Moving frame theory of zero-bias photocurrent on the surface of topological insulators

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The recent observation of zero-biased photocurrent on the surface of topological insulators allows to *identify* spin-orbit-coupled two-dimensional Dirac cones as ideal platforms for the manipulation of the tilt of Dirac cone. We show that the in-plane effective magnetic field $\tilde{B}$ implements a moving frame transformation on the topological insulators' helical surface states. As a result, photo-excited electrons on the surface undergo a Galilean boost proportional to the effective in-plane magnetic field $\tilde{B}$. The boost velocity is transversely proportional to $\tilde{B}$. This explains why the experimentally observed photocurrent depends linearly on $\tilde{B}$. Our theory, while consistent with the observation that at leading order the effect does not depend on the polarization of the incident radiation, at next leading order in $\tilde{B}$ predicts a polarization dependence in both parallel and transverse directions to the polarization. We also predict two induced Fermi-surface effects that can serve as further confirmation of our moving frame theory. Based on the estimated value $\zeta \approx 0.34$ of the tilt parameter for a magnetic field of $\tilde{B} \sim 3$T, our geometric picture qualifies the surface Dirac cone of magnetic topological insulators as an accessible platform for the synthesis and experimental investigation of strong analog gravitational phenomena.

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I. INTRODUCTION

Surface states of topological insulators (TIs) [1] feature a helical Dirac cone [2] where the two-dimensional Pauli matrices representing the associated Clifford algebra are realized by the actual spin of the electrons. This simple fact means that external magnetic influence can be employed to manipulate them in interesting ways. Ideally their Hamiltonian is given by $v(p_x\sigma_y - p_y\sigma_x)$. Breaking the time-reversal (TR) invariance by a magnetic field perpendicular to the surface gives rise to a $m\sigma_z$ term that opens up a gap, whereas in-plane magnetic field being a combination of σ_x and σ_y terms only shifts the location of the Dirac node, but does not open up a gap. The in-plane fields give rise to fascinating effects such as planar Hall effect [3,4]. It has been experimentally found that in presence of in-plane magnetic field, a zero-bias photocurrent flows in the sample whose direction is transverse to the in-plane magnetic field, and its magnitude is proportional to the amount of the in-plane magnetic field [5]. A similar photocurrent is observed at the zero external magnetic field [6]. This effect is a basis for the application of TIs in infrared optoelectronics [7]. The fact that σ matrices in the surface Dirac cone of TIs are actual spins, makes it plausible to use magnetic fields or magnetization to influence the Dirac cones. Can the Dirac cones be manipulated in other interesting ways? In this paper, by providing a theory for the observed zero-bias photocurrent on the surface of TIs that relies on the particle-hole asymmetry, we show that surface Dirac cones in TIs are a promising platform to manipulate Dirac cones in ways that resemble the way gravitational sources influence the null geodesics (i.e., cones) [8].

The above simple theory of helical Dirac cones ignores an important fact: The bulk band-edge states have two completely different characters with opposite parities [2], such as s and p character. As such, the particle-hole (PH) asymmetry is a genuine property of the helical Dirac cones as is evident from photoemission data both in bulk [1,9] and thin films of TIs [10,11] and must be incorporated into theoretical considerations [12]. The PH symmetry is represented by the fact that Pauli matrices $\sigma_{x,y}$ that construct the helical Dirac states anticommute with σ_z. The way to break PH symmetry is to add a term that does not anticommute with σ_z and hence it must be proportional to σ_0, the unit 2×2 matrix. A constant term proportional to σ_0 is ruled out as it redefines the energy axis. A linear term in $\vec{p}$ can be of the form $\vec{\zeta}^{(0)}.\vec{p}\sigma_0$ with the provision that since it is odd in $\vec{p}$, the other Dirac cone on the other surface must have the same form with opposite $\vec{\zeta}^{(0)}$ value. There can also be a term even in $\vec{p}$ that can be combined to give

$$H_0 = v(p_x\sigma_y - p_y\sigma_x) + \frac{p^2}{2m_*}\sigma_0 + \tau_z\vec{\zeta}^{(0)}.\vec{p}\,\sigma_0, \qquad (1)$$

where $\tau_z = \pm 1$ correspond to top and bottom surface and the PH asymmetry is represented by m_*^{-1}. The PH-symmetric situation corresponds to $m_*^{-1} \to 0$. Quadratic PH breaking term also appears in a microscopic derivation of the TI surface states [13] as well as in TI thin films [14]. Note that we are

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ignoring the trigonal warping terms [15,16] that are negligible in the experiment of Ref. [5].

