


Anisotropy-assisted magnon condensation in ferromagnetic thin filmsTherese Frostad,¹ Philipp Pirro,² Alexander A. Serga,² Burkard Hillebrands,²
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We theoretically demonstrate that adding an easy-axis magnetic anisotropy facilitates magnon condensation in thin yttrium iron garnet (YIG) films. Dipolar interactions in a quasi-equilibrium state stabilize room-temperature magnon condensation in YIG. Even though the out-of-plane easy-axis anisotropy generally competes with the dipolar interactions, we show that adding such magnetic anisotropy may even assist the generation of the magnon condensate electrically via the spin-transfer torque mechanism. We use analytical calculations and micromagnetic simulations to illustrate this effect. Our results may explain the recent experiment on Bi-doped YIG and open a pathway toward applying current-driven magnon condensation in quantum spintronics.

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Introduction. Magnon condensate with nonzero momentum at room temperature is a fascinating phenomenon first observed in 2006 [1]. Condensed magnons were observed at two degenerate magnon band minima of high-quality yttrium iron garnet (YIG), an easy-plane ferrimagnetic insulator with a very low magnetic dissipation [2–4], as the spontaneous formation of a quasi-equilibrium and coherent magnetization dynamics in momentum space [5–8]. To generate magnon condensate, nonequilibrium magnons must be pumped into the system by an *incoherent* stimulus such as parametric pumping [1,9–18], rapid cooling of thermal magnons [19–21], and spin-transfer torque [22–29]. Above a critical nonequilibrium magnon density, magnons may finally (quasi)thermalize to form a quasi-equilibrium magnon condensate at the bottom of magnon bands.

The study of magnon condensation is not only interesting from an academic point of view but also of great importance in various areas of emerging quantum technology and applied spintronics [15,30–33]. Therefore, it is crucial to clarify the intricate microscopic mechanisms at play and to present theoretical proposals to electrically control the generation of magnon condensate.

At low magnon densities, the interaction between magnons is weak, and they behave as free quasiparticles. But when the magnon population increases, as in magnon condensation experiments, the interactions between magnons become stronger and more crucial. Moreover, nonlinear magnon interactions facilitate the quasi-thermalization process of injected nonequilibrium magnons. A stable and steady

quasi-equilibrium magnon condensation requires an effective repulsive interaction between injected magnon quasiparticles. It is known that in a system mainly influenced by Heisenberg exchange interactions, interaction between magnons is attractive. However, it was shown that dipolar interactions may assist in the generation of a *metastable* double-degenerate magnon condensate in YIG [12–14,34–42].

Recently, it was theoretically shown that the quasi-thermalization time of magnon condensation is reduced in confined nanoscopic systems [43]. It was also demonstrated that the lateral confinement in YIG thin films enhances the dipolar interaction along the propagation direction of magnons and causes a deeper band depth, i.e., the difference between the ferromagnetic resonance (FMR) and magnon band minima. Increasing the lifetime of the magnon condensate was attributed to this enhancement of the band depth [43].

In another recent achievement in magnon condensation experiments, Divinsky *et al.* [22] found evidence of condensation of magnons by the spin-transfer torque mechanism. They introduced a small perpendicular magnetocrystalline anisotropy (PMA) through bismuth doping in the thin film of YIG, while the magnetic ground state still resides within the plane. This discovery opens a route toward electrical control of magnon condensation.

However, the interplay between the dipolar interactions, which was previously shown to be essential for the stability and thermalization of magnon condensation, and the counteracting out-of-plane easy-axis magnetic anisotropy is so far uncharted. In this Letter, we analyze the stability of condensate magnons in the presence of a PMA in YIG. We present simulations within the Landau-Lifshitz-Gilbert framework [44–46] that support our analytical calculations.

Model. We consider a thin ferromagnetic film in the y - z plane to model YIG. The magnetic moments are directed

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along the z direction by an in-plane external magnetic field of strength H_0 . The magnetic potential energy of the film contains contributions from the isotropic exchange interaction \mathcal{H}_{ex} , Zeeman interaction \mathcal{H}_Z , dipolar interaction \mathcal{H}_{dip} , and additionally a PMA energy \mathcal{H}_{an} in the x direction, normal to the film plane. YIG has a weak in-plane easy axis that can be neglected compared to the other energy scales in the system. The total spin Hamiltonian of the system reads

$$\mathcal{H} = \mathcal{H}_{\text{ex}} + \mathcal{H}_Z + \mathcal{H}_{\text{dip}} + \mathcal{H}_{\text{an}}. \quad (1)$$

The PMA energy is given by

$$\mathcal{H}_{\text{an}} = -K_{\text{an}} \sum_j (\mathbf{S}_j \cdot \hat{x})^2, \quad (2)$$

where $K_{\text{an}} > 0$ is the easy-axis energy, $\hbar\mathbf{S}_j$ is the vector of spin operator at site j , and \hbar is the reduced Planck constant. Details of the Hamiltonian can be found in the Supplemental Material (SM) [47].

The Holstein-Primakoff spin-boson transformation [48] allows us to express the spin Hamiltonian in terms of magnon creation and annihilation operators. The amplitude of the effective spin per unit cell in YIG at room temperature is large $S \approx 14.3 \gg 1$, [39,49,50], and thus we can expand the spin Hamiltonian in the inverse powers of the spin S , which is equivalent to the semiclassical regime. Up to the lowest order in nonlinear terms, the magnon Hamiltonian \mathcal{H} of a YIG thin film can be expressed as the sum of two components: \mathcal{H}_2 and \mathcal{H}_4 . The former represents a noninteracting magnon gas comprising quadratic magnon operators. The latter, on the other hand, constitutes nonlinear magnon interactions characterized by quartic magnon operators; see SM for details [47]. Note that three-magnon interactions are forbidden in our geometry by the conservation laws [51].

