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Letter

Loss of percolation transition in the presence of simple tracer-media interactions

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Random motion in disordered media is sensitive to the presence of obstacles which prevent atoms, molecules, and other particles from moving freely in space. When obstacles are static, a transition between confined motion and free diffusion occurs at a critical obstacle density: the percolation threshold. To test if this conventional wisdom continues to hold in the presence of simple tracer-media interactions from the type seen in recent experiments, we introduce the *Sokoban* random walk. Akin to the protagonist of an eponymous video game, the *Sokoban* has some ability to push away obstacles that block its path. While one expects this will allow the *Sokoban* to venture further away, we surprisingly find that this is not always the case. Indeed, as it moves—pushing obstacles around—the *Sokoban* always confines itself after traveling a characteristic distance that is set by the initial density of obstacles. Consequently, the percolation transition is lost. This finding breaks from the ruling ant in a labyrinth paradigm, vividly illustrating that even weak and localized tracer-media interactions cannot be neglected when coming to understand transport phenomena.

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Introduction. More than a century after their introduction to the readers of *Nature* by Karl Pearson [1], random walks continue to fascinate and draw attention [2,3]. While initially motivated by the theory of gambling [4] and financial speculation [5], random walks became important in the natural sciences following the pioneering works of Einstein [6], Smoluchowski [7], and others [8] on diffusion of atoms and molecules. In the time following the publication of these seminal works, random walks were further established as a versatile modeling tool [9–14], with applications in physics [15–22], chemistry [23–28], biology [29–33], movement ecology [34–39], and finance and economics [40,41].

One paradigmatic random walk is the "ant in a labyrinth" (AIL) [42], which was introduced by de Gennes as a simple model for diffusion in disordered media [43–45] (similar ideas were explored by Brandt [46], Kopelman [47], and Mitescu and Roussenq [48]). Consider an ant walking on a two-dimensional square lattice, where a fraction ρ of the lattice sites are occupied with obstacles (other sites are empty). Each time unit, the ant takes a step into an empty neighboring site that is chosen randomly. Given a specific obstacle density ρ, one can ask how does the mean squared displacement (MSD) of the ant depend on time? When ρ is small, i.e., most sites are empty, the ant's motion is almost unobstructed. In this case, the MSD scales linearly with time. On the other extreme, when ρ is large, most sites are occupied by obstacles and the ant's motion is highly restricted. In this limit the ant will find itself caged in the labyrinth, resulting in an MSD that saturates asymptotically. As it turns out, for large enough systems, the transition between restricted motion and free diffusion is sharp, occurring at a critical density $0 < \rho_c < 1$ [49].

The basic assumption in de Gennes' model is that obstacles comprising the media are immobile—namely, that the ant's motion has no affect on the distribution of obstacles around it. This assumption is fair when, e.g., considering a hydrogen atom diffusing in a solid [50]. Yet, it clearly breaks for active tracers, e.g., animals, microorganisms, and biological "machines" that plow their way through crowded environments. Recent controlled experimental studies of such scenarios revealed that the motion of active tracers in the presence of static—yet pushable—obstacles is inherently different than in a field of obstacles that cannot be moved. This happens because the tracer's motion directly affects obstacles in its path, thus altering the density field accordingly [51,52]. This, in turn, creates long-term memory [53], which could, e.g., alter first-passage times [51,52]. It is thus clear that tracer-media interactions are not negligible in such systems and that a different approach is required to describe and capture the emerging phenomena.

Model. We introduce a minimalist model to show that tracer-media interactions from the type mentioned above result in a drastic, qualitative change of transport properties (also see [51]). To this end, we consider a random walker that has *some* ability to push away obstacles that block its path. We imagine an $n \times n$ square arena where a fraction ρ of the available sites are occupied by obstacles. Taking n to be odd, we place a random walker at the center of this arena. The random walk then takes place according to the following rules which are illustrated in Fig. 1(a). The walker can move into an unoccupied neighboring site, placed horizontally or vertically relative to its position. In addition, even when a site

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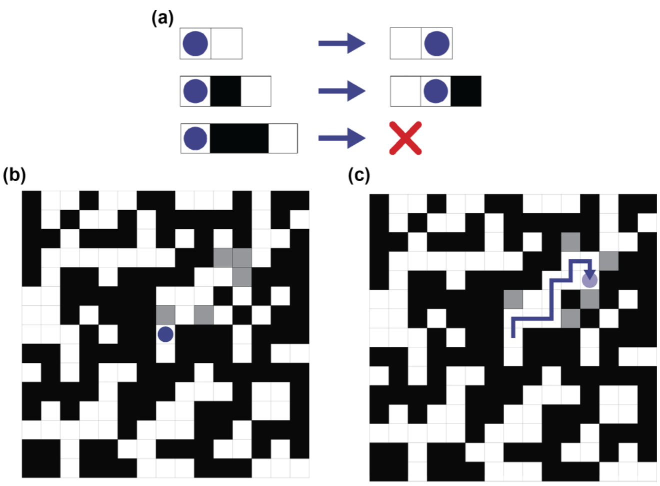

