

Bianisotropic exceptional points in an isolated dielectric nanoparticle

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(Received 22 June 2023; revised 3 November 2023; accepted 20 November 2023; published 16 January 2024)

Exceptional points are spectral singularities of open systems, where several eigenvalues and eigenvectors coalesce. In photonics, they are associated to remarkable phenomena, such as unidirectional scattering, enhanced sensing or chiral mode conversion. In this work, we study scattering of electromagnetic waves by a single dielectric nanoparticle and observe the appearance of exceptional points in its eigenvalue spectrum. Their existence is linked to breaking the mirror symmetry of the particle. Remarkably, they mark the onset from weak to strong coupling of the resonant modes. We discuss in detail the example of the electric and magnetic dipole modes supported by a silicon nanoparticle. We argue that any two modes of a resonant dielectric nanoparticle can merge to create an exceptional point, provided their resonant frequencies cross as functions of a parameter such as, e.g., aspect ratio, and their field distributions have opposite signs after a reflection in the transverse plane of the structure. The strongly coupled modes radiate as a mixture of electric and magnetic dipoles resulting in an intense bianisotropic response, being easily controlled by symmetry-breaking perturbations. We also study the effect of a dielectric substrate and demonstrate that the latter provides an additional mechanism to tune the position of exceptional points in the parameter space. Finally, we discuss applications of bianisotropic EPs, including their use for refractive-index sensing.

DOI: [10.1103/PhysRevResearch.6.013053](https://doi.org/10.1103/PhysRevResearch.6.013053)

I. INTRODUCTION

Open physical systems, such as systems interacting with an environment, are ubiquitous in nature. One of their most remarkable features is their ability to host spectral singularities where both eigenvalues and eigenvectors coalesce, known as *exceptional points* (EPs) [1,2]. At an EP, there is a collapse in the dimensionality of the eigenspectrum, resulting in a drastic increase in sensitivity to a small perturbation ϵ . Unlike Hermitian degeneracies (diabolical points), where two or more eigenfrequencies coincide [3], whose eigenfrequencies shift linearly with a perturbation, at an n th-order EP, the shift is proportional to $\epsilon^{1/n}$, where n is the number of coalescing eigenmodes [1,4]. This peculiarity renders EPs highly attractive for many applications, including biosensing schemes relying on resonance tracking [5].

EPs have no analog in Hermitian systems, such as those employed in conventional quantum mechanics. However,

other areas of wave physics are starting to explore the opportunities offered by operating at the vicinity of an EP. Besides their unique sensitivity, EPs are associated with abrupt phase transitions. In optics, this feature leads to a variety of counterintuitive phenomena, such as unidirectional invisibility, enhanced Sagnac effect in microcavities [6], laser mode selectivity, loss-induced revival of lasing, bulk Fermi arcs [7] and topologically protected chirality [8]. In the momentum space, they are associated with a half-integer polarization charge [9].

EPs are closely linked to parity-time (PT) symmetry, a notion inherited from quantum mechanics that tells us that systems invariant to parity inversion and time reversal can exhibit real spectra [10]. Indeed, it has been shown that EPs mark the transition between the PT-symmetric and PT-broken phase [11]. Most experimental realizations of EPs in photonics rely on this idea. Namely, by balancing gain and dissipation in optical microcavities, an EP emerges, drastically altering the response of the system [4,12].

Nevertheless, the presence of gain and loss or PT-symmetry is not necessary in order to reach an EP: they were recently demonstrated in plasmonic metasurfaces by balancing the losses of two plasmon modes [5,13]. Plasmonic EPs were utilized to develop an ultrasensitive scheme for anti-immunoglobulin G [5]. So far, the vast majority of works are

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restricted to wavelength-scale systems based on waveguides, metasurfaces and/or microresonators [4,5,7,8,14–16].

In recent years, resonant all-dielectric nanostructures have progressively replaced plasmonics in a broad range of applications in nano-optics [17,18]. This trend has been largely motivated by their negligible ohmic losses and their compatibility with complementary metal oxide semiconductor technology. Whereas plasmonic structures support plasmons, localized free electron oscillations at the surface of metallic particles, dielectric nanoparticles exhibit geometric resonances, concentrated within their volume. They are associated with the excitation of complex distributions of polarization currents [11]. Unlike conventional plasmonic particles [19], dielectric nanoparticles support a broad class of multipolar resonances of both electric and magnetic type, which can mutually interfere leading to remarkable phenomena such as the Kerker effect [20–24], nonradiating anapoles [25,26], spin to orbital angular momentum conversion [27], or magnetic light [28,29].

Intriguingly, even in the absence of heat dissipation, the modes of all-dielectric nanostructures are non-Hermitian in nature, as a result of radiation losses [30,31]. Leveraging non-Hermiticity in dielectric nanostructures has recently enabled the realization of quasibound states in the continuum in isolated cavities [32–34], and revealed a new superscattering regime accessible through modal interference [35].

Here, we aim at introducing non-Hermitian singularities as a prospective tool for dielectric resonant nanophotonics. Specifically, we study the conditions necessary for their realization and observation in a single dielectric nanoparticle with simple geometry. Critically, unlike the only early attempt [36], our design strategy is based solely on geometrical symmetry breaking [37,38], without changing the refractive index of the structure. This makes the EP realizable in a broad class of dielectric materials, including silicon. While we illustrate our approach with the lowest order electric and magnetic dipole modes (ED and MD modes) of a silicon nanodisk, the theory is general, allowing to design EPs from any two modes of a dielectric nanodisk fulfilling two conditions:

(1) Their resonant frequencies must cross as a function of aspect ratio,

(2) Their field distributions must have opposite mirror symmetry in the transverse plane of the structure.

In our example, we demonstrate that the EP corresponds to the onset of the strong coupling regime between the ED and MD modes. The modes radiate as a mixture of electric and magnetic dipoles, which result in a strong bianisotropic response, easily controlled by the symmetry breaking perturbation [Figs. 1(a) and 1(b)]. The latter can be probed experimentally, for instance, by measuring the asymmetry in backscattering from illuminations from top and bottom, as we calculate in Figs. 1(a) and 1(b).

Furthermore, we investigate the influence of a dielectric substrate, demonstrating how the latter provides an additional mechanism to tune the EP position in parameter space, facilitating the experimental implementation. Finally, we showcase the potential of bianisotropic EPs to detect small variations in the refractive index of the environment. The results confirm a much stronger sensitivity to perturbations in comparison with conventional ED and MD modes.

II. TWO-MODE MODEL OF A DIELECTRIC NANOPARTICLE

Consider a dielectric nanoparticle illuminated by a normal-incident plane wave. The plane wave excites several resonant modes in the particle. They can be understood as geometrical resonances, arising from the interference of traveling waves due to reflections from the cavity walls. They correspond to quasinormal modes (QNMs), solutions of Maxwell's equations with open boundary conditions in the absence of an exciting source [39,40].

For our purposes, we assume there are only two relevant modes in the spectral range of interest and denote them as $|m\rangle$ (as for MD), and $|p\rangle$ (as for ED).

Suppose that we perform a small perturbation in the shape of such particle. Then, the modes of the perturbed particle, $|u, d\rangle$, can be expressed as a linear combination of the original ones, e.g., $|u\rangle = C_m|m\rangle + C_p|p\rangle$. This yields a generalized eigenvalue problem [41–43] for the coefficients C_α :

$$\begin{pmatrix} \tilde{\omega}_p & 0 \\ 0 & \tilde{\omega}_m \end{pmatrix} \begin{pmatrix} C_p \\ C_m \end{pmatrix} = \tilde{\omega}_{u,d} \begin{pmatrix} 1 + V_{pp} & V_{pm} \\ V_{mp} & 1 + V_{mm} \end{pmatrix} \begin{pmatrix} C_p \\ C_m \end{pmatrix}. \quad (1)$$

Note that, since the modes leak energy to the environment, the eigenfrequencies $\tilde{\omega}_\alpha$ are complex, so that $\tilde{\omega}_\alpha = \omega_\alpha - i\gamma_\alpha$. ω_α is the resonant frequency of the mode, while γ_α is the loss rate. For an isolated mode, the loss rate is the half-width of the resonance peak.