Magnon dispersion of YIG with a finite PMA. The magnon dispersion in YIG is well known and has been studied extensively in both experimental and theoretical works [2,52,53]. Magnons traveling in the direction of the external magnetic field have the lowest energy. These so-called backward volume magnetostatic (BVM) magnons have a dispersion with double degenerate minima at finite wave vectors $q_z = \pm Q$. When pumping magnons into the thin film, the magnons may thermalize and eventually form a condensate state in these two degenerate minima with opposite wave vectors.

The noninteracting magnon Hamiltonian and the dispersion of BVM magnons, along the z direction, in the presence of a finite PMA reads

$$\mathcal{H}_2 = \sum_{q_z} \hbar\omega_{q_z} \hat{c}_{q_z}^\dagger \hat{c}_{q_z}, \quad (3a)$$

$$\hbar\omega_{q_z} = \sqrt{A_{q_z}^2 - B_{q_z}^2}, \quad (3b)$$

where $\hat{c}_{q_z}^\dagger$ (\hat{c}_{q_z}) are magnon creation (annihilation) operators, which are Bogoliubov bosons [47], and

$$A_{q_z} = D_{\text{ex}}q_z^2 + \gamma(H_0 + 2\pi M_S f_q) - K_{\text{an}}S, \quad (4a)$$

$$B_{q_z} = 2\pi M_S f_q - K_{\text{an}}S. \quad (4b)$$

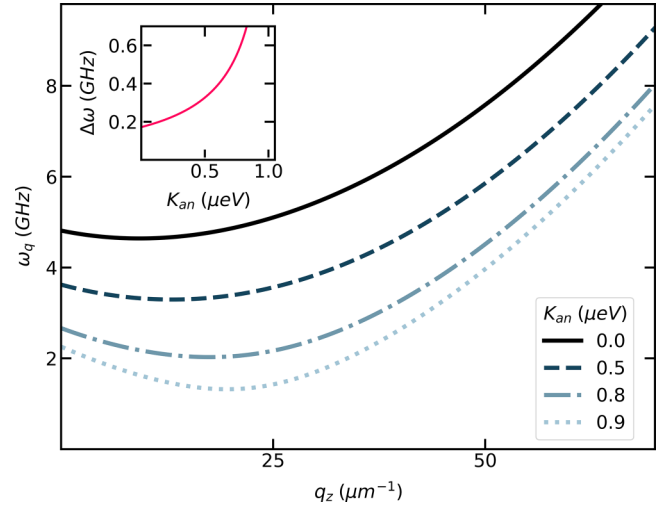


FIG. 1. The analytical dispersion of noninteracting BVM magnons in a YIG thin film for various PMA strengths, Eq. (3b). The inset shows the depth of the magnon band minima as a function of the PMA strength. We set the thickness $L_x = 50$ nm and the magnetic field in the z direction $H_0 = 1$ k Oe.

Here, $D_{\text{ex}} = J_{\text{ex}}Sa^2$ is the exchange constant, J_{ex} is the Heisenberg exchange coupling and $M_S = \gamma\hbar S/a^3$ is the saturation magnetization, where $\gamma = 1.2 \times 10^{-5} \text{ Oe}^{-1}$ is the gyromagnetic ratio and $a = 12.376 \text{ \AA}$ is the lattice constant of YIG. The form factor $f_q = (1 - e^{-|q_z|L_x})/(|q_z|L_x)$ stems from dipolar interactions in a thin magnetic film with thickness L_x [54,55].

Figure 1 shows the effect of PMA on the magnon dispersion of YIG. PMA decreases the FMR frequency $\omega_{q_z=0}$, in addition to a more significant decrease in the magnon band minima $\omega_{q_z=\pm Q}$. Therefore, the band depth $\Delta\omega = \omega_{q_z=0} - \omega_{q_z=\pm Q}$ is increased. The position of the band minima at $q_z = \pm Q$ is also shifted to larger momenta. In addition, the curvature of the minima increases as a function of the anisotropy strength. Above a critical PMA, K_{an}^c , the magnetic ground state is destabilized and the in-plane magnetic state becomes out of plane. We are interested in the regime in which the magnetic ground state remains in the plane, and thus the effective saturation magnetization is positive $M_{\text{eff}} = M_S - 2K_{\text{an}}/(\mu_0 M_S) > 0$.

The effect of PMA on magnon dispersion resembles the effect of confinement in the magnon spectra of YIG. In Ref. [43], it was shown that transverse confinement in a YIG thin film leads to an increase of the FMR frequency, the band depth, as well as shifting the band minima to higher momenta while the magnon band gap at the band minima is also increased. It was shown that this change of the spectrum in confined systems increases the magnon condensate lifetime. Therefore, we expect PMA to increase the magnon condensate lifetime and assist in the generation of magnon condensation.