The purpose of this paper is twofold: (i) To construct a theory the *zero-bias* photocurrent in magnetic [5] and nonmagnetic TIs [6] that relies on PH asymmetry and time-reversal (TR) symmetry for the observed zero-bias photocurrent response with additional higher order predictions. (ii) To reveal a moving frame picture of Galilean boost inherent in the Hamiltonian Eq. (1) once it is subjected to an external in-plane magnetic influence. The latter establishes an intriguing paradigm of synthesizing a large class of emergent space-time structures within the so called Painlevé-Gullstrand family of space-time metrics simply via engineering of the spin-orbit-coupled Dirac cones by external magnetic influence.

II. IN-PLANE MAGNETIC FIELD AS KNOB TO TUNE TILTING OF DIRAC CONE

Let us couple external magnetic field $\vec{B}$ to Eq. (1). This magnetic field in Ref. [5] is denoted by B_y. As can be seen in Figs. 3(a), 3(b) of Ref. [5], even at temperatures above the Curie temperature $T_c = 85$ K of the Cr atoms, the photocurrent continues to be linear in $\vec{B}$. For $T > T_c$, the magnetization $\vec{m}$ of the Cr atoms is zero and the entire photocurrent is caused by the external $\vec{B}$ field. This indicates that both the direct Zeeman coupling to external $\vec{B}$ and exchange coupling to magnetization $\vec{m}$ are important in the response of the Dirac cone. Therefore we introduce an effective magnetic field $\tilde{B} = \vec{B} + \vec{m}$ where the magnetization $\vec{m} = \vec{m}(T, \vec{B})$ of the Cr atoms is function of temperature and external magnetic field.

The Hamiltonian in the presence of $\tilde{B}$ and PH asymmetry will become

$$H_0 = v(p_x\sigma_y - p_y\sigma_x) + \frac{p^2}{2m_*}\sigma_0 + \tau_z\vec{\zeta}^{(0)}.\vec{p}\,\sigma_0 - g\mu_B\tilde{B}.\vec{\sigma}, \tag{2}$$

where μ_B is the Bohr magneton and g is the g factor that can be large due to the spin-orbit-coupled nature of the bands [17]. A magnetic field along the z direction will give a $\sim\tilde{B}_z\sigma_z$ term opening a gap. But we would like to understand the role of in-plane magnetic field components $\tilde{B}_a$ with $a = 1, 2$ for x, y directions, respectively. Upon coupling a planar B field, first of all the location of the Dirac node will be shifted. Such a shifting of Dirac cones is supported by *ab initio* calculations for Cr-doped Bi_2Se_3 even when the external field B is zero [18]. Secondly, driven by PH asymmetry, additional terms will also be generated as follows: By shifting the momenta according to

$$p_a \to p_a + g\frac{\mu_B}{v}\varepsilon_{ab}\tilde{B}_b, \quad a, b = 1, 2, \tag{3}$$

where ε_{ab} is totally antisymmetric tensor in planar indices 1,2 with $\varepsilon_{12} = 1$, we obtain

$$H_0 = v(p_x\sigma_y - p_y\sigma_x) + \frac{p^2}{2m_*}\sigma_0 + \tau_z\vec{\zeta}^{(0)}.\vec{p}\,\sigma_0 + \frac{g\mu_B}{m_* v}(p_x\tilde{B}_y - p_y\tilde{B}_x)\sigma_0 + \frac{(g\mu_B)^2}{2m_* v^2}\left(\tilde{B}_x^2 + \tilde{B}_y^2\right)\sigma_0, \tag{4}$$