FIG. 1. *Sokoban* random walk. Panel (a): laws of motion. The walk has two feasible moves: (i) it can step into an unoccupied site; (ii) it can step into an occupied site by pushing away an obstacle that occupied it one site forward, in its direction of motion. Only one obstacle can be pushed at a time. Thus, two obstacles in a row create a block. At each time step, the walker chooses between all feasible moves with equal probability. Panel (b): initial configuration of a 15×15 arena. Arenas are generated by randomly distributing obstacles, such that each site has probability ρ to be occupied. White squares indicate unoccupied sites. Black and gray squares indicate obstacles (these are identical, but distinguished here for clarity). Panel (c): a possible trajectory of the *Sokoban* random walk. Gray squares indicate obstacles that were pushed during the course of the walk.

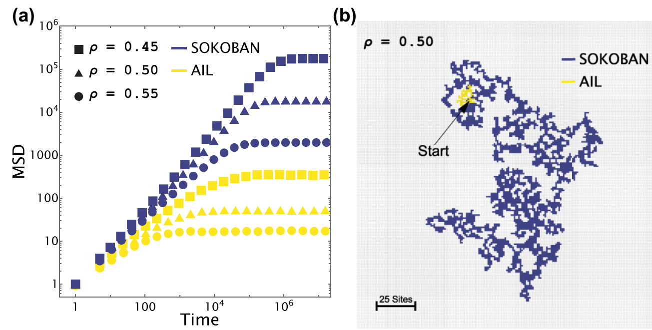

FIG. 2. *Sokoban* random walk vs AIL. Panel (a): mean squared displacement (MSD) of an AIL (yellow) and the *Sokoban* random walk (purple) as a function of time, for three different obstacle densities: $\rho = 0.45, 0.5, 0.55$. In the long time limit, the MSD of the *Sokoban* is orders of magnitude higher compared to the MSD of the AIL. Panel (b): trajectories of the AIL (yellow) and *Sokoban* (purple) random walks, starting in an *identical* arena with $\rho = 0.5$. The difference in MSD is evident.

is occupied by an obstacle, the walker can move into this site while pushing the obstacle one site forward, in its direction of motion. Yet, this can only be done provided that the next site (in the direction of motion) is vacant. Thus the walker cannot push more than one obstacle at a time. Finally, at each time step, the walker chooses between all feasible moves with equal probability.

The model presented herein is inspired by the video game *Sokoban* (Japanese for warehouse keeper), which was created in 1981 by Imabayashi. The premise of the game is simple: the player, playing as the keeper, pushes boxes around in a warehouse, in attempt to transport them to marked storage locations. The rules of the game are similar to the rules of the walk presented in Fig. 1. While being fairly simple to play, solving *Sokoban* puzzles turns out to be a difficult computational task. It was first proved to be NP-hard [54] and was later shown to be PSPACE-complete [55].

An illustration of a trajectory of the *Sokoban* random walk is given in panels (b) and (c) of Fig. 1. The initial configuration of the arena is given in panel (b) and the trajectory of the walk is illustrated in panel (c). Note that an AIL cannot push obstacles that stand in its path and would have thus been caged by the initial configuration of the arena. In contrast, the *Sokoban* was able to escape this cage by pushing some of the obstacles surrounding it (highlighted in gray). More generally, we expect that the ability to push obstacles will enable the *Sokoban* random walk to venture further away from its initial position when compared to an AIL that does not have this ability.

Monte Carlo simulations. To test this hypothesis, we simulate the *Sokoban* random walk and AIL, for a large number of randomly generated arenas that were moreover taken to be sufficiently large so as to completely avoid boundary effects. In Fig. 2(a) we plot the mean squared displacement, given by $\mathrm{MSD}(t) = \langle r^2(t) \rangle$, where $r(t)$ is the Euclidean distance to the initial position at time t and $\langle \cdot \rangle$ indicates an ensemble average over all generated walks. Plots are made for the *Sokoban* (purple) and AIL (yellow) random walks at three different obstacle densities. As expected, in the long time limit, the MSD of the *Sokoban* is significantly higher compared to the MSD of the AIL. As a result, the *Sokoban* explores larger portions of the arena as illustrated by the trajectories given in Fig. 2(b). Further illustration of the *Sokoban* walk is provided in a Supplemental video [56].

