Letter

Hayden-Preskill recovery in Hamiltonian systems

Yoshifumi Nakata [1,*] and Masaki Tezuka [2,†]

[1] *Yukawa Institute for Theoretical Physics, Kyoto University, Kitashirakawa, Sakyo-ku, Kyoto 606-8502, Japan*

[2] *Department of Physics, Kyoto University, Kitashirakawa, Sakyo-ku, Kyoto 606-8502, Japan*

(Received 20 March 2023; revised 1 April 2024; accepted 3 April 2024; published 25 April 2024)

Information scrambling refers to the unitary dynamics that quickly spreads and encodes localized quantum information over an entire many-body system and makes the information accessible from any small subsystem. While information scrambling is the key to understanding complex quantum many-body dynamics and is well-understood in random unitary models, it has been hardly explored in Hamiltonian systems. In this Letter, we investigate the information recovery in various time-independent Hamiltonian systems, including chaotic spin chains and Sachdev-Ye-Kitaev models. We show that information recovery is possible in certain, but not all, chaotic models, which highlights the difference between information recovery and quantum chaos based on the energy spectrum or the out-of-time-ordered correlators. We also show that information recovery probes transitions caused by the change of information-theoretic features of the dynamics.

DOI: 10.1103/PhysRevResearch.6.L022021

Introduction. A central challenge in modern physics is to characterize the dynamics in far-from-equilibrium quantum systems. The Hayden-Preskill protocol [1] offers an operational approach toward this goal and has been attracting much attention [2–29]. The protocol addresses if the information initially localized in a small subsystem can be recovered from other subsystems after unitary time evolution. If the unitary dynamics is sufficiently random, the information is rapidly encoded into the whole system, and information can be recovered from any small subsystem [1]. This phenomenon is can be thought of an information-theoretic manifestation of complex quantum dynamics and is called the *Hayden-Preskill recovery*. The unitary dynamics that leads to the Hayden-Preskill recovery is referred to as *information scrambling* [1].

The Hayden-Preskill recovery is of interdisciplinary interest: it is inspired by the information paradox of black holes [30–32], is formulated in the quantum information language, and is investigated by the technique of random matrix theory (RMT) [33,34]. Many related properties, such as entanglement generation [2,4], operator mutual information (OMI) [19,35–37], and out-of-time-ordered correlators (OTOCs) [26,38,39], have been intensely studied.

Despite this progress, information recovery in Hamiltonian systems has been rarely explored [40]. It is widely believed that the Hayden-Preskill recovery is possible in quantum chaotic systems, but the original analysis strongly relies on the random unitary assumption, which is unlikely to be satisfied even approximately by time-independent Hamiltonian dynamics [8,41]. Furthermore, quantum chaos is commonly characterized by eigenenergy statistics, which is a *static* property, but information recovery is about the *dynamical* properties. Thus, the relation between quantum chaos and information recovery is not *a priori* trivial.

In this Letter, we investigate in detail the information recovery in various Hamiltonian systems. We first provide a class of Hamiltonians that do not lead to information scrambling. This includes chaotic spin-1/2 chains, such as the Heisenberg model with random magnetic field and the mixed field Ising model. Notably, they saturate OTOCs for local observables but do not achieve the Hayden-Preskill recovery, demonstrating the difference between the saturation of OTOCs for local observables and information scrambling.

We then confirm information scrambling in the Sachdev-Ye-Kitaev (SYK) Hamiltonian [11,12,42,43], which is a canonical holographic dual to quantum gravity [13,14,44–46]. The SYK model is known to have scrambling features in many senses, such as saturation of OTOCs [11,12] for local observables, the maximum quantum Lyapunov exponent [47], and an RMT-like energy statistics [44,48,49]. Our result adds another scrambling feature to the model, that is, it achieves the Hayden-Preskill recovery. We also show that sparse variants [50] achieve the information recovery as well, possibly helping experimental realizations of the protocol.

We finally address the question whether the information recovery can reveal novel quantum many-body phenomena. Using a variant of SYK models, we affirmatively answer to this question: the information recovery can capture a transition that was previously unknown. The transition is caused by a drastic change of information-theoretic structures of the Hamiltonian dynamics. This is of interest as it characterizes complex quantum many-body dynamics from the quantum information viewpoint.

*yoshifumi.nakata@yukawa.kyoto-u.ac.jp

†tezuka@scphys.kyoto-u.ac.jp

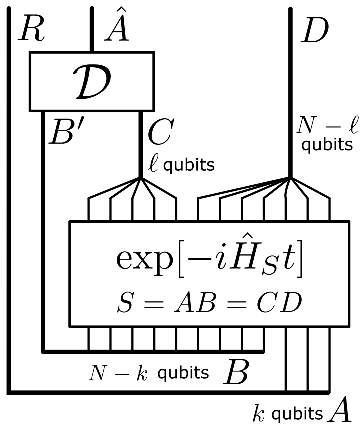

FIG. 1. A diagram of the Hayden-Preskill protocol. Time flows from bottom to top. Horizontal lines imply that the qubits connected by the line may be entangled. The initial states on AR and BB' are given by a maximally entangled state of k ebits, i.e., k EPR pairs, that keeps track of quantum information in A, and a purified state $|\xi(\beta)\rangle$ of a thermal state on B at the inverse temperature β, respectively. The system $S := AB$ undergoes Hamiltonian dynamics by $\hat{H}_S$, and then, is split into two subsystems, C of ℓ qubits and D of $N-\ell$ qubits. By applying a quantum channel $\mathcal{D}$ onto $B'C$, one aims to decode the quantum information in A, that is, to recover the k EPR pairs between $\hat{A}$ and R. This protocol has a natural interpretation in the context of information paradox [1]. See also a tutorial [51].

The Hayden-Preskill protocol. Given quantum many-body system S of N qubits, we encode quantum information into a local subsystem $A \subseteq S$ of k qubits ($k \ll N$). We then let the system S undergo the Hamiltonian time-evolution $U_{\hat{H}}(t) := e^{-i\hat{H}t}$ for some time t, where $\hat{H}$ is the Hamiltonian in S. After the time-evolution, the information is tried to be recovered from an ℓ-qubit subsystem $C \subseteq S$. Throughout our analysis, we assume $C \subseteq B := S \setminus A$ as far as $\ell \leqslant N-k$. The question is how large ℓ should be for a successful recovery.