The $V_{\alpha\beta}$ elements in Eq. (1) represent the perturbation. They are rigorously calculated as integrals of the modal fields over the surface of the nanoparticle [42]:

$$V_{\alpha\beta} = \Delta\varepsilon \int_S \mathbf{E}_\alpha^{(+)}(\mathbf{r}) \cdot \Delta V(\mathbf{r}) \mathbf{E}_\beta^{(-)}(\mathbf{r}) d^2\mathbf{r} = \langle \mathbf{E}_\alpha \Delta V \mathbf{E}_\beta \rangle. \quad (2)$$

The (+,−) signs in the superscripts indicate that the field must be evaluated directly outside or inside the surface of the particle, respectively. $\Delta\varepsilon$ is the permittivity contrast between the particle and the environment. The shape perturbation is contained in $\Delta V(\mathbf{r})$, which is the deformation undergone by the particle normal to its boundary. To ensure that the higher order modes can be safely neglected in the analysis, the latter need to fulfill the criteria [44] $|\tilde{\omega}_\alpha - \tilde{\omega}_\beta| \gg |\tilde{\omega}_\beta V_{\alpha\beta}|$, where α runs over all higher order modes, and $\beta = p, m$. Physically, this implies that their spectral separation needs to be much larger than their coupling with the modes of interest.

III. EXISTENCE OF EPs FOR A SINGLE DIELECTRIC NANOPARTICLE

We now explore the possibility to induce an EP in this system. First, we consider a simplified analytical form of Eq. (1), valid for modes with small loss rates [35]:

$$\mathcal{H}_0 \begin{pmatrix} C_p \\ C_m \end{pmatrix} = \tilde{\omega}_{u,d} \begin{pmatrix} C_p \\ C_m \end{pmatrix}, \quad (3)$$

$$\mathcal{H}_0 = \begin{pmatrix} \tilde{\omega}_p^{(1)} & \kappa_{pm} \\ \kappa_{mp} & \tilde{\omega}_m^{(1)} \end{pmatrix}, \quad (4)$$

where $\tilde{\omega}_\alpha^{(1)} = \tilde{\omega}_\alpha / (1 + V_{\alpha\alpha})$ and the coupling coefficients $\kappa_{\alpha\beta} \propto V_{\alpha\beta}$. \mathcal{H}_0 can now be interpreted as an effective

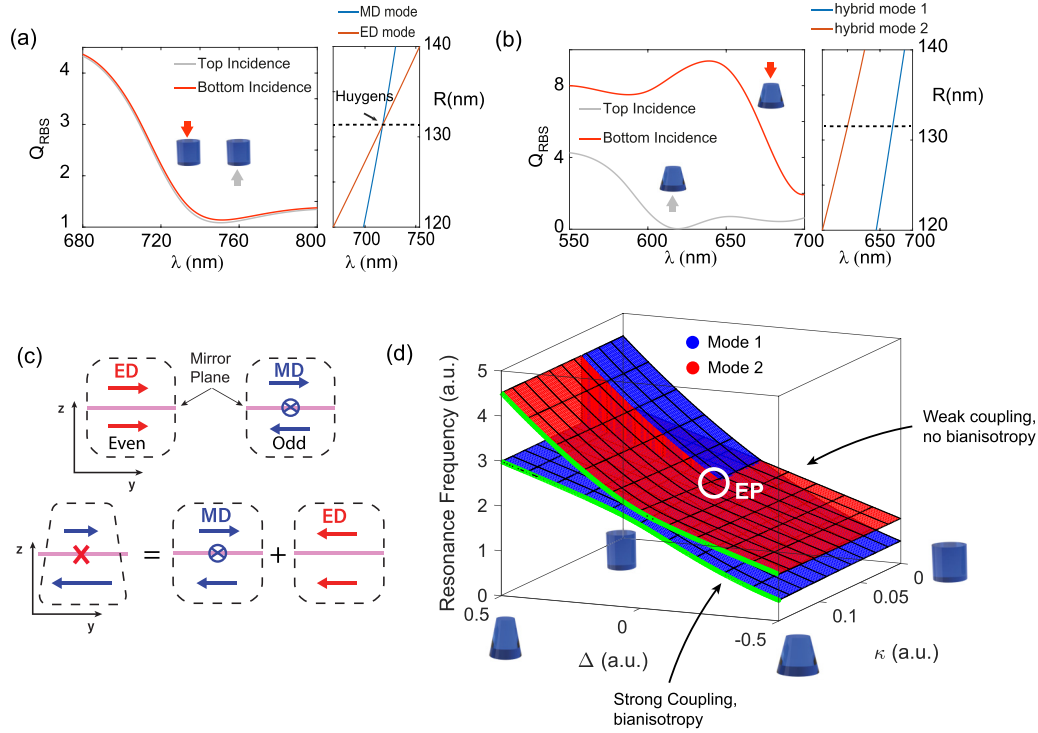


FIG. 1. Non-Hermitian transition from weak to strong bianisotropic response. (a) Right panel: Radar backscattering cross section (Q_{RBS}), for a weakly truncated silicon nanocylinder under normally incident plane wave illumination from top (red arrow in the left inset) and bottom (right inset). The top radius R has been optimized to induce a crossing between the electric and magnetic dipole modes (ED and MD modes), forming a Huygen's meta-atom, as shown in the left panel. The black dashed line indicates the chosen radius. (b) Same as (a), for a truncated nanocone with top radius 80 nm. The bottom radius is fixed to the one in (a). Due to the breaking of the vertical mirror symmetry, the modes are now coupled and possess a mixed bianisotropic character (hybrid modes 1 and 2). As a result, the back-scattering from the particle can be resonantly suppressed (enhanced) for illumination from the top (bottom). The decrease in the overall volume of the nanoparticle causes the resonances to blueshift. Parameters of the nanoparticles: height 100 nm, top radius 131 nm. (c) Top panel: symmetry of the ED and MD modes of a dielectric nanocylinder with respect to a horizontal mirror plane. Bottom panel: Effect of an out-of-plane perturbation on the MD mode, and equivalent decomposition into a sum of ED and MD contributions. (d) Evolution of the resonant frequencies of a prototypical non-Hermitian Hamiltonian, calculated with Eq. (5). Here, Δ denotes the detuning between the uncoupled modes and κ is the coupling coefficient, assumed to be real (see details in text). An EP marks the transition from weak to strong coupling regimes.

Hamiltonian dictating the evolution of the modes. The perturbed eigenfrequencies are found to be

$$\tilde{\omega}_{u,d} = E_{pm} \pm \sqrt{\Delta_{pm}^2 + \kappa_{pm}\kappa_{mp}}, \quad (5)$$

E_{pm} is the average of the two unperturbed eigenfrequencies, while Δ_{pm} is half their detuning, namely $\Delta_{pm} = (\tilde{\omega}_p - \tilde{\omega}_m)/2$.

Importantly, both real and imaginary parts of the perturbed eigenfrequencies become identical when the square root in Eq. (5) vanishes. This situation fully corresponds to an EP. If the leakage to the environment is small, and in the absence of ohmic loss, $\kappa_{pm} \approx \kappa_{mp}^*$. Two conditions are necessary to achieve the EP: (i) The resonant frequencies of the original modes must coincide so that $\omega_p = \omega_m$, and (ii) $2|\kappa_{pm}| \approx \pm(\gamma_p - \gamma_m)$. In conclusion, unlike conventional PT-symmetric structures, designing an EP in a passive, all-dielectric nanoparticle should require the ability to tune at least two parameters instead of one, in order to fulfill the two conditions above.

