



## Parrondo's paradox in network communication: A routing strategy

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The throughput and latency bottleneck in accessing system resources is prevalent in all communication systems. Likewise, communication overhead in modern computer systems is a vital limiting factor in their performance. In this Letter, we propose a routing strategy to improve communication in networks based on Parrondo's paradox. We show that random switching between the shortest-path algorithm and making the local optimum choice (greedy algorithm) yields a significant reduction in total transmission weight compared to when the shortest-path and greedy algorithms are operated separately. This effect recapitulates Parrondo's paradox, where two games/strategies are losing when played alone but create a winning outcome or optimum results when combined in a certain manner. The performance of the switching strategy is further validated under various parameters, and the results indicate that the effect is more remarkable with an increase in the number of packets and the number of nodes in the system. The proposed routing strategy enhances efficiency and scalability in modern computer and communication systems.

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Network science serves as a powerful tool to replicate real-world complex systems ranging from technological to social systems [1–4]. In general, the constituent elements of the system in the network's framework are represented as nodes, and the interactions between them are depicted as links. Over the past two decades, communication networks, including the internet, have become indispensable components of both the national economy and individual lives [5]. In communication networks, nodes represent the physical devices like computers, routers, or servers, and the links are akin to the connection between the devices. Further, with the increasing growth of network traffic, the internet capacity often becomes insufficient to deal with large numbers of data packets. The problem of improving communication networks involves increasing the number of packets generated per unit time to travel in a congestion-free state, minimizing travel costs and time, and reducing packet loss. It can be addressed via two ways: redesigning the network topology or advancing the routing strategy. However, the former approach demands a higher cost compared to the latter, making the advancement of the routing strategy more preferable.

In order to enhance network communication through optimizing routing strategy, a simple model of communication in networks with hierarchical branching [6] was proposed, where continuous phase transition between the free-flow and

the congested regime was described by an order parameter. Packet transmission was subsequently studied by the notion of load, capacities, and overload in networks [7–9]. Yan *et al.* [10] then proposed an efficient routing strategy in which the path between two nodes is defined as  $\sum_{i=1}^l k(x_i)^\beta$  where  $\beta$  is a tunable parameter,  $l$  is the path length, and  $k(x_i)$  represents the degree of the  $i$ th node in the path. Notably, the most efficient path is achieved when  $\beta$  is set to one. Recently, algorithms like random walk [11,12], integrating shortest-path and local routing strategy [13], next-nearest-neighbor [14], local routing strategy [15,16], link closing strategy [17], and edge-adding strategy [18] have been put forward by researchers to find the best routing strategy to enhance network communication. Efficient routing strategy has also been investigated in different types of network setup such as spatial networks [19], multilayer networks [20,21], and quantum networks [22,23].

In current network protocols, such as the transmission control protocol (TCP) and user datagram protocol (UDP), it is common practice to split data into numerous packets. This practice is employed when the packet size exceeds the maximum segment size (MSS) or maximum transmission unit (MTU) of the underlying network. The MSS is associated with the transmission control protocol (TCP) while the MTU constrains the size of data packets in both TCP and user datagram protocol (UDP). Hence, transmitting large-sized data involves fragmenting it into numerous smaller packets, which are subsequently transmitted separately across the network. These individual packets are later reassembled at the receiving end to reconstruct the original data [24–28]. Another reason for splitting data packets is the possibility of packet loss caused by device failures, disconnection, or data time out. In these scenarios, sending data again will incur additional latency and hence sending multiple packets tends to

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minimize the computational overhead [29–34]. Therefore, we are motivated to analyze multipacket data packet transmission instead of single-packet data transmission. In this work, we propose a routing strategy to enhance network communication based on Parrondo's paradox. It is well known that the latency bottleneck in data transmission is prevalent in almost all modern communication systems. We demonstrate that while individual constituent algorithms for packet transmission are suboptimal individually, switching between them randomly can lead to latency suppression and ultimately produce optimal results. This recapitulates the Parrondo effect, where two losing games can be combined in a certain manner to create a winning outcome or optimal results. Parrondo's paradox has been applied in different areas of research, including quantum game theory [35,36], computer science [37], social sciences [38,39], and evolutionary dynamics [40] among others [41].