The answer depends on the initial state in B as well as available resources in the recovery process. Here, we assume that the initial state in B is a thermal state $\xi^B(\beta)$ at inverse temperature β. Based on its eigendecomposition $\xi^B(\beta) = \sum_j p_j(\beta)|\psi_j\rangle\langle\psi_j|^B$, we introduce a purified state $|\xi(\beta)\rangle^{BB'} := \sum_j \sqrt{p_j(\beta)}\,|\psi_j\rangle^B \otimes |\psi_j\rangle^{B'}$ on the system BB'. When the subsystem B' is traced out, the marginal state on B is the original thermal state $\xi^B(\beta)$. We consider the scenario in which the subsystem B' can be used in the recovery process. This has a natural interpretation in the black hole information paradox [1] and is of considerable interest. See Fig. 1.

The recovery of quantum information is formally defined by introducing a virtual reference system R that keeps track of the quantum information. Denoting k Einstein-Podolsky-Rosen (EPR) pairs between A and R by $|\Phi\rangle^{AR}$, we set the initial state on the system $SB'R = ABB'R$ to $|\Psi(t=0,\beta)\rangle^{SB'R} = |\Phi\rangle^{AR} \otimes |\xi(\beta)\rangle^{BB'}$. The subsystem S undergoes the Hamiltonian time evolution by $\hat{H}_S$, resulting in the state $|\Psi(t,\beta)\rangle^{SB'R} = (I^{B'R} \otimes e^{-i\hat{H}_S t})\,|\Psi(t,\beta)\rangle^{SB'R}$ at time t. Let $\Psi^{CB'R}(t,\beta)$ be the marginal state on $CB'R$, which is given by taking the partial trace over D of $|\Psi(t,\beta)\rangle^{SB'R}$. In the following, we indicate the subsystem over which the partial trace is taken by omitting the subsystem from the superscript.

Following the standard convention [1], the recovery error is defined by

$$\Delta_{\hat{H}}(t,\beta) := \frac{1}{2}\min_{\mathcal{D}} \left\| |\Phi\rangle\langle\Phi|^{AR} - \mathcal{D}^{CB'\to A}(\Psi^{CB'R}(t,\beta)) \right\|_1. \quad (1)$$

Here, the minimum is taken over all possible quantum operations $\mathcal{D}$, namely, all completely positive and trace-preserving (CPTP) maps, from CB' to A, and $\|\rho\|_1 = \mathrm{Tr}\sqrt{\rho^\dagger\rho}$ is the trace norm. Due to the Holevo-Helstrom theorem [52], the trace norm between two quantum states characterizes how well they can be distinguished and is suitable to quantify the recovery error. We normalize $\Delta_{\hat{H}}(t,\beta)$ so that $0 \leqslant \Delta_{\hat{H}}(t,\beta) \leqslant 1$. See S1 [53].

Computing $\Delta_{\hat{H}}(t,\beta)$ is in general intractable due to the minimization over the CPTP maps. The *decoupling* condition provides a necessary and sufficient condition for the recovery in terms of the state $\Psi^{DR}(t,\beta)$ [54–56]. Using the condition, a calculable, and typically good, upper bound on $\Delta_{\hat{H}}(t,\beta)$ is obtained:

$$\Delta_{\hat{H}}(t,\beta) \leqslant \bar{\Delta}_{\hat{H}}(t,\beta) := \min\{1, \sqrt{2\Sigma_{\hat{H}}(t,\beta)}\}, \quad (2)$$

where $\Sigma_{\hat{H}}(t,\beta) := \|\Psi^{DR}(t,\beta) - \Psi^D(t,\beta)\otimes\pi^R\|_1/2$, and $\pi^R = I^R/2^k$ is the completely mixed state on R. See also S1 [53]. Note that the quantity $\Sigma_{\hat{H}}(t,\beta)$ is closely related to the mutual information between R and D, which is equivalent to the OMI [19], but the OMI leads to a worse bound than Eq. (2).

The recovery error $\Delta_{\hat{H}}(t,\beta)$ is also related to OTOCs. If one can compute OTOCs for all observables on the k-qubit subsystem A and the ℓ-qubit subsystem C, or all the $4^{k+\ell}$ operators that form an operator basis on AC, the recovery error could be evaluated [57,58]. However, this is computationally intractable as OTOCs for at least $4^{k+\ell}$ operators are needed. Note that the existing studies of OTOCs in Hamiltonian systems are mostly about the cases with $k=\ell=1$, and hence, do not provide much insight into the recovery error.

Random unitary model and information scrambling. The Hayden-Preskill protocol was understood well in a random unitary model, where the time evolution $e^{-i\hat{H}_S t}$ is replaced with a Haar random unitary. The model does not have a parameter corresponding to time t, and its recovery error $\Delta_{\mathrm{Haar}}(\beta)$ satisfies, with high probability,

$$\Delta_{\mathrm{Haar}}(\beta) \leqslant \bar{\Delta}_{\mathrm{Haar}}(\beta) := \min\{1, 2^{\frac{1}{2}(\ell_{\mathrm{Haar,th}}(\beta)-\ell)}\}, \quad (3)$$

where $\ell_{\mathrm{Haar,th}}(\beta) := \frac{1}{2}(N+k-H(\beta))$, and $H(\beta) = -\log[\mathrm{Tr}[(\xi^B(\beta))^2]]$ is the Renyi-2 entropy [1,24,56].

From Eq. (3), $\Delta_{\mathrm{Haar}}(\beta) \ll 1$ if $\ell \gg \ell_{\mathrm{Haar,th}}(\beta)$. In particular, $\ell_{\mathrm{Haar,th}}(0) = k$. Hence, if the system B is initially at infinite temperature, the k-qubit quantum information in A is recoverable with exponential precision from any subsystem of size larger than k, which is independent of N. This phenomena is referred to as the Hayden-Preskill recovery. Following the original proposal [1], we refer to the dynamics achieving the Hayden-Preskill recovery as information scrambling.