Nonlinear magnon interactions in the presence of PMA. To check the stability of condensate magnons at low energy, we should show that the interactions do not destabilize and destroy magnon condensation. To achieve this goal, we turn on the magnon interaction between the condensate magnons

at $q_z = \pm Q$. This magnon interaction consists of intra- and interband contributions, $\mathcal{H}_4 = \mathcal{H}_4^{\text{intra}} + \mathcal{H}_4^{\text{inter}}$, where

$$\mathcal{H}_4^{\text{intra}} = A(\hat{c}_Q^\dagger \hat{c}_Q^\dagger \hat{c}_Q \hat{c}_Q + \hat{c}_{-Q}^\dagger \hat{c}_{-Q}^\dagger \hat{c}_{-Q} \hat{c}_{-Q}), \quad (5a)$$

$$\begin{aligned} \mathcal{H}_4^{\text{inter}} = & 2B(\hat{c}_Q^\dagger \hat{c}_{-Q}^\dagger \hat{c}_Q \hat{c}_{-Q}) \\ & + C(\hat{c}_Q^\dagger \hat{c}_{-Q} \hat{c}_Q \hat{c}_{-Q} \hat{c}_Q^\dagger \hat{c}_{-Q} \hat{c}_Q \hat{c}_{-Q} \hat{c}_Q \\ & + \text{H.c.}) + D(\hat{c}_Q^\dagger \hat{c}_Q^\dagger \hat{c}_{-Q}^\dagger \hat{c}_{-Q}^\dagger + \text{H.c.}). \end{aligned} \quad (5b)$$

The intraband magnon interaction, parametrized by A , preserves magnon number. However, the interband magnon interaction includes both a magnon conserving contribution, parametrized by B , and nonconserving contributions, parametrized by C and D , see SM [47]. The interaction amplitudes are given by

$$\begin{aligned} A = & -\frac{\gamma\pi M_S}{SN}[(\alpha_1 + \alpha_3)f_Q - 2\alpha_2(1 - f_{2Q})] \\ & -\frac{D_{\text{ex}}Q^2}{2SN}(\alpha_1 - 4\alpha_2) + \frac{K_{\text{an}}}{2N}(\alpha_1 + \alpha_3), \end{aligned} \quad (6a)$$

$$\begin{aligned} B = & \frac{\gamma 2\pi M_S}{SN}[(\alpha_1 - \alpha_2)(1 - f_{2Q}) - (\alpha_1 - \alpha_3)f_Q] \\ & +\frac{D_{\text{ex}}Q^2}{2SN}(\alpha_1 - 2\alpha_2) + \frac{K_{\text{an}}}{N}(\alpha_1 + \alpha_3), \end{aligned} \quad (6b)$$

$$\begin{aligned} C = & \frac{\gamma\pi M_S}{2SN}\left[(3\alpha_1 + 3\alpha_2 + 4\alpha_3)f_Q - \frac{8}{3}\alpha_3(1 - f_{2Q})\right] \\ & +\frac{D_{\text{ex}}Q^2}{3SN}\alpha_3 + \frac{K_{\text{an}}}{4N}(3\alpha_1 + 3\alpha_2 + 4\alpha_3), \end{aligned} \quad (6c)$$

$$\begin{aligned} D = & \frac{\gamma\pi M_S}{2SN}[(3\alpha_1 + 3\alpha_2 + 4\alpha_3)f_Q - 2\alpha_2(1 - f_{2Q})] \\ & +\frac{D_{\text{ex}}Q^2}{2SN}\alpha_2 + \frac{K_{\text{an}}}{2N}(3\alpha_2 + \alpha_3). \end{aligned} \quad (6d)$$

Here, N is the total number of spin sites. The dimensionless parameters α_1 , α_2 , and α_3 are related to the Bogoliubov transformation coefficients, listed in the SM [47].

An off-diagonal long-range order characterizes the condensation state. The condensate state is a macroscopic occupation of the ground state and can be represented by a classical complex field. Therefore, to analyze the stability of the magnon condensate, we perform Madelung's transform $\hat{c}_{\pm Q} \rightarrow \sqrt{N_{\pm Q}}e^{i\phi_{\pm Q}}$, in which the macroscopic condensate magnon state is described with a coherent phase $\phi_{\pm Q}$ and a population number $N_{\pm Q}$ [39,40]. The total number of condensed magnons is $N_c = N_{+Q} + N_{-Q}$, while the distribution difference is $\delta = N_{+Q} - N_{-Q}$. N_c is set in the system by an external magnon pumping mechanism and is a constant. We also define the total phase as $\Phi = \phi_{+Q} + \phi_{-Q}$.

Finally, the macroscopic four-magnon interaction energy of condensed magnons is expressed as

$$\begin{aligned} \mathcal{V}_4(\delta, \Phi) = & \frac{N_c^2}{2}\left[A + B + 2C \cos \Phi \sqrt{1 - \frac{\delta^2}{N_c^2}} \right. \\ & \left. + D \cos 2\Phi - (B - A + D \cos 2\Phi) \frac{\delta^2}{N_c^2}\right]. \end{aligned} \quad (7)$$

This expression is similar to the one recently obtained without PMA [56], but the interaction amplitudes, Eq. (6),

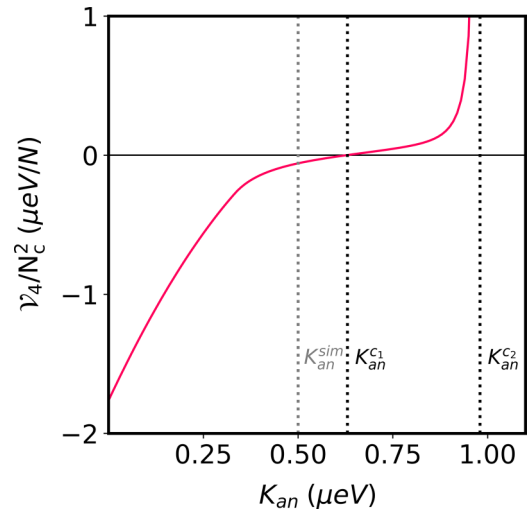


FIG. 2. The analytical nonlinear interaction energy of magnon condensate state, Eq. (7), as a function of the PMA strength. N and N_c are the total spins and condensate magnons, respectively. K_{an}^{c1} represents the critical value of the PMA at which the sign of nonlinear interaction energy is changed. On the other hand, K_{an}^{c2} corresponds to the critical value of PMA at which the in-plane magnetic ground state becomes unstable. We set $L_x = 50$ nm and $H_0 = 1$ k Oe. $K_{\text{an}}^{\text{sim}} = 0.5$ μeV denotes the PMA used in our micromagnetic simulations.

depend on the PMA through the Bogoliubov coefficients, see SM [47].