We consider weighted random networks  $G = \{V, E\}$ , consisting of a set of nodes  $V = \{v_1, v_2, v_3, \dots, v_N\}$  and links  $E = \{e_1, e_2, e_3, \dots, e_M\}$ , where  $N$  and  $M$  are sizes of  $V$  and  $E$ , respectively. Each link  $e_i$  has a resting weight  $w_i^\phi$  which represents the latency factor or time delay incurred when a single data packet is transmitted through the link in an unloaded state [42–44]. The weights of the links are taken from the uniform distribution over the interval  $[0,1]$ . Suppose  $n$  packets are sent from a randomly chosen source and all are intended for a common destination. They are sent  $\Delta T$  times apart regulated by the source node with a common destination and each packet moves one step ahead toward the destination at each time step. As the number of packets increases with time, the links in the network become overloaded and the latency factor over the links increases. The load of a link directly correlates with the time delay, as discussed in [45,46]. Let  $w_i(t)$  be the weight of the link  $e_i$  at time  $t$ , and it equals  $w_i(t) = w_i^\phi$ , until one packet is transmitted through the link. Whenever the next packet passes through, the link experiences an increment in the latency by an amount  $\epsilon_i$ . It is not difficult to argue that the links are subject to recovery from the load after sufficient time. Let  $\beta_i$  be the relaxation factor or decay constant associated with link  $e_i$  which describes the rate at which the link recovers from the load. Hence, the dynamics can be described by the following equation:

$$\frac{dw_i(t)}{dt} = -\beta_i(w_i(t) - w_i^\phi). \quad (1)$$

The weight of the link at time  $t$  can be obtained by solving Eq. (1),

$$w_i(t) = w_i^\phi + \exp(-\beta_i(t - t'))(w_i(t') - w_i^\phi), \quad (2)$$

where  $t'$  is the time at which the last packet was transmitted through the link  $e_i$ . For simplicity, we consider  $\beta_i = \epsilon_i = 0.1\forall i$  and  $\Delta T = 0.05$ . Equation (1) draws inspiration from the concept of delay gradient, a crucial element employed in congestion control mechanisms [47,48], where monitoring and responding to the derivatives of queuing with respect to time could aid in reducing latency. Furthermore, by examining Eq. (2), it becomes evident that the time delay experienced by packets when traveling across a particular link varies according to the link's state at a specific time, resembling the notion of delay variation [46,49].

The exact expression of the link weight for the  $n$ th packet passing through it can be analytically derived in terms of model parameters. When the time delay between two consecutive packets is  $\Delta T$ ,  $t - t'$  equals to  $m\Delta T$  where  $m$  is an integer. Let  $t_1, t_2, \dots, t_n$  be the time at which  $n$  packets pass through the link, respectively. Therefore,  $t_{i+1} - t_i$  is the time lag between the  $i$ th and  $(i + 1)$ th packets, and  $m_i = \frac{(t_{i+1} - t_i)}{\Delta T} \forall i \in \{1, 2, 3, \dots, n - 1\}$  is associated with  $i$ th and  $(i + 1)$ th packets. Through Eq. (1), it is easy to obtain  $w_i(1) = w_i^\phi$ ,  $w_i(2) = w_i^\phi + \epsilon$ , and  $w_i(3) = w_i^\phi + \epsilon(1 + \exp(-\beta m_2 \Delta T))$ . In this case, the weight  $w_i(n)$  of link when the  $n$ th packet passes through it is given by

$$w_i^\phi + \epsilon \times \left( 1 + \exp(-\beta \Delta T m_{n-1}) + \exp\left(-\beta \Delta T \sum_{i=n-2}^{n-1} m_i\right) + \dots + \exp\left(-\beta \Delta T \sum_{i=3}^{n-1} m_i\right) + \exp\left(-\beta \Delta T \sum_{i=2}^{n-1} m_i\right) \right).$$

Let  $S_i(n)$  be the weight when all the packets pass through the same link in consecutive time steps ( $m_1 = m_2 = m_3 = \dots = m_n = 1$ ). The above expression then becomes a geometric series and can be simplified as

$$S_i(n) = w_i^\phi + \epsilon \times \frac{1 - \exp(-(n - 1)\beta \Delta T)}{1 - \exp(-\beta \Delta T)}. \quad (3)$$