Hamiltonians without information scrambling. Due to the facts that the information scrambling occurs in the random unitary model and that quantum chaos can be characterized by RMT, information scrambling has been commonly studied in relation with quantum chaos. However, information

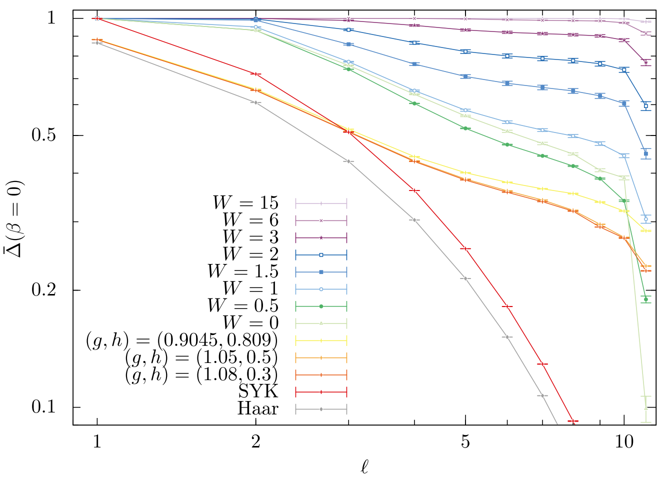

FIG. 2. Semilogarithmic plot of the late-time values of $\bar{\Delta}$ for $\hat{H}_{\rm XXZ}$, $\hat{H}_{\rm Ising}$, and $\hat{H}_{\rm SYK_4}$. To compare, the values for the Haar random unitary are also plotted. $k = 1$ for all models, and $N = 12$ ($N_{\rm q} = 12$) is chosen for the XXZ and Ising ($\rm SYK_4$) models. The dimension of the Haar random unitary is 2^{12}. Note that the conservation of the z component of spins (parity) is considered for the Heisenberg spin chain (SYK model). The values of (g, h) are those discussed in [61]. The averages for $t = (1, 2, \dots, 10) \times 10^6$ are plotted as the late-time value. For random-field average, 16 samples are taken.

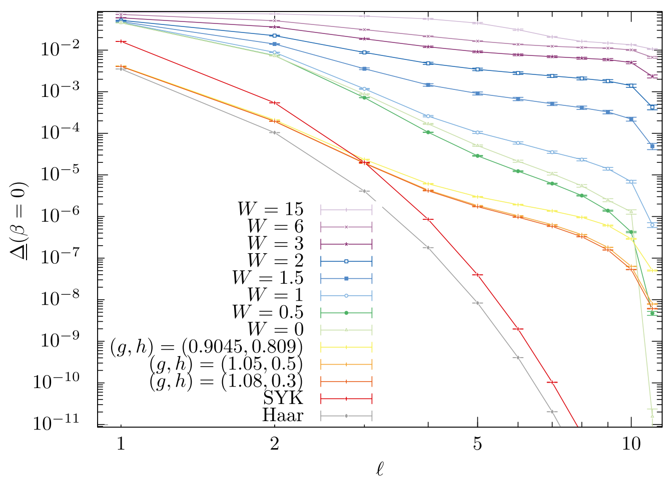

FIG. 3. Semilogarithmic plot of the late-time values of $\underline{\Delta}$ for $\hat{H}_{\rm XXZ}$, $\hat{H}_{\rm Ising}$ ($N = 12$), and $\hat{H}_{\rm SYK_4}$ ($N_{\rm q} = 12$). For the Hamiltonians with random field, averages over 16 samples are taken.

scrambling is not necessarily related to the quantum chaos in terms of the RMT-like energy statistics. For instance, we can analytically show that the dynamics of any commuting Hamiltonians is not information scrambling (see S2 [53]), while they can have RMT-like features [59,60] in the energy spectrum.

More illuminating instances are the spin-1/2 chains such as the Heisenberg with random magnetic field, $\hat{H}_{\rm XXZ} = \frac{1}{4}\sum_{j=1}^{N-1}(X_jX_{j+1} + Y_jY_{j+1} + J_zZ_jZ_{j+1}) + \frac{1}{2}\sum_{j=1}^{N} h_jZ_j$, where h_j are independently sampled from a uniform distribution in $[-W, W]$, and the mixed-field Ising with constant magnetic field, $\hat{H}_{\rm Ising} = -\sum_{j=1}^{N-1}(Z_jZ_{j+1}) - g\sum_{j=1}^{N} X_j - h\sum_{j=1}^{N} Z_j$. Here, X_j, Y_j, and Z_j denote the Pauli matrices on site j. In contrast to the fact that both have integrable-chaotic transitions by varying parameters [61–82], our numerical analysis reveals that the recovery errors are not as small as the random unitary model for any values of parameters at any time t.

This is demonstrated in Fig. 2, where the late-time values of $\bar{\Delta}_{\hat{H}}(t, \beta = 0)$ are plotted for these Hamiltonians. Hereafter, we set k to 1 in all numerics throughout the paper for the sake of computational tractability. It is observed that, by increasing ℓ, $\bar{\Delta}_{\hat{H}}$ decays inverse-polynomially or more slowly. We also provide in S3 and S4 [53] evidence that these values do not depend on the system size N. This implies that, although Fig. 2 is for $N = 12$ and $\ell \leqslant N - 1 = 11$, we can infer $\bar{\Delta}_{\hat{H}}$ for larger N and ℓ by extrapolation. By doing so, we may observe that $\bar{\Delta}_{\hat{H}}$ for $W \gtrsim 1$ may possibly stay nearly constant in the large-N limit unless $\ell \approx N$.

We can also investigate lower bounds on the recovery errors based on the mutual information, which we denote by $\underline{\Delta}_{\hat{H}}$ (see S1B [53] for the derivation). The bound is not tight, but we show in Fig. 3 that the lower bounds for $\hat{H}_{\rm XXZ}$ and $\hat{H}_{\rm Ising}$ scale similarly to those of the upper bounds. That is, they decay inverse polynomially or more slowly as ℓ increases and possibly stay almost constant if $W \gtrsim 1$ unless $\ell \approx N$.

As both upper and lower bounds scale similarly, we reasonably conclude $\Delta_{\hat{H}} = \Omega(1/\mathrm{poly}(\ell))$ in the large-N limit. This is in sharp contrast to the exponential decay of the recovery error in the random unitary model and implies that the dynamics of these Hamiltonians is not information scrambling in any parameter region.

More closely looking at Figs. 2 and 3, $\Delta_{\hat{H}}$ is likely to be dependent on the parameters of the Hamiltonians. It is known that $\hat{H}_{\rm XXZ}$ shows integrable-chaotic-MBL transitions as W increases and that the system is chaotic for $W \approx 0.5$ [66,70]. However, this chaotic transition does not seem to have strong consequence to information scrambling as both upper and lower bounds on $\Delta_{\hat{H}}$ for $W = 0.5$ are only slightly smaller than that in the integrable case with $W = 0$. For $\hat{H}_{\rm Ising}$, while the parameter $(g, h) = (1.08, 0.3)$ leads to the most chaotic feature in the entanglement structure [61], both upper and lower bounds on $\Delta_{\hat{H}}$ for that value can be worse than other parameters. This also indicates that information scrambling differs from quantum chaos and may not be able to be inferred from static features of Hamiltonians.

The fact that information scrambling is not observed in these systems does not contradict to the saturation of OTOCs for local, typically single-qubit, observables at late time when the parameters are appropriately set [83–88]. Our numerical results rather indicate that OTOCs for multiqubit observables are not saturated in such cases [57,58], which may be of independent interest.