where the terms in the second line are generated by shifting the Dirac node. The first, second, and fourth terms in the above equation are used in Ref. [4] to explain the planar Hall effect, whereas the last term in the second line is a shift in the chemical potential, but caused by in-plane magnetic field [19]. In fact our moving frame interpretation can be identified as underlying mechanism for planar Hall effect [20] as well as the electric-field effects induced by magnetic influences simply because in a moving frame, magnetic field appears to have an effect like an electric field. In addition this interpretation explains the zero-bias photocurrent induced by magnetic field. The latter becomes clear soon. Therefore, from Eq. (4) we obtain a magnetically driven tilting and chemical potential shift:

$$\zeta_a^{(1)} = \varepsilon_{ab}\frac{g\mu_B\tilde{B}_b}{m_* v^2}, \quad a, b = 1, 2, \quad \tilde{\mu} = \frac{|\zeta^{(1)}|g\mu_B|\tilde{B}|}{2}. \tag{5}$$

The first order term in $\tilde{B}$ is $v\vec{\zeta}^{(1)}.\vec{p}\,\sigma_0$, is responsible for tunability of the tilting and magnetic-field-induced gating of the Dirac cone at the surface of topological insulators. The intrinsic (zeroth order) tilt $\tau_z\vec{\zeta}^{(0)}$ that may exist even in the absence of $\tilde{B}$ can be either zero or nonzero. As we will see shortly, the data in both magnetic and nonmagnetic TIs support a nonzero $\vec{\zeta}^{(0)}$. Let us start by formulation of the linear term. The second order (in $\tilde{B}$) term is a magnetic-induced chemical potential. The above analysis shows that an in-plane magnetic field can be used to tune the tilting in the surface Dirac cone of topological insulators. In this way the total tilt will have a constant (background) term $\tau_z\vec{\zeta}^{(0)}$ and a tilt term $\zeta^{(1)}$ linear in the effective in-plane magnetic field as in Eq. (5).

III. TILT TERM AS MOVING FRAME

To see the physics arising from the tilting of the Dirac cone, let us for the moment forget the $\tilde{B}$-induced chemical potential in Eq. (4). Representing the total tilt as $\vec{\zeta} = \vec{\zeta}^{(0)} + \vec{\zeta}^{(1)}$, the Hamiltonian will be $v(p_x\sigma_y - p_y\sigma_x) + v\vec{\zeta}.\vec{p}\,\sigma_0$, whose eigenvalues are $\varepsilon_\pm = v\vec{\zeta}.\vec{p} \pm vp$ where $p = |\vec{p}|$ is the magnitude of momentum. This is the dispersion relation of a Dirac cone which is tilted along vector $\vec{\zeta}$ by an amount proportional to dimensionless number ζ. Already at this level one can immediately see that the parameter ζ changes the speed of right/left movers by opposite amounts. In order to see this clearly, consider one spatial dimension x where $\varepsilon_\pm = v(\zeta p_x \pm p_x)$. This means that the right-moving electrons' velocity is boosted to $(\zeta + 1)v$, while the left-moving electrons' velocity is $(1 - \zeta)(-v)$. Such a *velocity asymmetry* between right and left movers is naturally equivalent to viewing the movement of upright Dirac electrons in a frame moving with velocity $-v\vec{\zeta}$. The nontrivial fact is that such a moving frame effect can be generated by an in-plane magnetic field as in Eq. (5). Even more nontrivial fact is that as we will see in this paper, the boost velocity ζv can be as large as $1/3v$ (where the Fermi velocity v is the upper limit of speeds in the material) the mechanical analog of which would be to imagine a frame moving with a third of the speed of light.