The path followed by the packet originates from node  $v_i$  and destined to reach node  $v_j$  is represented by  $L(v_i \rightarrow v_j) : v_i \equiv x_1 \rightarrow x_2 \rightarrow \dots \rightarrow x_l \rightarrow \dots \rightarrow x_k \equiv v_j$ , where  $(x_l, x_{l+1}) \in E$  and  $x_l \in V$ . The weight of the path can be obtained by  $W(L) = \sum_{l=1}^{k-1} w_l$ , where  $w_l$  is the weight of the link  $(x_l, x_{l+1})$ . In communication networks, the shortest path is one of the most popular routing strategies, because the packet is forwarded along the path which minimizes the sum of the weight of the constituents links, that is,  $\min(W(L))$ . However, it is not always feasible to obtain the shortest path between two nodes because of the large size of the real-world communication networks [50]. Taking this into account, a routing strategy based on the local topological structural information of the network [50] was proposed. The packet is being forwarded based on the probability proportional to some tunable parameter over the degree of nodes.

Here, we introduce a routing strategy that enhances network communication by dynamically alternating between transmitting packets via the shortest path and making locally optimal choices. Specifically, the packet can follow the shortest path with probability  $\gamma$  or make the local optimum choice with probability  $1 - \gamma$  at each time step. To make the local optimum choice, the packet makes a transition from the current node  $v_i$  to one of its neighbors  $v_j$  with the probability

$$P(v_i \rightarrow v_j) = \frac{w_{ij}^{-1}}{\sum_j w_{ij}^{-1}}, \quad (4)$$

where the sum is taken over the neighbors of node  $i$  excluding the previously visited node. The approach is popularly known as the greedy algorithm in which the best available option is selected. We also assume that while following the shortest path, the packets have a global awareness of the network and follow the minimum weight path which is fixed for all pairs of

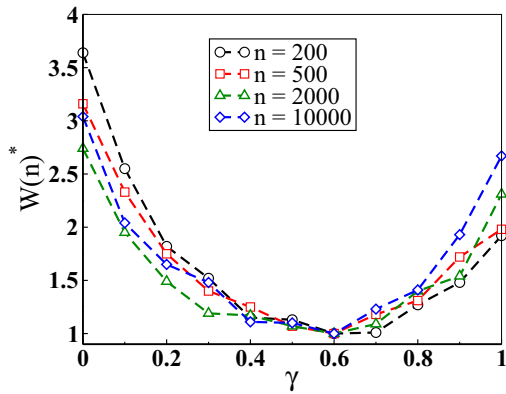


FIG. 1. Normalized total transmission weight  $W(n)^*$  for various values of  $\gamma$  and  $n$ . There are  $N = 100$  devices, and the results are averaged over 100 realizations of random networks with connection probability  $p = 0.5$ .

nodes at the initial construction of networks and the same for all the packets. Notably, routing protocols on the internet are inherently dynamic, allowing routers to update their routing tables in response to network events such as congestion and device failures. The dynamic feature through greedy algorithm involved in this work refers to forwarding packets to neighboring nodes stochastically, without considering optimal paths or network conditions.

The outcome of the switching strategy, quantified by the total transmission weight  $W(n)$  of  $n$  packets accumulated while traveling from source to destination, is explored across a range of values of  $\gamma$ . The normalized total transmission weight  $W(n)^* = \frac{W(n)}{\min(W(n))}$  for various  $n$  is shown in Fig. 1. It is found that the stochastic switching between the shortest-path approach ( $\gamma = 1$ ) and making the locally optimal choice ( $\gamma = 0$ ) could yield a significant reduction in total transmission weight compared to that of any individual strategy. The effect is more remarkable with a larger number of packets  $n$  because of the fixed shortest path between any two nodes, which remains the same for all packets transmitted from the source to the destination. Hence, the links along the path get congested, leading to accretion in  $W(n)$  with increasing  $n$ . On the contrary, the greedy algorithm performs poorly for both small and large  $n$  and accumulates a large value of  $W(n)$ . Therefore, under the switching strategy, the packets randomly take different paths along the shortest path, thereby bringing down the congestion and minimizing the value of  $W(n)$ . At the optimal value  $\gamma^*$ , where  $W(n)$  takes possible minimum value,  $W(n, \gamma = \gamma^*)$  is at least two times lower than  $W(n, \gamma = 0)$  and  $W(n, \gamma = 1)$  (Fig. 1). Overall, the stochastic switching between the two losing strategies could yield a lower  $W(n)$  for any values of  $\gamma \geq 0.2$ , thereby improving network communication.