Original and sparse SYK Hamiltonians. From these results, it is likely that more drastic Hamiltonians are needed to achieve the information scrambling. We next consider the SYK model, $\rm SYK_4$, of $2N_{\rm q}$ Majorana fermions:

$$\hat{H}_{\rm SYK_4} = \sum_{1\leqslant a_1<a_2<a_3<a_4\leqslant 2N_{\rm q}} J_{a_1a_2a_3a_4}\hat{\psi}_{a_1}\hat{\psi}_{a_2}\hat{\psi}_{a_3}\hat{\psi}_{a_4}, \quad (4)$$

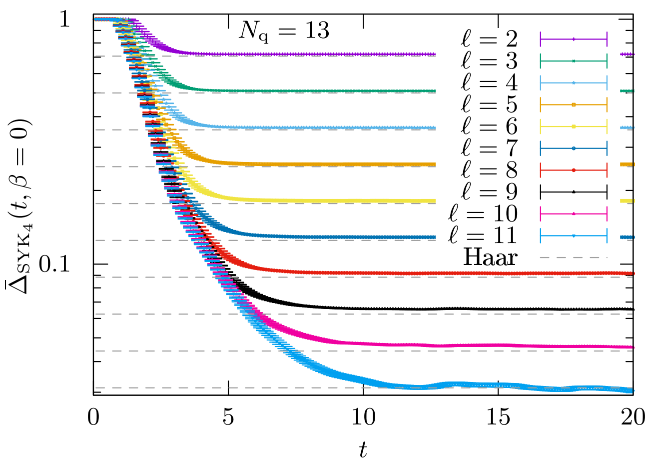

FIG. 4. Semilogarithmic plot of the value of $\bar{\Delta}_{\hat{H}_{\mathrm{SYK}_4}}(t, \beta = 0)$ against t for $N_q = 13$, and $2 \leqslant \ell < N_q - k$. Average over 64 samples is taken. For $\ell = 1$, $\bar{\Delta}_{\hat{H}_{\mathrm{SYK}_4}}(t, \beta = 0)$ is $\sim$1 for all t. The dashed lines represent $\bar{\Delta}'_{\mathrm{Haar}}(\beta = 0) = 2^{\frac{1}{2}(1-\ell)}$ given in Eq. (3) for $\ell = 2, 3, \ldots, N_q - 2$. For smaller values of N_q and finite β, see S5 [53].

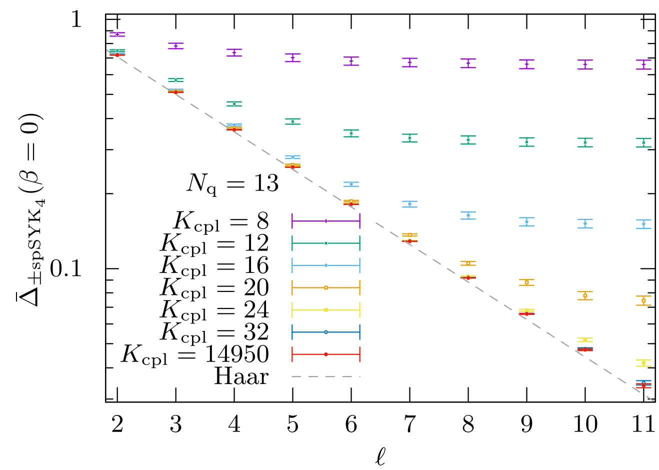

FIG. 5. The late-time value of $\bar{\Delta}_{\pm\mathrm{spSYK}_4}(t, \beta = 0)$ against ℓ is plotted for various numbers K_{cpl} of nonzero coupling constant. We set N_q to 13, and the number of samples is 64. The average for $t = (1, 2, \ldots, 10) \times 10^6$ is plotted as the late-time value in all figures.

with $\hat{\psi}_j$ being Majorana fermion operators. The couplings $J_{a_1a_2a_3a_4}$ are independently chosen at random from the Gaussian with average zero and $\sigma^2 = \binom{2N_q}{4}^{-1}$. Since the parity symmetry of SYK_4 leads to deviations in the information recovery [9,10,24,89], we focus on the even-parity sector and set $N = N_q - 1$. The recovery error of the corresponding random unitary model is given by $\bar{\Delta}'_{\mathrm{Haar}}(\beta) = \min\{1, 2^{(\ell_{\mathrm{Haar,th}}(\beta)-\ell)-\frac{1}{4}}\}$. See S5 [53] for details. We have also checked that the effect by the periodicity, characterized by $N_q \mod 4$, is negligible.

In Fig. 4, we numerically plot the upper bound on the recovery error, $\bar{\Delta}_{\mathrm{SYK}_4}(t, \beta = 0)$, against time t for various ℓ. It clearly shows that $\bar{\Delta}_{\mathrm{SYK}_4}$ quickly approaches $\bar{\Delta}'_{\mathrm{Haar}}$. This is also the case for $\beta > 0$. We estimate that $\bar{\Delta}_{\mathrm{SYK}_4}$ converges to $\bar{\Delta}'_{\mathrm{Haar}}$ before time $t = \mathcal{O}(\sqrt{N_q})$, which qualitatively supports the fast scrambling conjecture [2–4]. Hence, the SYK_4 dynamics, while differing from Haar random, has an excellent agreement with the prediction by RMT and achieves the Hayden-Preskill recovery. See also Figs. 3 and 2.

The situation remains the same even for a sparse simplification of SYK_4, spSYK_4. In spSYK_4, the number of nonzero random coupling constant is fixed to K_{cpl}. It recovers SYK_4 when $K_{\mathrm{cpl}} = \binom{2N_q}{4}$, but $K_{\mathrm{cpl}} = \mathcal{O}(N_q)$ is known to suffice to have chaotic features and to reproduce holographic properties [90,91].

In Fig. 5, we plot the upper bound on the recovery error for a further simplified sparse SYK model ($\pm\mathrm{spSYK}_4$), in which a half of the nonzero couplings is set to $1/\sqrt{K_{\mathrm{cpl}}}$ and the other half to $-1/\sqrt{K_{\mathrm{cpl}}}$ [92]. We observe that, when $K_{\mathrm{cpl}} \gtrsim 30 = \mathcal{O}(N_q)$, this simplification does not change the upper bound on the recovery error from that of the Haar value. Hence, $\pm\mathrm{spSYK}_4$ with $K_{\mathrm{cpl}} = \mathcal{O}(N_q)$ suffices to reproduce information-theoretic properties of SYK_4 as well as its chaotic features. As this number of nonzero couplings is substantially smaller than the original SYK_4, which has $K_{\mathrm{cpl}} = \mathcal{O}({N_q}^4)$, this would help experimental realizations of the Hayden-Preskill protocol in many-body systems. See S6 [53] for details.

