, J. High Energ. Phys. 11 (2016) 166.
[35] W. G. Unruh, Comment on "Proof of the quantum bound on specific entropy for free fields", Phys. Rev. D **42**, 3596 (1990).
[36] V. Eisler and Z. Zimborás, Area-law violation for the mutual information in a nonequilibrium steady state, Phys. Rev. A **89**, 032321 (2014).
[37] P. Ribeiro, Steady-state properties of a nonequilibrium Fermi gas, Phys. Rev. B **96**, 054302 (2017).
[38] S. Fraenkel and M. Goldstein, Entanglement measures in a nonequilibrium steady state: Exact results in one dimension, SciPost Phys. **11**, 085 (2021).
[39] P. Calabrese and J. Cardy, Time dependence of correlation functions following a quantum quench, Phys. Rev. Lett. **96**, 136801 (2006).

[40] V. Mukherjee, U. Divakaran, A. Dutta, and D. Sen, Quenching dynamics of a quantum XY spin-$\frac{1}{2}$ chain in a transverse field, Phys. Rev. B **76**, 174303 (2007).

[41] M. Fagotti and P. Calabrese, Evolution of entanglement entropy following a quantum quench: Analytic results for the XY chain in a transverse magnetic field, Phys. Rev. A **78**, 010306(R) (2008).

[42] V. Alba, M. Fagotti, and P. Calabrese, Entanglement entropy of excited states, J. Stat. Mech. (2009) P10020.

[43] L. Campos Venuti, N. T. Jacobson, S. Santra, and P. Zanardi, Exact infinite-time statistics of the Loschmidt echo for a quantum quench, Phys. Rev. Lett. **107**, 010403 (2011).

[44] K. Najafi, M. A. Rajabpour, and J. Viti, Light-cone velocities after a global quench in a noninteracting model, Phys. Rev. B **97**, 205103 (2018).

[45] K. Najafi, M. A. Rajabpour, and J. Viti, Return amplitude after a quantum quench in the XY chain, J. Stat. Mech. (2019) 083102.

[46] R. Jafari and A. Akbari, Dynamics of quantum coherence and quantum Fisher information after a sudden quench, Phys. Rev. A **101**, 062105 (2020).

[47] S. Bhattacharyya, S. Dasgupta, and A. Das, Signature of a continuous quantum phase transition in non-equilibrium energy absorption: Footprints of criticality on higher excited states, Sci. Rep. **5**, 16490 (2015).

[48] S. Roy, R. Moessner, and A. Das, Locating topological phase transitions using nonequilibrium signatures in local bulk observables, Phys. Rev. B **95**, 041105(R) (2017).

[49] P. Titum, J. T. Iosue, J. R. Garrison, A. V. Gorshkov, and Z.-X. Gong, Probing ground-state phase transitions through quench dynamics, Phys. Rev. Lett. **123**, 115701 (2019).

[50] A. Haldar, K. Mallayya, M. Heyl, F. Pollmann, M. Rigol, and A. Das, Signatures of quantum phase transitions after quenches in quantum chaotic one-dimensional systems, Phys. Rev. X **11**, 031062 (2021).

[51] J. Surace, L. Tagliacozzo, and E. Tonni, Operator content of entanglement spectra in the transverse field Ising chain after global quenches, Phys. Rev. B **101**, 241107(R) (2020).

[52] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevResearch.6.L032003 for additional results on two-point correlation functions, universality, and finite-size scaling analysis. It also discusses methods for calculating von Neumann entropy, mutual information, log-negativity and disjoint mutual information, and includes Refs. [78–89].

[53] R. R. P. Singh, M. B. Hastings, A. B. Kallin, and R. G. Melko, Finite-temperature critical behavior of mutual information, Phys. Rev. Lett. **106**, 135701 (2011).

[54] J. Wilms, J. Vidal, F. Verstraete, and S. Dusuel, Finite-temperature mutual information in a simple phase transition, J. Stat. Mech. (2012) P01023.

[55] F. C. Alcaraz and M. A. Rajabpour, Universal behavior of the Shannon and Rényi mutual information of quantum critical chains, Phys. Rev. B **90**, 075132 (2014).

[56] J.-M. Stéphan, Shannon and Rényi mutual information in quantum critical spin chains, Phys. Rev. B **90**, 045424 (2014).

[57] This notion of the mutual information correlation length is different from Ref. [11].

[58] P. Hayden, D. Leung, P. W. Shor, and A. Winter, Randomizing quantum states: Constructions and applications, Commun. Math. Phys. **250**, 371 (2004).

[59] A. Ben-Aroya and A. Ta-Shma, Quantum expanders and the quantum entropy difference problem, arXiv:quant-ph/0702129.

[60] M. B. Hastings, Entropy and entanglement in quantum ground states, Phys. Rev. B **76**, 035114 (2007).

[61] G. Vidal and R. F. Werner, Computable measure of entanglement, Phys. Rev. A **65**, 032314 (2002).

[62] M. B. Plenio, Logarithmic negativity: A full entanglement monotone that is not convex, Phys. Rev. Lett. **95**, 090503 (2005).