Importantly, it is not possible to reduce or scale  $W(n)$  indefinitely at  $\gamma^*$  for a fixed  $N$ . This is because the alternate paths available to the packets are also subject to congestion and there is a limit to  $\frac{W(n, \gamma=0)}{W(n, \gamma=\gamma^*)}$  and  $\frac{W(n, \gamma=1)}{W(n, \gamma=\gamma^*)}$ . We refer to the above observations as the Parrondo effect where both the shortest path approach ( $\gamma = 1$ ) and greedy algorithm ( $\gamma = 0$ ) are “losing” due to their large value of  $W(n)$  when they are

followed separately. However, the best possible results that can be considered as the winning outcome can be obtained if the two losing strategies are switched based on a stochastic scheme, which recapitulates the game-theoretic Parrondo’s paradox. Let  $l$  be the average number of hops between any pair of nodes along the shortest path. The exact expression for total transmission weight of  $n$  packets following the shortest-path approach ( $\gamma = 1$ ) can be expressed as

$$W(n) = \sum_{i=1}^l S_i(1) + \sum_{i=1}^l S_i(2) + \dots + \sum_{i=1}^l S_i(n), \quad (5)$$

and it can be rewritten based on Eq. (3),

$$W(n) = n \sum_{i=1}^l w_i^\phi + \frac{\epsilon l}{1 - \exp(-\beta \Delta T)} \times \left( n - 1 - \sum_{i=1}^{n-1} \exp(-i\beta \Delta T) \right).$$

The equation can be further simplified as,

$$W(n) = \left( n \sum_{i=1}^{i=l} w_i^\phi + \frac{\epsilon l}{1 - \exp(-\beta \Delta T)} \times \left( n - 1 - \frac{\exp(-\beta \Delta T) - \exp(-n\beta \Delta T)}{1 - \exp(-\beta \Delta T)} \right) \right). \quad (6)$$

It has been found that  $l$  scales as  $l \sim \ln(N)$  for both unweighted and weak disordered random networks [51]. Thus, we expect no significant change in  $W(n)$  with a change in network size  $N$  for fixed  $n$ , which will be verified later.

For a fixed network size  $N$ , there exists a total of  $\binom{N}{2}$  possible pairs of source and destination nodes. As the network size increases, the number of possible pairs grows proportionally with  $N^2$ . Consequently, for  $\gamma < 1$ , the value of the total transmission weight  $W(n)$  can exhibit significant fluctuations depending on the specific source and destination pairs and the configurations of the network. To illustrate this phenomenon, we present the probability distribution of  $W(n)$  in Fig. 2 for random realizations of source-destination pairs and network configurations. It can be found that the probability distribution  $P(W(n))$  follows a heavy-tailed pattern, with significant probabilities assigned to large values of  $W(n)$ . Hence, the optimal value of  $\gamma$ , denoted as  $\gamma^*$ , may vary from one realization to another. We also discovered that for values of  $\gamma' = \{0.4, 0.5, 0.6\}$ ,  $W(n, \gamma = \gamma')$  is smaller than  $W(n, \gamma)$  when  $n \gg 100$ . Therefore,  $\gamma' = \{0.4, 0.5, 0.6\}$  represents the optimal solution for the switching strategy. In order to determine the optimal  $\gamma^*$ , the expected value of  $W(n)$ , denoted as  $\langle W(n) \rangle$ , is obtained through  $\int W(n)P(W(n))dW$ . It is noteworthy that  $\langle W(n, \gamma = 0.6) \rangle$  is smaller than  $\langle W(n, \gamma) \rangle$  for  $n \gg 100$ , where  $\gamma \in (0, 1)/0.6$ . This finding suggests that  $\gamma = 0.6$  is the preferable choice for optimizing the total transmission weight in packet transmission.

