## **Letter**

## **Observation of highly correlated ultrabright biphotons through increased atomic ensemble density in spontaneous four-wave mixing**

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The pairing ratio, a crucial metric assessing a biphoton source's ability to generate correlated photon pairs, remains underexplored despite theoretical predictions. This study presents experimental findings on the pairing ratio, utilizing a double- $\Lambda$  spontaneous four-wave mixing biphoton source in cold atoms. At an optical depth (OD) of 20, we achieved an ultrahigh biphoton generation rate of up to  $1.3 \times 10^7$  per second, with a successful pairing ratio of 61%. Increasing the OD to 120 significantly improved the pairing ratio to 89%, while maintaining a consistent biphoton generation rate. This achievement, marked by high generation rates and robust biphoton pairing, holds great promise for advancing efficiency in quantum communication and information processing. Additionally, in a scenario with a lower biphoton generation rate of  $5.0 \times 10^4$  per second, we attained an impressive signal-to-background ratio of 241 for the biphoton wavepacket, surpassing the Cauchy-Schwarz criterion by approximately  $1.5 \times 10^4$  times.

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*Introduction*. Temporally correlated biphotons have recently garnered considerable attention in the fields of optical quantum computing and quantum communication, thanks to their exceptional nonclassical properties. Of particular significance is their role as heralded single-photon sources, which have found applications in diverse domains, including quantum cryptography  $[1-4]$ , quantum metrology  $[5-8]$ , and quantum imaging [\[9–12\]](#page-4-0). Among the various biphoton sources, the spontaneous four-wave mixing (SFWM) mechanism, distinguished by its operation near atomic resonance, stands out for its ability to conveniently manipulate bandwidth and serves as a bridge between different quantum devices, thus attracting significant interest.

The operation of SFWM near resonance accommodates diverse energy level configurations, including the double-  $\Lambda$  scheme [\[13–15\]](#page-4-0) and cascade-transition scheme [\[16–18\]](#page-4-0). This proximity to atomic resonance enables the generation of bright biphotons with low optical power [\[19,20\]](#page-4-0), as well as the production of narrowband biphotons [\[21,22\]](#page-4-0). Especially for the double- $\Lambda$  scheme, characterized by its intrinsic

--type electromagnetically induced transparency (EIT) structure [\[23–](#page-4-0)[27\]](#page-5-0), it not only significantly suppresses the generation of noise photons [\[28–31\]](#page-5-0) but also provides a wide bandwidth tuning capability [\[32–34\]](#page-5-0). This facilitates its direct application in conjunction with quantum devices [\[35–38\]](#page-5-0) or reshaping of the biphoton waveforms [\[39–41\]](#page-5-0). Furthermore, the  $\Lambda$  structure supports convenient implementation in twoor three-level atomic systems [\[42–46\]](#page-5-0).

Despite the numerous remarkable achievements of the  $double- $\Lambda$  SFWM scheme, there is a frequently overlooked$ concern: limited atomic density hinders spontaneously emitted photons from achieving perfect coherence through the stimulated four-wave mixing (FWM) process [\[47,48\]](#page-5-0). Incoherent emissions not involved in the stimulated FWM process can significantly diminish the biphoton pairing capacity, referred to as the pairing ratio. Unfortunately, while the Heisenberg-Langevin operator theory is predictive in understanding the concept of the pairing ratio [\[49\]](#page-5-0), there is currently a lack of relevant research and investigation in experimental studies.

In this Letter, we present the thorough investigation of the pairing ratio using the double- $\Lambda$  SFWM in a cold  ${}^{87}Rb$ ensemble. This configuration, chosen for its inherent EIT effect, allows for easy control of the biphoton bandwidth. We achieved an exceptionally high biphoton generation rate of approximately  $1.3 \times 10^7$  per second at a low optical depth (OD) of 20, with the pairing ratio measured at 0.61. Under the same generation rate, a ratio of 0.89 was observed at a high

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<span id="page-1-0"></span>

FIG. 1. Diagram of the double-A SFWM system and experimental setup. M, mirror; L, lens; PBS, polarizing beam splitter; QWP, quarter-wave plate; SMF, single-mode fiber; MMF, multimode fiber; EFS, etalon filter set; SPCM, single-photon counting module. The inset shows the relevant energy levels of the  $87Rb$  atom.

