Coherent states in microwave-induced resistance oscillations and zero resistance states

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We investigate irradiated high-mobility two-dimensional electron systems (2DES) under low or moderated magnetic fields. These systems present microwave-induced magnetoresistance oscillations (MIRO) which, as we demonstrate, reveal the presence of coherent states of the quantum harmonic oscillator. We also show that the principle of minimum uncertainty of coherent states is at the heart of MIRO and zero-resistance states (ZRS). Accordingly, we are able to explain, based on coherent states, important experimental evidence of these photo-oscillations, such as their physical origin, their periodicity with the inverse of the magnetic field and their peculiar oscillations minima and maxima positions in regards of the magnetic field. Thus, remarkably enough, we come to the conclusion that 2DES, under low magnetic fields, become a system of quasiclassical states or coherent states and MIRO would be the smoking gun of the existence of these peculiar states in these systems.

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I. INTRODUCTION

The first idea of coherent states or quasiclassical states was introduced by Schrödinger [1] describing minimum uncertainty constant-shape Gaussian wave packets of the quantum harmonic oscillator. They were constructed by the quantum superposition of the stationary states of the harmonic oscillator. These wave packets displaced harmonic oscillating similarly as their classical counterpart [1]. Later on, Glauber [2] applied the concept of coherent states to the electromagnetic field being described by a sum of quantum field oscillators for each field frequency or mode. These coherent states of electromagnetic radiation introduced by Glauber are extensively used nowadays in quantum optics. Coherent states [3–6] are also an essential and powerful tool in condensed matter when describing the dynamics of quantum systems that are very close to a classical behavior. One remarkable example of this consists of one electron under the influence of a moderate and constant magnetic field (B). The quantum mechanical solution of this problem leads us to Landau states which are mere stationary states of the quantum harmonic oscillator. Under low or moderate values of B, this system can be described by an infinite superposition of Landau states, i.e., a coherent state. The resulting wave packet oscillates classically at the cyclotron frequency (w_c) inside the quadratic potential keeping constant the Gaussian shape (see Fig. 1) and complying with the minimum uncertainty condition.

The discovery of microwave-induced magnetoresistance oscillations (MIRO) [7–9] two decades ago led to a great deal of theoretical works back then as the displacement model [10], the inelastic model [11], and the microwave-driven electron orbits model [12–15]. According to the latter, Landau states, under radiation, spatially and harmonically oscillate with the guiding center at the radiation frequency (w) performing classical trajectories. In this swinging motion electrons are scattered by charged impurities giving rise to oscillations in the irradiated magnetoresistance, i.e., MIRO.

In this letter we demonstrate that the electron dynamics and magnetotransport in high-mobility 2DES is governed by the coherent states of the quantum harmonic oscillator. In fact, we conclude that 2DES under low or moderate B become a systems of coherent states and when irradiated, MIRO [7–9] brings to light the peculiar nature of these states. In other words, irradiated coherent states of the quantum harmonic oscillator are at the heart of MIRO. Accordingly, we incorporate the concept of coherent states to the microwave-driven electron orbit model [12–15]. Thus, a remarkable obtained result is that the time τ (evolution time [16]) it takes a scattered electron to jump between coherent states to give significant contributions to the current has to be equal to the cyclotron period $T_c = 2\pi/w_c$. For different values of τ, the contribution turns out negligible. This result holds in the dark and under radiation where τ will play an essential role. Thus, MIRO is mainly dependent on τ along with w. τ also determines the peculiar B-dependent MIRO extrema position and explains the periodicity of MIRO with the inverse of B. Thus, MIRO finally reveals that coherent states of the quantum harmonic oscillator are present in high-mobility 2DES when under low B playing a lead role in magnetotransport both in the dark and under radiation. On the other hand, coherent states minimize the Heisenberg uncertainty principle and then, in our model, this would establish which states can be reached by scattering.

