

Vertical velocity of a small sphere in a sheared granular bed

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Small particles fall through sheared beds of larger particles in settings ranging from geophysics to industry, but the study of large-to-small size ratios R , spanning the trapping threshold R_t , has been neglected. In simulations of noncohesive spheres for $R < R_t$, the small-sphere vertical velocity v_p first increases with shear rate $\dot{\gamma}$ as trapping time decreases, but v_p then decreases as velocity fluctuations frustrate downward mobility. For $R > R_t$, v_p is constant at low $\dot{\gamma}$, but again decreases at high $\dot{\gamma}$. We model these behaviors and discuss analogies with electron transport in solids.

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Introduction. The movement of solid particles through porous media, i.e., *percolation*, is relevant in many scientific and engineering contexts. In granular materials, smaller particles fall through void spaces between larger particles. In geophysics, the percolation of solids, such as that for soil erosion or sediment transport in rivers, influences environmental sustainability, landscape dynamics, and ecological habitats. In biological systems, percolation occurs as the movement of nutrients through biological tissues and organs. In industry, percolation occurs in filtration systems for separation and in packed beds for absorption. The percolation dynamics in these processes strongly depend on the relative particle size and pore structure (static and dynamic) of the porous media.

In mixtures of size-disperse granular materials, small-particle percolation leads to demixing (segregation), which is a critical issue in various natural [1–3] and industrial [4,5] processes. Recent studies have advanced models of granular segregation (see, e.g., [6,7]), but nearly all consider mixtures with large-to-small particle diameter ratios, $R \lesssim 2$, where interparticle contacts are enduring [8–10]. In these cases, segregation can be characterized by a concentration-dependent mean vertical velocity (i.e., the *percolation velocity* [11]) v_p , which increases monotonically with both R and shear rate $\dot{\gamma}$.

For larger R , where the small particles are referred to as *fine* particles or *finer*, the v_p dependence on $\dot{\gamma}$ and R in sheared flows changes significantly. In particular, for $R \gtrsim 2$ and low fines concentration, v_p dramatically increases with increasing R [11,12], while, in contrast, v_p is nearly R independent at larger fines concentration (above 10%) for $2 \lesssim R \lesssim 4$ [13]. Here we focus on fine-particle percolation in uniform shear

flow in the zero-concentration limit, where an increasing tendency toward *free sifting* or *spontaneous interparticle percolation* [14–17] (i.e., the downward motion of fines at $\dot{\gamma} = 0$) with increasing R leads to qualitative changes in the dependence of v_p on $\dot{\gamma}$ and other parameters. To focus on the effects of $\dot{\gamma}$ and R , we consider noncohesive particle interactions, noting that cohesive forces profoundly affect the statics and dynamics of granular material when they are large relative to particle weight.

Free sifting has been primarily studied in static beds for $R > R_t$ [16,18–21], where the free-sifting threshold R_t is $R_{t0} = (2/\sqrt{3} - 1)^{-1} \approx 6.46$ for rigid monodisperse bed spheres [22], but is larger in polydisperse mixtures of “soft” particles [23]. Free sifting can also occur for $R < R_t$ in randomly packed static beds when a subset of pore throats—the minimum opening between neighboring bed spheres—exceeds the fine-particle diameter. In this case, motion is *transient* since a fine particle inevitably encounters an impassible pore throat [24] and is trapped. Despite the ubiquity of fines in industrial solids processing [25,26], their potential for affecting the mobility of various geophysical flows [27–29], and their importance in sediment infiltration that shapes rivers [30,31], few studies address the movement, or percolation, of fines through sheared granular beds [11,12,32].

In this Letter, we show that fine-particle percolation is pronounced and unavoidably coupled with shear for $R < R_t$ because fines that would be trapped in a static bed are repeatedly remobilized by shear-induced particle rearrangements. In past work, the complexity of this problem and the limited parameter space that was explored produced puzzling inconsistencies regarding the dependence of v_p on $\dot{\gamma}$ [32] and R [11–13]. Here we resolve these issues by characterizing the small-sphere vertical velocity in a larger-sphere bed for size ratios spanning the free-sifting threshold ($2 \leq R \leq 10$) and spatially uniform shear rates covering the quasistatic and rapid dense flow regimes [33]. Our results reveal a nonmonotonic dependence of v_p on $\dot{\gamma}$, provide relations for predicting v_p in the