We now look at the total interaction energy and amplitudes of condensate magnons in more detail. Figure 2 shows the effective interaction potential of condensate magnons as a function of the PMA. In a critical PMA strength, K_{an}^{c1} , the sign of the interaction changes. This means that below K_{an}^{c1} the interaction reduces the total energy of condensate magnons, while above K_{an}^{c1} the interaction increases its energy. This critical anisotropy is well below the critical magnetic anisotropy strength K_{an}^{c2} that destabilizes the in-plane magnetic ground state. In the following, we consider a PMA strength below the critical anisotropy $K_{\text{an}} < K_{\text{an}}^{c1}$.

The interacting potential energy of the condensate magnons, Eq. (7), has five extrema, $\partial \mathcal{V}_4(\delta_i, \Phi_i) = 0$, at

$$\delta_1 = 0, \quad \Phi_1 = 0; \quad (8a)$$

$$\delta_2 = 0, \quad \Phi_2 = \pi; \quad (8b)$$

$$\delta_3 = 0, \quad \Phi_3 = \cos^{-1}\left(-\frac{C}{D}\right); \quad (8c)$$

$$\delta_4 = N_c \left[1 - \left(\frac{C}{B - A + D}\right)^2\right]^{\frac{1}{2}}, \quad \Phi_4 = 0; \quad (8d)$$

$$\delta_5 = \delta_4, \quad \Phi_5 = \pi. \quad (8e)$$

$\delta_i = 0$ indicates condensate states with symmetric magnon populations in the two magnon band minima, while $\delta_i \neq 0$ represents states with nonsymmetrical magnon populations. Whether any of these extrema represent the actual minimum of the interacting potential energy, i.e., $\partial^2 \mathcal{V}_4(\delta_i, \Phi_i) > 0$, depends on the system parameters. Finding these minima allows us to construct the phase diagram for magnon condensate.

TABLE I. The material parameters used in the micromagnetic simulations.

Parameter	Symbol	Value
Saturation magnetization	$4\pi M_S$	1.75 kOe
Effective spin	S	14.3
Exchange constant	D_{ex}	$0.64 \text{ eV}\text{\AA}^2$
Gilbert damping parameter	α	10^{-3}

Phase diagram for magnon condensate. Now, we theoretically explore the (meta)stability of the magnon condensate as a function of the film thickness L_x and the magnetic field strength H_0 , applied along the z direction in the plane, using the YIG spin parameters, see Table I. We characterize a (meta)stable state of a magnon condensate in the phase diagram as one that minimizes the interaction potential \mathcal{V}_4 while satisfying the condition $\mathcal{V}_4 < 0$, ensuring a reduction in the total magnon condensate energy at $q_z = \pm Q$ through interactions.

First, we present the phase diagram for magnon condensation in YIG, in the absence of PMA, in Fig. 3(a). The thinner films are expected to have a symmetric distribution of magnons between the two magnon band minima, the state with $\delta_1 = 0$, and only thicker films with larger applied magnetic fields tend to have nonsymmetric magnon populations, the state with $\delta_4 \neq 0$. This phase diagram is in agreement with previous studies [39,56].

Next, we add a PMA, with strength $K_{\text{an}} = 0.5 \mu\text{eV}$, and plot the phase diagram of the magnon condensate in Fig. 3(b) for different thicknesses. Compared to the case without PMA, we see that the condensate magnons can only be stabilized for thinner films, and within our material parameters, we do not have any metastable condensation above 90 nm since the sign of the total interaction energy becomes positive. In addition, we have a richer phase diagram in the presence of PMA. PMA tends to push the magnon condensate within our material parameters toward a more nonsymmetric population distribution between the two magnon band minima, states with $\delta_4 \neq 0$ and $\delta_5 \neq 0$. Since both minima are degenerate, there is an oscillation of magnon population between these two minima. In very thin films, less than 30 nm, we may have a symmetric condensate magnon state, $\delta_1 = 0$, in our system.

This phase diagram shows that in the presence of a PMA, magnon condensate can be still survived as a metastable state. In addition, as we discussed earlier, a PMA increases the band depth and reduces the curvature of noninteracting magnon dispersion, see Fig. 1, leading to an enhancement of the condensate magnon lifetime. Thus, we expect that introducing a small PMA into a thin film of YIG facilitates the magnon condensation process. It is worth mentioning that the stabilizing condensed magnons in thinner films with finite PMA is not a real problematic issue since the injection of magnons into the system by electrical means is more efficient in thin films.