II. THEORETICAL MODEL

We first obtain an expression for the coherent states of a radiation-driven quantum harmonic oscillator. The starting point is the exact solution of the time-dependent Schrödinger equation of a quantum harmonic oscillator under a time-dependent force. This corresponds to the electronic wave function for a 2DES in a perpendicular B, a dc electric field E_{DC}, and microwave (MW) radiation which is considered semiclassically. The total hamiltonian H can be written as

$$\begin{aligned} H &= \frac{P_x^2}{2m^*} + \frac{1}{2}m^* w_c^2 (x - X(0))^2 - eE_{dc}X(0) \\ &\quad + \frac{1}{2}m^* \frac{E_{dc}^2}{B^2} - eE_0 \cos wt (x - X(0)) \\ &\quad - eE_0 \cos wt X(0) \\ &= H_1 - eE_0 \cos wt X(0), \end{aligned} \tag{1}$$

where the corresponding wave function solution is given by [12,17,18]

$$\Psi_n(x,t) = \phi_n(x - X(0) - x_o(t)) e^{-iw_c(n+1/2)t} e^{\frac{i}{\hbar}\Theta(t)}, \tag{2}$$

where

$$\begin{aligned} \Theta(t) &= \left[m^* \frac{dx_o(t)}{dt} x - \int_0^t L dt' \right] \\ &\quad + X(0) \left[-m^* \frac{dx_o(t)}{dt} x + \int_0^t E_0 \cos wt' dt' \right]. \end{aligned} \tag{3}$$

$X(0)$ is the guiding center of the driven-Landau state, E_0 the MW electric field intensity, ϕ_n is the solution for the Schrödinger equation of the unforced quantum harmonic oscillator and $x_0(t)$ is the classical solution of a forced harmonic oscillator:

$$x_0(t) = \frac{eE_o}{m^* \sqrt{(w_c^2 - w^2)^2 + w^2 \gamma^2}} \sin wt = A \sin wt, \tag{4}$$

where γ is a phenomenologically introduced damping factor for the electronic interaction with acoustic phonons and L is the classical Lagrangian. Apart from phase factors, the wave function turns out to be the same as a quantum harmonic oscillator (Landau state) where the center is driven by $x_0(t)$. Thus, all driven-Landau states harmonically oscillate in phase at the radiation frequency.

A coherent state denoted by $|\alpha\rangle$ is defined as the eigenvector of the annihilation operator $\hat{a}$ with eigenvalue α and can be expressed as a superposition of quantum harmonic oscillator states [16]

$$|\alpha\rangle = \sum_n c_n(\alpha)|\phi_n\rangle = e^{-|\alpha|^2/2} \sum_n \frac{\alpha^n}{\sqrt{n!}}|\phi_n\rangle. \tag{5}$$

The coherent state $|\alpha\rangle$ can be also obtained with the displacement operator $D(\alpha)$ [16] acting on the quantum harmonic oscillator ground state $|\phi_0\rangle$, $|\alpha\rangle = D(\alpha)|\phi_0\rangle$, where the unitary operator $D(\alpha)$ is defined by $D(\alpha) = e^{\alpha a^\dagger - \alpha^* a}$. The coherent state in the position representation or wave function then reads $\psi_\alpha(x) = \langle x|D(\alpha)|\phi_0\rangle$. We observe, according to the obtained MW-driven wave function [Eq. (2)], that the irradiated Landau-level structure remains unchanged with respect to the undriven situation; same Landau-level index and energy. Then, we conclude that the system is quantized, in the same way as the unforced quantum harmonic oscillator [18]. Thus, we can construct the driven-coherent states based on driven-Landau states similarly as if they were undriven [16]:

$$\begin{aligned} |\psi_\alpha(x,t)\rangle &= e^{\frac{i}{\hbar}\Theta(t)} e^{-iw_c t/2} e^{-|\alpha|^2/2} \\ &\quad \times \sum_n \frac{(\alpha e^{-iw_c t})^n}{\sqrt{n!}} |\phi_n(x - X(0) - x_o(t))\rangle. \end{aligned} \tag{6}$$

Now applying the displacement operator, we can calculate the wave function corresponding to the coherent state of the MW-driven quantum oscillator:

$$\begin{aligned} \psi_\alpha(x,t) &= e^{\frac{i}{\hbar}\Theta(t)} e^{-iw_c t/2} \langle x|D(\alpha)|\phi_0(x - X(0) - x_o(t))\rangle \\ &= e^{\frac{i}{\hbar}\Theta(t)} e^{i\vartheta_\alpha} e^{-iw_c t/2} e^{\frac{i}{\hbar}\langle p\rangle(t)x} \phi_0[x - X(0) - x_o(t) \\ &\quad - \langle x\rangle(t)], \end{aligned} \tag{7}$$

where

$$\begin{aligned} &\phi_0[x - X(0) - x_o(t) - \langle x\rangle(t)] \\ &\quad = \left(\frac{mw_c}{\pi\hbar}\right)^{1/4} e^{-\left[\frac{x - X(0) - x_o(t) - \langle x\rangle(t)}{2\Delta x}\right]^2}. \end{aligned} \tag{8}$$