The impact of increasing the network size  $N$  on  $W(n)$  is then explored under  $\gamma \in \{0, 0.6, 1\}$  and  $n \in \{50, 500, 2000\}$  in Fig. 3. For the greedy algorithm ( $\gamma = 0$ ),  $W(n)$  shows an increasing trend and fits well linearly with  $N$  for all  $n$ .

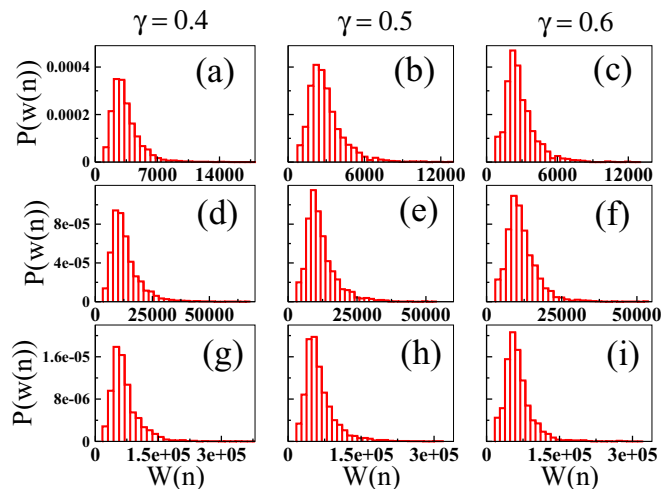


FIG. 2. Probability distribution of  $W(n)$  for  $\gamma \in \{0.4, 0.5, 0.6\}$  in three columns and (a)–(c)  $n = 200$ , (d)–(f)  $n = 500$ , and (g)–(i)  $n = 2000$ . Results are averaged over 2500 realizations.

In order to quantify the increment in  $W(n)$  with  $N$ , that is,  $\Delta W(n)/\Delta N$ , the log-log curve fitting  $\log(W(n)) = m \times \log(N) + C$  is performed. Since  $W(n)$  increases significantly with  $n$ , we normalize it by dividing it by its maximum value to ensure a consistent scale across different values of  $N$ . In this case, it can be easily obtained that  $m = 0.73 \pm 0.02$  for all  $n$ . Hence,  $\Delta W(n)/\Delta N$  remains the same for all  $n$ . For the shortest path approach ( $\gamma = 1$ ),  $W(n) \approx c$  where  $c$  is the constant which is in accordance with Eq. (6). Intriguingly, for  $\gamma = 0.6$ ,  $W(n)$  does not display significant change when  $N < 1000$ , but shows a decreasing trend after  $N > 1000$  for all  $n$ . Furthermore,  $W(n, \gamma = 0)^*$  and  $W(n, \gamma = 1)^*$  against  $N$  are applied to quantify the degree of enhancement of the proposed random switching strategy with the system size in Fig. 4. It is evident that  $W(n, \gamma = 0)^*$  increases drastically with the increase in network size, especially for  $N \geq 1000$ , and  $W(n, \gamma = 1)^*$  is at least ten times smaller than  $W(n, \gamma = 0)$  for all  $n$ . Thus, with the increase in network size, the greedy algorithm becomes more inefficient. In addition,  $W(n, \gamma = 1)^*$  depicts an increasing trend with the increase in  $N$ , and  $W(n, \gamma = \gamma^*)$  is six times smaller than  $W(n, \gamma = 1)$  for large  $N$  and  $n$ . We expect that further increment in  $N$  and  $n$  can lead to a significant improvement in the reduction of total transmission

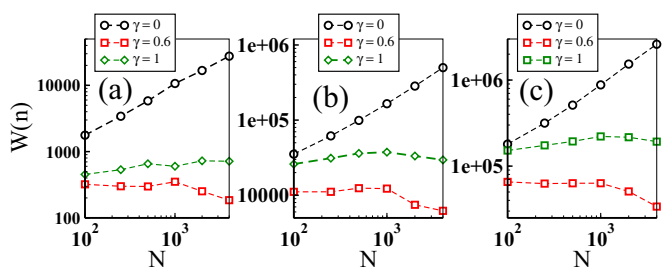


FIG. 3.  $W(n)$  against  $N$  for  $\gamma \in \{0, 0.6, 1\}$  and (a)  $n = 50$ , (b)  $n = 500$ , and (c)  $n = 2000$ . The results are averaged over 100 realizations for  $N < 1000$  and 30 realizations for  $N \geq 1000$ . For all  $N$ ,  $p = 0.5$  remains fixed.