All densities in Fig. 2 were taken to be above the 2D site-percolation threshold, namely, the critical density $\rho_c \simeq 0.407$ [57] above which the AIL's MSD saturates at long times. The fact that for these densities the *Sokoban* is able to explore larger portions of the arena hints that the critical density for this walk should be higher than, or equal to, the percolation threshold; thus allowing the *Sokoban* to roam unbounded when the density of obstacles drops below ρ_c. However, when simulating the *Sokoban* for $\rho < \rho_c$, we surprisingly find that its MSD still saturates in the long time limit. An example is given in panels (a)–(c) of Fig. 3, where we take $\rho = 0.4$, and present a time evolution of a typical *Sokoban* trajectory. Snapshots are taken for $t = 10^5,\ 10^6$, and 10^7. For $t \gtrsim 10^7$ the walk does not visit new sites, indicating it is indeed confined (see Fig. S1 [56]).

Further evidence that the *Sokoban* random walk dynamically confines itself at densities lower than the percolation threshold comes from extensive numerical simulations that we perform for this system. Defining the exploration radius $r_\infty = \lim_{t\to\infty} \sqrt{\mathrm{MSD}(t)}$, i.e., the level at which the square root of the MSD saturates, we plot this quantity as a function of ρ for the *Sokoban* random walk and AIL [Fig. 3(d)]. As