A prototypical example of the eigenfrequency evolution near a passive EP is displayed in Fig. 1(d). The latter has

been calculated with Eq. (5), assuming $\kappa_{pm} = \kappa_{mp}^* = \kappa$, where κ is real. This assumption is well justified for well-confined modes, since their fields can then be made approximately real, and this renders the coupling coefficients almost real by virtue of Eq. (2). Suppose we have $\omega_p = \omega_m$. From the discriminant in Eq. (5), three regimes can be distinguished. When $4\kappa^2 < (\gamma_p - \gamma_m)^2$, the square root becomes imaginary, hence the loss rates avoid the crossing, but the real part of the eigenfrequencies can cross. This is the weak coupling regime. When $4\kappa^2 = (\gamma_p - \gamma_m)^2$, condition (ii) is fulfilled, and an EP appears in the dispersion. Finally, increasing the coupling beyond this point $4\kappa^2 > (\gamma_p - \gamma_m)^2$, the square root becomes real, and the resonant frequencies avoid the crossing, while the loss rates can cross, marking the onset of the strong coupling regime.

The former theoretical discussion gives no hints as whether the EP can be reached in practice in a real physical system. In the following, taking as an example the electric and magnetic dipole modes, (ED and MD modes), we rigorously show that EPs can be supported by a single dielectric nanoparticle. As depicted in Fig. 1(d), the detuning can be controlled by varying height or radius, while the coupling can be increased

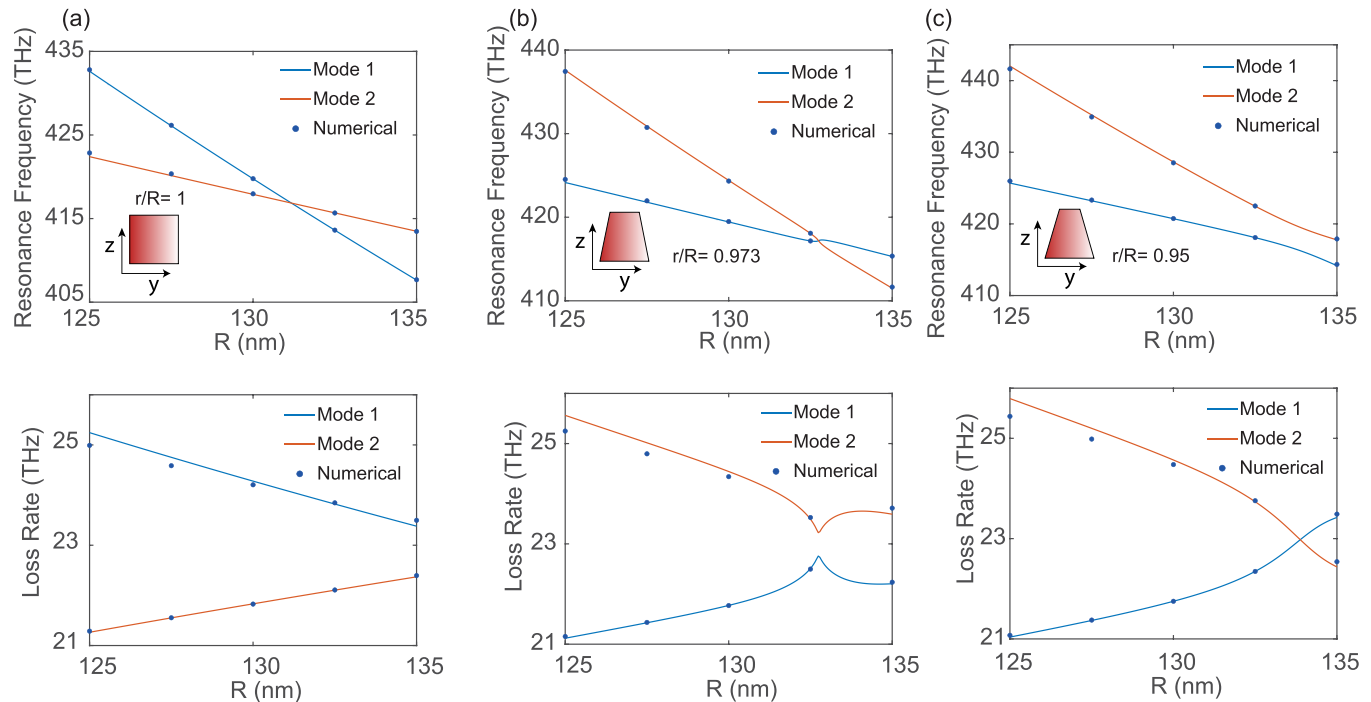


FIG. 2. Comparison between numerical results and the solution of Eqs. (1) for different coupling regimes. No fitting has been performed. In all cases, the height of the nanoparticle is fixed at $H = 100$ nm. Refractive index is kept at $n_p = 4$. The numerical results have been obtained with the eigenfrequency solver integrated in COMSOL Multiphysics. More details on the simulation setup can be found in Appendix A 1. (a) Upper panel: crossing of the real parts of the two resonant modes in the weak coupling regime, before transitioning through an EP ($r/R = 1$). The scheme depicts a cross section of unperturbed geometry. Lower panel: avoided crossing of the imaginary parts. (b) same as (a) for $r/R = 0.95$, near the EP. (c) Avoided crossing of the real parts in the strong coupling regime, after transitioning through the EP (upper panel), and crossing of the imaginary parts (lower panel). The schemes in (b) and (c) depict cross sections of the truncated cone (over-deformed for a better visualization). In all plots, lines of different colors indicate different QNMs obtained with Eq. (1). Only when $r/R = 1$ we can associate to each of them a pure electric or magnetic character.

with the conicity. Hence, we introduce a design strategy that enables full control over the detuning and coupling strength between multipolar modes, providing clear guidelines for the realization of Eps at the nanoscale.

We start by investigating the lowest order multipolar modes of a dielectric nanocylinder. A cylinder has full rotational symmetry along its principal axis and hosts a horizontal mirror plane perpendicular to it, which we label as σ_z . Note that, in consequence, the electromagnetic modes supported by the cylinder can be divided into even or odd with respect to σ_z . Namely, if we denote by $\hat{\sigma}_z$ an operator that reflects the electromagnetic field along σ_z , its action over a mode of the cylinder is $\hat{\sigma}_z \mathbf{E}_\alpha = \pm \mathbf{E}_\alpha$, depending on whether the mode is even or odd. A pictorial representation of the ED mode and the MD mode is shown in Fig. 1(c). It can be clearly seen that the first is even with respect to σ_z , while the second is odd.

As shown in Fig. 2(d), the resonant frequencies of the ED and MD modes [45,46] can be overlapped by varying the radius. In a simplified view, this can be understood by interpreting the ED mode as a standing wave formed between the lateral walls of the cylinder, and the MD mode as a Fabry-Perot mode that develops between the top and bottom walls of the cavity [26]. An increase in the separation between the lateral walls is then expected to strongly shift the ED resonance, but affect less the MD resonance, so that both can be brought together [see for instance the right panel in Fig. 1(a),

or Fig. 2(a)]. Thus, condition (i) can be easily fulfilled for the two modes. Interestingly, the diabolical point formed by the ED and the MD modes has been actively exploited for the realization of a so-called Huygen's meta-atom [46,47]. Such a scatterer is characterized by resonantly enhanced forward scattering due to far field interference between the ED and MD modes. This effect can be confirmed by the small values of backward scattering efficiency near resonance [Fig. 1(a)].

Unfortunately, condition (ii) cannot be realized, since the ED and MD modes cannot couple. They have different sign under $\hat{\sigma}_z$, so that

$$\kappa_{md} \propto \langle \mathbf{E}_m, \mathbf{E}_d \rangle = \langle \mathbf{E}_m \hat{\sigma}_z^\dagger \hat{\sigma}_z \mathbf{E}_d \rangle = -\langle \mathbf{E}_m, \mathbf{E}_d \rangle = 0. \quad (6)$$

In Eq. (6), we have used the fact that $\hat{\sigma}_z$ is unitary, and the perturbation $\Delta V = \Delta R$ can be taken out of the integral; it is just a constant equal to the difference between the radius of the original cylinder and the perturbed one. Importantly, Eq. (6) explains why EPs have not been yet implemented in all-dielectric nanophotonics: all even and odd modes of a nanodisk excited by a normally incident plane wave follow the same rule, and therefore cannot be brought to coalesce.