[63] J. Eisert, V. Eisler, and Z. Zimborás, Entanglement negativity bounds for fermionic Gaussian states, Phys. Rev. B **97**, 165123 (2018).

[64] V. Eisler and Z. Zimborás, On the partial transpose of fermionic Gaussian states, New J. Phys. **17**, 053048 (2015).

[65] W. K. Wootters and B. D. Fields, Optimal state-determination by mutually unbiased measurements, Ann. Phys. **191**, 363 (1989).

[66] J. Eisert, M. Cramer, and M. B. Plenio, *Colloquium*: Area laws for the entanglement entropy, Rev. Mod. Phys. **82**, 277 (2010).

[67] In contrast with the mutual information, the entanglement entropy generally diverges with the ultraviolet cutoff.

[68] F. Ares, J. G. Esteve, F. Falceto, and E. Sánchez-Burillo, Excited state entanglement in homogeneous fermionic chains, J. Phys. A: Math. Theor. **47**, 245301 (2014).

[69] P. Calabrese and J. Cardy, Quantum quenches in $1+1$ dimensional conformal field theories, J. Stat. Mech. (2016) 064003.

[70] S. Sachdev, *Quantum Phase Transitions*, 2nd ed. (Cambridge University Press, United Kingdom, 2011).

[71] A. Iucci and M. A. Cazalilla, Quantum quench dynamics of the Luttinger model, Phys. Rev. A **80**, 063619 (2009).

[72] M. A. Cazalilla and M.-C. Chung, Quantum quenches in the Luttinger model and its close relatives, J. Stat. Mech.: Theory Exp. (2016) 064004.

[73] A. Browaeys and T. Lahaye, Many-body physics with individually controlled Rydberg atoms, Nat. Phys. **16**, 132 (2020).

[74] M. Morgado and S. Whitlock, Quantum simulation and computing with Rydberg-interacting qubits, AVS Quantum Sci. **3**, 023501 (2021).

[75] G. Pagano, A. Bapat, P. Becker, K. S. Collins, A. De, P. W. Hess, H. B. Kaplan, A. Kyprianidis, W. L. Tan, C. Baldwin, L. T. Brady, A. Deshpande, F. Liu, S. Jordan, A. V. Gorshkov, and C. Monroe, Quantum approximate optimization of the long-range Ising model with a trapped-ion quantum simulator, Proc. Natl. Acad. Sci. USA **117**, 25396 (2020).

[76] A. Elben, B. Vermersch, C. F. Roos, and P. Zoller, Statistical correlations between locally randomized measurements: A toolbox for probing entanglement in many-body quantum states, Phys. Rev. A **99**, 052323 (2019).

[77] T. Brydges, A. Elben, P. Jurcevic, B. Vermersch, C. Maier, B. P. Lanyon, P. Zoller, R. Blatt, and C. F. Roos, Probing Rényi entanglement entropy via randomized measurements, Science **364**, 260 (2019).

[78] E. Barouch and B. M. McCoy, Statistical mechanics of the XY model. II. Spin-correlation functions, Phys. Rev. A **3**, 786 (1971).

[79] E. L. Basor and K. E. Morrison, The Fisher-Hartwig conjecture and Toeplitz eigenvalues, Linear Algebra Its Appl. **202**, 129 (1994).

[80] F. Franchini and A. G Abanov, Asymptotics of Toeplitz determinants and the emptiness formation probability for the XY spin chain, J. Phys. A: Math. Gen. **38**, 5069 (2005).

[81] P. Calabrese, J. Cardy, and E. Tonni, Entanglement entropy of two disjoint intervals in conformal field theory, J. Stat. Mech. (2009) P11001.

[82] M. Fagotti and P. Calabrese, Entanglement entropy of two disjoint blocks in XY chains, J. Stat. Mech. (2010) P04016.

[83] V. Alba, L. Tagliacozzo, and P. Calabrese, Entanglement entropy of two disjoint blocks in critical Ising models, Phys. Rev. B **81**, 060411(R) (2010).

[84] A. Coser, E. Tonni, and P. Calabrese, Partial transpose of two disjoint blocks in XY spin chains, J. Stat. Mech. (2015) P08005.

[85] H. Shapourian, K. Shiozaki, and S. Ryu, Many-body topological invariants for fermionic symmetry-protected topological phases, Phys. Rev. Lett. **118**, 216402 (2017).

[86] H. Shapourian, K. Shiozaki, and S. Ryu, Partial time-reversal transformation and entanglement negativity in fermionic systems, Phys. Rev. B **95**, 165101 (2017).

[87] M. Headrick, Entanglement Rényi entropies in holographic theories, Phys. Rev. D **82**, 126010 (2010).

[88] T. Kuwahara and K. Saito, Area law of noncritical ground states in 1D long-range interacting systems, Nat. Commun. **11**, 4478 (2020).

[89] F. Franchini, *An Introduction to Integrable Techniques for One-Dimensional Quantum Systems* (Springer, United Kingdom, 2017), Vol. 940.