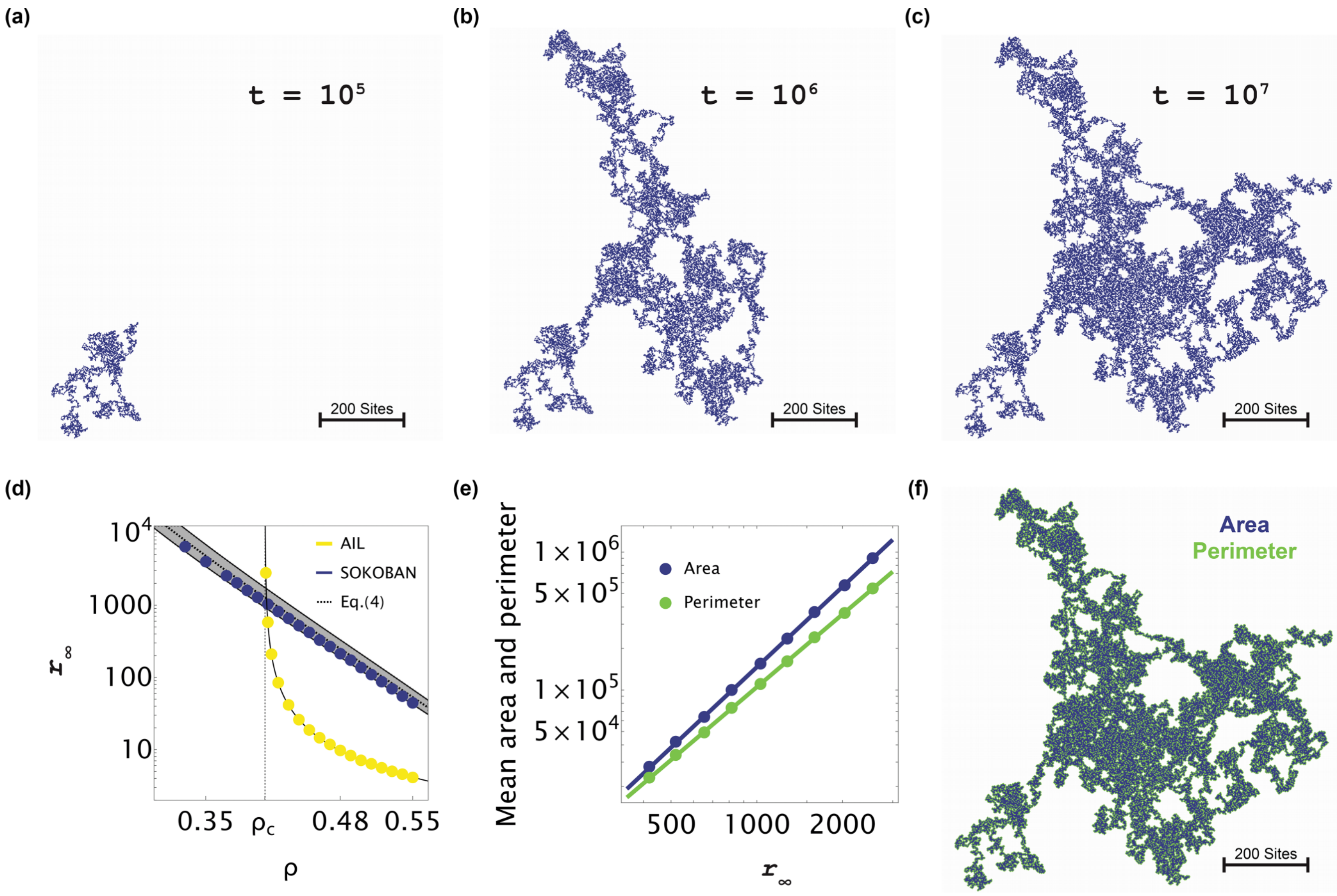

FIG. 3. Loss of percolation transition in the *Sokoban* random walk. Panels (a)–(c): sites visited by the *Sokoban* random walk, in an arena with an obstacle density of $\rho = 0.4$, for $t = 10^5, 10^6, 10^7$. By time $t = 10^7$ the *Sokoban* random walk is effectively caged and does not visit new sites (see comparison with $t = 10^8$ in Fig. S1 [56]). Panel (d): the exploration radius $r_\infty = \lim_{t\to\infty}\sqrt{\mathrm{MSD}(t)}$ for the *Sokoban* random walk (purple markers) and AIL (yellow markers) vs the obstacle density ρ (log-log scale). Markers coming from simulations show that, while the exploration radius of the AIL diverges as $\rho \to \rho_c$, this radius remains finite for the *Sokoban* random walk. Simulations are in excellent agreement with Eq. (4) whose prediction is given by a dashed line surrounded by a grayed out sleeve, which indicates an error of $\pm 2.5\%$ in the estimates made for the parameters γ and C. Panel (e): the mean number of sites $\langle \mathscr{A} \rangle$ visited by the *Sokoban* and mean number of perimeter sites $\langle \mathscr{P} \rangle$ follow the power-law relations of Eqs. (2) and (3) [note the log-log scale; also see panel (f)]. Markers come from simulations and best fits yield the following estimates of the relevant parameters: $A \simeq 0.2277$, $B \simeq 0.5736$, $\alpha \simeq 1.936$, and $\beta \simeq 1.754$. Panel (f): area (purple) and perimeter (green) of the trajectory from panel (c). The area of the trajectory is defined as the number of visited sites. The perimeter of the trajectory is taken as the double layer of unvisited sites that surround area sites (share a mutual edge).

expected, for the AIL: r_∞ diverges when ρ approaches $\rho_c \simeq 0.407$ from above, indicating the existence of a critical density beyond which the walk is no longer confined. However, for the *Sokoban* random walk we find that r_∞ is finite for all values of ρ sampled.

The results presented in panels (a)–(d) of Fig. 3 assert that the critical density of the *Sokoban* random walk cannot be higher than the percolation threshold. Thus, if such critical density even exists, it must be lower than the percolation threshold. Alternatively, it is possible that the *Sokoban* random walk does not have a critical density and that this walk dynamically confines itself at every positive obstacle density $\rho > 0$. One way to try and find out will be to simulate this system for increasingly smaller obstacle densities, which in turn requires increasingly larger arenas. However, this brute force approach is limited computationally and quickly runs into trouble.

Scaling approach. Instead, we take a scaling approach seeking better physical understanding for why the *Sokoban* random walk might be dynamically confining itself. We first note that in order for confinement to occur the *Sokoban* must push surrounding obstacles until it eventually creates a cage from which it cannot escape; e.g., see Fig. 1(c) and the Supplemental video [56]. Visited sites within the cage will be surrounded by a double layer of obstacles (perimeter) that prevents the walker from accessing additional sites. This double layer is required since the *Sokoban* will otherwise be able to push its way out and breach the perimeter.

To further proceed, we formulate a sufficient condition that leads to caging. We define $\mathscr{A}$ to be the area covered by the *Sokoban* trajectory, i.e., the total number of sites visited in the long time limit. In addition, we let $\mathscr{P}$ stand for the number of sites in the double-layered perimeter that surrounds visited sites. For example, the double-layered perimeter of the sites visited by the trajectory in Fig. 3(c) is indicated in Fig. 3(f). Recall that area and perimeter sites were initially occupied by obstacles with the same probability ρ. Thus, on average, $\mathscr{A}\rho$ area sites were initially occupied while $\mathscr{P}(1-\rho)$ perimeter sites were vacant. Now, to guarantee caging, we demand

$$\mathscr{A}\rho = \mathscr{P}(1-\rho). \tag{1}$$

In other words, Eq. (1) states that the *Sokoban* will surely get caged when $\mathscr{A}\rho$ obstacles are pushed clear from its path to occupy the $\mathscr{P}(1-\rho)$ perimeter sites that were initially empty.