The only way of coupling the ED and MD modes is by breaking the mirror symmetry in some fashion. For example, as shown in Fig. 2(c), the cylinder can be transformed into a truncated cone. This asymmetric perturbation drastically alters the field distributions of the original modes. Its effect can

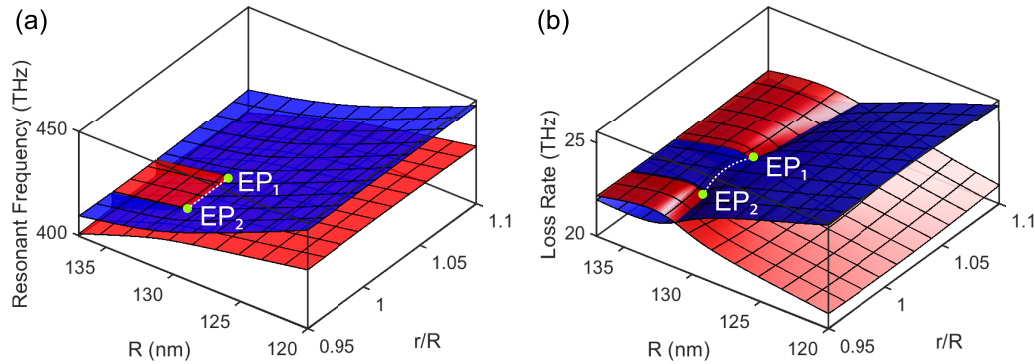


FIG. 3. Calculated Riemann surfaces of the two eigenfrequencies in the vicinity of a pair of EPs. To obtain them, two parameters of the structure need to be tuned: (i) the radius of a silicon cylinder in the vicinity of the Kerker condition (where the ED and MD modes overlap), (ii) the conicity of the particle (ratio between top and bottom radii). (a) Real part, (b) imaginary part (loss rate). White dashed lines indicate the region in parameter space displaying a bulk Fermi arc. Green dots correspond to the EPs, where both real and imaginary parts coalesce.

be graphically visualized by decomposing the perturbed field into the sum of the original and the perturbation [Fig. 1(c), lower panel]. Remarkably, the new modes are of mixed electric and magnetic nature, enabling them to mutually couple.

Having identified the key symmetry that needs to be broken gives us freedom to control at will the parameters entering in Eq. (5). The detuning can be set with the bottom radius R , while coupling can be induced by modifying the top-to-bottom ratio r/R .

In the next step, we calculate with the help of Eqs. (1) (which do not introduce any assumptions on the coefficients), the evolution of the eigenfrequencies as a function of the two parameters. In this case, $\Delta V(z) = z\Delta r/H + \Delta R$, where the origin of the z coordinate is taken at the bottom of the nanoparticle, H is the height of the unperturbed cylinder and $\Delta r = r - R$. Note that $\Delta V(z)$ allows us to investigate both radial deformations and variations in conicity. To demonstrate the accuracy of the analytical method, Fig. 2 compares the results of the latter with numerical simulations in COMSOL Multiphysics. Good agreement is observed in both real and imaginary parts for the weak, EP and strong coupling regimes. We clarify that the term strong coupling here strictly refers to the strong interaction between the Mie modes of the bare resonator, unlike other platforms exploiting the coupling between Mie modes and excitons, forming the so-called Mie-exciton polaritons [48,49].

Having confirmed the validity of our model, we calculate with the latter the evolution of resonant frequencies and loss rates of the two modes as functions of the two parameters, Fig. 3. Starting with a silicon nanocylinder with height 100 nm, increasing or decreasing r/R leads to the transition from a crossing of the resonant frequencies to an avoided crossing, a signature of strong coupling. The opposite occurs for the loss rates. Marking the transition from one regime to another, second order EPs can be found [green dots in Figs. 3(a) and 3(b)]. The two eigenfrequency sheets, more commonly known as Riemann surfaces, are seen to intersect each other. The Riemann surface for each mode is cut at an EP, giving rise to rich topological effects. Importantly, for a constant ΔR , $\Delta V(z)$ can take different sign depending on whether the top radius r is increased or decreased. The two situations correspond to the two possible ways of fulfilling

condition (ii). Thus, EPs in a single dielectric nanoparticle *always come in pairs*. They are connected in parameter space by an open arc along which the resonant frequencies of the two modes are degenerate but display different loss rates. This phenomenon is analogous to a bulk Fermi arc [9], connecting EPs of periodic systems in momentum space. Since the underlying physics is identical, we extend the same terminology to the case under study.

Figure 4 shows a projection of the Riemann surfaces as a function of r/R , near (blue lines), and far (orange lines) from the EP pair. After the EPs, there is an abrupt transition and the resonant frequencies split with a dependence $f_0 \propto \sqrt{r/R}$. Drastic changes in the loss rates are also observed. We remark an apparent asymmetry between the conditions to reach EPs for $r/R > 1$ and $r/R < 1$. This is simply because the bulk Fermi arc is not at constant R , but blueshifts slightly for decreasing r/R due to the change in the overall volume of the nanoparticle.

In this section, we have confirmed the possibility to realize EPs in a single dielectric nanoparticle. For the sake of simplicity, ohmic losses have been neglected. However, introducing a small extinction coefficient does not prevent the EP, only shifts it in the parameter space, and increases the loss rates of the involved QNMs. We have verified this in Fig. 9 of the Appendix.

IV. BIANISOTROPIC RESPONSE IN THE REGIME OF STRONG COUPLING

Most relevant physical phenomena associated with EPs do not occur exactly at the singularity but in the vicinity of it. For instance, a loop around an EP in parameter space leads to mode interchange. Two loops lead to a recovery of the original mode and an accumulated 4π non-Hermitian geometrical phase [8]. Other interesting phenomena are associated to the transition from the weak to the strong coupling regime [50,51]. In this section, we demonstrate how, after transitioning through the EP, the new hybrid modes naturally lead to a strong bianisotropic response in scattering [52,53].

To do so, we calculate the multipolar decomposition in the weak and strong coupling regimes (Fig. 5). Since the perturbation of the cylindrical shape breaks the mirror

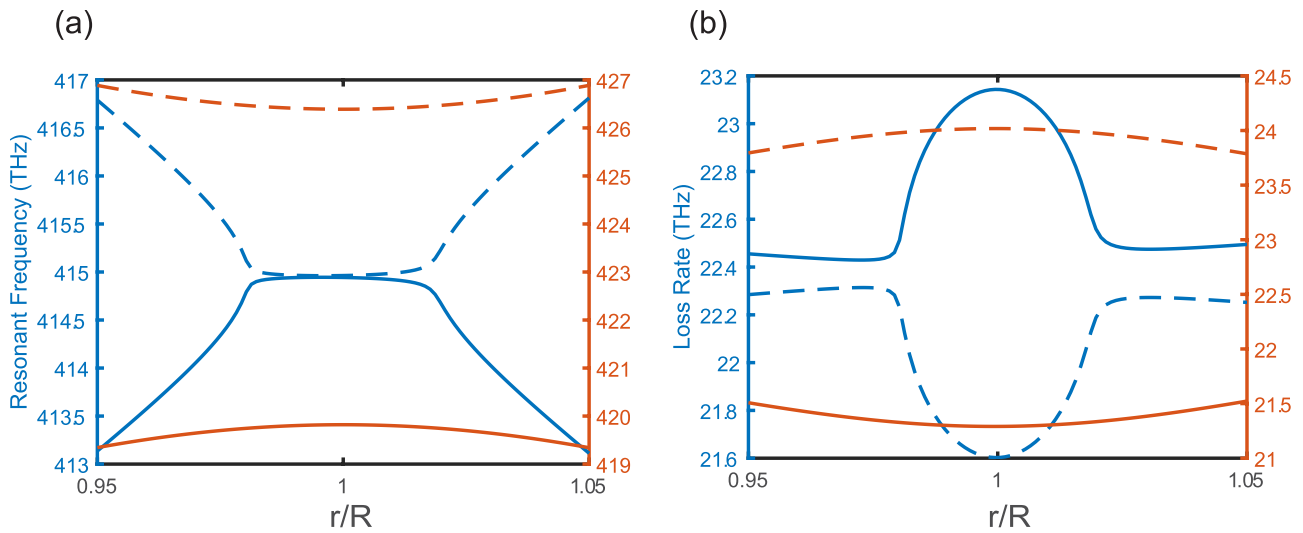


FIG. 4. Projections of the Riemann sheets in the vicinity of the bulk Fermi arc (blue lines), and far from it (orange lines), calculated for two different bottom radii. Dashed and solid lines indicate the path traced by each of the two QNMs. Near the bulk Fermi arc, the two resonant frequencies are identical, until a critical value of conicity (the EP), where a drastic change in the behavior can be observed.