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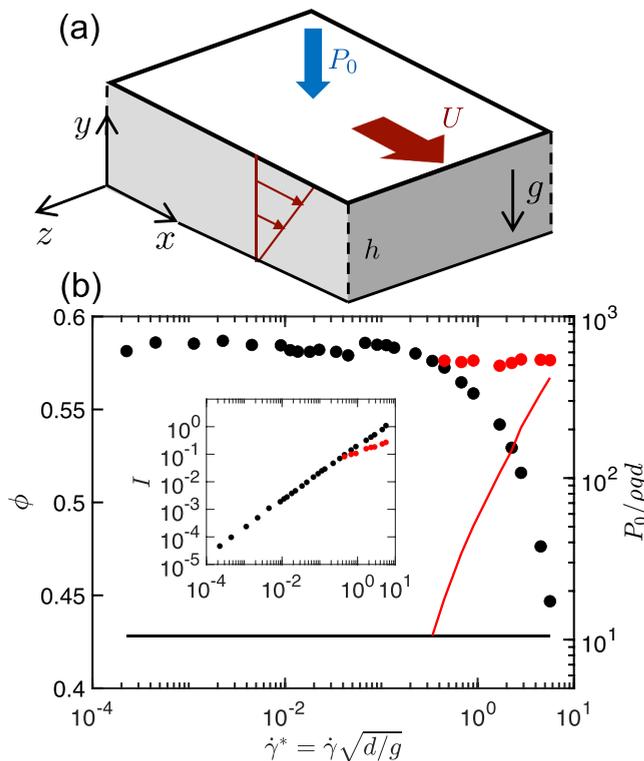


FIG. 1. Simulation conditions. (a) Single fine particles (not shown) percolate under gravity in a confined granular shear flow with a controlled spatially uniform shear rate $\dot{\gamma}$ (see text). (b) Bed particle packing density ϕ vs nondimensional shear rate $\dot{\gamma}^* = \dot{\gamma}\sqrt{d/g}$ for overburden pressure P_0 , held constant at 1.2 kPa (black line and circles) or gradually increased above 1.2 kPa for $\dot{\gamma}^* > 0.3$ to maintain nearly constant ϕ (red curve and circles). Inset: Inertial number $I = \dot{\gamma}d/\sqrt{P/\rho}$ vs $\dot{\gamma}^*$ for constant (black) and increasing (red) P_0 .

low- and high-shear-rate regimes, and add insight into the dominant physics in each regime. We also address similarities between gravitational-field-driven fine-particle transport and electric-field-driven conduction of electrons, where R_t delineates the boundary between semiconductorlike ($R < R_t$) and metal-like ($R > R_t$) transport regimes and $\dot{\gamma}$ acts similarly to temperature.

Methods. LIGGGHTS [34], a discrete element method code, is used to simulate single spheres of diameter d_f in a confined dense flow of bed particles of diameter d , with a prescribed horizontal velocity varying linearly with depth [35] as depicted in Fig. 1(a). The domain is periodic in the streamwise (x) and spanwise (z) directions and confined in the depthwise direction (y) by two horizontal walls formed by particles with diameter d positioned on a planar-square lattice with a unit cell length of $1.2d$ and two-dimensional packing density of 0.545. The horizontal position of each wall particle is randomized within its unit cell. A constant downward force on the top wall, which is otherwise free to move vertically and in the spanwise direction, sets the bed overburden pressure P_0 , which is increased with increasing $\dot{\gamma}$ to maintain a constant volume fraction $\phi \approx 0.58$ of bed particles, as shown in Fig. 1(b) and discussed in further detail below. The bottom wall is stationary, while the top wall is translated in the streamwise direction with velocity $U = \dot{\gamma}h(t)$, where the

time-dependent bed height $h(t)$ accommodates dilation due to shear and varies in time by $\sim \pm 0.1\%$. The flow domain is $20d$ long, $10d$ wide, and $60d$ deep, comprising approximately 12 500 large particles (gray region) for most of the simulations. Bed particles have a uniform diameter dispersity of $\pm 10\%$ to minimize layering [36]. To ensure homogeneous shear in the presence of gravity, a small streamwise stabilizing force $-k(\dot{\gamma}y_i - u_i)$, where u_i and y_i are the streamwise velocity and height of bed particle i , respectively, and $k = 0.1$ kg/s, is applied to each bed particle (but not the percolating fine particles) at each simulation time step [35,37–39]. This approach produces the desired homogeneous shear flow without altering the flow rheology [40] or segregation [41].