To see how such a boost on Dirac electrons gives rise to a nontrivial space-time geometry, let us combine the energy and momentum to define $1 + 2$ dimensional covariant momentum

$p_\mu = (-\varepsilon/v, p_x, p_y)$. Then taking the $\vec{\zeta}.\vec{p}$ term to the left side of the dispersion relation and squaring both sides (upon which $\pm$ disappears), the resulting equation being quadratic form in p_μ can be written as [21]

$$g^{\mu\nu} p_\mu p_\nu = 0, \tag{6}$$

where the contravariant components of the *emergent space-time metric* can be immediately read off:

$$g^{\mu\nu} = \begin{pmatrix} -1 & -\zeta_x & -\zeta_y \\ -\zeta_x & 1-\zeta_x^2 & -\zeta_x\zeta_y \\ -\zeta_y & -\zeta_x\zeta_y & 1-\zeta_y^2 \end{pmatrix}. \tag{7}$$

The inverse of the above metric gives the covariant components

$$g_{\mu\nu} = \begin{pmatrix} -1+\zeta^2 & -\zeta_x & -\zeta_y \\ -\zeta_x & 1 & 0 \\ -\zeta_y & 0 & 1 \end{pmatrix}, \tag{8}$$

where $\zeta^2 = \zeta_x^2 + \zeta_y^2$. The compact representation of Eq. (8) is

$$ds^2 = -(vdt)^2 + (d\vec{x} - \vec{\zeta} v dt)^2. \tag{9}$$

For the upright Dirac cone where there is no left/right velocity asymmetry, the tilt parameter is $\zeta = 0$ and the invariant space-time distance is $ds^2 = -(vdt)^2 + (d\vec{x})^2$. The relation between the latter and the invariant distance Eq. (9) in presence of a tilt is given by the Galilean transformation $(vt, \vec{x}) \to (vt, \vec{x} - \vec{\zeta} vt)$, where the boost parameter $\vec{\zeta}$ according to Eq. (5) is controlled by the effective in-plane magnetic field. Therefore the surface states of topological insulators subjected to in-plane magnetic field are solid-state realizations of moving frame where the frame velocity $-v\vec{\zeta}$ is tunable by in-plane magnetic field. Larger effective in-plane magnetic fields $\tilde{B}$ corresponds to faster moving frames. The direction of movement of the frame is always transverse to $\tilde{B}$.

Our moving frame interpretation is the most natural way in which optical absorption becomes a source of photocurrent. In the absence of tilt, the states in the upper branch of the Dirac cone are populated by the absorption of light. But since left/right mover's velocities are symmetric, there will be no net current. In this way the photocurrent is canceled out and remains inert. However, in presence of tilting, right and left movers' velocity will not be balanced anymore. The circular constant-energy surfaces will be deformed to elliptic constant-energy surfaces, the occupation of which is only set by their (constant) energy. In this way a net current arises from the velocity imbalance.

In the following we will use our moving frame picture based on Hamiltonian Eq. (4) to explain features of the experiment in Refs. [5,6]. Our theory further predicts that in second order in $\vec{\zeta}$, interesting polarization dependence and a novel $\tilde{B}$-induced Drude peak appears.

IV. PHOTOCURRENT IN MOVING FRAME THEORY

Equipped with moving frame picture let us explain essential features of the photocurrent experiment in Refs. [5,6]. The response of the helical electron system in the surface is controlled by an *effective* magnetic field $\tilde{B}$ that is sum of externally applied $\vec{B}$ and a term $\vec{m}$ arising from the Cr magnetic moments. The boost velocity is given by joint effect of these two terms and a possibly nonzero intrinsic $\vec{\zeta}^{(0)}$. The latter can be either extrinsically induced by external influence [22] or can be due to a built-in nematicity imposed on Dirac electrons [23].

Transversality of current. The first and foremost aspect of the observed photocurrent indicated in Fig. 1(b) of Ref. [5] is that when in-plane magnetic field is applied in y direction, the photocurrent is collected in x direction. This immediately follows from Eq. (5). This equation further predicts that if the in-plane field is applied along the x direction, the corresponding current will be collected in the $-y$ direction. This distinguishes our mechanism from a generic theory of photogalvanic effect for reduced symmetry at the surface [24,25]. Note that in Eq. (5) the PH asymmetry m_*^{-1} plays the all-important role of creating *velocity asymmetry*. Hence within our theory, the photocurrent can be enhanced in more PH asymmetric (larger m_*^{-1}) bands.