Probing transitions by the Hayden-Preskill protocol. Yet another SYK model attracting much attention is the SYK_{4+2} model [93]. The Hamiltonian is

$$\hat{H}_{\mathrm{SYK}_{4+2}}(\theta) = \cos\theta\, \hat{H}_{\mathrm{SYK}_4} + \sin\theta\, \hat{H}_{\mathrm{SYK}_2}, \tag{5}$$

where $\hat{H}_{\mathrm{SYK}_2} = i\sum_{1\leqslant b_1<b_2\leqslant 2N_q} K_{b_1b_2}\hat{\psi}_{b_1}\hat{\psi}_{b_2}$, and $\theta \in [0, \pi/2]$ is a mixing parameter. The coupling constants $\{K_{b_1b_2}\}$ satisfy $K_{b_2b_1} = -K_{b_1b_2}$ and are normalized for the variance of eigenenergies of $\hat{H}_{\mathrm{SYK}_{4+2}}(\theta)$ to be unity.

The SYK_{4+2} model has a peculiar energy-shell structure in the sense of the local density of states in Fock space, which shows drastic changes by varying θ. Accordingly, the range of $\theta \in [0, \pi/2]$ is divided into four regimes I, II, III, and IV [94,95]. In I, only one energy-shell is dominant in the whole Hilbert space, and it is quantum chaotic. As θ increases the size of the energy-shell becomes diminished, and $\mathcal{O}(\mathrm{poly}(N_q))$ energyfshells appear in II and III. The energy statistics remains RMT-like in these two regimes. Characterizing physics in II and III has been under intense investigations [96]. In IV, the number of energy-shell approaches $\mathcal{O}(\exp(N_q))$, and Fock-space localization is observed.

Based on the Hayden-Preskill protocol in SYK_{4+2}, each regime can be operationally characterized. In Fig. 6, we plot the late time values of the upper bound $\bar{\Delta}_{\mathrm{SYK}_{4+2}}(t, \beta = 0)$ of the recovery error against $\tan\theta$, in which two characteristic values of θ, $\tan\theta_1 \approx 0.5$ and $\tan\theta_2 \approx 20$, are observed. The first plateau ($\theta \in [0, \theta_1)$) corresponds to the regime I. As $\bar{\Delta}_{\mathrm{SYK}_{4+2}} \approx \bar{\Delta}'_{\mathrm{Haar}}$ in this regime, the system is information scrambling. The second one ($\theta \in (\theta_1, \theta_2]$) correspond to II and III, where $\bar{\Delta}_{\mathrm{SYK}_{4+2}}$ is substantially larger than $\bar{\Delta}'_{\mathrm{Haar}}$, which seems to be the case even in the large N_q limit (see S7 [53]). The third plateau, $\theta \in (\theta_2, \pi/2]$, corresponds to IV, where the system is almost SYK_2.

For sufficiently small and large θ, the behavior of $\bar{\Delta}_{\mathrm{SYK}_{4+2}}$ can be naturally understood. For small θ, the model is approximately SYK_4. As we have observed above, the

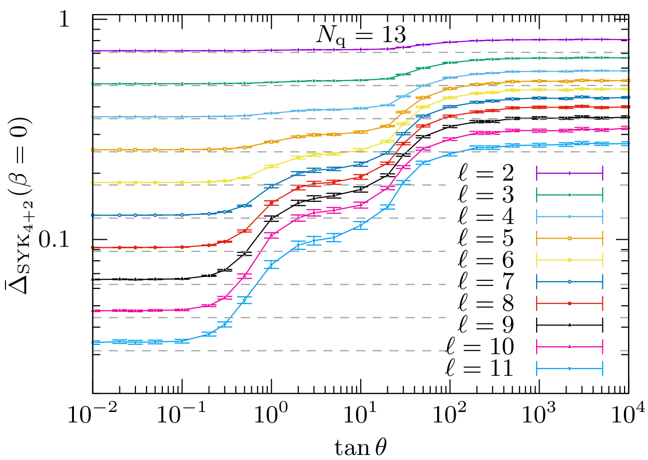

FIG. 6. The late-time value of $\bar{\Delta}_{\mathrm{SYK}_{4+2}}(t, \beta = 0)$ plotted for various ℓ against the value of δ. N_q is set to 13. The number of samplings is 64. The lines connecting the data points are guide to the eye. The horizontal dashed lines indicate $\bar{\Delta}'_{\mathrm{Haar}}$ for various ℓ.

dynamics of SYK_4 quickly achieves the Hayden-Preskill recovery. Hence, this should also be the case in the regime I. In contrast, for sufficiently large θ, the model is almost SYK_2 and the Fock-space localization occurs. Thus, information recovery should not be possible, resulting in the absence of information scrambling in the regime IV. In contrast, $\bar{\Delta}_{\mathrm{SYK}_{4+2}}$ is smoothly changing for the intermediate values of θ, which is seemingly in tension with the division of the regimes II and III in terms of the energy-shell structure.

To understand the intermediate plateau, we shall recall that, in II and III, transitions from one energy shell to the other are strongly suppressed, which effectively results in the division of the whole Hilbert space into $\mathcal{O}(\mathrm{poly}(N_q))$ energy shells [94]. Additionally, it is known that the dynamics in each energy shell seems to be approximately Haar random *within* the subspace [95]. The intermediate plateau of $\bar{\Delta}_{\mathrm{SYK}_{4+2}}$ can be explained from these common features in II and III. Since the whole Hilbert space is effectively divided into smaller ones, within which the dynamics remains still Haar random, the unitary dynamics in II and III induces *partial decoupling* [97] rather than decoupling. In this case, the recovery error is given in the form of $2^{\ell'_{\mathrm{th}}-\ell} + \Delta_{\mathrm{rem}}$ [24]. Here, $\ell'_{\mathrm{th}} \approx \ell_{\mathrm{Haar,th}} + \mathcal{O}(\sqrt{k})$ and Δ_{rem} quantifies the amount of information that cannot be recovered unless $\ell \approx N_q$. As we set $k = 1$ in our numerics, ℓ'_{th} is hardly observed in our analysis. In contrast, Δ_{rem} is clearly observed as an intermediate plateau. It is known that Δ_{rem} is inverse proportional to the standard deviation of energy in D. As the standard deviation of energy in B shall be $\mathcal{O}(\sqrt{N_q})$, that in D is at least $\mathcal{O}(\sqrt{N_q})$. Hence, we can qualitatively estimate that $\Delta_{\mathrm{rem}} = O(1/\sqrt{N_q})$, which remains nearly constant unless $\ell \approx N_q$.