After the sheared bed of large particles reaches the steady state, fine particles with identical diameter d_f are randomly positioned just above ($1.02 < y/h < 1.03$) the porous top wall of the bed. All fine particles are given an initial streamwise velocity matching the streamwise velocity of the moving top wall, which allows them to quickly fall beneath the wall when they are simultaneously released. In order to examine the zero-concentration limit and, thereby, avoid the complications associated with fine-particle jamming, fine particles interact with bed particles but not with each other. The number of fine particles depends on R and varies from $\sim 10^3$ ($R = 2$) to $\sim 10^4$ ($R = 10$), which keeps the fine-particle volume concentration below 1%. This low concentration prevents the multiple, simultaneously percolating, fine particles from significantly altering the flow kinematics and rheology of the bed particles, and allows the dynamics of multiple independent fine particles to be computed in one simulation, which is computationally efficient and provides sufficient data for accurate statistics. Doubling the number of fine particles in additional simulations, covering $3 \leq R \leq 7$ and $1 \leq \dot{\gamma} \leq 40$ s $^{-1}$, does not alter the flow kinematics, the bed rheology, or the measured value of v_p . Simulation times vary with R and $\dot{\gamma}$, and are typically of the order of 10 s to ensure that most fine particles reach the lower wall. Simulation times for smaller R at lower $\dot{\gamma}^* = \dot{\gamma}\sqrt{d/g}$ are increased so that the fraction of fine particles remobilized by shear-induced rearrangement is sufficient for accurate measurement of v_p . For example, the simulation time is increased up to 100 s for $R < 6$ and $\dot{\gamma}^* < 10^{-3}$. Fine-particle motion is characterized by the mean vertical velocity, $v_p = \langle v_i \rangle$, or *percolation velocity*, and the root-mean-square (rms) velocity, $v_{\text{rms}} = \langle \sqrt{(v_p - v_i)^2} \rangle$, where v_i is the vertical velocity of fine particle i and brackets indicate ensemble and temporal averaging.

The majority of the simulations presented in this Letter use the following parameters: bed particle average diameter $d = 5$ mm with 10% uniform polydispersity, acceleration due to gravity $g = 9.81$ m s $^{-2}$, restitution coefficient $e = 0.8$, friction coefficient $\mu = 0.5$, and densities of bed (ρ) and fine particle (ρ_f) are both 2500 kg m $^{-3}$. Additionally, d , g , e , and ρ_f are varied in other simulations to explore their effects on the percolation velocity. The simulations use the Hertz contact model with a Young's modulus of 5×10^7 Pa and a Poisson's ratio of 0.4. The simulation time step is 5×10^{-6} s to ensure numerical stability even for the largest size ratios and highest shear rates that are examined.

Previous studies indicate that decreasing ϕ increases v_p in both static [24] and flowing [42] systems. Increasing $\dot{\gamma}^*$