OD of 120. This demonstrates that a high OD is advantageous at the same generation rate. Additionally, at a relatively low biphoton generation rate of  $5.0 \times 10^4$  per second, the signalto-background ratio for the biphoton wavepacket reached 241, exceeding the Cauchy-Schwarz criterion by approximately  $1.5 \times 10^4$  times.

*Experimental setup*. We trapped cold <sup>87</sup>Rb atoms using a standard magneto-optical trap. After optically pumping them to the ground state  $|5S_{1/2}, F = 1\rangle$ , as illustrated in Fig. 1, we irradiated the atomic ensemble with a far-detuned driving field characterized by a Rabi frequency  $\Omega_d$  and a nearly resonant coupling field denoted by  $\Omega_c$ . Synchronization between these fields was achieved through injection locking with an external cavity diode laser (not shown). The driving field, with detuning  $\Delta_d$ , operated on the  $\sigma^+$  transition  $|5S_{1/2}, F = 1\rangle \rightarrow$  $|5P_{3/2}, F = 2\rangle$ , effectively suppressing incoherent fluorescence from one-photon absorption. This allowed for primary emission of Stokes photons via spontaneous Raman scattering [\[50\]](#page-5-0). Subsequently, the nearly resonant coupling field, with detuning  $\Delta_c$ , acted on the  $\sigma^+$  transition  $|5S_{1/2}, F = 2\rangle \rightarrow$  $|5P_{3/2}, F = 2\rangle$ , inducing the emission of anti-Stokes photons. We used an elongated atomic ensemble to enhance the stimulated FWM effect and boosting specific direction scattering probability.

In the SFWM experiment, biphotons were generated using a 10 µs driving pulse in each 2.5 ms cycle. To prevent laser leakage, we adopted a backward configuration where the driving and coupling beams counter-propagated, each with  $1/e^2$ full widths of 250 and 310 µm. The corresponding optical powers of the  $1 \Gamma$  driving and coupling fields were approximately 7.5 and 11.1  $\mu$ W. Both fields drove  $\sigma^+$  transitions, resulting in generated photon pairs exhibiting  $\sigma^+$  polarization, propagating in opposite directions and passing through respective etalon filter sets (EFS). The intersection angle between the Stokes (anti-Stokes) and driving (coupling) beams was set at 1.7◦ for our experiment. Each EFS consisted of two etalons, each with an extinction ratio of roughly 30 dB and an approximate bandwidth of 100 MHz, separated by an optical isolator. The total extinction ratios of the Stokes and anti-Stokes channels were 114 dB and 124 dB, respectively.

Biphotons were detected using fiber-coupled single-photon counting modules (SPCM-AQRH-13-FC). Upon detection, an 8 ns pulse was emitted from the SPCM toward the time-of-flight multiscaler (TOF, MCS6A-4T8, not shown). In the coincidence count experiment, we measured the time difference between Stokes and anti-Stokes photons. When generated as a correlated pair, they arrived at the SPCMs within the correlation time, contributing to a nonflat biphoton wavepacket. The TOF generated a histogram of coincident counts based on these data points, providing insight into the source of biphotons.

*Pairing ratio*. We utilized the Heisenberg-Langevin operator theory (refer to Sec. I A in the Supplemental Material [\[51\]](#page-5-0)) to analyze the biphoton generation and the associated pairing ratio properties in double- $\Lambda$  SFWM [\[49\]](#page-5-0). The photon generation rate, given by  $R = \frac{c}{L} \langle \hat{a}^\dagger \hat{a} \rangle$  (refer to Sec. I B in the Supplemental Material [\[51\]](#page-5-0)) and derived from the annihilation operator  $\hat{a}$ , leads to the following expressions:

$$
R_s = \int \frac{d\omega}{2\pi} \left( |B|^2 + \sum_{jk,j'k'} \int_0^L dz P_{jk}^* \mathcal{D}_{jk^{\dagger},j'k'} P_{j'k'} \right)
$$
  