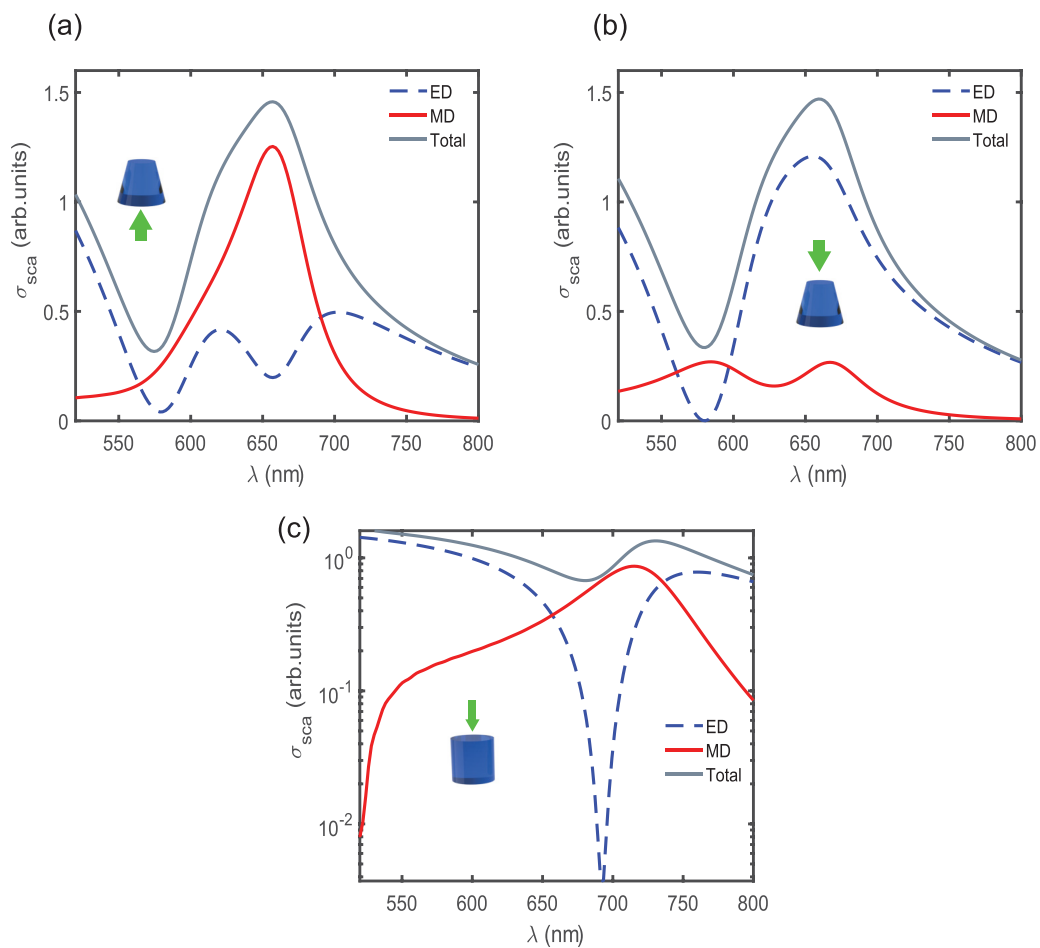


FIG. 5. Bianisotropy and strong coupling. (a) Multipolar decomposition of the scattering cross section for a truncated cone with $R = 130$ nm, $r/R = 0.6$, $H = 100$ nm, under incident plane wave illumination from the bottom, as depicted in the scheme. (b) Multipolar decomposition of the scattering cross section for the same truncated cone as in (a), with illumination from top. (c) Multipolar decomposition of an unperturbed nanocylinder with $R = 130$ nm. The vertical axis is shown in log-scale for better visualization. The multipole contributions have been obtained semi-analytically from full wave simulations in COMSOL Multiphysics, following the procedure in Ref. [56].

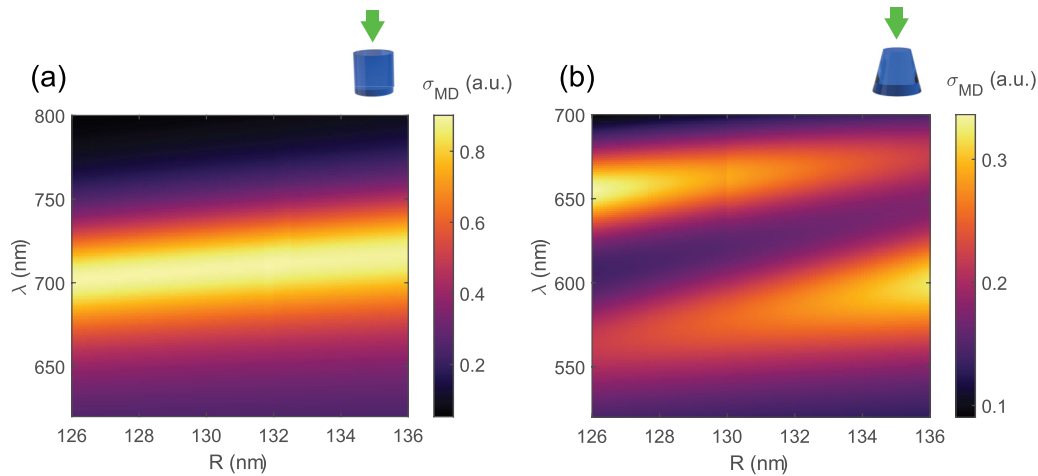


FIG. 6. Multipole decomposition before and after the transition through an EP. (a) and (b) Magnetic dipole scattering cross section before (a) and after (b) the transition through the EP, for different radii (ratio $r/R = 0.6$). In the second case, a clear split in the MD resonance can be observed in the strong coupling regime, in contrast to the weak coupling regime.

symmetry, in the strong coupling regime, we observe the emergence of enhanced omega-type bianisotropy [54]. This is a result of the enhanced mixing of the multipolar content of each mode: i.e., there is no longer a pure ED or MD mode, and exciting any of the two modes results in scattering of both electric and magnetic dipolelike fields.

As shown in Figs. 5(a) and 5(b), illumination from the top of the nanoparticle or from bottom results in a drastically different multipolar response, albeit preserving the same total scattering cross section [53–55]. For comparison, the multipolar decomposition of the unperturbed cylinder is shown in Fig. 5(c). As we show in Appendix A 3, the total scattering cross section must be preserved for any reciprocal scatterer with negligible ohmic losses. However, this restriction does not apply to the individual multipole contributions to the cross section.

Interestingly, with illumination from the top, two MD peaks can be clearly seen. However, the ED cross section resembles qualitatively the unperturbed nanocylinder [inset of Fig. 5(a)]. Conversely, when the nanocone is illuminated from below, the multipolar spectrum displays the reversed behavior, namely, the ED cross section splits up, while only a single MD peak can be observed.

In Fig. 6(a), we clarify the origin of the split in the MD cross section. The latter is associated with the repulsion of the modes in the strong coupling regime. To verify this, the MD cross section is plotted as a function of the bottom radius R for $r/R = 1$ and $r/R = 0.6$ (Fig. 6). In the strong coupling regime, the two peaks follow the shift in the resonant frequencies of the hybrid modes. Conversely, in the weak coupling regime (nanocylinder), only one peak is appreciable, corresponding to the MD mode.