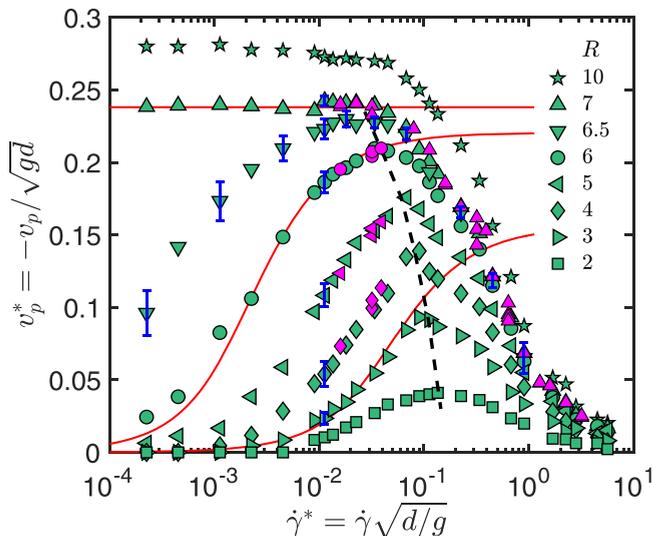


FIG. 2. Scaled fine-particle vertical velocity, $v_p^* = -v_p/\sqrt{gd}$, vs scaled shear rate, $\dot{\gamma}^* = \dot{\gamma}\sqrt{d/g}$, for various particle-size ratios R , gravitational accelerations g , and bed particle diameters d , with restitution coefficient $e = 0.8$. The volume fraction of the large particles is kept nearly constant ($0.57 < \phi < 0.58$) by increasing the overburden pressure P_0 as $\dot{\gamma}$ is increased. Magenta symbols indicate data for $d \in \{2.5, 10, 15\}$ mm and $g \in \{4.91, 19.62, 29.43\}$ m/s² at $\dot{\gamma} = 1$ s⁻¹ for $R \leq 6$ and for $1 \leq \dot{\gamma} \leq 100$ s⁻¹ for $R = 7$ that validate the scaling. The dashed black curve approximates the maximum v_p^* location for different R . Error bars for selected cases indicate the standard error of v_p . Red curves are predictions of the low-shear-rate regime model [Eq. (2)] for $R = 3, 6, \text{ and } 7$.

with constant $P_0 = 1200$ Pa $\sim 10\rho gd$ [horizontal black line in Fig. 1(b)] increases the bed dilation, which decreases ϕ [black circles in Fig. 1(b)] for $\dot{\gamma}^* > 0.3$, while the inertial number of the bed particles, $I = \dot{\gamma}d/\sqrt{P/\rho}$, increases linearly [inset of Fig. 1(b)]. In the expression for I , P is the normal stress of bed particles averaged over a $40d$ -thick layer in the middle of the bed. For $\dot{\gamma}^* \gtrsim 1$, I tends towards dilute collisional flow [43]. To minimize changes in the pore size distribution due to shear-driven variation in ϕ [24] and to keep the maximum ϕ value closer to that for dense flow, P_0 is increased by a factor of 40 with increasing $\dot{\gamma}^*$ over the range $0.3 < \dot{\gamma}^* < 4$, as indicated in the figure. This procedure keeps ϕ in the range $0.57 < \phi < 0.58$ [red curve and red circles in Fig. 1(b)] and the inertial number in the dense flow range $10^{-4} \lesssim I \lesssim 0.3$.

Vertical velocity. Figure 2 plots the scaled fine-particle vertical velocity, $v_p^* = -v_p/\sqrt{gd}$, versus the scaled shear rate, $\dot{\gamma}^*$ for $2 \leq R \leq 10$. As in static beds, fines always percolate downward on average, even at the largest $\dot{\gamma}^*$, and v_p^* increases monotonically with increasing R for all $\dot{\gamma}^*$. However, the $\dot{\gamma}^*$ dependence of v_p^* is strongly R dependent. First, for size ratios in the static-bed passing regime ($R > 6.5 > R_t$ here due to overburden-pressure-driven deformation and polydispersity of bed particles that decreases the minimum pore throat diameter relative to rigid monodisperse bed particles [23]), v_p^* initially remains constant at its static-bed value as $\dot{\gamma}^*$ is increased from zero. Hence, $v_p \propto \sqrt{gd}$ for $R \gtrsim R_t$. However, above $\dot{\gamma}^* \gtrsim 0.03$, v_p^* decreases with increasing $\dot{\gamma}^*$. Second, for size ratios in the static-bed trapping regime ($R \leq 6.5$), v_p^*

increases from zero with increasing $\dot{\gamma}^*$, similar to segregation with $R \lesssim 2$ [8–10,12]. However, v_p^* reaches a maximum near $\dot{\gamma}^* \sim 0.1$ and then decreases toward zero with further increase in $\dot{\gamma}^*$. Note that a previously observed $\dot{\gamma}$ independence of v_p for $R \approx 2.5$ [32] results from that study's limited shear-rate range, $0.04 < \dot{\gamma}^* < 0.14$, which serendipitously brackets the peak in v_p about which v_p is nearly constant (e.g., see $R = 2$ data in Fig. 2). To test the nondimensionalization of v_p and $\dot{\gamma}$, Fig. 2 also includes data where d and g differ from the values used in the other simulations. This additional set of data (magenta symbols) overlays the data for $d = 5$ mm and $g = 9.81$ m/s² at the corresponding R values, indicating that the scaling of v_p and $\dot{\gamma}$ is correct in both low- and high- $\dot{\gamma}$ regimes.