Linearity in low-fields. This can be immediately seen in our theory as follows: The photoexcitation places an electron on the conduction surface state. In the absence of boost, each state $+\vec{p}$ and its time-reversed partner $-\vec{p}$ having opposite group velocities, cancel each other's effect. Hence the optical excitation will not be able to produce a net current. But in a moving frame there will be a net (group) velocity and hence current imbalance of $2v\vec{\zeta}$. The approximate linear dependence of the photocurrent in Ref. [5] can be understood by ignoring the intrinsic $\tau_z\vec{\zeta}^{(0)}$: Within a ballistic picture, one immediately obtains that the induced photocurrent is proportional to $\varepsilon_{ab}\tilde{B}_b$.

Zero-field photo-current. As can be seen in Fig. 2(b) of Ref. [5], even at zero-applied B_y there is a small but nonzero photocurrent. Within our theory, this can be naturally attributed to an intrinsic tilt $\vec{\zeta}^{(0)}$. A similar intrinsic tilt in the parent nonmagnetic compound is clearly visible in the photoemission data of Ref. [6]. Within this picture, the contribution of the intrinsic tilt to the photocurrent in Ref. [5] can be on the scale of $\sim$20% of the photocurrent induced by $B_y = 5$T. The presence of τ_z in our theory immediately implies that probing the other surface of TI slab will give the opposite zero-field photocurrent. The physical cause of nonzero $\vec{\zeta}^{(0)}$ is a separate question. It could arise from an externally (or perhaps in interacting systems, spontaneously) induced nematicity of Dirac electrons whereby two opposite directions in space are preferred over the other directions [22].

Dependence on the polarization. The polarization independence in general grounds is expected when the surface Dirac cones enjoy a rotational invariance [26]. In general the polarization of light responds to anisotropic environments [27]. Therefore the lack of dependence of the photocurrent on polarization in Ref. [5] can only be understood when the total tilt $\vec{\zeta}$ is negligible [28]. To see how polarization dependence can arise in moving frame, let us start with the corresponding metric Eq. (7) for which the Kubo formula for undoped Dirac electrons gives (in units of $v = 1$)

$$\Pi^{\mu\nu}(q) = \pi(q)[q^2 g^{\mu\nu} - q^\mu q^\nu], \quad \pi(q) = -\frac{1}{16\sqrt{q^2}}, \tag{10}$$

where $q_\mu = (-\omega, \vec{q})$ is the three-momentum and $q^2 = q^\mu q_\mu = \vec{q}^2 - (\omega - \vec{\zeta}.\vec{q})^2$ [29], reflecting that the underlying space-time structure is imprinted in the optical response as well. The above response function satisfies the Ward identity $q_\mu \Pi^{\mu\nu} = 0$ guaranteeing that the photon remains massless. If for the time being we ignore the dependence Eq. (5) of chemical potential on the $\tilde{B}$, the above expression gives a polarization-dependent current as follows: Assume that the Cartesian coordinate 1 denotes the direction of polarization and the direction transverse to polarization in the xy plane is denoted by 2. Then the conductivity tensor derived from the above $\langle j^\mu j^\nu \rangle$ expression for the optical processes with $q_\mu = (-\omega, \vec{0})$ is [30]

$$\sigma^{ab} = \frac{e^2}{16\hbar} g^{ab} = \frac{e^2}{16\hbar}[\delta^{ab} - \zeta^a \zeta^b], \tag{11}$$

where ζ^1 is the component of $\vec{\zeta}$ along the polarization. In the absence of tilt, the above expression reduces to the diagonal conductivity of graphene type [31–33]. Denoting the angle between the polarization of the incident light (direction 1) and the direction of $\vec{\zeta}$ by θ, the conductivity tensor at zero chemical potential will become

$$\sigma_{\mu=0} = \frac{e^2}{16\hbar}\begin{pmatrix} 1 - \zeta^2 \sin^2\theta & \zeta^2 \sin\theta\cos\theta \\ \zeta^2 \sin\theta\cos\theta & 1 - \zeta^2\cos^2\theta \end{pmatrix}. \tag{12}$$