Micromagnetic simulation of magnon condensate. To validate our theoretical predictions and illustrate the facilitation of magnon condensate formation by incorporating a PMA, we conducted a series of micromagnetic simulations. Simulations were performed using MUMAX³ [57], which solves the

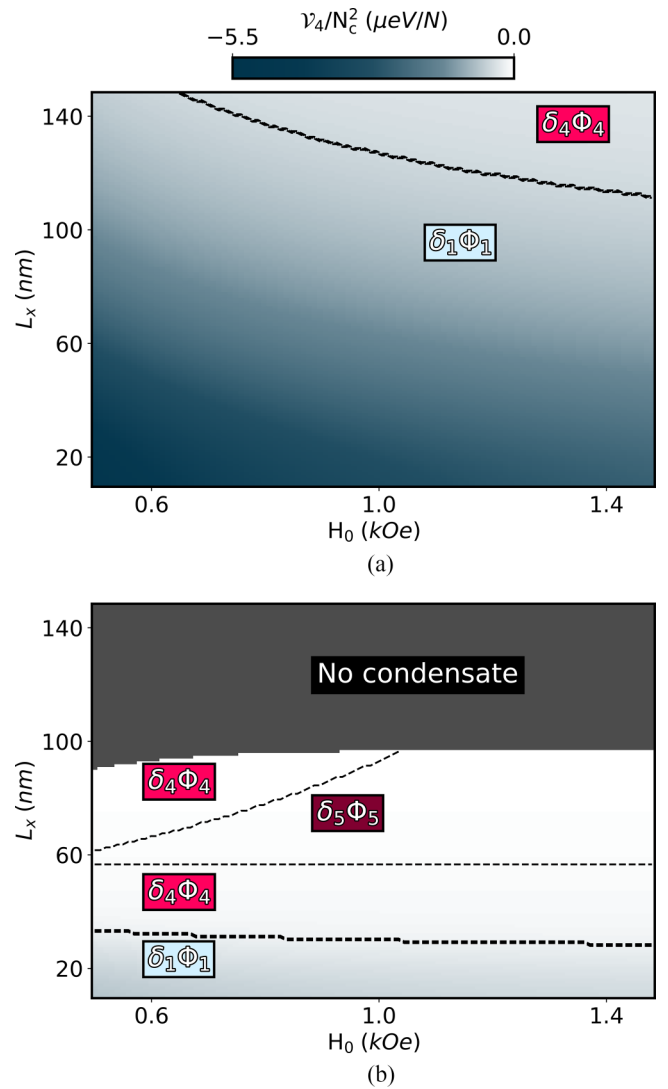


FIG. 3. The theoretical phase diagram of the condensate magnons in the absence (a) and presence (b) of PMA. We plot the magnon interaction energy \mathcal{V}_4/N_c^2 , Eq. (7), as a function of the film thickness L_x and the magnetic field strength H_0 , applied along the z direction. Different states are labeled based on different extrema listed in Eq. (8). The dashed black lines indicate the boundaries between the different condensate phases, Eq. (8). We set $K_{\text{an}} = 0.5 \mu\text{eV}$ in (b).

semiclassical Landau-Lifshitz-Gilbert (LLG) equation that describes magnetization's precessional motion; see SM [47]. In the limit of large S , it is important to note that we enter the semiclassical regime and hence the LLG equation may effectively capture and accurately describe the spin dynamics of the system. In our ferromagnetic thin-film simulation, magnons are excited via spin-transfer torque at zero temperature, eliminating thermal magnons. The nonequilibrium magnons in the film are the result of injection of spin current across the sample surface [22]. Optimal spin torque strength ensures that the magnon population reaches the critical density required for forming condensed magnons; see SM for simulation details [47]. By introducing spin torque into the system, we excite magnons with varying wave vectors and

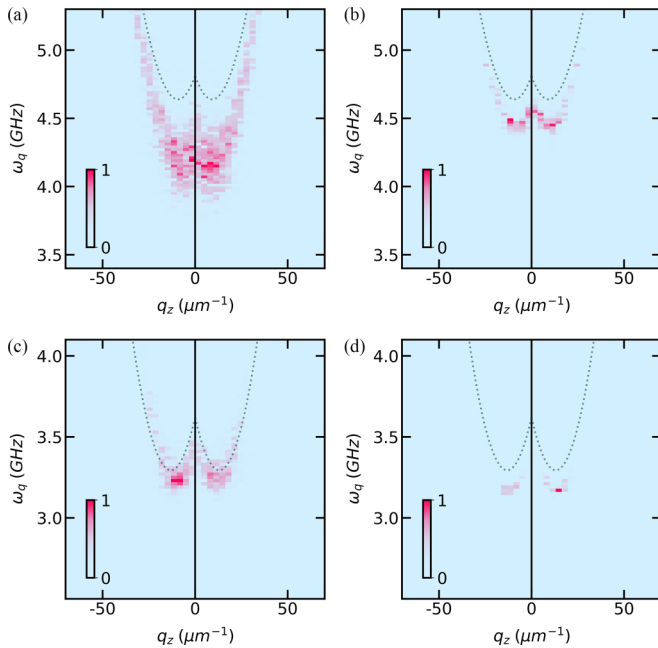


FIG. 4. Micromagnetic simulation of nonequilibrium magnon distribution injected by spin torque mechanism for a YIG thin film with a thickness of $L_x = 50$ nm and lateral sizes of $L_y = L_z = 5$ μm , in the presence of an external magnetic field along the z direction $H_0 = 1$ kOe. (a) and (b) show magnon distributions of the initial nonequilibrium injected magnons and final quasi-equilibrium magnon condensate steady state, respectively, when $K_{\text{an}} = 0$. (c) and (d) show magnon distributions of initial nonequilibrium excited magnons and final quasi-equilibrium magnon condensate steady state, respectively, when $K_{\text{an}} = 0.5$ μeV . The dotted line indicates the analytical dispersion relation of noninteracting magnons, Eq. (3b). Because of magnon-magnon interactions, the simulated magnon dispersion has a nonlinear spectral shift compared to the analytical noninteracting magnon dispersion. Although the duration of magnon pumping by spin-transfer torque is the same in the absence or presence of the PMA, the critical torque amplitude is lower in the presence of PMA.

frequencies, as illustrated in Figs. 4(a) and 4(c). A portion of these nonequilibrium magnons undergoes thermalization through nonlinear magnon-magnon interactions, leading to the establishment of a stable and quasi-equilibrium state of condensed magnons located at the minima of the magnon

band spectra, $\pm Q$, as depicted in Figs. 4(b) and 4(d). This sharp peak in the number of magnons at the band minima is a signature of magnon condensate.