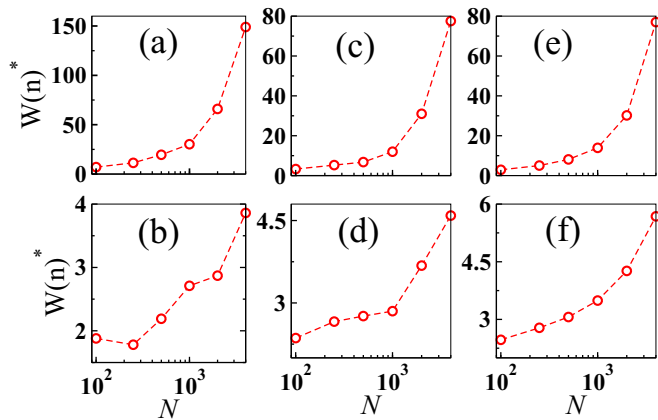


FIG. 4.  $W(n)^*$  against  $N$  for  $\gamma = 0$  (upper panel) and 1 (lower panel) when (a), (b)  $n = 50$ ; (c), (d)  $n = 500$ ; and (e), (f)  $n = 2000$ . The results are averaged over 100 realizations for  $N < 1000$  and 30 realizations for  $N \geq 1000$ . For all  $N$ ,  $p = 0.5$  remains fixed.

weight even of an order of magnitude compared to the two losing approaches. Overall, network communication can be greatly enhanced by switching between two losing strategies for large values of  $N$  and  $n$ .

The robustness of these results with the change in model parameters is then explored. The results of  $W(n)^*$  against  $\gamma$  for various connection probabilities  $p$  are presented in Fig. 5. With the increase in  $p$ ,  $W(n, \gamma = 0)^*$  shows a decreasing trend due to the denser network and a larger number of available paths for packets from source to destination. Therefore, the greedy algorithm significantly increases the transmission weights due to the random movements of packets and congestion caused by the smaller number of available paths, rendering it to be increasingly inefficient with the decrease in  $p$ . On the contrary, there is no significant impact of change in  $p$  on  $W(n, \gamma = 1)^*$ . Nevertheless, Parrondo’s paradox is still in play.

The impact of change in time delay ( $\Delta T$ ) on  $W(n)^*$  is discussed (Fig. 6). Apparently,  $W(n, \gamma)$  will decrease with the increase in  $\Delta T$  because the links have longer time intervals to relax from the congestion. However, we are more interested in  $W(n, \gamma = 0, 1)^*$  to envisage and quantify the efficacy of the switching strategy. It can be found that  $W(n, \gamma = 0)^*$  shows an increasing trend for  $\Delta T > 0.05$ . This is because the links along the shortest path get free from congestion and it is not pragmatic to choose the greedy algorithm over the switching

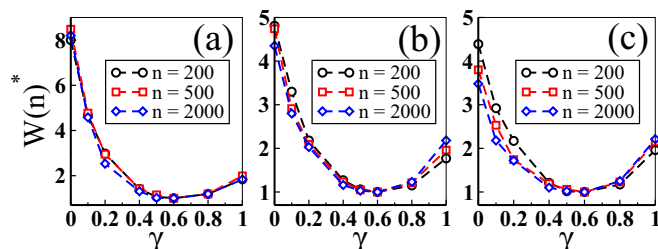


FIG. 5.  $W(n)^*$  against  $\gamma$  for various values of connection probabilities when (a)  $p = 0.1$ , (b)  $p = 0.2$ , and (c)  $p = 0.3$ . The results are averaged over 100 realizations.