$\langle x\rangle(t)$ and $\langle p\rangle(t)$ are the position and momentum mean values, respectively [16], $\langle x\rangle(t) = \sqrt{\frac{2\hbar}{m^* w_c}}|\alpha_0|\cos(w_c t - \varphi)$, and $\langle p\rangle(t) = -\sqrt{2m^*\hbar w_c}|\alpha_0|\sin(w_c t - \varphi)$ where we have used that $\alpha = |\alpha_0|e^{-(iw_c t - \varphi)}$. Δx is the position uncertainty and the global phase factor $e^{i\vartheta_\alpha} = e^{\alpha^{*2} - \alpha^2}$. Then, the wave packet associated with $\Psi_\alpha(x,t)$ is therefore given by

$$|\Psi_\alpha(x,t)|^2 = |\phi_0[x - X(0) - x_o(t) - \langle x\rangle(t)]|^2. \tag{9}$$

Thus, according to the above, the microscopic physical description of a high-mobility 2DES under low or moderate B would consist of constant-shaped Gaussian wave packets harmonically displacing with w_c in the undriven case and with w_c and w under radiation.

To calculate the longitudinal magnetoresistance R_{xx}, we first obtain the longitudinal conductivity σ_{xx} following a semi-classical Boltzmann model [19–21]

$$\sigma_{xx} = 2e^2 \int_0^\infty dE \rho_i(E)(\Delta X_0)^2 W_I \left(-\frac{df(E)}{dE}\right), \tag{10}$$

with E being the energy, $\rho_i(E)$ the Landau states density of the initial coherent state, and W_I is the electron-charged impurities scattering rate. We consider now that the scattering takes place between coherent states of quantum harmonic oscillators. Thus, ΔX_0 is the distance between the guiding centers of the scattering-involved coherent states.

We first study the dark case and according to the Fermi's golden rule, W_I is given by

$$W_I = N_i \frac{2\pi}{\hbar} |\langle\psi_{\alpha'}|V_s|\psi_\alpha\rangle|^2 \delta(E_{\alpha'} - E_\alpha), \tag{11}$$

where N_i is the number of charged impurities, ψ_α and $\psi_{\alpha'}$ are the wave functions corresponding to the initial and final coherent states respectively, V_s is the scattering potential for charged impurities [20] $V_s = \sum_q V_q e^{iq_x x} = \sum_q \frac{e^2}{2S\epsilon(q+q_{TF})} e^{iq_x x}$, S being the sample surface, ϵ the dielectric constant, q_{TF} is the Thomas-Fermi screening constant [20], and q_x the x

component of $\overrightarrow{q}$, the electron momentum change after the scattering event. E_α and $E_{\alpha'}$ stand for the coherent states initial and final energies, respectively.

The averaging on the impurities distribution has been considered in a very simple approach following Askerov [21], Ando *et al.*[20], and J. H. Davies [22]. Thus, if the concentration of impurities is not too high, and they are randomly distributed in the sample, the interferences caused by the impurity centers can be neglected. Then, we have ignored those interferences and assume that the scattering due to each impurity is independent of the others. As a result, the total scattering is equal to the scattering rate for one impurity center multiplied by the total number of impurities N_i.

