

Enhancing exotic quantum fluctuations in a strongly entangled cavity BEC system

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We show that the strong coupling of a quantum light field and correlated quantum matter induces exotic quantum fluctuations in the matter sector. We determine their spectral characteristics and reveal the impact of the atomic s -wave scattering. In particular, we derive the dissipative Landau and Beliaev processes from the microscopic Hamiltonian using imaginary-time path integrals. By this, their strongly sub-Ohmic nature is revealed analytically. A competition between damping and antidamping channels is uncovered. Their intricate influence on physical observables is quantified analytically and the Stokes shift of the critical point is determined. This illustrates the tunability of the quantum matter fluctuations by exploiting strong light-matter coupling.

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Introduction. When an ultracold atomic gas is placed in an optical cavity, the strong light-matter interaction can give rise to a rich landscape of quantum phases [1–5]. A prominent example is the second-order nonequilibrium Dicke quantum phase transition [6]. It can be observed by strongly coupling an atomic Bose-Einstein condensate (BEC) to a cavity, pumped with a retroreflected laser beam, which is adjusted perpendicularly to the cavity axis. Above a critical pump strength, the atomic density, in a spontaneous \mathbb{Z}_2 symmetry-breaking process, switches from a homogeneous state to one of two possible density gratings, which resemble the black or white fields of a checkerboard lattice. Each density grating fulfills the Bragg condition to maximize coherent scattering [7] of photons from the pump field into the cavity and vice versa. The resulting interference between the pump and intracavity light fields leads to a two-dimensional light shift potential, which stabilizes the prevailing density grating.

A two-mode description of the atomic field operator can explain this phenomenon [8]. The photon scattering couples the zero-momentum state of the condensate to a symmetric superposition of four finite-momentum condensate states $|\pm k, \pm k\rangle$, which carry two quanta of photon momentum $\hbar k$, one along the cavity, and one along the pump axis [1]. Eventually, condensate fluctuations trigger the transition. At zero temperature, quantum fluctuations determine the fate of the condensate [8]. The scattering of the pump photons creates polaritons that involve both the collective oscillations of the condensate and the intracavity field [8]. The excitation

spectrum provides a soft mode whose frequency vanishes as the critical point is approached [8].

Even if dissipation by cavity loss is included, the criticality survives [9], yet with the critical exponent modified from $1/2$ to 1 . Due to the quantum correlations between the cavity photons and the atoms, this dissipation channel enables nondestructive measurement of the atomic state [10–12] by detecting the outgoing photons. Hence, fluctuations in the matter and light sectors can be studied *in situ*.

Commonly, damping of the excited atomic state is introduced phenomenologically [2]. On the microscopic level, damping of quasiparticles in a superfluid occurs as Landau [13–17] and Beliaev damping [18–20]. Within a Born-Markov approximation, the damping rate of the polariton soft mode is known [21]. Landau damping vanishes at zero temperature and grows monotonously with a vanishing soft mode frequency. Beliaev damping peaks when the polariton decays into two phonons close to the edge of the Brillouin zone [21] which induces a nonanalytical point in the fluctuation spectrum of the condensate [22]. The Born-Markov approximation breaks down when the two significant poles of the polariton Green's function form an avoided crossing [21]. Indeed, phenomenological sub-Ohmic baths do modify the critical exponent [23]. Sub-Ohmic reservoirs induce strong non-Markovian dynamics and come along with exotic quantum-colored noise features [24]. They originate due to charge noise in superconducting qubits [25], quantum dots [26], quantum impurity systems [27,28], and nanomechanical oscillators [29], and rule the ultraslow glass dynamics [30]. Sub-Ohmic dissipative quantum dynamics is notoriously difficult to calculate [31].

In this Letter, we formulate a generalized Bogoliubov theory to microscopically derive the colored spectra of the quantum statistical fluctuations. They show exotic, strongly sub-Ohmic features, yielding significant non-Markovian dynamics. An imaginary-time path integral allows us to describe

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the formation of the dissipative two-mode polariton in an exact analytic fashion. Finally, we determine the influence of damping on the physical observables, illustrating thereby the possibility to control and enhance the quantum fluctuations in the system.