The numerical simulations confirm the supportive role of PMA in the condensation process. First, there is a reduction in the threshold of spin-transfer torque necessary to inject the critical magnon density into the system, enabling the system to attain said critical magnon density even at lower torque amplitudes. Second, the final condensate magnons in the presence of the PMA are more localized around the band minima than in the absence of PMA. Simulations also indicate that PMA shifts the population of condensate magnons from a symmetric distribution between two band minima to a non-symmetric distribution, Fig. 4. This agrees with the analytical phase diagram in Fig. 3(b).

Summary and concluding remarks. Dipolar interactions are assumed to be relevant to stabilizing the magnon condensate within YIG. The presence of a PMA is expected to counteract dipolar interactions. In this Letter, we show that even at intermediate strengths of the PMA field, a magnon condensate state can exist as a metastable state. We note that the anisotropy increases the band depth and curvature of the magnon dispersion. These adjustments to the magnon spectrum are expected to facilitate magnon condensate formation. From the calculations of effective magnon-magnon interactions and minimizing the interaction potential at the band minima, we find the magnon condensate phase diagram. We demonstrate that the inclusion of PMA results in a magnon condensate with a more intricate phase diagram compared to when PMA is absent. A finite PMA has the tendency to drive the magnon condensate towards a nonsymmetric magnon population at band minima in thinner films and lower magnetic fields, as compared to the absence of PMA. Micromagnetic simulations within the LLG framework confirm our analytical results and analyses.

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[1] S. O. Demokritov, V. E. Demidov, O. Dzyapko, G. A. Melkov, A. A. Serga, B. Hillebrands, and A. N. Slavin, Bose–Einstein condensation of quasi-equilibrium magnons at room temperature under pumping, *Nature (London)* **443**, 430 (2006).
 [2] V. Cherepanov, I. Kolokolov, and V. L’vov, The saga of YIG: Spectra, thermodynamics, interaction and relaxation of magnons in a complex magnet, *Phys. Rep.* **229**, 81 (1993).
 [3] L. Soumah, N. Beaulieu, L. Qassym, C. Carrétéro, E. Jacquet, R. Lebourgeois, J. Ben Youssef, P. Bortolotti, V. Cros, and A. Anane, Ultra-low damping insulating magnetic thin films get perpendicular, *Nat. Commun.* **9**, 3355 (2018).

[4] C. Dubs, O. Surzhenko, R. Thomas, J. Osten, T. Schneider, K. Lenz, J. Grenzer, R. Hübner, and E. Wendler, Low damping and microstructural perfection of sub-40nm-thin yttrium iron garnet films grown by liquid phase epitaxy, *Phys. Rev. Mater.* **4**, 024416 (2020).
 [5] S. Demokritov, Comment on “Bose–Einstein condensation and spin superfluidity of magnons in a perpendicularly magnetized yttrium iron garnet film,” *JETP Lett.* **115**, 691 (2022).
 [6] T. B. Noack, V. I. Vasyuchka, A. Pomyalov, V. S. L’vov, A. A. Serga, and B. Hillebrands, Evolution of room-temperature