The V_s matrix element is given by [19–21]

$$|\langle\psi_{\alpha'}|V_s|\psi_\alpha\rangle|^2 = \sum_q |V_q|^2 |I_{\alpha,\alpha'}|^2 \tag{12}$$

and the term $I_{\alpha,\alpha'}$ [19–21]:

$$I_{\alpha,\alpha'} = \int_{-\infty}^{\infty} \psi_{\alpha'}(x - X'(0) - \langle x'\rangle(t')) \times e^{iq_x x}\psi_\alpha(x - X(0) - \langle x\rangle(t))dx. \tag{13}$$

After lengthy algebra we obtain an expression for $I_{\alpha,\alpha'}$:

$$|I_{\alpha,\alpha'}| = e^{-\frac{[X'(0)-X(0)+\langle x'\rangle(t')-\langle x\rangle(t)]^2}{8(\Delta x)^2}} e^{-\frac{q_X^2(t)2(\Delta x)^2}{4}}, \tag{14}$$

where $q_x(t)$is given by

$$\begin{aligned} q_x(t) &= q_x + \sqrt{2m\hbar w_c}/\hbar[|\alpha_0'|\sin(w_c t') - |\alpha_0|\sin(w_c t)] \\ &= q_x + 2\sqrt{2m\hbar w_c}/\hbar|\alpha_0|\cos(w_c(t+\tau/2))\sin(w_c\tau/2)). \end{aligned} \tag{15}$$

On the other hand,

$$\langle x'\rangle(t') - \langle x\rangle(t) \simeq \sqrt{\frac{2\hbar}{mw_c}}|\alpha_0| 2\sin\left(w_c\left(t+\frac{\tau}{2}\right)-\varphi\right) \times \sin\left(-w_c\frac{\tau}{2}\right), \tag{16}$$

where t and t' are the initial and final times for the scattering event and τ is the evolution time between coherent states. Thus, $t' = t + \tau$. We have considered also that for low values of B, $|\alpha_0'| \simeq |\alpha_0|$. Developing the above exponential we can finally get to

$$|I_{\alpha,\alpha'}| \propto e^{-2|\alpha_0|^2 \sin^2\left(w_c\left(t+\frac{\tau}{2}\right)-\varphi\right)\sin^2\left(w_c\frac{\tau}{2}\right)}. \tag{17}$$

For typical experimental values of B, $|\alpha_0|^2 > 50$ and thus, $I_{\alpha,\alpha'} \to 0$. Accordingly, the scattering rate and conductivity would be negligible, too. Nonetheless, there is an important exception when τ equals the cyclotron period T_c: $\tau = \frac{2\pi}{w_c}$. In other words, the scattered electron begins and ends in the same position in the Landau orbit. Only in this case $I_{\alpha,\alpha'} \neq 0$. Thus, only scattering processes fulfilling the previous condition of τ will efficiently contribute to the current. The rest of the contributions can be neglected. Finally the expression of $I_{\alpha,\alpha'}$ reads [19]

$$|I_{\alpha,\alpha'}| = e^{-\left(\frac{(X'(0)-X(0))^2}{8(\Delta x)^2} + \frac{q_X^2(\Delta x)^2}{2}\right)} = e^{-\left(\frac{q^2 2(\Delta x)^2}{4}\right)}, \tag{18}$$

where $X'(0) - X(0) = [-q_y 2(\Delta x)^2]$ [19]. This, in turn, leads us to a final expression for W_I:

$$W_I = \frac{n_i e^4}{2\pi\hbar\epsilon^2}\int \frac{e^{-q^2(\Delta x)^2}}{(q+q_{TF})^2}(1-\cos\theta)\delta(E_{\alpha'} - E_\alpha)d^2q, \tag{19}$$

where n_i is the charged impurity density and θ is the scattering angle. The density of initial Landau states $\rho_i(E)$ can be obtained by using the Poisson sum rules to get to [23] $\rho_i(E) = \frac{m^*}{\pi\hbar^2}[1 - 2\cos(\frac{2\pi E}{\hbar w_c}e^{-\pi\Gamma/\hbar w_c})]$. Finally, gathering all terms and solving the energy integral, we obtain an expression for σ_{xx} that reads

$$\begin{aligned} \sigma_{xx} = {} & \frac{n_i e^6 m^*}{2\pi^3\hbar^3\epsilon^2}(\Delta X_0)^2 \frac{1}{\hbar w_c}\left(\frac{1+e^{-\pi\Gamma/\hbar w_c}}{1-e^{-\pi\Gamma/\hbar w_c}}\right) \\ & \times\left(1 - \frac{2\chi_s}{\sinh(\chi_s)}\cos\left(\frac{2\pi E_F}{\hbar w_c}\right)e^{-\pi\Gamma/\hbar w_c}\right) \\ & \times\int \frac{e^{-q^2(\Delta x)^2}}{(q+q_{TF})^2}(1-\cos\theta)d^2q, \end{aligned} \tag{20}$$

where $\chi_s = 2\pi^2 k_B T/\hbar w_c$, k_B being the Boltzmann constant, E_F the Fermi energy, and Γ the Landau level width. To obtain R_{xx} we use the relation $R_{xx} = \frac{\sigma_{xx}}{\sigma_{xx}^2+\sigma_{xy}^2} \simeq \frac{\sigma_{xx}}{\sigma_{xy}^2}$, where $\sigma_{xy} \simeq \frac{n_e e}{B}$ and $\sigma_{xx} \ll \sigma_{xy}$, n_e being the 2D electron density.