The value of $\dot{\gamma}^*$ where v_p^* begins to drop decreases with increasing R (dashed curve), e.g., $\dot{\gamma}^* \approx 0.14$ for $R = 2$, while $\dot{\gamma}^* \approx 0.03$ for $R = 6.5$. This sensitivity to R along with the decrease in v_p with increasing $\dot{\gamma}$ for $\dot{\gamma}^* \gtrsim 0.1$ is due to increasing fine-particle velocity fluctuations (characterized by v_{rms}), which frustrate percolation and increase with increasing R or $\dot{\gamma}^*$, as described later. In static beds, a similar decrease in v_p is observed with increasing e for both $4 \leq R \leq R_t$ [24] and $R > R_t$ [16,18,20,24].

Figure 2 and previous work in static beds [24] suggest that fine-particle percolation in sheared beds depends on three mechanisms: geometric trapping, which is possible when $R < R_t$; bed particle rearrangement due to shear; and fine-particle velocity fluctuations, which frustrate percolation. The basics are as follows. First, fines with $R < R_t$ that are trapped remobilize due to shear-driven bed rearrangements at a rate that is proportional to $\dot{\gamma}$ and increases with R , since passable voids are generated at a higher rate for smaller fines. Second, the average time to pass a passable void increases with increasing excitation of the fines, i.e., v_{rms} . Consequently, at high-shear rates, where trapping times are short for $R < R_t$ and v_{rms} is large ($v_{\text{rms}} \propto \dot{\gamma}$) for all R , v_p decreases with increasing $\dot{\gamma}$.

Low-shear-rate regime. To better understand the dominant physics and develop a model for v_p in this regime, we start with the percolation depth model for $R < R_t$ in static beds,

$p(\Delta y) \propto P_p^{\frac{\Delta y}{d}}$, where $p(\Delta y)$ is the probability that a fine particle falls a distance Δy or more from its starting height without becoming trapped, and P_p represents the probability of a fine passing through a randomly selected pore throat, which is equivalent to the fraction of constrictions with diameters larger than d_f [24]. In static beds, $p(\Delta y)$ is the proportion of fine-particle trajectories that exceed a depth $\geq \Delta y$, assuming that the passage of fines through consecutive pore throats is independent [24,44]. Since untrapped fines percolate with mean velocity $v_{p,s} = -c_1\sqrt{gd}$ [24], $p(\Delta y)$ can be reformulated as a function of time using $\Delta y = -v_{p,s}t$ as $p(t) \propto P_p^{\frac{-v_{p,s}t}{d}}$, where $p(t)$ is the probability that a fine particle is untrapped after time t . The average vertical velocity over t is then

$$v_p = \frac{v_{p,s}}{t} \int_0^t p(t') dt' \propto \frac{d}{t \ln P_p} \left(1 - P_p^{\frac{-v_{p,s}t}{d}}\right). \quad (1)$$

For sheared systems, we assume that the time interval between significant bed rearrangements scales as $t_b = c_2(R)\dot{\gamma}^{-1}$. Substituting t_b for t in Eq. (1) gives v_p as a function of shear

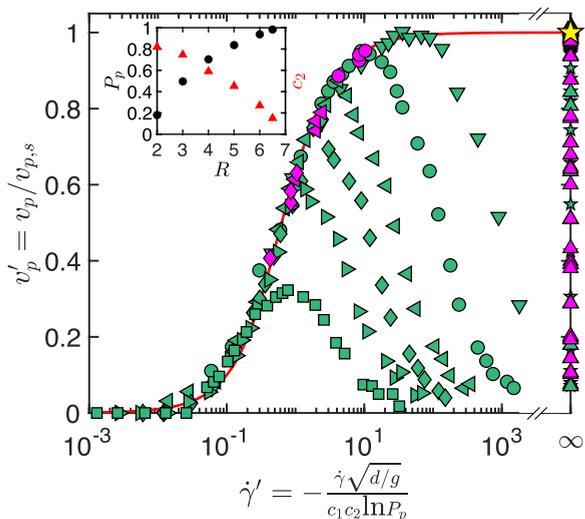


FIG. 3. Fine-particle vertical velocity scaled by the vertical velocity of untrapped fines in a static bed, $v_p/v_{p,s}$, vs rescaled shear rate $\dot{\gamma}'$, showing the collapse of all data (symbols) from Fig. 2 in the low-shear-rate regime onto the prediction of Eq. (3) (red curve). Data include varying R (symbols), and g and d (magenta) for $\phi \approx 0.58$ and $e = 0.8$. Passing regime data ($R > 6.5$) fall on the right boundary since $\dot{\gamma}' = \infty$. Inset: P_p (left: black circle) and c_2 (right: red triangle) vs R .