- magnon gas: Toward a coherent Bose-Einstein condensate, *Phys. Rev. B* **104**, L100410 (2021).
- [7] D. Snoke, Coherent questions, *Nature (London)* **443**, 403 (2006).
- [8] R. E. Troncoso and A. S. Núñez, Dynamics and spontaneous coherence of magnons in ferromagnetic thin films, *J. Phys.: Condens. Matter* **24**, 036006 (2012).
- [9] P. Anderson and H. Suhl, Instability in the motion of ferromagnets at high microwave power levels, *Phys. Rev.* **100**, 1788 (1955).
- [10] H. Suhl, The non-linear behaviour of ferrites at high signal level, *Proc. IRE* **44**, 1270 (1956).
- [11] V. E. Demidov, O. Dzyapko, M. Buchmeier, T. Stockhoff, G. Schmitz, G. A. Melkov, and S. O. Demokritov, Magnon kinetics and Bose-Einstein condensation studied in phase space, *Phys. Rev. Lett.* **101**, 257201 (2008).
- [12] S. M. Rezende, Theory of coherence in Bose-Einstein condensation phenomena in a microwave-driven interacting magnon gas, *Phys. Rev. B* **79**, 174411 (2009).
- [13] P. Nowik-Boltyk, O. Dzyapko, V. Demidov, N. Berloff, and S. Demokritov, Spatially non-uniform ground state and quantized vortices in a two-component Bose-Einstein condensate of magnons, *Sci. Rep.* **2**, 482 (2012).
- [14] A. A. Serga, V. S. Tiberkevich, C. W. Sandweg, V. I. Vasyuchka, D. A. Bozhko, A. V. Chumak, T. Neumann, B. Obry, G. A. Melkov, A. N. Slavin *et al.*, Bose-Einstein condensation in an ultra-hot gas of pumped magnons, *Nat. Commun.* **5**, 3452 (2014).
- [15] D. A. Bozhko, A. A. Serga, P. Clausen, V. I. Vasyuchka, F. Heussner, G. A. Melkov, A. Pomyalov, V. S. L'vov, and B. Hillebrands, Supercurrent in a room-temperature Bose-Einstein magnon condensate, *Nat. Phys.* **12**, 1057 (2016).
- [16] C. Sun, T. Nattermann, and V. L. Pokrovsky, Unconventional superfluidity in yttrium iron garnet films, *Phys. Rev. Lett.* **116**, 257205 (2016).
- [17] O. Dzyapko, P. Nowik-Boltyk, B. Koene, V. E. Demidov, J. Jersch, A. Kirilyuk, T. Rasing, and S. O. Demokritov, High-resolution magneto-optical Kerr-effect spectroscopy of magnon Bose-Einstein condensate, *IEEE Magn. Lett.* **7**, 1 (2016).
- [18] I. Borisenko, B. Divinskiy, V. Demidov, G. Li, T. Nattermann, V. Pokrovsky, and S. Demokritov, Direct evidence of spatial stability of Bose-Einstein condensate of magnons, *Nat. Commun.* **11**, 1691 (2020).
- [19] M. Schneider, T. Brächer, D. Breitbach, V. Lauer, P. Pirro, D. A. Bozhko, H. Yu. Musiienko-Shmarova, B. Heinz, Q. Wang, T. Meyer *et al.*, Bose-Einstein condensation of quasiparticles by rapid cooling, *Nat. Nanotechnol.* **15**, 457 (2020).
- [20] M. Schneider, D. Breitbach, R. O. Serha, Q. Wang, M. Mohseni, A. A. Serga, A. N. Slavin, V. S. Tiberkevich, B. Heinz, T. Brächer, B. Lägél, C. Dubs, S. Knauer, O. V. Dobrovolskiy, P. Pirro, B. Hillebrands, and A. V. Chumak, Stabilization of a nonlinear magnonic bullet coexisting with a Bose-Einstein condensate in a rapidly cooled magnonic system driven by spin-orbit torque, *Phys. Rev. B* **104**, L140405 (2021).
- [21] C. Safranski, I. Barsukov, H. Lee *et al.*, Spin caloritronic nanoscillator, *Nat. Commun.* **8**, 117 (2017).
- [22] B. Divinskiy, H. Merbouche, V. E. Demidov, K. Nikolaev, L. Soumah, D. Gouéré, R. Lebrun, V. Cros, J. B. Youssef, P. Bortolotti, A. Anane, and S. O. Demokritov, Evidence for spin current driven Bose-Einstein condensation of magnons, *Nat. Commun.* **12**, 6541 (2021).
- [23] M. Schneider, D. Breitbach, R. O. Serha, Q. Wang, A. A. Serga, A. N. Slavin, V. S. Tiberkevich, B. Heinz, B. Lägél, T. Brächer, C. Dubs, S. Knauer, O. V. Dobrovolskiy, P. Pirro, B. Hillebrands, and A. V. Chumak, Control of the Bose-Einstein condensation of magnons by the spin Hall effect, *Phys. Rev. Lett.* **127**, 237203 (2021).
- [24] D. Breitbach, M. Schneider, B. Heinz, F. Kohl, J. Maskill, L. Scheuer, R. O. Serha, T. Brächer, B. Lägél, C. Dubs, V. S. Tiberkevich, A. N. Slavin, A. A. Serga, B. Hillebrands, A. V. Chumak, and P. Pirro, Stimulated amplification of propagating spin waves, *Phys. Rev. Lett.* **131**, 156701 (2023).
- [25] V. E. Demidov, S. Urazhdin, E. R. J. Edwards, M. D. Stiles, R. D. McMichael, and S. O. Demokritov, Control of magnetic fluctuations by spin current, *Phys. Rev. Lett.* **107**, 107204 (2011).
- [26] V. Demidov, S. Urazhdin, G. De Loubens, O. Klein, V. Cros, A. Anane, and S. Demokritov, Magnetization oscillations and waves driven by pure spin currents, *Phys. Rep.* **673**, 1 (2017).
- [27] S. A. Bender, R. A. Duine, and Y. Tserkovnyak, Electronic pumping of quasiequilibrium Bose-Einstein-condensed magnons, *Phys. Rev. Lett.* **108**, 246601 (2012).
- [28] S. A. Bender, R. A. Duine, A. Brataas, and Y. Tserkovnyak, Dynamic phase diagram of dc-pumped magnon condensates, *Phys. Rev. B* **90**, 094409 (2014).
- [29] Y. Tserkovnyak, S. A. Bender, R. A. Duine, and B. Flebus, Bose-Einstein condensation of magnons pumped by the bulk spin Seebeck effect, *Phys. Rev. B* **93**, 100402(R) (2016).
- [30] K. Nakata, K. A. van Hoogdalem, P. Simon, and D. Loss, Josephson and persistent spin currents in Bose-Einstein condensates of magnons, *Phys. Rev. B* **90**, 144419 (2014).
- [31] S. N. Andrianov and S. A. Moiseev, Magnon qubit and quantum computing on magnon Bose-Einstein condensates, *Phys. Rev. A* **90**, 042303 (2014).
- [32] M. Mohseni, V. I. Vasyuchka, V. S. L'vov, A. A. Serga, and B. Hillebrands, Classical analog of qubit logic based on a magnon Bose-Einstein condensate, *Commun. Phys.* **5**, 196 (2022).
- [33] Y. M. Bunkov, A. N. Kuzmichev, T. R. Safin, P. M. Vetoshko, V. I. Belotelov, and M. S. Tagirov, Quantum paradigm of the foldover magnetic resonance, *Sci. Rep.* **11**, 7673 (2021).
- [34] O. Dzyapko, I. Lisenkov, P. Nowik-Boltyk, V. E. Demidov, S. O. Demokritov, B. Koene, A. Kirilyuk, T. Rasing, V. Tiberkevich, and A. Slavin, Magnon-magnon interactions in a room-temperature magnonic Bose-Einstein condensate, *Phys. Rev. B* **96**, 064438 (2017).
- [35] S. Demokritov, V. Demidov, O. Dzyapko, G. Melkov, and A. Slavin, Quantum coherence due to Bose-Einstein condensation of parametrically driven magnons, *New J. Phys.* **10**, 045029 (2008).
- [36] V. E. Demidov, O. Dzyapko, S. O. Demokritov, G. A. Melkov, and A. N. Slavin, Observation of spontaneous coherence in Bose-Einstein condensate of magnons, *Phys. Rev. Lett.* **100**, 047205 (2008).
- [37] S. M. Rezende, Theory of microwave superradiance from a Bose-Einstein condensate of magnons, *Phys. Rev. B* **79**, 060410(R) (2009).
- [38] I. S. Tupitsyn, P. C. E. Stamp, and A. I. Burin, Stability of Bose-Einstein condensates of hot magnons in yttrium iron garnet films, *Phys. Rev. Lett.* **100**, 257202 (2008).