One important condition that features coherent states is that they minimize the Heisenberg uncertainty principle. Thus, for the time-energy uncertainty relation [16], $\Delta t\Delta E = h$. For our specific problem, $\Delta t = \tau$ that implies $\Delta E = \hbar w_c$, ΔE being the energy difference between scattering-involved coherent states. Thus, we obtain two conditions for the scattering between coherent states to take place, first $\tau = \frac{2\pi}{w_c}$, and second the energy difference equals $\hbar w_c$. There are also physical reasons that endorse the latter especially in high-mobility samples where the levels are very narrow in terms of states density. In these systems the only efficient contributions to scattering are the ones corresponding to aligned Landau levels (see Fig. 2), i.e., when $\Delta E = n \times \hbar w_c$. The most intense of them is when $n = 1$ that corresponds to the closest in distance coherent states or smallest value of ΔX_0 [see Eq. (18)]. This agrees with the condition that when $n = 1$, the Heisenberg uncertainty principle is minimized. The two conditions discussed above hold in the dark and under radiation. For the latter case, MIRO reveals the important role played by τ in the based-on-coherent states magnetotransport processes.

When we turn on the light, the term that is going to be mainly affected in the σ_{xx} expression is the distance between the coherent states guiding centers, i.e., ΔX_0. This average distance now turns into ΔX_{MW} [24–28]:

$$\begin{aligned} \Delta X_{MW} &= X'_{MW} - X_{MW} \\ &= \Delta X_0 - A(\sin w(t+\tau) - \sin wt) \\ &\quad + \sqrt{\frac{2\hbar}{m^* w_c}}|\alpha_0|(\cos w_c(t+\tau) - \cos w_c t). \end{aligned} \tag{21}$$

If we consider, on average, that the scattering jump begins when the MW-driven oscillation is at its midpoint, ($wt = 2\pi n$, n being a positive integer), and being $\tau = 2\pi/w_c$, we

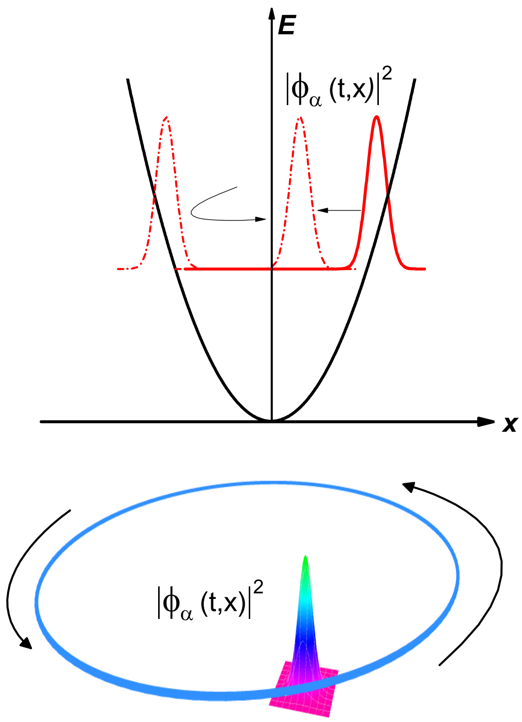

FIG. 1. Schematic diagrams of coherent states: The probability density of the coherent state is a constant-shaped Gaussian distribution, whose center oscillates in a harmonic potential similarly as its classical counterpart. The lower part exhibits the 2D approach.