rate, bed structure (via P_p) and its variation (via c_2), bed particle diameter, and gravitational acceleration:

$$v_p = \frac{d\dot{\gamma}}{c_2 \ln P_p} \left(1 - P_p^{c_1 c_2 \sqrt{\frac{g}{\dot{\gamma}^2 d}}} \right). \quad (2)$$

This relation is alternatively expressed as

$$v'_p = \frac{v_p}{v_{p,s}} = \dot{\gamma}' [1 - \exp(-1/\dot{\gamma}')], \quad (3)$$

where $\dot{\gamma}' = -C\dot{\gamma}\sqrt{d/g}$ with $C^{-1} = c_1 c_2 \ln P_p$ as the single model parameter. Equation (2) exhibits the appropriate limiting behaviors under its assumption that v_{rms} is small: (i) as $\dot{\gamma} \rightarrow \infty$ ($t_b \rightarrow 0$), $v_p \rightarrow v_{p,s} \propto \sqrt{gd}$ for all R ; (ii) as $\dot{\gamma} \rightarrow 0$, $v_p \propto d\dot{\gamma}$ for the trapping regime ($R < R_t$) as in most shear-driven percolation models for small R [8–10,12] and is independent of g ; (iii) in the passing regime ($R > R_t$, $P_p = 1$), $v_p \propto \sqrt{gd}$ independent of $\dot{\gamma}$ as in (i).

To compare Eq. (2) to our data, we determine P_p by characterizing the pore throat size distribution using Delaunay triangulation [24,45,46]. For $\phi \approx 0.58$, P_p is nearly independent of the shear rate for $\dot{\gamma}^* \lesssim 0.1$, and increases from 0.17 to 0.93 as R is increased from 2 to 6. From [24], $c_1 = 0.09\sqrt{R}$ for $\phi \approx 0.58$ and $e = 0.8$. Fits of Eq. (2) to the simulation results for three R values obtained by adjusting the one free parameter c_2 match the simulation data at low $\dot{\gamma}^*$, as shown in Fig. 2 (solid curves). The inset in Fig. 3 indicates that P_p increases with R and c_2 decreases with R , as expected.

All data in Fig. 2 are compared to the universal form of the model [Eq. (3)] in Fig. 3, which plots the vertical velocity scaled by the untrapped vertical velocity from the static bed, $v'_p = v_p/v_{p,s}$, versus the rescaled shear rate $\dot{\gamma}' = -C\dot{\gamma}^*$. Data for all R as well as varying g and d (magenta) collapse onto

the model prediction (red curve) in the low-shear-rate region. For free-sifting cases ($R > 6.5$), $\dot{\gamma}' = \infty$ since $P_p = 1$, and the corresponding symbols fall on the far right of Fig. 3 and approach $v'_p = 1$ (yellow star) in the low-shear-rate regime. *High-shear-rate regime.* When v'_p for $\dot{\gamma}^* \gtrsim 0.1$ is plotted versus $\dot{\gamma}^*$ on a logarithmic scale in Fig. 4(a), it is clear that $v'_p \propto 1/\dot{\gamma}^*$ for $\dot{\gamma}^* \gtrsim 0.4$ and various R , g , and d . This behavior is related to increasing fine-particle velocity fluctuations, which frustrate percolation. To demonstrate the relation between $\dot{\gamma}^*$ and v_{rms} , Fig. 4(b) plots the scaled vertical root-mean-square velocity fluctuations of fines, $v_{rms}^* = v_{rms}/\sqrt{gd}$, versus $\dot{\gamma}^*$ for various R . For context, v_{rms}^* for bed particles (\times) increases linearly with $\dot{\gamma}^*$, as would be expected from the corresponding increase in interparticle collisions. Similarly, for $\dot{\gamma}^* \gtrsim 0.4$, $v_{rms}^* \propto \dot{\gamma}^*$ for all R , indicating that fine-particle velocity fluctuations are driven by the bed-particle velocity fluctuations in the high- $\dot{\gamma}^*$ regime. In comparison, for $\dot{\gamma}^* \lesssim 0.1$, gravity-driven fluctuations dominate, so that v_{rms}^* is either constant (free sifting, $R > 6.5$) or decreases slower than $\dot{\gamma}^*$ (trapping, $R \leq 6.5$) because v_{rms}^* is an average over the trapped (smaller bed-driven fluctuations) and untrapped (larger gravity-driven fluctuations) states. Simulations with different g and d values at $R = 7$ [magenta triangles in Fig. 4(b)] confirm the scaling of v_{rms} with $\dot{\gamma}$ and indicate that v_{rms} is g independent where $v_{rms}^* \propto \dot{\gamma}^*$, but proportional to \sqrt{gd} where v_{rms}^* is constant.