- [39] F. Li, W. M. Saslow, and V. L. Pokrovsky, Phase diagram for magnon condensate in yttrium iron garnet film, *Sci. Rep.* **3**, 1372 (2013).
- [40] H. Salman, N. G. Berloff, and S. O. Demokritov, Microscopic theory of Bose-Einstein condensation of magnons at room temperature, in *Universal Themes of Bose-Einstein Condensation*, edited by N. P. Proukakis, D. W. Snoke, and P. B. Littlewood (Cambridge University Press, Cambridge, 2017), Chap. 25, p. 493.
- [41] J. Hick, F. Sauli, A. Kreisel, and P. Kopietz, Bose-Einstein condensation at finite momentum and magnon condensation in thin film ferromagnets, *Eur. Phys. J. B* **78**, 429 (2010).
- [42] V. E. Demidov, O. Dzyapko, S. O. Demokritov, G. A. Melkov, and A. N. Slavin, Thermalization of a parametrically driven magnon gas leading to Bose-Einstein condensation, *Phys. Rev. Lett.* **99**, 037205 (2007).
- [43] M. Mohseni, A. Qaiumzadeh, A. A. Serga, A. Brataas, B. Hillebrands, and P. Pirro, Bose-Einstein condensation of nonequilibrium magnons in confined systems, *New J. Phys.* **22**, 083080 (2020).
- [44] M. Lakshmanan, The fascinating world of the Landau-Lifshitz-Gilbert equation: An overview, *Philos. Trans. R. Soc. London A* **369**, 1280 (2011).
- [45] L. Landau and E. Lifshitz, 3 - On the theory of the dispersion of magnetic permeability in ferromagnetic bodies, in *Perspectives in Theoretical Physics*, edited by L. P. Pitaevski (Pergamon, Amsterdam, 1992), pp. 51-65.
- [46] T. L. Gilbert, A phenomenological theory of damping in ferromagnetic materials, *IEEE Trans. Magn.* **40**, 3443 (2004).
- [47] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevResearch.6.L012011> for more details on the magnon Hamiltonian and simulation.
- [48] T. Holstein and H. Primakoff, Field dependence of the intrinsic domain magnetization of a ferromagnet, *Phys. Rev.* **58**, 1098 (1940).
- [49] S. Streib, N. Vidal-Silva, K. Shen, and G. E.W. Bauer, Magnon-phonon interactions in magnetic insulators, *Phys. Rev. B* **99**, 184442 (2019).
- [50] H. Maier-Flaig, S. Klingler, C. Dubs, O. Surzhenko, R. Gross, M. Weiler, H. Huebl, and S. T. B. Goennenwein, Temperature-dependent magnetic damping of yttrium iron garnet spheres, *Phys. Rev. B* **95**, 214423 (2017).
- [51] A. D. Boardman and S. A. Nikitov, Three- and four-magnon decay of nonlinear surface magnetostatic waves in thin ferromagnetic films, *Phys. Rev. B* **38**, 11444 (1988).
- [52] A. J. Princep, R. A. Ewings, S. Ward, S. Tóth, C. Dubs, D. Prabhakaran, and A. T. Boothroyd, The full magnon spectrum of yttrium iron garnet, *npj Quantum Mater.* **2**, 63 (2017).
- [53] A. A. Serga, A. V. Chumak, and B. Hillebrands, YIG magnonics, *J. Phys. D: Appl. Phys.* **43**, 264002 (2010).
- [54] F. J. Buijnsters, L. J. Van Tilburg, A. Fasolino, and M. I. Katsnelson, Two-dimensional dispersion of magnetostatic volume spin waves, *J. Phys.: Condens. Matter* **30**, 255803 (2018).
- [55] H. Zabel and M. Farle, *Magnetic Nanostructures: Spin Dynamics and Spin Transport* (Springer, New York, 2012), Vol. 246.
- [56] C. Sun, T. Nattermann, and V. L. Pokrovsky, Bose-Einstein condensation and superfluidity of magnons in yttrium iron garnet films, *J. Phys. D: Appl. Phys.* **50**, 143002 (2017).
- [57] A. Vansteenkiste, J. Leliaert, M. Dvornik, M. Helsen, F. Garcia-Sanchez, and B. Van Waeyenberge, The design and verification of MuMax3, *AIP Adv.* **4**, 107133 (2014).

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