Erratum: Heterogeneity can markedly increase final outbreak size in the SIR model of epidemics [Phys. Rev. Research 6, 012010 (2024)]

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In our Letter, we have developed a theory on the dependence of the final outbreak size in the SIR (susceptible-infected-recovered) model of epidemics on the population network's heterogeneity strength and *average* reproductive number, which was denoted by R_0. Our analysis showed that as a function of the order parameter R_0, there exists a critical R_0^c, above which the maximal outbreak size is obtained at zero heterogeneity, but below which, the maximal outbreak size is obtained at nonzero heterogeneity. However, in the published Letter, we have misnamed the order parameter and called it "*basic* reproductive number" instead of "*average* reproductive number," even though the mathematical definition we have used throughout the text was consistent with the latter. As such, we mistakenly denoted the order parameter by R_0, even though R_0 is usually reserved for the basic (rather than the average) reproduction number and is defined in a different manner, especially in the case of heterogeneous networks [1].

Let us briefly explain how the average reproduction number, which we here denote by Λ, differs from R_0—the basic reproduction number. The former, which was the actual order parameter used in the published Letter, is defined as $\Lambda = \beta k_0/\gamma$ where k_0 is the degree distribution's mean, β is the contact's infectious rate and γ is the recovery rate per individual. In contrast, assuming $k_0 \gg 1$, R_0 is defined as $R_0 = \beta\langle k^2\rangle/(\gamma k_0)$, where $\langle k^2\rangle = k_0^2 + \sigma^2$ is the second moment of the degree distribution, while σ^2 is the distribution's variance [1]. Notably, in the limit of weakly heterogeneous networks, $\sigma \ll k_0$, one has $\langle k^2\rangle \simeq k_0^2$, and therefore $\Lambda \simeq R_0$. However, for strongly heterogeneous networks, i.e., when $\sigma = \mathcal{O}(k_0)$, the two order parameters strongly differ.

To avoid confusion, when reading the published Letter, one should replace R_0 by Λ throughout the text. Also, in the *Discussion* section, the first part of the paragraph on the implications of the phase transition on realistic scenarios (until the sentence on positively-correlated networks) does not necessarily apply. This is because the various diseases that are specified are characterized via their R_0 value rather than Λ, and one cannot directly compare the values of these parameters without specifying the underlying network topology.

[1] R. Pastor-Satorras, C. Castellano, P. V. Mieghem, and A. Vespignani, Epidemic processes in complex networks, Rev. Mod. Phys. **87**, 925 (2015).

Correction: Some wording erroneously referred to a corrected version of the Letter and has been modified so that the Erratum is self-contained.