\n
$$
= \int d\omega \widetilde{R}_s(\omega),
$$
  
\n
$$
R_{as} = \int \frac{d\omega}{2\pi} \left( |C|^2 + \sum_{jk,j'k'} \int_0^L dz Q_{jk} \mathcal{D}_{jk,j'k^{\dagger}} Q_{j'k}^* \right)
$$
  
\n
$$
= \int d\omega \widetilde{R}_{as}(\omega),
$$
\n(2)

where  $\mathcal{D}_{jk^{\dagger},j^{\prime}k^{\prime}}$  and  $\mathcal{D}_{jk,j^{\prime}k^{\prime}}$  are diffusion coefficients (refer to Sec. IC in the Supplemental Material [\[51\]](#page-5-0)), while  $R_{s(as)}$ represents the spectrum of Stokes (anti-Stokes) photons. The total photon generation rates,  $R_s$  and  $R_{as}$ , comprise two components: correlated photons produced by the stimulated FWM process with coefficients *B* and *C*, and uncorrelated photons due to vacuum field fluctuations, represented by an integral term with diffusion coefficients. The pairing ratio,  $r_p$ , denotes the ratio of the correlated photons to the total generated photons. Under ideal conditions, *Rs* and *Ras* are nearly equal. However, in experiments, *Ras* is slightly smaller due to phase mismatch and ground state decoherence. Therefore, the biphoton generation rate  $R_B$  is contingent on  $R_{as}$ .

In the SFWM process within the atomic ensemble, a spontaneously emitted Stokes photon from one atom may interact with nearby atoms, triggering stimulated Raman scattering. Unlike spontaneous Raman scattering, the Stokes photon generated by stimulated Raman scattering shares the same direction as the incident Stokes photon, enhancing directionality. This collective enhancement effect establishes paired correlation with the anti-Stokes photon through the stimulated FWM process, reflected in coefficients *B* and *C*. However, as SFWM relies on vacuum field fluctuations, the generated photons exhibit isotropic (uncorrelated) nature, represented by the integral term with diffusion coefficients in Eqs. (1) and (2). While the paired correlations of these biphotons can be established through the stimulated FWM process, it requires a sufficiently high OD within the atomic ensemble.

<span id="page-2-0"></span>*Coincidence count rate*. In biphoton systems, the normalized Glauber second-order cross-correlation function  $g_{s-as}^{(2)}(\tau)$ is often used alongside the photon generation rate. This function is a crucial parameter for evaluating the temporal correlation between biphotons. The derived theoretical expression is as follows:

$$
g_{s-as}^{(2)}(\tau) = 1 + \frac{1}{R_s R_{as}} \left| \int \frac{d\omega}{2\pi} e^{-i\omega \tau} \right|
$$

$$
\times \left( B^* D + \sum_{j k, j' k'} \int_0^L dz P_{jk}^* \mathcal{D}_{j k^{\dagger}, j' k'} \mathcal{Q}_{j' k'} \right) \right|^2. \quad (3)
$$