From this perspective, we can understand the two transitions as reflections of the changes of decoupling properties. In I, the combined regime over II and III, and IV, the SYK_{4+2} dynamics leads to full, partial, and no decoupling, respectively. Accordingly, each regime has qualitatively different behaviors in the information recovery. The emerging difference between II and III in the energy-shell picture should be an artifact due to the fact that the energy shell is viewed in the Fock basis, which is not necessarily physically intrinsic to the system.

Summary and discussions. In this Letter, we have studied the information recovery in various Hamiltonian systems and have shown that information scrambling in the sense of information recovery does not always coincide with quantum chaos. Spin chains are unlikely to be information scrambling, while they are quantum chaotic in energy spectrum and saturate OTOCs for local observables. In contrast, the (sparse) SYK models are information scrambling and have the latter two properties. We have also demonstrated a potential use of the information recovery protocol to find new transitions caused by a information-theoretic mechanics.

It is unknown if any local spin models can be information scrambling since the family of SYK models, the only models that are information scrambling in our analysis, does not have spatially local interactions. It will be also of interest to further explore the direction of characterizing various quantum phases in the information-theoretic manner.

Acknowledgments. The work was supported by Grants-in-Aid for Transformative Research Areas (A) No. JP21H05182, No. JP21H05183, and No. JP21H05185 from MEXT of Japan. The work of M.T. was partly supported by Grants-in-Aid No. JP20H05270 and No. JP20K03787 from MEXT of Japan. Y.N. was supported by JST, PRESTO Grant No. JPMJPR1865, Japan, and by JSPS KAKENHI Grant No. JP22K03464, Japan. Part of the computation in this paper was conducted using the Supercomputing Facilities of the Institute for Solid State Physics, University of Tokyo. The authors thank Satyam S. Jha for collaboration in the early stage of the work. M.T. also thanks M. Hanada, C. Hotta, and N. Iizuka for valuable discussions.

[1] P. Hayden and J. Preskill, Black holes as mirrors: Quantum information in random subsystems, J. High Energy Phys. 09 (2007) 120.
[2] Y. Sekino and L. Susskind, Fast scramblers, J. High Energy Phys. 10 (2008) 065.
[3] L. Susskind, Addendum to fast scramblers, arXiv:1101.6048.
[4] N. Lashkari, D. Stanford, M. Hastings, T. Osborne, and P. Hayden, Towards the fast scrambling conjecture, J. High Energy Phys. 04 (2013) 022.
[5] S. H. Shenker and D. Stanford, Black holes and the butterfly effect, J. High Energy Phys. 03 (2014) 067.
[6] S. H. Shenker and D. Stanford, Stringy effects in scrambling, J. High Energy Phys. 05 (2015) 132.
[7] D. A. Roberts and D. Stanford, Diagnosing chaos using four-point functions in two-dimensional conformal field theory, Phys. Rev. Lett. **115**, 131603 (2015).
[8] D. A. Roberts and B. Yoshida, Chaos and complexity by design, J. High Energy Phys. 04 (2017) 121.
[9] B. Yoshida, Soft mode and interior operator in the Hayden-Preskill thought experiment, Phys. Rev. D **100**, 086001 (2019).
[10] J. Liu, Scrambling and decoding the charged quantum information, Phys. Rev. Res. **2**, 043164 (2020).

[11] A. Kitaev, "Hidden correlations in the Hawking radiation and thermal noise.", talk at KITP (2015), http://online.kitp.ucsb.edu/online/joint98/kitaev/.

[12] A. Kitaev, "A simple model of quantum holography.", talks at KITP (2015), http://online.kitp.ucsb.edu/online/entangled15/kitaev/ and http://online.kitp.ucsb.edu/online/entangled15/kitaev2/.

[13] K. Jensen, Chaos in AdS_2 holography, Phys. Rev. Lett. **117**, 111601 (2016).

[14] J. Maldacena and D. Stanford, Remarks on the Sachdev-Ye-Kitaev model, Phys. Rev. D **94**, 106002 (2016).

[15] S. Sachdev, Bekenstein-hawking entropy and strange metals, Phys. Rev. X **5**, 041025 (2015).

[16] M. Blake, Universal charge diffusion and the butterfly effect in holographic theories, Phys. Rev. Lett. **117**, 091601 (2016).

[17] C. W. von Keyserlingk, T. Rakovszky, F. Pollmann, and S. L. Sondhi, Operator hydrodynamics, OTOCs, and entanglement growth in systems without conservation laws, Phys. Rev. X **8**, 021013 (2018).

[18] V. Khemani, A. Vishwanath, and D. A. Huse, Operator spreading and the emergence of dissipative hydrodynamics under unitary evolution with conservation laws, Phys. Rev. X **8**, 031057 (2018).

[19] P. Hosur, X.-L. Qi, D. A. Roberts, and B. Yoshida, Chaos in quantum channels, J. High Energy Phys. 02 (2016) 004.

[20] F. Pastawski, B. Yoshida, D. Harlow, and J. Preskill, Holographic quantum error-correcting codes: Toy models for the bulk/boundary correspondence, J. High Energy Phys. 06 (2015) 149.

[21] F. Pastawski, J. Eisert, and H. Wilming, Towards holography via quantum source-channel codes, Phys. Rev. Lett. **119**, 020501 (2017).

[22] T. Kohler and T. Cubitt, Toy models of holographic duality between local Hamiltonians, J. High Energy Phys. 08 (2019) 017.

[23] P. Hayden and G. Penington, Learning the alpha-bits of black holes, J. High Energy Phys. 12 (2019) 007.

[24] Y. Nakata, E. Wakakuwa, and M. Koashi, Black holes as clouded mirrors: the Hayden-Preskill protocol with symmetry, Quantum **7**, 928 (2023).

[25] Y. Nakata, T. Matsuura, and M. Koashi, Constructing quantum decoders based on complementarity principle, arXiv:2210.06661.

[26] Y. Cheng, C. Liu, J. Guo, Y. Chen, P. Zhang, and H. Zhai, Realizing the Hayden-Preskill protocol with coupled Dicke models, Phys. Rev. Res. **2**, 043024 (2020).

[27] K. A. Landsman, C. Figgatt, T. Schuster, N. M. Linke, B. Yoshida, N. Y. Yao, and C. Monroe, Verified quantum information scrambling, Nature (London) **567**, 61 (2019).