Model. We consider the internal state transition ω_A of N_C atoms which is transversely pumped by a laser with the frequency ω_P far detuned by $\Delta_A = \omega_P - \omega_A$. The pump mode function is $h(\mathbf{r}) = h_P \cos ky$ with the Rabi frequency h_P . The single-mode cavity with frequency ω_C hosting the atoms is described by the cavity photon field operator a and the detuning $\Delta_{C,0} = \omega_P - \omega_C$. It is unoccupied in the absence of scattered pump photons in the longitudinal cavity mode with the mode function $g(\mathbf{r}) = g_C \cos kx$. The amplitude $g_C = \sqrt{U_0 \Delta_A}$ depends on the cavity frequency shift by a single atom U_0 and Δ_A . In addition, we include the s -wave scattering between the condensate atoms by the pseudopotential U and consider the effective Hamiltonian with the atomic internal ground state field operators $\psi(\mathbf{r})$ in the form [32] (with $\hbar = 1$)

$$\begin{aligned} H &= -\Delta_{C,0} a^\dagger a + \int d\mathbf{r} \psi^\dagger(\mathbf{r}) H_A^{(1)} \psi(\mathbf{r}), \\ H_A^{(1)} &= -\frac{1}{2m} \nabla^2 + \frac{U}{2} \psi^\dagger(\mathbf{r}) \psi(\mathbf{r}) \\ &\quad + \frac{1}{\Delta_A} [h^2(\mathbf{r}) + g^2(\mathbf{r}) a^\dagger a + g(\mathbf{r}) h(\mathbf{r}) (a + a^\dagger)]. \quad (1) \end{aligned}$$

Two types of light-matter coupling occur. First, the coupling between the occupation numbers induces a dynamic frequency shift characterized by U_0 and the periodicity π/k along the cavity axis. Below the Dicke phase transition, this coupling is much smaller than the atomic recoil energy, $U_0 \langle a^\dagger a \rangle \ll \omega_R = k^2/(2m)$ and we include its zeroth-order contribution $\Delta_C = -\Delta_{C,0} + U_0 N_C/2$. The second type of coupling is due to the atoms scattering the pump photons into the cavity and vice versa. It thus depends on both the single-atom frequency shift U_0 and the pump Rabi frequency h_P . The combination of the pump and cavity mode function yields a $2\pi/(\sqrt{2}k)$ periodicity along the diagonal of the cavity and the pump direction.

Mode expansion and Bogoliubov rotation. Focusing on the regime below the critical pump strength we expand the atomic field operators in the nonlocalized regime in terms of the two-mode Bloch functions with the momentum operators b_p and c_p [21], i.e.,

$$\begin{aligned} \psi(\mathbf{r}) &= \frac{1}{\sqrt{V_{2D}}} \sum_{p \in \mathcal{P}} e^{ipr} (b_p + 2 \cos kx \cos ky c_p), \\ \mathcal{P} &= \left\{ \left(\frac{2\pi j_x}{L_x}, \frac{2\pi j_y}{L_y} \right)^T \mid j_x, j_y \in \mathbb{Z} \wedge |p_x|, |p_y| < \frac{k}{2} \right\}, \quad (2) \end{aligned}$$

with $V_{2D} = L_x L_y$ being the atomic condensate volume. This expansion is motivated by the special role of the zero-momentum condensate mode b_0 and the checkerboard mode $c \equiv c_0$ with momentum $\sqrt{2}k$ for the phase transition [1,8]. Commonly, s -wave scattering is weak, and a macroscopic number n_0 of atoms occupies the condensate mode b_0 . Therefore, we can rewrite the condensate operators in the spirit

of Bogoliubov theory as the total atom number reduced by fluctuations, i.e.,

$$b_0, b_0^\dagger = \sqrt{n_0} = \sqrt{N_C - c^\dagger c - \sum_{p \neq 0} (b_p^\dagger b_p + c_p^\dagger c_p)}. \quad (3)$$

To generalize Bogoliubov theory, we include operators of $\mathbf{p} \neq 0$ up to quadratic order and keep all terms of at least $\mathcal{O}(\sqrt{N_C})$. The latter condition corresponds to including those interaction processes which involve the condensate mode, i.e., among others umklapp processes are neglected. Then, we apply the Bogoliubov rotation

$$\begin{aligned} b_p &\rightarrow b_p \cosh \alpha_{b,p} + b_{-p}^\dagger \sinh \alpha_{b,p}, \\ c_p &\rightarrow c_p \cosh \alpha_{c,p} + c_{-p}^\dagger \sinh \alpha_{c,p}. \quad (4) \end{aligned}$$