The integral term on the right-hand side of Eq. (3) reveals the correlation of biphotons. This term is equivalent to the wavepacket of the anti-Stokes single photon, conditioned on the postselection of a Stokes single photon. This correlation provides valuable information for evaluating the biphoton source. For instance, the peak signal-to-background ratio, denoted as  $r_{SB}$ , is defined as the maximum value of  $[g_{s-as}^{(2)}(\tau) - 1]$ . It serves as a standard metric for assessing the nonclassicality of a biphoton source. In the case of a classical field, the Cauchy-Schwarz inequality universally applies:  $[g_{s-s}^{(2)}(\tau)]^2 [g_{s-s}^{(2)}(0)g_{as-as}^{(2)}(0)]^{-1} \leq 1$ . The normalized autocorrelation functions of the Stokes and anti-Stokes fields can be derived as  $g_{s-s}^{(2)}(\tau) = 1 + R_s^{-2} |\int d\omega \widetilde{R}_s e^{-i\omega \tau}|^2$  and  $g_{as-as}^{(2)}(\tau) =$ <br>  $g_{as-as}^{(2)}(\tau) = 1 + R_s^{-2} |\int d\omega \widetilde{R}_s e^{-i\omega \tau}|^2$  and  $g_{as-as}^{(2)}(\tau) =$  $1 + R_{as}^{-2} \int d\omega \tilde{R}_{as} e^{-i\omega \tau}$ <sup>2</sup>. These equations indicate that both the Staling and anti-Staling fields while the small states with the Stokes and anti-Stokes fields exhibit thermal states, with  $g_{s-s}^{(2)}(0) = g_{as-as}^{(2)}(0) = 2$ . Nonclassical behavior is observed when  $r_{SB} > 1$ . Additional details and initial proofs of the thermal field distributions for both the Stokes and anti-Stokes fields can be found in Sec. I D in the Supplemental Material [\[51\]](#page-5-0).

In our data processing, we introduced the coincidence count rate  $R_C$  to facilitate the acquisition of  $r_p$  and  $R_B$ .  $R_C$  is calculated as  $R_s R_{as} g_{s-as}^{(2)}(\tau) \Delta T + R_{env}$  (refer to Secs. II A and II B in the Supplemental Material  $[51]$ ), where  $R_{env}$  accounts for environmental background count rates, arising from laser leakage or SPCM dark counts. For data processing, we used a time bin of  $\Delta T = 1/R_s$  to tally Stokes photons, enabling the postselection of a single Stokes photon. This ensures that the background and correlated regions of the coincidence count rate correspond to  $R_B + R_{env}$  and  $r_p$ , respectively.

*Biphoton bandwidth*. Figure 2 presents the experimental *R*<sub>C</sub> for various coupling field conditions. The time bin for detected anti-Stokes photons,  $\Delta \tau = 6.4$  ns, aligns with the time interval between experimental data points in Fig. 2. In Figs.  $2(a)$  and  $2(b)$ , we set the coupling Rabi frequencies  $\Omega_c$  to  $4 \Gamma$  and  $1 \Gamma$ , respectively. With the OD fixed at 15, both cases yielded a measured biphoton generation rate  $R_B$ of approximately  $3.4 \times 10^5$  s<sup>-1</sup>. The delay time in Fig. 2(b) is significantly longer than that in  $2(a)$ . This delay arises from two intrinsic properties of the double- $\Lambda$  SFWM system: the damped Rabi oscillation with a period denoted as  $\tau_R = 2\pi/\sqrt{|\Omega_c|^2 - \Gamma^2/4}$ , and the delay time attributed to the EIT effect denoted as  $\tau_{\text{EIT}} = \text{FOD}/|\Omega_c|^2$  [\[52\]](#page-5-0). Both characteristic times are influenced by the coupling field. The damped Rabi oscillation periods in Figs.  $2(a)$  and  $2(b)$  are calculated as 42 and 192 ns, respectively. As  $\Omega_c$  decreases, the EIT



FIG. 2. Biphotons with controllable bandwidth. The red lines represent the theoretical curves, while the black circles indicate the experimental data points. The time bin for detecting the anti-Stokes photons is  $\Delta \tau = 6.4$  ns. Other parameters are OD = 15,  $\Omega_d = 1\Gamma$ ,  $\Delta_d = 10\Gamma$ ,  $\gamma_{21} = 0.001\Gamma$ ,  $\Delta kL = 0.37\pi$ , (a)  $\Omega_c = 4\Gamma$ ,  $\Delta_c = 0\Gamma$ , (b)  $\Omega_c = 1\Gamma$ ,  $\Delta_c = 0\Gamma$ , (c)  $\Omega_c = 1\Gamma$ ,  $\Delta_c = 1\Gamma$ , (d)  $\Omega_c = 1\Gamma$ ,  $\Delta_c =$  $3<sub>\Gamma</sub>$ 