The above expression is frequency independent and suggests that the dependence on polarization appears at the second order of the total tilting parameter $\vec{\zeta} = \vec{\zeta}^{(0)} + \vec{\zeta}^{(1)}$. This gives a term of the form $\zeta^{(0)}\zeta^{(0)}$, $\zeta^{(0)}\zeta^{(1)} = \zeta^{(0)}\tilde{B}$ and a second-order term of the form $\zeta^{(1)}\zeta^{(1)} \propto \tilde{B}^2$. Hence the approximate independence on polarization only holds for small $\tilde{B}$ and small $\zeta^{(0)}$. Carefully re-visiting the experiment by looking for effect that are second order in the above small quantities will reveal polarization dependence of the above form.

Second-order Drude weight. At the limit of extremely low-energy photons $\omega \to 0$, due to the moving frame velocity, the Drude weight will be accessible in the photocurrent which is given by [34]

$$\frac{e^2\mu\delta(\hbar\omega)}{4\hbar\zeta^2}[1 - \sqrt{1-\zeta^2}] \approx \frac{e^2}{16\hbar}\frac{(\mu_B\tilde{B})^2}{m_* v^2}\delta(\hbar\omega). \tag{13}$$

The above formula offers two advantages: (i) It is a unique prediction of moving frame theory and the observation of the above Drude weight will be additional support of our scenario. (ii) The measurement of the Drude weight at $T > T_c$ in units of $\sigma_0 = e^2/(16\hbar)$ as a function of $\tilde{B} = B_y$ can be used to directly determine the coefficient $1/(m_* v^2)$ of the PH symmetry breaking.

Magnetic field induced gating. In our theory, the chemical potential itself depends on the tilting parameter Eq. (5). The conductivity tensor for nonzero μ is given by [34–36]

$$\sigma^{11} = \frac{e^2}{16\pi\hbar}\begin{cases} \theta_X - \cos(2\theta)\sin(2\theta_X)\Theta(1 - X^2) \\ \pi\Theta(-X - 1) \end{cases} \tag{14}$$

where $X = (2\mu - \hbar\omega)/\hbar\omega\zeta$ defines $\theta_X = \arccos X$ and θ as before is the angle between the $\vec{\zeta}$ and the polarization of the incident light [34,35,37]. The expression for σ^{22} can be obtained by replacing $\theta \to \theta + \pi/2$. These corrections arise in the range $|X| < 1$ or equivalently

$$\frac{2\mu}{1+\zeta} < \hbar\omega < \frac{2\mu}{1-\zeta}. \tag{15}$$

The upper and lower limit of the above inequality define twice the aphelion and perihelion of the constant-energy ellipse at energy μ. The width of the above interval is proportional to the eccentricity of the constant-energy ellipse multiplied by the chemical potential. Since in the present problem the magnetization induced μ [Eq. (5)] itself is proportional to $\tilde{B}^2$, the energy range in which the above correction appears will have a second-order term $\propto \zeta^{(0)}\tilde{B}^2$ (as well as a third-order term $\propto \tilde{B}^3$) contribution. Therefore the corrections arising from the $\tilde{B}$-induced chemical potential Eq. (5) will induce a deviation from universal conductivity of Dirac electrons over a small energy range around $2\tilde{\mu} \propto \tilde{B}^2 \sim (\zeta^{(1)})^2$. The photon energies reported in Fig. 2(c) of Ref. [5] are above 100 meV. Observation of the above Fermi-level effects requires lower-energy photons. The above gating effect that relies on $\tilde{B}$ (an hence on ζ) can give rise to an *equilibrium* current away from the transient photoexcited peak. In the following we use this effect to estimate the tilt parameter in a self-consistent way.