The coefficients depend on the atomic interaction nU with the atom density $n = N_C/V_{2D}$ as well as on the respective band $\omega'_{b,p} = \mathbf{p}^2/(2m) \equiv \omega_p$ and $\omega'_{c,p} = \omega_p + k^2/m + h_P^2/(4\Delta_A) = \omega_p + 2\omega_R - \bar{\omega}_P$. The last term with the pump strength $\bar{\omega}_P = -h_P^2/4\Delta_A$ follows from the $h^2(\mathbf{r})$ term in the two-dimensional field Hamiltonian Eq. (1). The coefficients read for $v = b, c$: $\tanh 2\alpha_{v,p} = nU/(\omega'_{v,p} + nU)$. After the Bogoliubov rotation, the new quasiparticle frequencies are

$$\omega_{v,p} = \sqrt{(\omega'_{v,p})^2 + 2nU\omega'_{v,p}}. \quad (5)$$

Finally, we abbreviate the coefficients as

$$\begin{aligned} \phi_{1p} &= \cosh \alpha_{b,p} \cosh \alpha_{c,p}, & \phi_{2p} &= \sinh \alpha_{b,p} \sinh \alpha_{c,p}, \\ \theta_{1p} &= \cosh \alpha_{b,p} \sinh \alpha_{c,p}, & \theta_{2p} &= \sinh \alpha_{b,p} \cosh \alpha_{c,p}, \quad (6) \end{aligned}$$

and their combinations as $\phi_p = \phi_{1p} + \phi_{2p}$ as well as $\theta_p = \theta_{1p} + \theta_{2p}$. Moreover, the zero-momentum coefficient follows as $\phi_0 = \cosh \alpha - \sinh \alpha$ with $\alpha \equiv \alpha_{c,0}$. By this, we obtain the effective Hamiltonian

$$\begin{aligned} H &= \Delta_C a^\dagger a + \omega_0 c^\dagger c + \lambda_0 (a + a^\dagger)(c + c^\dagger) + H_\beta + H_{\text{int}}, \\ H_\beta &= \sum_{p \neq 0} (\omega_{b,p} b_p^\dagger b_p + \omega_{c,p} c_p^\dagger c_p), \quad (7) \end{aligned}$$

with $\omega_0 = \omega_{c,0}$ and $\lambda_0 = \sqrt{N_C} \phi_0 \lambda$. The interaction Hamiltonian

$$\begin{aligned} H_{\text{int}} &= \lambda (a + a^\dagger) K + \eta \phi_0 (c + c^\dagger) K + \eta (c \cosh \alpha - c^\dagger \sinh \alpha) \bar{K}^\dagger \\ &\quad + \eta (c^\dagger \cosh \alpha - c \sinh \alpha) \bar{K} \quad (8) \end{aligned}$$

describes the coupling of the cavity mode and the checkerboard mode to the atomic fluctuations with $\mathbf{p} \neq 0$. Its coupling parameters are $\lambda = \sqrt{-U_0 \bar{\omega}_P}$ to the cavity and $\eta = 2\sqrt{N_C} U/V_{2D}$ to the checkerboard mode. The two combinations of fluctuation operators are summarized as K and \bar{K} . We distinguish between the terms describing Landau and Beliaev processes $K = K^L - K^B$ and $\bar{K} = -\bar{K}^L + \bar{K}^B$ with

$$\begin{aligned} K^L &= \sum_{p \neq 0} \phi_p [b_p^\dagger c_p + \text{H.c.}], & K^B &= \sum_{p \neq 0} \theta_p [c_{-p} b_p + \text{H.c.}], \\ \bar{K}^L &= \sum_{p \neq 0} [\theta_{2p} c_p b_p^\dagger + \theta_{1p} c_p^\dagger b_p], \\ \bar{K}^B &= \sum_{p \neq 0} [\phi_{1p} c_{-p} b_p + \phi_{2p} b_p^\dagger c_{-p}^\dagger]. \quad (9) \end{aligned}$$