V. SUBSTRATE-MEDIATED CONTROL OF EXCEPTIONAL POINTS

Now, we investigate the conditions necessary to design a bianisotropic EP in a practical experimental setup. As shown

in Fig. 3(a), the latter occurs at very small perturbations of the cylindrical shape (less than 5% change in the radial ratio). This can pose impediments for fabrication. A way to overcome this issue is acting further on the parameters entering condition (ii). Until now, only $\kappa \propto \Delta r/R$ was used. Since the difference between the loss rates of the electric and magnetic dipole modes in the cylinder is originally small, a small κ (and therefore a small perturbation of the top radius) is required to compensate it and reach the EP. We propose to introduce a substrate as an additional tuning mechanism. As we explain in the next paragraph, the latter can modify the difference between the loss rates $\Delta\gamma$ [Fig. 7(a)]. By doing so, the deformation Δr to achieve the EP can be increased to experimentally accessible values.

The underlying physical mechanism can be understood regarding the ED and MD modes as standing waves of different geometrical origin [26]. As mentioned earlier, the ED mode appears from oscillations between the lateral walls (Mie mode), while the MD mode can be associated with oscillations between the top and bottom walls of the nanocylinder (Fabry-Perot mode). It is thus expected that a modification in the index contrast of the bottom wall of the cavity affects more strongly the MD mode. The eigenfrequency of a Fabry-Perot mode is given, approximately, by Ref. [26] $\omega_\ell = c/n_p H[\pi\ell + i \ln(r_{\text{top}}r_{\text{bott}})]/2$, where ℓ is the number of maxima of the standing wave, n_p is the particle refractive index, and r_{top} , r_{bott} are the reflection coefficients from the top and bottom walls, respectively. The latter expression is derived assuming no coupling with other modes takes place due to the substrate. For high-index nanoparticles deposited on top of dielectric substrates, the assumption is well fulfilled, since the modal fields are strongly confined within the high-index region. From the latter expression, we see that a change in the substrate index affects only the imaginary part of the eigenfrequency, i.e., the loss rate of a Fabry Perot mode evolves approximately as $\gamma \propto \ln(r_{\text{top}}r_{\text{bott}})$. Conversely, the loss rates of Mie modes are not strongly affected.

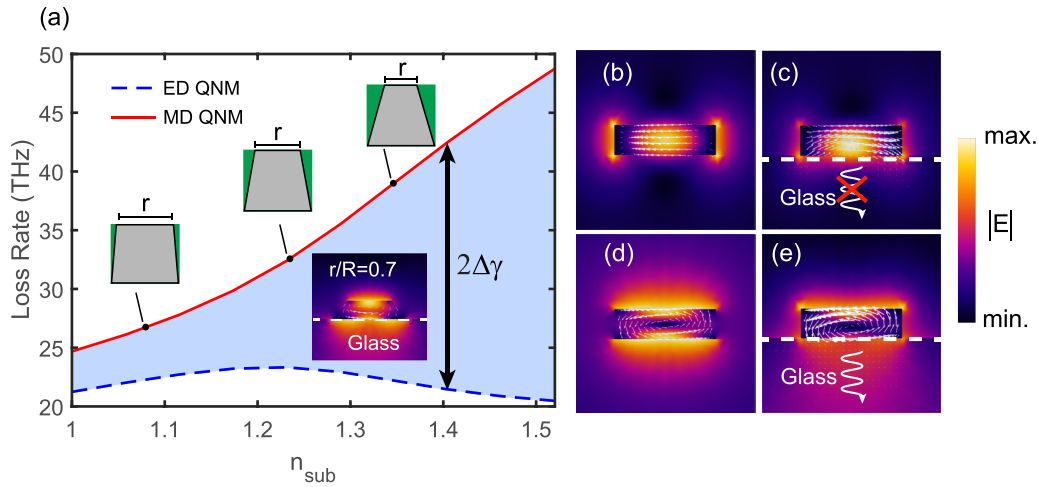


FIG. 7. Substrate engineering of exceptional points. By controlling the substrate index, the difference between the loss rates of the two involved resonant modes can be tuned, thus modifying the critical ratio r/R where the EP will take place. (a) Loss rates of the ED and MD modes of the cylinder as a function of the substrate index. The schematic insets depict how the ratio r/R needed to reach the EP is increased with the substrate contrast. The field inset shows the near field of the EP when the particle is deposited on glass. (b) and (c) Field distribution (electric field norm) of the ED mode of the cylinder in a homogeneous environment (air) (b) and on top of a glass substrate (c). (d) and (e) Same as (c) and (d), but for the MD mode.

The qualitative arguments given above can be exploited as design rules to control the difference between the loss rates of both modes entering condition (ii). To verify our theory, we calculate the loss rates of the ED and MD modes in the nanocylinder for different substrate index contrasts [Fig. 7(a)]. We confirm that the ED mode maintains an approximately constant loss rate, while the MD mode almost doubles it when varying the substrate from pure water to glass ($n_g = 1.52$). Figures 7(b)–7(e) display the field distributions of the ED and MD modes of the nanocylinder in air environment [Figs. 7(b) and 7(d)] and when the resonator is placed on top of a glass substrate [Figs. 7(c) and 7(e)]. In agreement with our physical picture, the ED mode shows no significant changes besides an overall redistribution of the near field. In contrast, the MD mode leaks strongly to the substrate, resulting in an increase of its loss rate.

As a result of the above, the conicity required to reach the EP increases, as depicted in Fig. 8(a). On top of glass, the EP occurs near $r/R = 0.7$, an almost 30% change with respect to the nanoparticle in an homogeneous environment.

Summarizing, we have shown that a dielectric substrate can be used to control the loss rates of Fabry-Perot modes, enabling fine tuning of the EP condition. Specifically, with an increasing substrate index, the EP condition is fulfilled at larger conicities. This mechanism can be exploited to design EPs in nanoparticles with experimentally accessible geometries.

VI. DESIGN OF LOCAL REFRACTIVE-INDEX SENSORS

EPs in microcavities have attracted a great deal of interest for the development of ultrasensitive sensor technology [5,14]. Specifically, they can improve significantly the performance of sensing schemes based on the detection of resonance splittings. The basic idea is the following; in a system where two or more modes are degenerate (the sensor), a perturbation ϵ (the target signal) lifts the degeneracy resulting in a

splitting of the resonant frequencies, which can be detected and quantified. For conventional degeneracies (that is, systems where two resonances are simply brought together), the split is linear with the perturbation. However, owing to the topology of the Riemann sheets surrounding the EP, the separation between the resonant frequencies turns out to be in the order of $\sqrt{\epsilon}$. For a sufficiently small perturbation, the sensitivity of an EP is larger than that of a conventional degeneracy [4,5].

In this section, we study the benefits that EPs supported by single nanoparticles could offer for all-optical nanosensors with miniature footprints. Specifically, we envision the possibility to detect local changes in the refractive index of a surrounding environment.

To assess the potential of bianisotropic EPs in biosensing, we first optimize our truncated nanocone placed on top of a glass substrate and embedded in water [refer to scheme in Fig. 8(a)].

Emulating a conventional experimental protocol [57], we introduce small variations in the refractive index of the surrounding environment (n_{env}), ranging from pure water to a concentrated solution of ethylene glycol 9:1 ($n_{\text{env}} = 1.34$), as shown in Fig. 8(a). The change in n_{env} introduces a small perturbation to the system's modes, making them depart from the EP. We envision a setup capable of tracking the evolution of the split between the resonant frequencies, Δf . Experimentally, the eigenfrequencies can be extracted with a pole fitting of, e.g. forward scattering [5], or by means of a principal component analysis [58].