effect causes anti-Stokes photons to propagate slowly. The EIT delay times in Figs.  $2(a)$  and  $2(b)$  are 25 and 398 ns, respectively. The overall delay time is determined by the larger of  $\tau_R$  and  $\tau_{\text{EIT}}$ , i.e., max( $\tau_R$ ,  $\tau_{\text{EIT}}$ ). Consequently, the behavior of the biphoton wavepacket in Fig.  $2(b)$ , where  $\tau_{\text{EIT}}$ dominates, exhibits characteristics reminiscent of slow light, with the slow light effect noticeable in the trailing edge of the biphoton wavepacket. Conversely, in Fig. 2(a), where  $\tau_R$ surpasses  $\tau$ <sub>EIT</sub>, subtle oscillatory features are present within the biphoton wavepacket.

Figures  $2(c)$  and  $2(d)$  demonstrate how changes in coupling detuning  $\Delta_c$  affect the biphoton bandwidth. All experimental parameters were consistent with those in Fig.  $2(b)$ , except for the  $\Delta_c$ . A shorter delay time in Fig. 2(c) is observed due to the introduction of  $\Delta_c = 1\Gamma$ , which reduces the effective OD and shortens the EIT-induced delay. Conversely, with  $\Delta_c = 3\Gamma$  in Fig. 2(d), the tail lengthens again. This behavior is attributed to damped Rabi oscillations, where a larger  $\Delta_c$  weakens the interaction between the coupling field and the atomic medium, requiring more time to convert spinwave excitations into anti-Stokes photons. The values of  $R<sub>B</sub>$  in Figs. 2(c) and 2(d) are  $3.4 \times 10^5$  s<sup>-1</sup> and  $3.1 \times 10^5$  s<sup>-1</sup>, respectively. This demonstrates that by detuning the coupling field, we can control the biphoton bandwidth without significantly reducing  $R<sub>B</sub>$ . Further discussions can be found in Sec. II C in the Supplemental Material [\[51\]](#page-5-0).

*High-purity biphotons*. Figure [3\(a\)](#page-3-0) illustrates high-purity biphoton generation achieved with parameters  $\Omega_d = 0.5 \Gamma$ ,  $\Omega_c = 4 \Gamma$ , and OD = 10. The theoretical  $R_B$  is calculated as  $5.0 \times 10^4$  s<sup>-1</sup>. In experiments,  $R_B$  was determined by subtracting the total measured background count rate,  $R_{\text{tot}} =$  $1.6 \times 10^5$  s<sup>-1</sup>, from the environmental background count rate,  $R_{env} = 1.1 \times 10^5 \text{ s}^{-1}$ . This yielded an experimental  $R_B$  of approximately  $5.0 \times 10^4$  s<sup>-1</sup>, in close agreement with the theoretical prediction. In Fig.  $3(b)$ , we theoretically calculated  $R_B$ to be  $1.9 \times 10^6$  s<sup>-1</sup> at  $\Omega_d = 3$   $\Gamma$ . The experimentally observed

<span id="page-3-0"></span>



FIG. 3. High-purity biphotons. The red lines represent the theoretical curves, while the black circles indicate the experimental data points. The time bin for detecting the anti-Stokes photons is  $\Delta \tau$  = 1.6 ns. The remaining parameters are set to  $OD = 10$ ,  $\Omega_c = 4 \Gamma$ ,  $\Delta_d = 10 \Gamma$ ,  $\gamma_{21} = 0.001 \Gamma$ ,  $\Delta k = 0.37\pi$ , with (a)  $\Omega_d = 0.5 \Gamma$  and (b)  $\Omega_d = 3 \Gamma$ . (c) The peak signal-to-background ratio  $r_{SB}$  versus  $\Omega_d$ . The black squares represent the experimental data, and the black line is the curve fitted to these experimental data points. (d) The biphoton generation rate  $R_B$  and pairing ratio  $r_p$  as a function of  $\Omega_d$ . The experimental data points for  $R_B$  and  $r_p$  are represented by the unfilled blue and solid magenta circles, respectively. The theoretical curves for  $R_B$  and  $r_p$  are depicted by the blue and magenta lines, respectively.