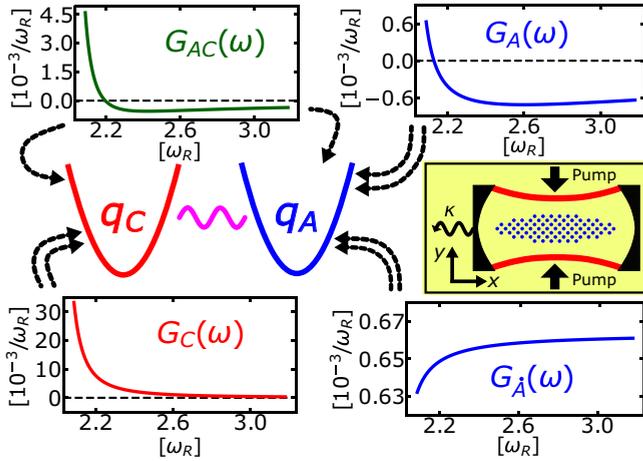


FIG. 1. Zero-temperature Beliaev spectral densities $G_x(\omega)$ describing the quasiparticle quantum dissipative baths acting on the cavity (q_C , red) and atom checkerboard (q_A , blue) mode, respectively. The shaded plot shows a sketch of the cavity BEC setup with the identical color coding for the quantum occupation of the cavity mode and the checkerboard pattern atomic fluctuation in the cigar-shaped BEC. In addition the loss through one of the cavity mirrors κ is displayed which will be included in the theoretical model at a latter stage. The parameters are $\bar{\omega}_p = 0.01\omega_R$, $nU = 0.1\omega_R$, $U_0 = -10^{-3}\omega_R$, $N_C = 5 \times 10^4$, $L_x = 60 \mu\text{m}$, $L_y = 11 \mu\text{m}$, $m = m_{\text{Rb87}}$, and $\omega_R = 2\pi \times 3.56 \text{ kHz}$. The spectral densities show strong sub-Ohmic forms and deviate from an Ohmic bath $G(\omega) \propto \omega$.

When in Eq. (8) an excitation in the cavity or the checkerboard mode is created or annihilated, Landau processes describe the corresponding annihilation (creation) of a phonon in one band and the corresponding creation (annihilation) in the other band with identical quasimomentum \mathbf{p} . Instead, Beliaev processes create or annihilate simultaneously a phonon in both bands with opposite \mathbf{p} . For both types, the Bogoliubov rotation induces a momentum dependent coupling parameter ϕ_p or θ_p .

Landau and Beliaev quasiparticle damping. To reveal the influence of the quantum fluctuations on the system of the cavity and the checkerboard mode, we interpret the $\mathbf{p} \neq 0$ modes as a dissipative bath and employ the imaginary time-path integral formalism. It relies on the correlator $\langle H_{\text{int}}(\tau)H_{\text{int}}(\tau') \rangle_\beta$ [33]. The expectation value of any mix of Landau and Beliaev operators vanishes, which naturally separates the two damping channels. The displacements of the system modes $q_C = (a + a^\dagger)/\sqrt{2\Delta_C}$ and $q_A = (c + c^\dagger)/\sqrt{2\omega_0}$ are used to describe its degrees of freedom, with the influence functional $S_{\text{infl}}[q_C, q_A] = -\frac{1}{2} \int_0^\beta d\tau \int_0^\beta d\tau' \langle H_{\text{int}}(\tau)H_{\text{int}}(\tau') \rangle_\beta$ with the inverse temperature $\beta = 1/T$ ($k_B = 1$) and with

$$\begin{aligned} & \langle H_{\text{int}}(\tau)H_{\text{int}}(\tau') \rangle_\beta \\ &= \kappa_C(\tau - \tau') \Delta_C q_C(\tau) q_C(\tau') \\ &+ \kappa_{AC}(\tau - \tau') \sqrt{\Delta_C \omega_0} [q_C(\tau) q_A(\tau') + q_A(\tau) q_C(\tau')] \\ &+ \kappa_A(\tau - \tau') \omega_0 q_A(\tau) q_A(\tau') + \kappa_A(\tau - \tau') \frac{1}{\omega_0} \dot{q}_A(\tau) \dot{q}_A(\tau'). \end{aligned} \quad (10)$$