V. DISCUSSIONS

Let us use the data in Ref. [5] to estimate the value of tilt that is induced by external magnetic influence. The current density per electron that arises from moving frame picture is $2e\zeta v$ where the Fermi velocity is $v = 4.28 \times 10^5$ ms^{-1} and $e = 1.6 \times 10^{-19}$ C. The length of the sample used in the experiment along which the current is carried is $L_x = 5.6 \times 10^{-3}$ m. The thickness $L_z = 2 \times 10^{-9}$ m and the lateral dimension drop out from the end estimate of ζ as follows. To estimate the total number of electrons, since we do not have enough data to estimate the number of photoexcited electrons, we rely on the magnetic-field-induced gating. Due to such a nonzero gating effect, even in the absence of incident photons the resulting Fermi surface can give rise to a nonzero current. This can be interpreted in two ways: (i) One explanation is that in tilted Dirac cone the group velocity of left/right-moving Dirac particles will not be identical anymore. That is why the off-peak current can be attributed to such gating effect. (ii) Zero steady-state current is a property of nonmoving frame. Such a zero-current state, in a moving frame will appear like a current-carrying state. This leads to an off-peak current (i.e., away from the *transient* photoexcitation peak) that in the present experiment is on the scale of 10^{-7} A. Our theory at $\tilde{B} = 3$T gives $\tilde{\mu} = g\zeta\mu_B\tilde{B}/2 = \zeta g 17.34 \times 10^{-5}$ eV. Since the above energy scale is smaller than the thermal energy scale at a typical 20 K of the experiment, assuming a circular Fermi surface, the total number N of the electrons will be given by

$$N = \frac{1}{4\pi}\left[\frac{g\tilde{\mu}\zeta}{\hbar v} \times 10^{-3}\text{m}\right]^2 \approx 3 \times 10^4 (\zeta g)^2,$$

where we have used the laser spot size 1×10^{-3} m. From this a three-dimensional current density $j = 2\zeta e v N (L_x L_y L_z)^{-1}$ passing through the cross section $L_y L_z$ gives a current that depends only on L_x. Using the value of the g factor of $g = 19.4$

[17] for the in-plane fields the current will be given by

$$I = 24.61 \times 10^{-6}\zeta^3\text{A} \approx 10^{-6}\text{A}$$

that gives $\zeta^{(1)} \sim 0.34$. This is comparable to the tilt values of borophene compounds [23]. A similar tilting of ∼0.5 can be inferred from the photoemission data in the spin-orbit-coupled Dirac cones of transition metals induced by a magnetic layer. Our estimate of ζ based on the above gating effects can be larger for larger fields. Our theory further accounts for the zero-field photocurrent in Fig. 2(c) of Ref. [5]. The value of zero-field photocurrent in the peak region is about 20% of its value at the field of 5 T. Therefore our estimate of the tilt at 3 T gives a lower bound for the intrinsic tilt $\zeta^{(0)}$ being ∼20% of the induced tilt. We therefore expect $\zeta^{(0)} \gtrsim 0.08$.

Our theory explains both intrinsic and $\vec{B}$-induced zero-bias photocurrent on the surface of topological insulators. We further predict nonlinear effects in the form of polarization dependence and a nontrivial Drude weight, as well as deviations from universal optical absorption of Dirac electrons in $\omega \to 0$ limit. Both effects can be used to infer information about the breaking of PH symmetry that is represented by m_*^{-1} on which our theory is built. The above additional predictions rely on the covariance of the optical response of a generalized Minkowski space-time [29]. The above covariance allows for the propagation of transverse electric (TE) on the surface of TIs that solely arises from nonzero ζ [38]. Since the TE modes cannot propagate in other conducting media, the magnetization-enabled tilt can be used as a switch for the TE modes.