 $R_{\rm B}$ , obtained from measurements of  $R_{\rm tot} = 2.0 \times 10^6$  s<sup>-1</sup> and  $R_{env} = 1.2 \times 10^5 \text{ s}^{-1}$ , also closely matches theoretical prediction. As  $\Omega_d$  increases, both  $R_B$  and  $R_{tot}$  rise significantly. However, this also leads to a notable decrease in  $r_{SB}$ , as shown in Fig. 3(c). At  $\Omega_d = 0.5 \Gamma$ , we observed an experimental  $r_{SB} = 241$ , surpassing the Cauchy-Schwarz criterion by a factor of approximately  $1.5 \times 10^4$ . If the *R*<sub>env</sub> in our experiment could be completely eliminated, it would lead to a more pronounced violation of the Cauchy-Schwarz criterion, exceeding the normal level by a factor of  $5.9 \times 10^4$ .

Figure  $3(d)$  shows the variation of  $R_B$  and  $r_p$  with different  $\Omega_d$  values. At  $\Omega_d = 0.5 \Gamma$  and  $\Omega_d = 3 \Gamma$ , the corresponding  $r_p$  values are 0.63 and 0.59, respectively. These experimental  $r_p$  values were determined based on the area under the correlated biphoton wavepacket. The  $r_p$  obtained from the area and those obtained from Eqs. [\(1\)](#page-1-0) and [\(2\)](#page-1-0) are equivalent, as detailed in Sec. II D in the Supplemental Material [\[51\]](#page-5-0). In the SFWM process, atomic ensembles play a crucial role in collectively enhancing the correlation between the Stokes and anti-Stokes fields along the applied light direction. Therefore, with a fixed OD, while increasing  $\Omega_d$  can boost  $R_\text{B}$ , the limited density of atomic ensembles constrains their ability to produce correlated photon pairs, leading to a slight decrease in *rp*.

*Highly correlated biphotons*. Figure 4(a) showcases the generation of ultrabright biphotons using specific parameters  $\Omega_d = 3 \Gamma$ ,  $\Omega_c = 4 \Gamma$ ,  $\Delta_d = 5 \Gamma$ , and OD = 20, resulting in a remarkable theoretical  $R_B$  of 1.3 × 10<sup>7</sup> s<sup>-1</sup>. This exceeds rates reported in the literature for the double- $\Lambda$  SFWM scheme. Under these conditions, the experimental total background count rate was  $R_{\text{tot}} = 1.3 \times 10^7 \text{ s}^{-1}$ , with environmental



FIG. 4. Highly correlated ultrabright biphotons. The red lines represent the theoretical curves, while the black circles indicate the experimental data points. The time bin for detecting the anti-Stokes photons is  $\Delta \tau = 1.6$  ns. The remaining parameters are set to  $\Omega_d =$ 3  $\Gamma$ ,  $\gamma_{21} = 0.001 \Gamma$ ,  $\Delta k = 0.37\pi$ , with (a)  $OD = 20$ ,  $\Omega_c = 4 \Gamma$ ,  $\Delta_d = 5 \Gamma$ , and (b) OD = 120,  $\Omega_c = 8.8 \Gamma$ ,  $\Delta_d = 14.9 \Gamma$ . (c) The peak signal-to-background ratio  $r_{SB}$  versus OD. The black squares represent the experimental data, and the black line is the curve fitted to these experimental data points. (d) The biphoton generation rate  $R_B$  and pairing ratio  $r_p$  as a function of OD. The experimental data points for  $R_B$  and  $r_p$  are represented by the unfilled blue and solid magenta circles, respectively. The theoretical curves for  $R_B$  and  $r_p$ are depicted by the blue and magenta lines, respectively.