The correlators are sorted depending on the system modes they couple to, as shown in Fig. 1 (see Supplemental

Material [34]). We label the ensuing dissipation channels by $x \in \{C, AC, A, \dot{A}\}$. The kernels $\kappa_x(\tau)$ are related to the spectral densities $G_x(\omega)$ of the respective quasiparticle damping channel as

$$\kappa_x(\tau) = \int_0^\infty d\omega G_x(\omega) D_\omega(\tau), \quad (11)$$

with the free thermal Green's function $D_\omega(\tau) = [1 + n(\omega)]e^{-\omega\tau} + n(\omega)e^{\omega\tau}$ with $n(\omega)$ being the Bose-Einstein distribution. The spectral density is composed of the two parts as $G_x(\omega) = G_x^L(\omega)\Theta(\omega_0 - \omega) + G_x^B(\omega)\Theta(\omega - \omega_0)$, separated by the border ω_0 of the frequency support (see below). Here, $\Theta(x)$ is the Heaviside function.

The spectral densities follow in the discrete form as $G_x^{L/B}(\omega) = \sum_{\mathbf{p} \neq 0} \gamma_x f_x^{L/B}(\omega_p) \mathcal{N}_p^{L/B} \delta(\omega - \omega_p^{L/B})$, with the coupling parameters $(\gamma_C, \gamma_{AC}, \gamma_A, \gamma_{\dot{A}}) = (\lambda^2, \lambda\eta\phi_0, \eta^2\phi_0^2/2, \eta^2/2\phi_0^2)$ and the combinations of Bose-Einstein distributions for Landau and Beliaev damping as $\mathcal{N}_p^L = n(\omega_{b,p}) - n(\omega_{c,p})$ and $\mathcal{N}_p^B = 1 + n(\omega_{b,p}) + n(\omega_{c,p})$, respectively. The functions $f_x^{L/B}(\omega_p)$ are given in the Supplemental Material [34].

All Landau processes share the same finite frequency range $\omega_p^L = \omega_{c,p} - \omega_{b,p} \in (2\omega_R - \bar{\omega}_p, \omega_0)$. The finite support results from the Brillouin zone of the phonon bands in Eq. (2). Beliaev processes exist in the range $\omega_p^B = \omega_{c,p} + \omega_{b,p} \in (\omega_0, 3\omega_R - \bar{\omega}_p + 2nU)$. We find exact analytic expressions for the spectral densities of the dissipative Landau and Beliaev bath in the continuum limit in the form

$$G_x^{L/B}(\omega) = \frac{V_{2D}}{2\pi} m \gamma_x f_x^{L/B}(\omega) \frac{\mathcal{N}^{L/B}(\omega)}{|g'_{L/B}(\omega)|}. \quad (12)$$

The details of the derivation and the explicit forms of $f_x^{L/B}(\omega)$, $\mathcal{N}^{L/B}(\omega)$, and $g'_{L/B}(\omega)$ are given in the Supplemental Material [34]. They encompass combinations of the coefficients in Eq. (6) and the quasiparticle frequencies, Eq. (5). They depend on temperature, the s -wave scattering nU and the strength of the transversal pump $\bar{\omega}_p$ and can thus be tuned from outside.

Landau processes are inherently thermal and vanish at zero temperature, such that the quantum fluctuations only include Beliaev processes. The Beliaev spectral densities are shown in Fig. 1. Two channels of the Beliaev damping, both the κ_{AC} and, more pronounced, the κ_A consist of several processes as follows from Eq. (10). Evidently, processes compete with each other, thereby providing damping or antidamping. This is reflected in the spectral densities with positive (damping) or negative (antidamping) spectral weights in different frequency regions. The possibility to tune the bath spectra alters the crossover frequency that separates damping from antidamping.