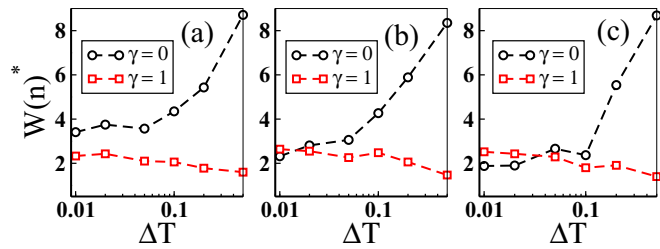


FIG. 6.  $W(n)^*$  against  $\Delta T$  for  $\gamma = \{0, 1\}$  when (a)  $n = 200$ , (b)  $n = 500$ , and (c)  $n = 2000$ . The results are averaged over 100 realizations.

strategy for large  $\Delta T$ . On the other hand, the change of  $\Delta T$  has no significant impact on  $W(n, \gamma = 1)^*$ . However, the switching strategy still remains the more favorable approach with  $2 - c \leq W(n, \gamma = 1)^* \leq 2 + c$ , where  $c \approx 0.5$  for  $0.01 \leq \Delta T \leq 0.5$ .

In physical situations, links may recover faster or slower compared to the increment in the latency due to the congestion, leading to different relaxation factors  $\beta$ . Therefore, the parameter  $\eta = \beta/\epsilon$  is defined to further study the effect of inequalities between  $\beta$  and  $\epsilon$  on  $W(n)$ .

Specifically, the links recover slower when  $\eta < 1$  and faster when  $\eta > 1$  as compared to the increment in latency. The results of  $W(n)^*$  against  $\gamma$  for various values of  $\eta < 1$  and  $\eta > 1$  are presented in Fig. 7. For  $\eta < 1$ , it can be found that Parrondo's paradox is still in play, where  $W(n)$  takes a possible minimum value and is at least two times smaller than  $W(n, \gamma = 0, 1)$ . In addition, there is no significant impact of  $\eta$  on  $W(n)^*$  when  $\eta < 1$ .  $W(n, \gamma = 0)^*$  increases as links recover faster ( $\eta > 1$ ), because it is always feasible for packets to consider the shortest path while the greedy algorithm becomes increasingly inefficient for  $\eta > 1$ . Remarkably, the switching strategy maintains its superiority over the shortest-path approach within the range  $1 < \eta \leq 6$ , even in scenarios where link recovery is expedited.

We have proposed a routing strategy to enhance network communication based on Parrondo's paradox. We have shown that stochastic switching between the shortest-path and greedy

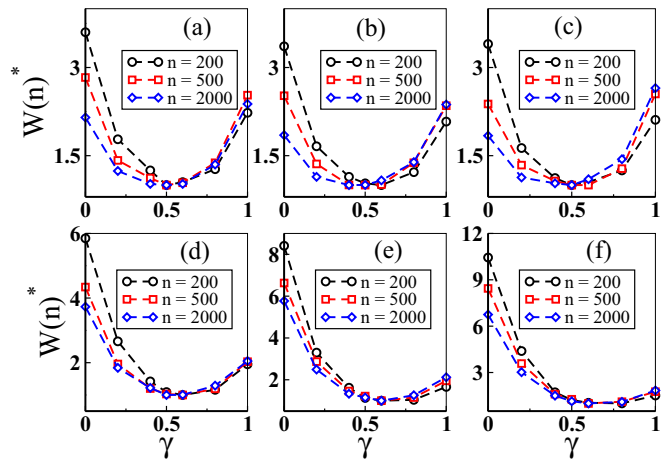


FIG. 7.  $W(n)^*$  against  $\gamma$  for various values of  $\eta$  when (a)  $\eta = 0.5$ , (b)  $\eta = 0.25$ , (c)  $\eta = 0.16$ , (d)  $\eta = 2$ , (e)  $\eta = 4$ , and (f)  $\eta = 6$ . The results are averaged over 100 realizations.

approaches in the routing strategy of packet transmission can result in a significant reduction in the total transmission weight. Both algorithms are "losing" as they do not yield optimal results individually. If the two strategies are combined in a certain manner, the best possible winning outcome can then be obtained. In addition, we have found that the effect is more notable with the increase in the number of packets as well as the number of nodes in the system. We have also validated the performance of the approach under many different parameters. It is noteworthy that the predominant majority of existing literature has been constructed with a singular focus on the properties of nodes but our work presents a distinctive approach: a model built fundamentally upon the attributes of links. This perspective has presented a profound shift in our understanding and design of packet transmission frameworks.

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