For all $\dot{\gamma}^*$, v_{rms}^* is larger for larger R and approaches a limiting curve for large R , as Fig. 4(b) shows. Additional simulations with fine-particle density varied by two orders of magnitude (250 kg m^{-3} to $2.5 \times 10^4 \text{ kg m}^{-3}$) at constant R change v_{rms} by $<7\%$ (v_p is also minimally affected), indicating that the increase in v_{rms} with increased R is due to the decreased fine-particle diameter (i.e., smaller fines are less constrained by bed particles than larger fines) rather than decreased fine-particle mass.

Having demonstrated the linear dependence of v_{rms} on the shear rate at high $\dot{\gamma}$, Fig. 4(c) tests our hypothesis that the mean vertical velocity decreases with increasing v_{rms} . Indeed, the data show that $v'_p \propto 1/v_{rms}^*$ when $v_{rms}^* \gtrsim 0.4$ for all R . In dimensional form, $v_p \propto gd/v_{rms} \propto g/\dot{\gamma}$, where the linear dependence of v_p versus $\dot{\gamma}$ on g alone at high $\dot{\gamma}$ contrasts with the low-shear-rate scaling of v_p with \sqrt{gd} in the free-sifting regime and with d alone in the trapping regime. Equally significant, Fig. 4(c) also includes data for various values of the restitution coefficient between the bed and fine particles, $0.2 \leq e \leq 1$, for $R = 5$ and 7. Low restitution reduces v_{rms} for large $\dot{\gamma}$ such that different combinations of e and $\dot{\gamma}$ producing the same v_{rms} yield the same v_p . Hence, v_{rms} determines v_p in the high-shear-rate regime ($\dot{\gamma}^* \gtrsim 0.4$).

Discussion. This study of gravity-driven percolation of single noncohesive fine particles in sheared granular beds reveals different dominant physics at low- and high-shear rates. For low-shear rates, $\dot{\gamma}\sqrt{d/g} \lesssim 0.1$, as $\dot{\gamma}$ increases from zero, bed-particle rearrangements due to shear reduce fine-particle trapping to increase the mean vertical velocity v_p . A statistical model of this mechanism [Eq. (2) or (3)] accurately predicts v_p for a wide range of conditions. In the high-shear-rate regime, $\dot{\gamma}\sqrt{d/g} \gtrsim 0.1$, increasing $\dot{\gamma}$ increases the fine-particle velocity fluctuations which frustrate percolation such that $v_p \propto 1/v_{rms} \propto 1/\dot{\gamma}$ (Fig. 4).