The spectral densities diverge at the borderline of Landau and Beliaev damping at $\omega = \omega_0$, where the excluded checkerboard mode ($|\mathbf{p}| = 0$) lies. The thermal distributions $\mathcal{N}^{L/B}(\omega)$ have a pole of order 1 at ω_0 . The term $f_x^{L/B}(\omega)/|g'_{L/B}(\omega)|$ does not diverge at this point. Although being finite, $\omega = \omega_0$ is still a singular point forming a cusp. In accordance with their frequency ranges, the Landau functions $f^L/|g'_L|$ form the left side (for $\omega < \omega_0$) and the Beliaev functions $f^B/|g'_B|$ the right side (for $\omega > \omega_0$) of each cusp along the frequency axis. We find that $\lim_{\omega \rightarrow \omega_0} \frac{d}{d\omega} f_x^{L/B}(\omega)/|g'_{L/B}(\omega)| \propto |\omega - \omega_0|^{-1/2}$ (for common parameters $\omega'_{c,0} \gg nU$) [34]. Importantly, it follows

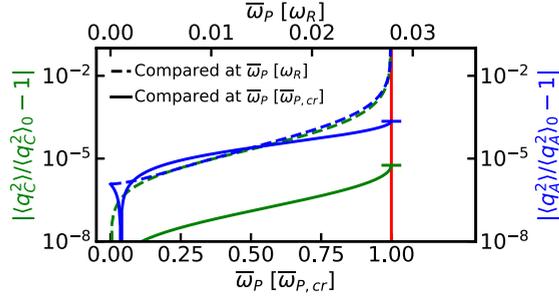


FIG. 2. Relative difference between the quantum fluctuations in the photon (green, left) and the atom (blue, right) sector with quasiparticle damping and without the quasiparticle damping for varying pump strengths. The upper axis is scaled to the recoil frequency and the result shown as dashed lines thus includes the effect of the Stokes shift. On the other hand, the lower axis is scaled in units of the critical pump strength and accordingly the result shown by the solid lines does not include the Stokes shift. The red vertical marks the critical point of the modified Dicke phase transition. The temperature is $\beta\omega_R = 1.71 \times 10^3$ such that thermal fluctuations are negligible and quantum fluctuations prevail. Parameters for the recoil resolved experimental setup of Refs. [35,36] are used: $\Delta_C = 2\omega_R$, $nU = 1.6 \times 10^{-2}\omega_R$, $\kappa = 1.25\omega_R$, $\omega_D = 10^9\omega_R$, $n_{\max} = 10^4$, and the remaining parameters are as in Fig. 1.

that the spectral function at zero temperature near the singular point ω_0 is characterized by $G_x^B(\omega) \propto (\omega - \omega_0)^{1/2}$ with the spectral exponent $s = 1/2$. This sub-Ohmic behavior has been introduced phenomenologically previously [23] and is now derived microscopically in the present Letter. Sub-Ohmic quantum fluctuations commonly induce strong, exotic non-Markovian real-time dynamics.

Quantum fluctuations in the atom and photon sectors. With the imaginary-time influence functional, observables of the fluctuations can be calculated. The partition function Z follows after a Matsubara expansion in β . We consider $\langle q_C^2 \rangle = -(\beta\Delta_C)^{-1} \partial_{\Delta_C} \ln Z$ for the cavity and $\langle q_A^2 \rangle = -(\beta\omega_0)^{-1} \partial_{\omega_0} \ln Z$ for the atomic sector. We additionally include a weak cavity photon loss by an Ohmic bath with damping constant κ , as determined experimentally [2,9], and a Drude cutoff ω_D . We investigate the strongly entangled system when approaching the critical point from below by increasing the atom-cavity coupling $\lambda = \sqrt{-U_0\bar{\omega}_p}$. In Fig. 2, we compare the atom and cavity fluctuations with $\langle \cdot \rangle$ and without $\langle \cdot \rangle_0$ Landau and Beliaev damping. Close to the critical point $\bar{\omega}_{p,cr}$, both relative differences diverge. This behavior is generic also for thermal fluctuations. The fluctuations in the cavity and checkerboard modes differ near $\bar{\omega}_p = 0$. Clearly, without pumping, the empty cavity does not couple to the atomic fluctuations. Naturally, the two dissipation channels κ_A and $\kappa_{\tilde{A}}$ are coupled to the checkerboard mode by the s -wave scattering U .

