# Erratum: Braiding Majorana corner modes in a second-order topological superconductor [Phys. Rev. Research 2, 032068(R) (2020)]

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We became aware of two errors in our original paper. The first is of typographical nature and occurs in the expression of the bulk Bogolyubov-de Gennes Hamiltonian in Eq. (1). The bottom-right block $-\hat{h}_{\boldsymbol{k}}^*$ should be, in fact, $-\sigma_3\hat{h}_{\boldsymbol{k}}^*\sigma_3$, resulting in the corrected Eq. (1):

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \hat{h}_{\boldsymbol{k}} & \hat{\Delta}_{\boldsymbol{k}} \\ \hat{\Delta}_{\boldsymbol{k}}^* & -\sigma_3\hat{h}_{\boldsymbol{k}}^*\sigma_3 \end{pmatrix}. \tag{1}$$

This change does not have any effect on the results of the paper, all of them are based on the correct form of $\mathcal{H}(\boldsymbol{k})$.

The second error concerns the quantization of $\phi_{xy}$ and $\phi_{yx}$, the complex phases of the eigenvalues of the nested Wilson loop operator (WLO). They have been introduced by the following incorrect statements in the section *Computing topological invariants*:

*In our 2D model, using the nested WLOs we obtain two* $\mathbb{Z}_2$ *corner topological invariants* $\phi_{xy}, \phi_{yx} \in \{0, \pi\}$*, which are quantized by the effective chiral symmetry introduced in Eq. (2b) (confirmed by numerical simulations).*

For arbitrary time instances during the adiabatic cycle, the phases $(\phi_{xy}, \phi_{yx})$ are not quantized by any symmetry present in the model. Therefore, throughout the paper they are inaccurately referred to as topological invariants, in general. Their approximate values are indeed either $(0, \pi)$ or $(\pi, 0)$, alternating as displayed in Fig. 3. The chiral symmetry $\tilde{C}$, introduced in Eq. (2b), is not sufficient to quantize the nested WLO eigenvalues $\phi_{xy}$ and $\phi_{yx}$, as proven in Appendix D of Ref. [1]. Having been unaware of that result at the time our research was conducted, we relied, as stated above, on numerical simulations, which showed $\phi_{xy}$ and $\phi_{yx}$ remained very close to 0 or $\pi$ even under the influence of $\tilde{C}$-symmetric perturbations. The shortcomings and subtleties of such numerical arguments became clear recently after our systematic reevaluation of the role of the symmetries in our model.

The quantization is rigorously established if the system possesses a mirror symmetry [1], which is the case at four special points of our cycle, according to Table I of the Supplemental Material of our original paper. More precisely, at the time instances when $t \in \{0, T/2\}$, we obtain $\phi_{xy} = 0$ due to the mirror symmetry $\mathcal{M}_y$, while $\mathcal{M}_x$ protects the equality $\phi_{yx} = 0$ for $t \in \{T/4, 3T/4\}$. At all such points $t \in \{nT/4 \mid n \in \mathbb{N}\}$, our improved numerical analysis indicates convergence of the other phase toward $\pi$ in the thermodynamic limit ($|\phi - \pi| \sim 10^{-10}$). The constraints imposed on the nested WLO by the symmetries listed in the aforementioned Table I are necessary to preserve this feature and practically restrict the phase $\phi$ in question to $\pi$, but are insufficient for a strict quantization. Nevertheless, a value $\phi \to \pi$ is associated with the existence of Majorana zero modes (MZMs) localized at adjacent corners, as illustrated in Fig. 4 of our original paper and explained in the next paragraph. An alternative, strictly quantized topological invariant was proposed in Ref. [2] for an anisotropic generalization of our model, with MZMs perfectly localized on the corner sites. Such a system is topologically equivalent to the one in our publication, and its boundary on whose end sites the two MZMs respectively reside can be interpreted as a nontrivial Kitaev chain decoupled from the bulk [2]. The nested Pfaffian $\boldsymbol{Q} \equiv (Q_x, Q_y)$ introduced therein takes the same value as $(e^{i\phi_{yx}}, e^{i\phi_{xy}})$ at the four special time instances and is protected by the particle-hole symmetry $\mathcal{P}$, thereby rigorously capturing the edge topology of this family of Hamiltonians.

Within the approach used in our publication, the nested WLO eigenvalue $\phi \neq 0$ indicates that the bulk Wannier functions are displaced within the unit cell, and that Wannier edge modes (WEMs) are formed at the boundaries perpendicular to the displacement direction. A calculation as described in Sec. V of Ref. [1] confirms, for instance when $t \in \{T/4, 3T/4\}$, that $\phi_{xy} \to \pi$ is indeed consistent with the existence of one WEM at each $y =$ const. edge of a system infinite in the $x$ direction. In this case, one finds as well that only the WEM localized at the bottom edge has a nontrivial Wannier center along $x$, and thus gives rise to the spatially well-separated corner states pictured in Figs. 4(c) and 4(g) when all boundaries are open. In the closely related model of Ref. [2], this configuration corresponds to a nontrivial Kitaev chain along the bottom $y$ edge and is thus characterized by $Q_y = -1$ (while $Q_x = 1$ is trivial). An analogous bulk-boundary argument is valid for $t \in \{0, T/2\}$, when $\phi_{yx} \to \pi$ ($Q_x = -1$ in Ref. [2]) and the topological Majorana corner states share the left $x$ edge [Figs. 4(a) and 4(e)].

Away from the four special points, the MZM transient between adjacent corners is not maximally localized, and the deviations of $(\phi_{xy}, \phi_{yx})$ from $(0, \pi)$ or $(\pi, 0)$ are accordingly more noticeable, although numerically small, of the order of 0.1%. Still, the Hamiltonians characterized by these two (approximate) values can be adiabatically connected to the appropriate special configurations with $t \in \{nT/4 \mid n \in \mathbb{N}\}$, without closing the bulk or Wannier gaps. For arbitrary $t$, $(\phi_{xy}, \phi_{yx}) \approx (0, \pi)$ or $(\pi, 0)$ thus indicates whether the system is closer to a configuration with corner states sharing the left $x$ or bottom $y$ edge, respectively (see Fig. 4 of our original paper). This geometrical information is equivalently conveyed by the nested Pfaffian of Ref. [2] through the values $\boldsymbol{Q} = (-1, 1)$ or $(1, -1)$, respectively. Additionally, $\boldsymbol{Q} = (-1, -1)$ is associated to the system when the MZMs are closest to diagonally opposite corners ($t \in \{nT/4 + T/8 \mid n \in \mathbb{N}\}$), which our WLO-based approach does not distinguish.

Details of this discussion and the numerical analysis are provided in Ref. [3].

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[1] W. A. Benalcazar, B. A. Bernevig, and T. L. Hughes, Electric multipole moments, topological multipole moment pumping, and chiral hinge states in crystalline insulators, Phys. Rev. B **96**, 245115 (2017).

[2] P. P. Poduval, T. L. Schmidt, and A. Haller, Perfectly localized Majorana corner modes in fermionic lattices, Phys. Rev. B **108**, 035124 (2023).

[3] T. E. Pahomi, Features of boundary modes in quasi-two-dimensional topological superconductors, Ph.D. Thesis, ETH Zurich, 2024, Chapter 3.