[28] A. R. Brown, H. Gharibyan, S. Leichenauer, H. W. Lin, S. Nezami, G. Salton, L. Susskind, B. Swingle, and M. Walter, Quantum gravity in the lab. I. Teleportation by size and traversable wormholes, PRX Quantum **4**, 010320 (2023).

[29] S. Nezami, H. W. Lin, A. R. Brown, H. Gharibyan, S. Leichenauer, G. Salton, L. Susskind, B. Swingle, and M. Walter, Quantum gravity in the lab. II. Teleportation by size and traversable wormholes, PRX Quantum **4**, 010321 (2023).

[30] S. W. Hawking, Black hole explosions? Nature (London) **248**, 30 (1974).

[31] S. W. Hawking, Particle creation by black holes, Commun. Math. Phys. **43**, 199 (1975).

[32] S. W. Hawking, Breakdown of predictability in gravitational collapse, Phys. Rev. D **14**, 2460 (1976).

[33] F. Haake, *Quantum Signatures of Chaos* (Springer, Berlin, 2001).

[34] M. Mehta, *Random Matrices* (Elsevier Science, Amsterdam, 2004).

[35] P. Caputa, J. Simón, A. Štikonas, T. Takayanagi, and K. Watanabe, Scrambling time from local perturbations of the eternal BTZ black hole, J. High Energy Phys. 08 (2015) 011.

[36] L. Nie, M. Nozaki, S. Ryu, and M. T. Tan, Signature of quantum chaos in operator entanglement in 2d CFTs, J. Stat. Mech. (2019) 093107.

[37] K. Goto, M. Nozaki, S. Ryu, K. Tamaoka, and M. T. Tan, Scrambling and recovery of quantum information in inhomogeneous quenches in two-dimensional conformal field theories, Phys. Rev. Res. **6**, 023001 (2024).

[38] A. I. Larkin and Y. N. Ovchinnikov, Quasiclassical method in the theory of superconductivity, Sov. Phys. JETP **28**, 1200 (1969).

[39] H. Shen, P. Zhang, R. Fan, and H. Zhai, Out-of-time-order correlation at a quantum phase transition, Phys. Rev. B **96**, 054503 (2017).

[40] V. Balasubramanian, A. Kar, C. Li, O. Parrikar, and H. Rajgadia, Quantum error correction from complexity in brownian SYK, J. High Energy Phys. 08 (2023) 071.

[41] J. Cotler, N. Hunter-Jones, J. Liu, and B. Yoshida, Chaos, complexity, and random matrices, J. High Energy Phys. 11 (2017) 048.

[42] S. Sachdev and J. Ye, Gapless spin-fluid ground state in a random quantum Heisenberg magnet, Phys. Rev. Lett. **70**, 3339 (1993).

[43] S. Sachdev, Holographic metals and the fractionalized fermi liquid, Phys. Rev. Lett. **105**, 151602 (2010).

[44] J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, Black holes and random matrices, J. High Energy Phys. 05 (2017) 118.

[45] G. Sarosi, AdS_2 holography and the SYK model, in Proceedings of XIII Modave Summer School in Mathematical Physics — PoS(Modave2017) (Sissa Medialab, Trieste, 2018) arXiv:1711.08482.

[46] D. A. Trunin, Pedagogical introduction to the Sachdev–Ye–Kitaev model and two-dimensional dilaton gravity, Phys. Usp. **64**, 219 (2021).

[47] J. Maldacena, S. H. Shenker, and D. Stanford, A bound on chaos, J. High Energy Phys. 08 (2016) 106.

[48] Y.-Z. You, A. W. W. Ludwig, and C. Xu, Sachdev-Ye-Kitaev model and thermalization on the boundary of many-body localized fermionic symmetry-protected topological states, Phys. Rev. B **95**, 115150 (2017).

[49] A. M. García-García and J. J. Verbaarschot, Spectral and thermodynamic properties of the Sachdev-Ye-Kitaev model, Phys. Rev. D **94**, 126010 (2016).

[50] S. Sachdev, Statistical mechanics of strange metals and black holes, ICTS Newsletter **8**, 1 (2022).

[51] S. Xu and B. Swingle, Scrambling dynamics and out-of-time-ordered correlators in quantum many-body systems, PRX Quantum **5**, 010201 (2024).