As a comparison, in Fig. 8(a) we also plot the evolution of Δf in a conventional diabolical point formed by overlapping the resonant frequencies of the MD and ED modes of the original nanocylinder. The latter shows the expected linear dependence with n_{env} , originated by a distortion of the evanescent fields of the subwavelength modes. In stark contrast, near the EP, Δf demonstrates a square-root behavior. To further

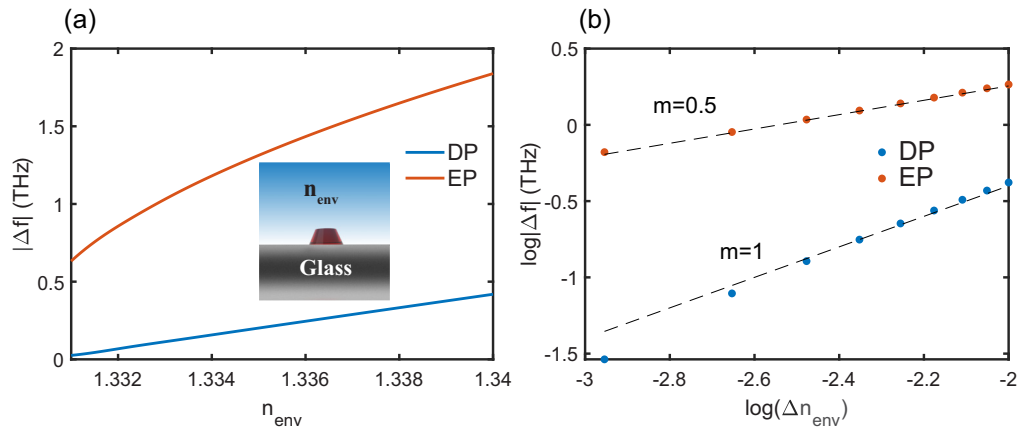


FIG. 8. Refractive index sensitivity near a bianisotropic exceptional point. The nanocone dimensions are $R = 191$ nm, $r/R = 0.575$, $H = 100$ nm. (a) Split between the resonance frequencies of the two modes when collapsed in an EP or simply brought together by tuning the radius of the nanocylinder, forming a diabolical point (DP). The local refractive index is varied from pure water to a solution of ethylene glycol in water ($n_{\text{env}} = 1.34$), emulating realistic experimental conditions [57]. (b) Log-Log plot of (a) and linear fit (dashed lines). The slopes (m) show a linear dependence of the modes at the DP with Δn_{env} , while a square-root dependence ($m = 0.5$), is confirmed for the EP.

confirm the power laws, Fig. 8(b) displays a log-log plot of Δf as a function of the change in the environment refractive index, Δn_{env} . The dashed lines correspond to a linear fit, with slope m . The diabolical point displays a slope of 1, whereas the EP is well fitted with $m = 0.5$.

Importantly, due to the square-root asymptotic for small Δn_{env} , frequency splits near the EP are much larger than those of a conventional dielectric disk optimized at a diabolical point. In consequence, we anticipate optical biosensing schemes based on bianisotropic EPs to be more sensitive to local environment changes than previously reported devices implemented with conventional ED and MD resonances.

VII. CONCLUSION AND OUTLOOK

We have predicted the existence of non-Hermitian singularities, EPs, in a single dielectric nanoparticle, subject solely to geometrical perturbations. To do so, we have proposed and validated a modal theory which evidences the critical role of vertical symmetry breaking to achieve the EP. The latter is general and tells us that any two resonant modes of a dielectric disk whose eigenfrequencies cross with the aspect ratio and have opposite sign with respect to a mirror reflection perpendicular to the z axis can be brought to an EP by a continuous perturbation that breaks the mirror plane of the disk. Our theory can be applied to other structures beyond the case of a disk. For instance, we have also designed an EP in a rectangular dielectric nanowire, by introducing a hole that breaks the mirror symmetry (Appendix A 4).

Critically, our approach can be implemented for conventional dielectric materials in the near-IR part of the visible spectra, such as silicon. As an example, we demonstrate the formation of a bianisotropic EP between the ED and MD modes by transforming a nanodisk into a truncated nanocone. We confirm a transition from weak to strong coupling mediated by the EP, and omega-type bianisotropy arising in the strong coupling regime. In the latter case, the avoided crossing can be observed in the multipolar decomposition of the

scattering cross section. For the first time to our knowledge, we study the role of a dielectric substrate and build a physical picture on its influence on the EP. Interestingly, we suggest theoretically and verify numerically that the latter can be used to bring the EP to experimentally accessible geometrical parameters.

Finally, we propose to utilize bianisotropic EPs to detect small local changes in the refractive index of an aqueous environment. The idea is based on tracking the split between the resonant frequencies of the two modes. The numerical results demonstrate that much larger sensitivities can be obtained for the EP in comparison with the ED and MD modes of a nanodisk. This is due to the anomalous square-root dispersion characteristic of EPs, in contrast to the conventional linear one. The results suggest a route towards novel biosensing schemes based on single dielectric nanoparticles tuned near an EP. We remark, however, that the performance of future EP sensors implemented with dielectric nanoparticles will be limited by noise as well as fabrication imperfections, same as in other platforms [59].

Thus, this work sets the basis for bringing together all-dielectric nanophotonics and EP physics, paving the way for exciting applications and new uncharted effects. For instance, isolated dielectric nanoparticles supporting EPs can be combined in clusters, which should enable the realization of higher-order EPs [14]. Another intriguing direction is the study of the interplay between EPs and other scattering anomalies commonly encountered in dielectric nanostructures, such as bound states in the continuum [60], lattice resonances or supercavity modes. All these are associated with high quality factors, which could drastically improve the performance of EP-based dielectric biosensors [1].

Beyond sensing, EPs play a crucial role in the emerging field of non-Hermitian topological physics [10]. By revealing the existence of non-Hermitian singularities in realistic Si nanostructures, all-dielectric nanophotonics presents itself as an ideal playground for testing and validating ideas uprooted in the latter.

ACKNOWLEDGMENTS

The authors thank A. Bogdanov for fruitful discussions. The research was supported by Priority 2030 Federal Academic Leadership Program. The authors gratefully acknowledge the financial support from the Ministry of Science and Higher Education of the Russian Federation (Agreement No. 075–15–2022–1150). The numerical simulations of exceptional points were partially supported by the Russian Science Foundation Grant No. 23–72–00037. V.B. acknowledges the support of the Latvian Council of Science, project: NEO-NATE, No. lzp-2022/1–0553. Y.K. acknowledges a support from the Australian Research Council (Grant No. DP210101292), as well as the International Technology Center Indo-Pacific (ITC IPAC) and Army Research Office under Contract No. FA520921P0034.

APPENDIX: METHODS AND ADDITIONAL EXAMPLE

1. Details on the numerical models

The numerical simulations of the eigenmode spectra of all the structures in the main article were performed with COMSOL Multiphysics © using its finite-element method eigenmode solver, which is based on the Arnoldi iteration method. Two numerical models were developed:

Model (i): nanoparticle embedded in a homogeneous medium. The latter is placed at the center of the simulation domain and surrounded by a sphere (refractive index n_m), with radius in the order of a third of the resonant wavelengths of interest, (hereon denoted as λ for brevity). The air sphere is enclosed by a spherical perfectly matched layer (PML) with thickness in the order of λ . At the external boundary of the PML, the perfect electric conductor boundary condition is imposed. All domains except the PML are meshed with a tetrahedral mesh, with a maximum mesh size $d_{m,p} = \lambda/5n_{m,p}$, where p denotes “nanoparticle”. The PML is meshed with a swept mesh with at least 5 mesh elements across.

Model (ii): Nanoparticle embedded in a medium, deposited on top of a substrate with refractive index n_{sub} . To introduce a substrate, the simulation domain, excluding the nanoparticle, is subdivided in an upper (\mathcal{S}_+) and a lower (\mathcal{S}_-) hemisphere with their respective spherical PML regions, and the nanoparticle is placed on top of \mathcal{S}_- . \mathcal{S}_- is then assigned the refractive index n_{sub} . \mathcal{S}_+ is assigned the refractive index n_m . The physical domain of \mathcal{S}_- is meshed with a tetrahedral mesh with a maximum mesh size $d_s = \lambda/5n_{\text{sub}}$. The PML domain in \mathcal{S}_- is meshed with a swept mesh with at least 10 mesh elements across, keeping a maximum mesh size d_s . The nanoparticle mesh settings and the boundary conditions are kept the same as in model (i).