Role of the Stokes shifts. The presence of the quasiparticle dissipative baths induces a Stokes shift to the two-mode system, leading also to a shift of the critical point. This can be seen in Fig. 2, where the fluctuations are finite exactly at the “naked” critical point. For these parameters, the relative deviation is 2.3×10^{-4} in the pump strength. The Stokes shift can be derived analytically from the $n = 0$ term in the

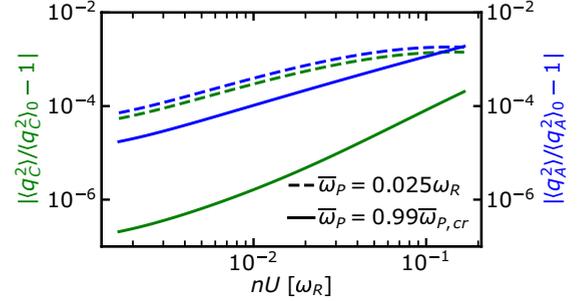


FIG. 3. Relative difference between the quantum fluctuations in the photon (green, left) and the atom (blue, right) sector in dependence of the atom-atom interaction nU for two different values of $\bar{\omega}_p$ and for the same parameters as in Fig. 2.

Matsubara expansion of the observables. We find

$$\lambda_{cr}^2 = \frac{\Delta_C^2 + \kappa^2}{\Delta_C} \frac{\omega_0 - R_A}{\tilde{R}_C(\omega_0 - R_A) + (2\phi_0\sqrt{N_C} - \tilde{R}_{AC})^2}. \quad (13)$$

The reorganization energies $R_x = \int_0^\infty d\omega G_x(\omega)$ shift the critical point. The tilde indicates that the λ dependence has been removed from the spectral densities involving the cavity sector before computing R_x . In an exact treatment, R_x depend on the pumping, and, with them, their reorganization energy. Similarly, Δ_C depends on the single atom-cavity shift U_0 . This renders Eq. (13) implicit. Without the quasiparticle bath, $R_x = 0$ the known critical point of an open Dicke phase transition is recovered [9]. In addition, the quantum fluctuations of the system with and without the atomic quasiparticle damping differ even if we correct for the Stokes shift. This can be seen as the difference between the solid and dashed lines in Fig. 2. However, the deviation saturates close to the critical point. The critical exponent is unchanged despite the sub-Ohmic characteristics of the quasiparticle baths. The sub-Ohmic signature is not relevant for the polariton soft mode exactly at the phase transition and thus does not imprint on the critical exponent [22,23].

Impact of s -wave scattering. Finally, the obvious channel to control the atomic quantum fluctuations is the s -wave scattering. The change of the quantum fluctuations for increasing U is displayed in Fig. 3. For low temperature, an increase in nU already yields enhanced damping because the pump strength is held constant. Yet, the critical pump strength is shifted even without the bath, as seen from Eq. (13), due to the dependence of the checkerboard mode ω_0 on nU , leading correspondingly to an increase of the necessary pump strength $\bar{\omega}_p$. If we correct for the Stokes shift which grows approximately linear with U [34], the influence becomes more pronounced. Additionally, with larger nU the difference of influence seen on the cavity and checkerboard mode after correcting for the shift of the critical point is greatly reduced. In general, the strength of the fluctuations is tuned over two orders of magnitude.

Conclusions. Starting from a microscopic Hamiltonian, we have derived in closed analytic form the spectral characteristics of the quantum fluctuations of the atom and photon sector of a cavity BEC system in the presence of atomic s -wave scattering. The atomic quantum fluctuations show

strong sub-Ohmic signatures of the quasiparticle damping, with the ensuing strongly non-Markovian dynamics. Furthermore, different channels of the Beliaev processes compete, leading to a peculiar spectral behavior with competing damping and antidamping. We identify two channels to control and enhance the fluctuations, the atomic s -wave scattering and the strength of the transversal pump. We also provide analytic expressions for the observables of the fluctuations and determine the Stokes shift of the critical atom-cavity coupling in the presence of quasiparticle damping. The controllability of the quantum fluctuations and access to the dynamics bring detailed understanding of sub-Ohmic quantum baths relevant not only in cavity BEC but furthermore in such fields as superconducting qubits, quantum information, nanomechanical and glassy systems, and quantum dots. In these systems the

time-nonlocal dynamics can be exploited given an understanding of their spectral properties of which we provide with these results.

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