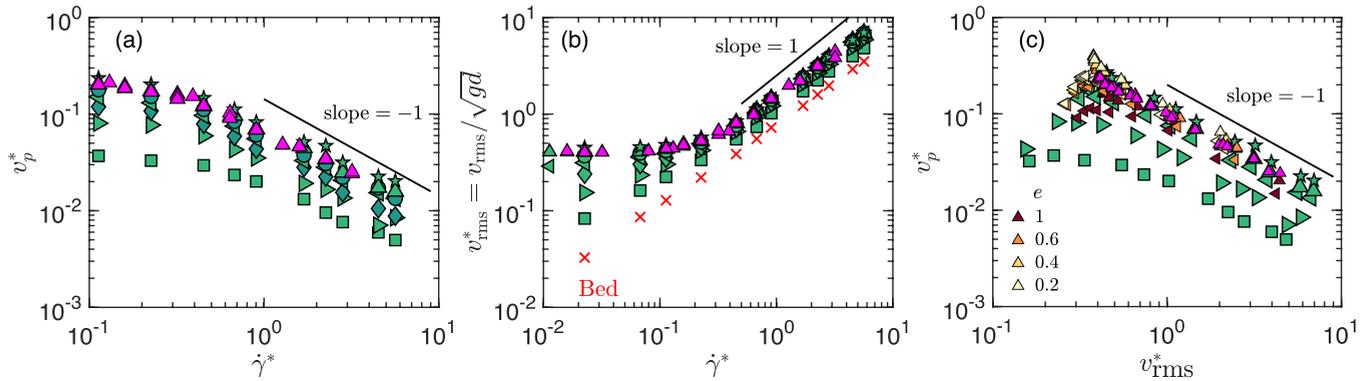


FIG. 4. (a) Scaled fine-particle vertical velocity, $v_p^* = -v_p/\sqrt{gd}$, varies inversely with scaled shear rate, $\dot{\gamma}^* = \dot{\gamma}\sqrt{d/g}$. (b) Scaled fine-particle rms velocity fluctuations, $v_{rms}^* = v_{rms}/\sqrt{gd}$, increase linearly with $\dot{\gamma}^*$ for $\dot{\gamma}^* \gtrsim 0.4$. Bed particle data (\times) shown for comparison. (c) v_p^* varies inversely with v_{rms}^* for various shear rates, and restitution coefficients e (colors as indicated). Data in (a)–(c) include various size ratios $2 \leq R \leq 10$ (symbols), and g and d values (magenta) as in Fig. 2.

Beyond its relevance to macroscale percolation, gravitational-field-driven transport of fines in sheared large-particle beds shares intriguing similarities with electric-field-driven conduction in solids, where conductivity is the product of mobility and carrier concentration; see, e.g., [47]. For $R > R_t$, the granular system is metal-like in that fine particles are never trapped and their mobility decreases with increasing v_{rms} [Fig. 4(c)]. The $v_p^* \propto 1/v_{rms}^*$ dependence for fine-particle mobility is analogous to lattice scattering in the Drude model [48] for electron transport due to an electric field E . In the model, the mean electron momentum is $p = m_e v = qE\tau$, where m_e and q are the electron mass and charge, respectively, and τ is the time between collisions with lattice particles. Replacing v with v_p , qE/m_e with g , and τ with d/v_{rms} (since the fine-particle mean free path is proportional to d) yields the observed scaling $v_p \propto gd/v_{rms}$ for fines. The near-constant v_p^* region evident in Fig. 2 for $R \in \{7, 10\}$ and $\dot{\gamma}^* \lesssim 0.1$ occurs because $v_{rms} \neq 0$, even at $\dot{\gamma}^* = 0$ in static beds, due to $O(\sqrt{gd})$ velocity fluctuations during free sifting. It is only when $\dot{\gamma}^* > 0.1$ that shear-induced fluctuations of $O(d\dot{\gamma})$ become significant enough relative to \sqrt{gd} that v_p decreases.

For $R < R_t$, the granular system is semiconductorlike in that the fraction of time that fine particles are untrapped increases with $\dot{\gamma}$, analogous to the increase in electron concentration in the conduction band of an n -type semiconductor with increasing temperature due to ionization. For sufficiently large $\dot{\gamma}^*$, fines are effectively never trapped, resembling the

saturation region in doped semiconductors where carrier concentration is constant, and $v_p \propto 1/v_{rms}$ as for the metal-like $R > R_t$ case due to enhanced scattering. The $R = 6.5$ case is notable in that with increasing $\dot{\gamma}^*$, “saturation” occurs before v_{rms} begins to increase [see above and Fig. 4(b)] and mobility to decrease, resulting in a plateau in v_p for $0.01 < \dot{\gamma}^* < 0.05$.

This work only begins to explore the intriguing physics of fine-particle percolation in driven granular systems, and many questions and challenges remain. For instance, our model and scalings accurately capture the dependence of v_p on $\dot{\gamma}$, d , and g , but understanding how to incorporate e and R as well as cohesion in expressions for v_p^* and v_{rms} is likely to be nontrivial, although analogies with conduction in solids may be fruitful, as outlined above. Additionally, extending our single-fine-particle-limit results to finite fine-particle concentrations, c_f , would also be valuable and potentially analogous to strongly correlated electron systems. Preliminary heap flow simulations with $R > 4$ and global c_f up to 30% exhibit high-shear regions with local $c_f < 5\%$, where insights from the single-fine-particle limit are likely applicable, but also low-shear regions, where fines pack densely around large particles, forming a continuous fine-particle phase that greatly reduces their vertical mobility.

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