Our theory also accounts for a similar transient photocurrent on the surface of the parent *nonmagnetized* Sb_2Te_3 [6]. This has been attributed to "asymmetry between the transient population of opposite parallel momenta" [6]. However, as can be seen in Fig. 1(f) of Ref. [6], even without the asymmetric population, the photoemission from the photoexcited bands in this study show a clear *intrinsic* velocity asymmetry. This indicates a possible *intrinsic tilt* $\zeta^{(0)}$ even without the application of in-plane magnetic field in the surface Dirac cone. In a similar Sb-doped system $(Bi_{1-x}Sb_x)_2Te_3$, a systematic dependence of the photocurrent on the angle of incidence of the radiation is dubbed photon-drag effect, meaning that the momentum $\vec{q}_{||}$ of the light is transferred to electrons to cause the photocurrent [39]. However, again a substantial intrinsic tilting is clearly visible as velocity imbalance along the x direction [39]. Within our theory, the contribution of the intrinsic tilt $\vec{\zeta}^{(0)}$ can be separated from the photon-drag effect by illuminating the other surface of the TI where the intrinsic tilt must reverse its sign. Such an intrinsic tilting can also be seen as nonzero photocurrent at zero magnetic field in Fig. 2(c) of Ref. [5]. In fact the data in this figure suggest that the zero-field contribution to the photocurrent is comparable to the photocurrent induced by a file of $B_y \sim 1$ T. The subtraction of photocurrents at fields $B_y = 5$ T and $B_y = -5$ T in Fig. 3(c) of Ref. [5] amounts to subtraction of the background $\zeta^{(0)}$ effect, thereby giving a perfect coincidence between the *antisymmetrized photocurrent* and the magnetization in Fig. 3(c) of Ref. [5].

VI. OUTLOOK AND FURTHER POSSIBILITIES

The fact that moderate magnetic fields of few Tesla are able to induce tilting values of few tens of percent means that these fields can induce moving frame velocities that are a significant fraction of the Fermi velocity that sets the upper limit of speeds v. This resembles a situation where a body moves at, e.g., 0.34 of the speed of light. As such, our synthetic space-time setup in spin-orbit-coupled 2D Dirac cones that arises from our moving frame interpretation allows to design a number of experiments to investigate certain (even strong) gravitational effects on the tabletop: (i) The extrinsically induced tilt $\vec{\zeta}$ depends on the joint effect of B_y and the magnetization. The critically enhanced fluctuations of $\vec{m}$ near the Curie temperature can be employed to produce larger photocurrents and therefore better detectors of infrared radiation [40,41]. Apart from such a device application, this implies that inhomogeneously heated sample would imply inhomogeneously tilted Dirac cone, and therefore a nontrivial fabric for the emergent space-time. In particular, a temperature gradient below T_c would imprint a tilt profile with nonzero $\vec{\nabla}.\vec{\zeta}$, or a domain wall configuration of $\vec{m}$ would imply a nonzero $\vec{\nabla} \times \vec{\zeta}$. The latter would be a "gravitomagnetic field" *that points along z direction* [42] by simulating a metric [Eq. (8)] that corresponds to a rotating gravitational source. This can be a fascinating opportunity offered by spin-orbit-coupled Dirac cone of the surface of TIs. (ii) A magnetic superstructure on the surface of a topological insulator [43] arranged with wave vector $\vec{q} = (q, 0)$ can be used to synthesize a background geometry with gravitational density wave (akin to the familiar spin- and charge-density wave states [44]). The spin polarization of the stripes can be aligned with an external in-plane $\vec{B}$ field to generate $\zeta_{x(y)} = \zeta^{(0)}_{x(y)} + \zeta^{\max}_{x(y)} \cos(qx)$ profile that can be used to investigate gravitational grating on the tabletop [45]. In a more generic setting, any form of spatially inhomogeneous magnetic-field texture, directly imprints the corresponding texture on the fabric of the space-time [Eq. (8)] and would allow to study effects of fluctuations and disorder on the fabric of space-time that is not easy to measure for the space-time in the cosmos. (iii) Torsion in condensed-matter systems can naturally arise from defects of underlying lattice [46]. This can be combined with the magnetic manipulation of the Dirac cone in order to synthesize solid-state space-times with nonzero torsion [47]. Note that the present space-time geometry is distinct from quantum geometry [48,49] that deals with distance in the space of wave functions rather than the space-time.

The ability to influence the properties of synthetic space-time by magnetic fields on the surface Dirac cones rises hopes to shed light on the role of galactic and extragalactic magnetic fields on the fabric of space-time [50].

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