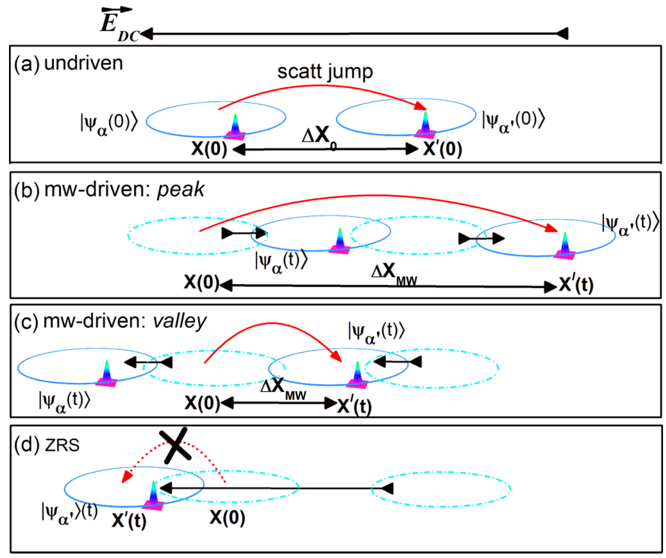

FIG. 3. Schematic diagrams for electron scattering between coherent states in the dark (undriven) and with radiation (MW driven). (a) Undriven scattering. The average distance (advanced distance) between initial $|\psi_\alpha\rangle$ and final coherent state $|\psi_{\alpha'}\rangle$ is $\Delta X(0)$. This distance mainly determines R_{xx}. (b) MW-driven scattering giving rise to peaks. Now the average advanced distance is larger because the final state, minimizing the Heisenberg uncertainty principle, is farther than the dark position due to the swinging motion of the driven-coherent states. (c) MW-driven scattering giving rise to valleys. When the final coherent state is closer we obtain MIRO valleys. (d) Situation when MW power is high enough and the states go backwards. In this scenario the final state ends up behind the initial-state dark position and the scattering jump cannot take place.

end up having

$$\Delta X_{MW} = \Delta X_0 - A \sin 2\pi \frac{w}{w_c}. \tag{22}$$

This result affects dramatically σ_{xx} and in turn R_{xx}. Now photo-oscillations rise according to ΔX_{MW} and its built-in sine function. In Fig. 3 we present schematic diagrams for the different situations regarding MIRO peaks and valleys and zero-resistance states (ZRS). In the undriven scenario an electron in the initial coherent state scatters with charged impurities and jumps to the final coherent state, minimizing the Heisenberg uncertainty principle. The latter condition determines what coherent states can be connected via scattering. On average, the advanced distance is $\Delta X_0 = X_0' - X_0$ [see Fig. 3(a)]. When the light is on, depending on the term $A \sin 2\pi \frac{w}{w_c}$, sometimes the minimum-uncertainty final state will be further away than in the dark regarding the initial state position. Thus, on average, $\Delta X_{MW} > \Delta X_0$ and R_{xx} will be larger, giving rise to peaks [see Fig. 3(b)]. On the other hand, other times the final coherent state will be closer and $\Delta X_{MW} < \Delta X_0$ and R_{xx} will be smaller, giving rise to valleys [see Fig. 3(c)]. Finally, when the driven coherent states are going backward and the radiation power is large enough, the final state, minimizing the uncertainty principle, will be behind the initial state in the dark [see Fig. 3(d)]. However, the scattered electron can only effectively jump forward due to the dc electric field direction and the final coherent state can never be reached; in the forward direction there is no final coherent state fulfilling the minimum uncertainty condition and the scattering cannot be completed. Thus, the system reaches the ZRS scenario where the electron remains in the initial coherent state.

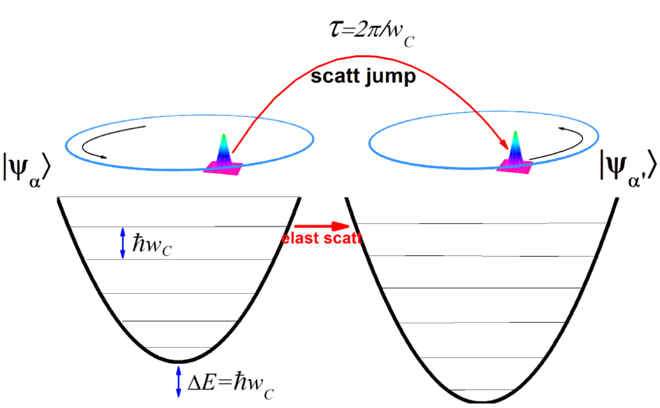

FIG. 2. Schematic diagram of scattering process between coherent states Ψ_α and $\Psi_{\alpha'}$. The scattering is quasielastic. The probability density for both coherent states is a constant-shaped Gaussian wave packet. The process evolution time τ is the cyclotron period, i.e., $\tau = 2\pi/w_c = T_c$. ΔE is the energy difference between the coherent states.