[52] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition* (Cambridge University Press, Cambridge, 2010).
[53] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevResearch.6.L022021 for the upper and lower bounds on the recovery error and additional discussions on the Hayden-Preskill recovery by a Haar random dynamics, in commuting Hamiltonian models, in the Heisenberg model with random magnetic field, in the Ising model with chaotic spin chain, in the pure SYK_4 model, in the sparse SYK_4 model, and in the SYK_{4+2} model, which includes Refs. [98–101].
[54] P. Hayden, M. Horodecki, A. Winter, and J. Yard, A decoupling approach to the quantum capacity, Open Syst. Inf. Dyn. **15**, 7 (2008).
[55] F. Dupuis, M. Berta, J. Wullschleger, and R. Renner, One-shot decoupling, Commun. Math. Phys. **328**, 251 (2014).
[56] F. Dupuis, The decoupling approach to quantum information theory, arXiv:1004.1641.
[57] B. Yoshida and A. Kitaev, Efficient decoding for the Hayden-Preskill protocol, arXiv:1710.03363.
[58] B. Yoshida and N. Y. Yao, Disentangling scrambling and decoherence via quantum teleportation, Phys. Rev. X **9**, 011006 (2019).
[59] H. G. Katzgraber and F. Krząkała, Temperature and disorder chaos in three-dimensional ising spin glasses, Phys. Rev. Lett. **98**, 017201 (2007).
[60] G. Gur-Ari, R. Mahajan, and A. Vaezi, Does the SYK model have a spin glass phase? J. High Energy Phys. 11 (2018) 070.
[61] J. F. Rodriguez-Nieva, C. Jonay, and V. Khemani, Quantifying quantum chaos through microcanonical distributions of entanglement, arXiv:2305.11940.
[62] L. F. Santos, Integrability of a disordered Heisenberg spin-1/2 chain, J. Phys. A **37**, 4723 (2004).
[63] K. Kudo and T. Deguchi, Level statistics of XXZ spin chains with a random magnetic field, Phys. Rev. B **69**, 132404 (2004).
[64] L. Viola and W. G. Brown, Generalized entanglement as a framework for complex quantum systems: Purity versus delocalization measures, J. Phys. A **40**, 8109 (2007).
[65] M. Žnidarič, T. Prosen, and P. Prelovšek, Many-body localization in the Heisenberg XXZ magnet in a random field, Phys. Rev. B **77**, 064426 (2008).
[66] A. Pal and D. A. Huse, Many-body localization phase transition, Phys. Rev. B **82**, 174411 (2010).
[67] A. D. Luca and A. Scardicchio, Ergodicity breaking in a model showing many-body localization, Europhys. Lett. **101**, 37003 (2013).
[68] F. Alet and N. Laflorencie, Many-body localization: An introduction and selected topics, C. R. Phys. **19**, 498 (2018).
[69] D. A. Abanin, E. Altman, I. Bloch, and M. Serbyn, Colloquium: Many-body localization, thermalization, and entanglement, Rev. Mod. Phys. **91**, 021001 (2019).
[70] D. J. Luitz, N. Laflorencie, and F. Alet, Many-body localization edge in the random-field Heisenberg chain, Phys. Rev. B **91**, 081103 (2015).
[71] A. Morningstar and D. A. Huse, Renormalization-group study of the many-body localization transition in one dimension, Phys. Rev. B **99**, 224205 (2019).
[72] J. Šuntajs, J. Bonča, T. Prosen, and L. Vidmar, Quantum chaos challenges many-body localization, Phys. Rev. E **102**, 062144 (2020).
[73] P. Sierant, D. Delande, and J. Zakrzewski, Thouless time analysis of anderson and many-body localization transitions, Phys. Rev. Lett. **124**, 186601 (2020).
[74] M. Kiefer-Emmanouilidis, R. Unanyan, M. Fleischhauer, and J. Sirker, Evidence for unbounded growth of the number entropy in many-body localized phases, Phys. Rev. Lett. **124**, 243601 (2020).
[75] T. Chanda, P. Sierant, and J. Zakrzewski, Many-body localization transition in large quantum spin chains: The mobility edge, Phys. Rev. Res. **2**, 032045 (2020).
[76] D. Sels and A. Polkovnikov, Dynamical obstruction to localization in a disordered spin chain, Phys. Rev. E **104**, 054105 (2021).
[77] M. Kiefer-Emmanouilidis, R. Unanyan, M. Fleischhauer, and J. Sirker, Slow delocalization of particles in many-body localized phases, Phys. Rev. B **103**, 024203 (2021).
[78] A. Morningstar, L. Colmenarez, V. Khemani, D. J. Luitz, and D. A. Huse, Avalanches and many-body resonances in many-body localized systems, Phys. Rev. B **105**, 174205 (2022).
[79] D. Sels, Bath-induced delocalization in interacting disordered spin chains, Phys. Rev. B **106**, L020202 (2022).
[80] P. Sierant and J. Zakrzewski, Challenges to observation of many-body localization, Phys. Rev. B **105**, 224203 (2022).
[81] R. Ghosh and M. Žnidarič, Resonance-induced growth of number entropy in strongly disordered systems, Phys. Rev. B **105**, 144203 (2022).
[82] M. C. Bañuls, J. I. Cirac, and M. B. Hastings, Strong and weak thermalization of infinite nonintegrable quantum systems, Phys. Rev. Lett. **106**, 050405 (2011).
[83] J. Li, R. Fan, H. Wang, B. Ye, B. Zeng, H. Zhai, X. Peng, and J. Du, Measuring out-of-time-order correlators on a nuclear magnetic resonance quantum simulator, Phys. Rev. X **7**, 031011 (2017).
[84] R. Fan, P. Zhang, H. Shen, and H. Zhai, Out-of-time-order correlation for many-body localization, Sci. Bull. **62**, 707 (2017).
[85] J. Riddell and E. S. Sørensen, Out-of-time ordered correlators and entanglement growth in the random-field XX spin chain, Phys. Rev. B **99**, 054205 (2019).
[86] J. Lee, D. Kim, and D.-H. Kim, Typical growth behavior of the out-of-time-ordered commutator in many-body localized systems, Phys. Rev. B **99**, 184202 (2019).
[87] S. Xu and B. Swingle, Accessing scrambling using matrix product operators, Nat. Phys. **16**, 199 (2020).
[88] R. K. Shukla, A. Lakshminarayan, and S. K. Mishra, Out-of-time-order correlators of nonlocal block-spin and random observables in integrable and nonintegrable spin chains, Phys. Rev. B **105**, 224307 (2022).
[89] H. Tajima and K. Saito, Universal limitation of quantum information recovery: Symmetry versus coherence, arXiv:2103.01876.
[90] S. Xu, L. Susskind, Y. Su, and B. Swingle, A sparse model of quantum holography, arXiv:2008.02303.
[91] A. M. García-García, Y. Jia, D. Rosa, and J. J. M. Verbaarschot, Sparse Sachdev-Ye-Kitaev model, quantum chaos, and gravity duals, Phys. Rev. D **103**, 106002 (2021).

[92] M. Tezuka, O. Oktay, E. Rinaldi, M. Hanada, and F. Nori, Binary-coupling sparse Sachdev-Ye-Kitaev model: an improved model of quantum chaos and holography, Phys. Rev. B **107**, L081103 (2023).

[93] A. M. García-García, B. Loureiro, A. Romero-Bermúdez, and M. Tezuka, Chaotic-integrable transition in the Sachdev-Ye-Kitaev model, Phys. Rev. Lett. **120**, 241603 (2018).

[94] F. Monteiro, T. Micklitz, M. Tezuka, and A. Altland, Minimal model of many-body localization, Phys. Rev. Res. **3**, 013023 (2021).

[95] F. Monteiro, M. Tezuka, A. Altland, D. A. Huse, and T. Micklitz, Quantum ergodicity in the many-body localization problem, Phys. Rev. Lett. **127**, 030601 (2021).

[96] D. K. Nandy, T. Čadež, B. Dietz, A. Andreanov, and D. Rosa, Delayed thermalization in the mass-deformed Sachdev-Ye-Kitaev model, Phys. Rev. B **106**, 245147 (2022).

[97] E. Wakakuwa and Y. Nakata, One-shot randomized and nonrandomized partial decoupling, Commun. Math. Phys. **386**, 589 (2021).

[98] M. M. Wilde, *Quantum Information Theory* (Cambridge University Press, Cambridge, 2013).

[99] R. Alicki and M. Fannes, Continuity of quantum conditional information, J. Phys. A **37**, L55 (2004).

[100] A. Winter, Tight uniform continuity bounds for quantum entropies: Conditional entropy, relative entropy distance and energy constraints, Commun. Math. Phys. **347**, 291 (2016).

[101] I. Dumitriu and A. Edelman, Matrix models for beta ensembles, J. Math. Phys. **43**, 5830 (2002).