2. Influence of ohmic losses

Realistic dielectric materials display small amounts of ohmic losses. Therefore, it is necessary to study the influence of loss in the proposed design. Figure 9 displays the effect of introducing a small extinction coefficient k on the dispersion near the EP. The value chosen for k is typical for Si in the near-IR part of the visible. It can be seen that the resonance frequencies are almost unaffected, while the loss rates of both

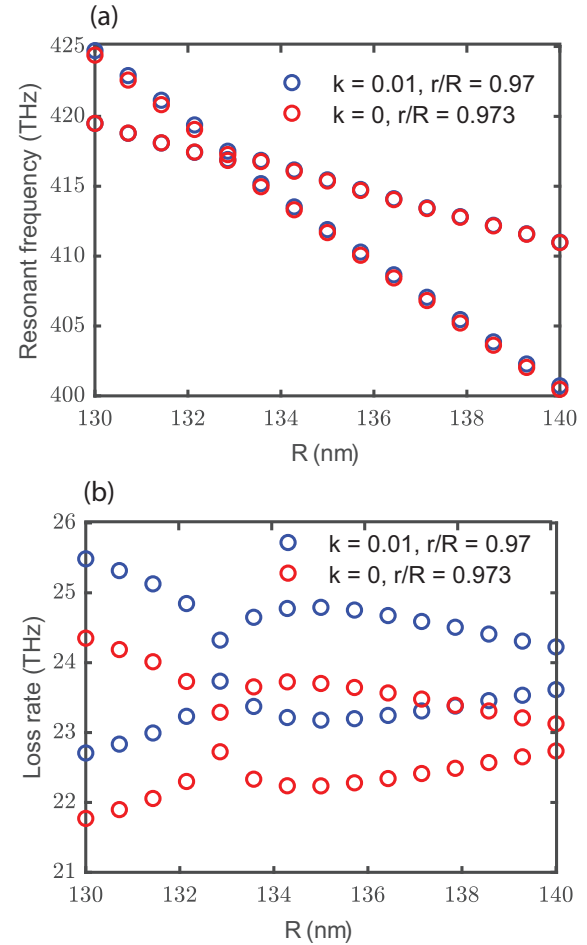


FIG. 9. (a) Resonant frequencies in the vicinity of the EP for a truncated cone, with refractive index $n = 4 + ik$ with $k = 0$ and $k = 0.01$. (b) Loss rates for the two cases in (a). Height of the nanoparticles is fixed to 100 nm.

modes increase for all particle radii studied. However, the EP dispersion can still be observed.

3. Dipolar model of the bianisotropic response

In this section, we follow the approach in Ref. [53] and provide a simple model that explains the main qualitative features of the bianisotropic response of the truncated cone, in the dipolar approximation. We assume, without loss of generality, that our nanoparticle is illuminated by a normally incident, x-polarized plane wave with amplitude $|E_x^{\text{inc}}|$. Only the electric and magnetic dipole moments have non-negligible contributions to the cross section. Hence, we can write:

$$\frac{p_x^\pm}{\epsilon_0} = \alpha_{ee} E_x^{\text{inc}} \pm \alpha_{em} Z_0 H_y^{\text{inc}}, \quad (\text{A1})$$

$$Z_0 m_y^\pm = \alpha_{me} E_x^{\text{inc}} \pm \alpha_{mm} Z_0 H_y^{\text{inc}}, \quad (\text{A2})$$

where the α_{ij} are the elements of the (magnetolectric) polarizability tensor and Z_0 is the vacuum impedance. The \pm signs are selected depending whether we have illumination from the top (+) or from the bottom (−). Imposing reciprocity, it can be shown that $\alpha_{em} = -\alpha_{me}$. With the help of this model, the

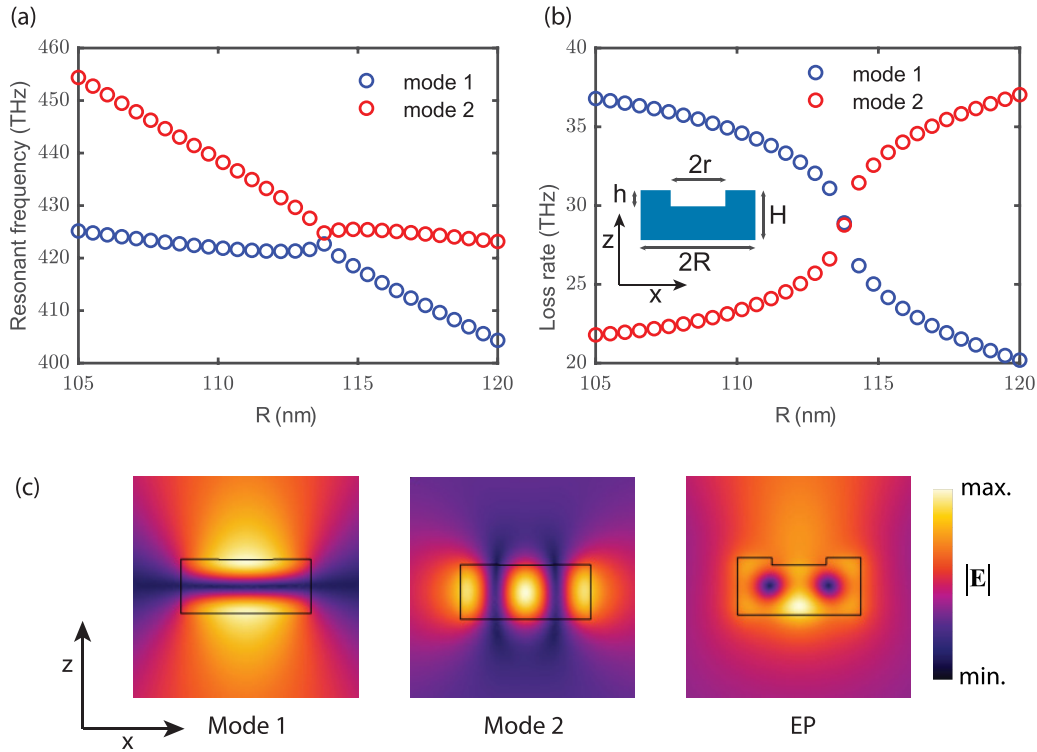


FIG. 10. (a) Resonant frequencies in the vicinity of an EP designed in a perturbed rectangular nanowire. (b) Loss rates for the two cases in (a) Inset: sketch of the geometry. (c) From left to right, electric field norms in the x - z plane of mode 1 and mode 2 in the unperturbed nanowire ($h = 0$) and near the EP ($h = 13.43$ nm, $R = 113.8$ nm). Structure parameters for panels (a) and (b): $n = 4$, $H = 100$ nm, $r = 50$ nm, $h = 13.43$ nm.

extinction cross section gives

$$\sigma_{\text{ext}} = k \text{Im}(\alpha_{ee} + \alpha_{mm}). \quad (\text{A3})$$

We can see from Eq. (A3) that σ_{ext} does not depend on the magnetoelectric components α_{em} , α_{me} . Instead, it only depends on the conventional response, which does not change sign with the direction of illumination. In the absence of losses $\sigma_{\text{ext}} = \sigma_{\text{sca}}$. As a result, the total scattering cross section does not change by reversing the direction of illumination. However, if we decompose σ_{sca} into its ED and MD contribution, we get

$$\sigma_{\text{sca}} = \sigma_p + \sigma_m = \frac{k^4}{6\pi} (|\alpha_{ee} \pm \alpha_{em}|^2 + |-\alpha_{em} \pm \alpha_{mm}|^2). \quad (\text{A4})$$

$\sigma_{p,m}$ do depend on the magnetoelectric terms, and can change significantly upon reversing the illumination direction.

4. Bianisotropic EP in a perturbed rectangular nanowire

The approach described in the main text is general and applies beyond the case of a truncated cone. As an additional example, here we report the emergence of a bianisotropic EP in a perturbed rectangular nanowire. The geometry is sketched in Fig. 10(b). It consists of a rectangular nanowire with height H , radius R and refractive index $n = 4$. Two modes (mode 1 and mode 2) are selected since they cross and mode 1 (2) is odd (even) in the unperturbed nanowire. Their field distributions are shown in the two leftmost panels of Fig. 10(c). To induce the EP, we drill a hole of radius r and height h . All the geometrical parameters are given in the caption of Fig. 10.

Keeping r constant, we gradually increase h and study the mode dispersions as a function of R . The EP can be found when $h \approx 13.43$ nm and $R \approx 113.8$ nm.

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