III. RESULTS

In Fig. 4 we present calculated results of the irradiated R_{xx} vs B for a radiation frequency of 103 GHz and $T = 1$ K. The

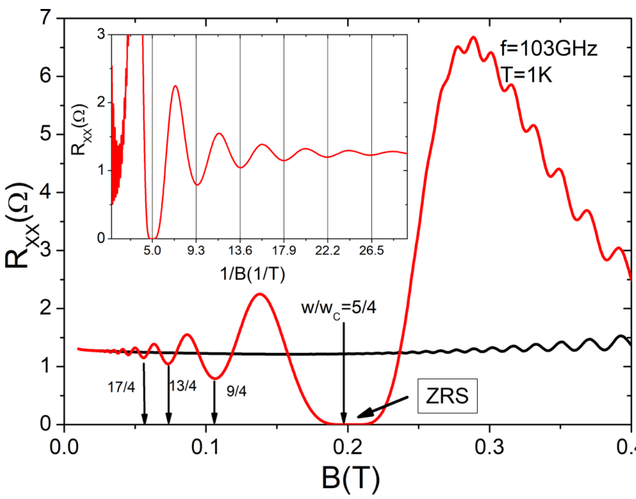

FIG. 4. Calculated magnetoresistance as a function of B, for a radiation frequency of 103 GHz and $T = 1$ K. The dark case is also exhibited. Minima positions are indicated with arrows corresponding to $\frac{w}{w_c} = j + \frac{1}{4}$, j being a positive integer. Zero resistance states are obtained around $B \simeq 0.2T$. Inset: irradiated magnetoresistance showing periodicity vs $1/B$.

dark case is also exhibited. In our simulations all results have been based on experimental parameters corresponding to the experiments by Mani *et al.* [7]. We obtain clear MIRO where the minima positions are indicated with arrows and, as in experiments [7], correspond to $\frac{w}{w_c} = j + \frac{1}{4}$, j being a positive integer. Minima positions show a clear 1/4-cycle shift, which is a universal property that features MIRO and shows up in any experiment about MIRO irrespective of the sort of carrier [29] and platform [30]. In the minima corresponding to $j = 1$, ZRS are found. Now with the help of our present model based on coherent states we can explain such a peculiar value for the minima position. Thus, it is straightforward to check out that if we substitute equation $\frac{w}{w_c} = j + \frac{1}{4}$ in ΔX_{MW} [Eq. (22)], we would obtain minima values of the latter and in turn of R_{xx}. Therefore, from the minima-positions relation we can obtain the value $2\pi/w_c$ which would be the "smoking gun" that would reveal the presence of coherent states of quantum harmonic oscillators sustaining the magnetorresistance of high-quality 2DES. Another evidence of the latter would be the MIRO periodicity with the inverse of B (see inset of Fig. 4) that would be explained by the presence of τ in the argument of the sine function.

IV. SUMMARY

Summing up, we have demonstrated that magnetoresistance in a high-mobility 2DES under MW radiation can be explained in terms of the coherent states of the quantum harmonic oscillator. When irradiated, these systems give rise to MIRO that reveals the presence of these quasiclassical states in high-quality samples when under low B. These MW-driven coherent states have been used to calculate irradiated magnetoresistance, finding that the principle of minimum uncertainty of coherent states is crucial to understand MIRO and their properties and zero-resistance states. We conclude that any experiment on irradiated magnetoresistance of 2D systems, regardless of carrier and platform [29,31], show that MIRO reveals the existence of coherent states of the quantum harmonic oscillator. We expect that dealing with even higher mobility samples ($\mu > 10^7$), it would be possible to achieve the quantum superposition of coherent states yielding, for instance in the case of two, even and odd coherent states of the quantum harmonic oscillator. Then, when irradiated, we expect that MIRO would evolve showing striking results revealing the presence of coherent states superposition [32,33].

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