background at  $R_{env} = 2.3 \times 10^5 \text{ s}^{-1}$ , accounting for only 1.8% of the total. Thus, in this high  $R<sub>B</sub>$  scenario, the primary source of background count arises from the high photon generation rate rather than environmental factors. Furthermore, in this scenario, the measured  $r_{SB}$  was 2.4, surpassing the Cauchy-Schwarz criterion by a factor of 2.9, while  $r_p$  was only 0.61. Although increasing the coupling power can enhance  $r_{SB}$ , as demonstrated in Fig. [2,](#page-2-0) it does not lead to corresponding improvements in  $r_p$ . To enhance both  $r_p$  and  $r_{SB}$ , we further increased the OD. In addition to OD = 20, we measured the biphoton wavepacket at  $OD = 40, 60, 80, 100,$ and 120. We fine-tuned  $\Omega_c$  to maintain a consistent biphoton bandwidth, while keeping  $\Omega_d = 3 \Gamma$  constant and adjusting  $\Delta_d$  to maintain a theoretical *R*<sub>B</sub> of 1.3 × 10<sup>7</sup> s<sup>−1</sup>. Specific parameters can be found in Sec. IIE in the Supplemental Material [\[51\]](#page-5-0).

In Fig. 4(b), we present the scenario with  $OD = 120$ ,  $\Omega_c = 8.8 \Gamma$ , and  $\Delta_d = 14.9 \Gamma$ . Here, the measured  $r_{SB}$  at 4.2 exceeds the Cauchy–Schwarz criterion by 6.8 times. An evident positive correlation emerges between increased OD and enhanced  $r_p$ , resulting in a higher  $r_{SB}$  due to augmented coincidence receptions [Fig. 4(c)]. This enhancement stems from the increased accumulation of biphoton correlations along a specific direction at higher OD values. Photons generated at higher OD levels are more likely to encounter subsequent atoms, amplifying the collective enhancement through the stimulated FWM process. Furthermore, while  $r_{SB}$  can also be improved by increasing  $\Omega_c$ , this approach does not enhance  $r_p$ , and therefore cannot improve the generation rate of temporally correlated biphotons. Figure  $4(d)$  illustrates the relationships between  $R_B$  and  $r_p$  with OD. At OD = 120,

<span id="page-4-0"></span>we observed the highest  $r_p$  of 0.89, indicating a significant improvement in correlated photon pair generation. The experimental  $R_{\rm B} = 1.3 \times 10^7 \,\rm s^{-1}$  signifies the successful generation of approximately  $1.2 \times 10^7$  pairs of correlated photons per second. Additionally, the Fourier transform of  $(R_C - R_{tot})$ reveals a biphoton bandwidth of approximately 24 MHz, resulting in a spectral brightness of the biphoton source at  $5.4 \times 10^5$  s<sup>-1</sup>MHz<sup>-1</sup>, surpassing the highest achieved by submegahertz biphoton sources [\[53\]](#page-5-0). These results highlight the crucial role of high OD in SFWM-based biphoton sources, allowing for higher values of  $R_B$  and  $r_p$ . This enables the generation of a large quantity of high-quality correlated photon pairs for use in various quantum systems.

*Conclusion*. Our investigation into the biphoton pairing ratio, utilizing the double- $\Lambda$  SFWM in cold  $^{87}$ Rb atoms, revealed a marginal decrease with higher biphoton generation rates. Nonetheless, this trend can be effectively addressed by elevating the atomic ensemble density. The highest

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pairing ratio observed was 0.89 at an OD of 120, accompanied by an ultrabright biphoton generation rate of up to  $1.3 \times$ 10<sup>7</sup> s−1, surpassing previously reported rates achieved via the double- $\Lambda$  SFWM scheme. Furthermore, our experiment demonstrated the highest signal-to-background ratio of the biphoton wavepacket at 241, achieved at a low biphoton generation rate of  $5.0 \times 10^4$  s<sup>-1</sup>. This outstanding performance exceeded the Cauchy-Schwarz criterion by approximately  $1.5 \times 10^4$  times. These results underscore the capability of the double-A SFWM scheme in advancing biphoton sources for future quantum technologies.

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