Due to the inherent randomness of the *Sokoban* walk the $\mathscr{A}$ and $\mathscr{P}$ defined above are random, taking different values with every realization. However, averaging over many realizations we observe that these quantities obey a power-law scaling

$$\langle\mathscr{A}\rangle \simeq A r_\infty^\alpha \tag{2}$$

and

$$\langle\mathscr{P}\rangle \simeq B r_\infty^\beta, \tag{3}$$

as shown in Fig. 3(e). Underlying these relations is what seems like a fractal shape of the *Sokoban's* trajectories (Fig. S2 [56]). Substituting $\mathscr{A}$ and $\mathscr{P}$ in Eq. (1) by their averages and rearranging we obtain

$$r_\infty = \left(\frac{1-\rho}{C\rho}\right)^{1/\gamma}, \tag{4}$$

where $\gamma = \alpha - \beta$ and $C = A/B$.

Equation (4) conveys a relation between the exploration radius, r_∞, of the *Sokoban* and the obstacle density ρ. To test it, we fit the simulations data in Fig. 3(e) and estimate the parameters $\{A, B, \alpha, \beta\}$ which govern the power-law scalings of $\langle\mathscr{A}\rangle$ in Eq. (2) and $\langle\mathscr{P}\rangle$ in Eq. (3). Using these estimates we obtain $\gamma \simeq 0.182$ and $C \simeq 0.397$. Substituting these numbers back into Eq. (4), we plot the predicted relation [dashed line, Fig. 3(d)]. We compare this prediction to direct estimates of the mean exploration radii that were obtained from the asymptotic MSDs at different obstacle densities [markers, Fig. 3(d)]. Very good agreement is found between the prediction of Eq. (4) and data coming from simulations.

While the agreement between Eq. (4) and data coming from simulations is very good, it is not perfect. One source of error comes from the conservative assumption that was made while writing Eq. (1). Namely, that the *Sokoban* gets caged only when all visited (area) sites are empty and all perimeter sites are occupied. Yet, we find that caging usually occurs earlier, with some visited sites still occupied by obstacles and some perimeter sites still empty. This can happen as the *Sokoban* may get trapped in a small microenvironment that becomes isolated from the rest of the arena after caging occurs. However, modifying Eq. (1) to state that the *Sokoban* gets caged when a fraction $f_{\mathscr{A}}$ of the obstacles that resided in visited sites were pushed to occupy a fraction $f_{\mathscr{P}}$ of perimeter sites that were initially vacant, $\mathscr{A} f_{\mathscr{A}}\rho = \mathscr{P} f_{\mathscr{P}}(1-\rho)$, yields $C = A f_{\mathscr{A}}/B f_{\mathscr{P}}$ in Eq. (4) and does not change γ.

Interestingly, for the obstacle densities examined here, we find that the average fraction $f_{\mathscr{A}}$ is only slightly larger than $f_{\mathscr{P}}$, thus explaining the slight overestimate in the theoretical prediction of r_∞ compared with simulations data [Fig. 3(d)]. Whether larger deviations from Eq. (4) arise for smaller obstacle densities is currently unknown, but cannot be entirely ruled out since $f_{\mathscr{A}}$ and $f_{\mathscr{P}}$ also show some dependence on ρ.

Discussion and outlook. In this paper we introduced a model for random walks in disordered media. Contrary to the canonical "ant in a labyrinth" AIL model, the *Sokoban* random walk considered herein actively interacts and modifies its surroundings by pushing obstacles in the course of its motion. We studied the dynamics of the *Sokoban* using extensive Monte Carlo simulations, measured its MSD, and compared it to that obtained for an AIL in the presence of obstacles. At obstacle densities above the percolation threshold, we find that the *Sokoban* typically roams much further than an AIL that cannot push away obstacles that block its path. However, at obstacle densities that are close to the percolation threshold and lower, there is a striking change of trend: while the AIL becomes unbounded, the *Sokoban* random walk remains confined (caged).

A conservative sufficient condition regarding the onset of caging was used, in tandem with fractal scaling laws, to derive Eq. (4) which relates the density of obstacles to the asymptotic root MSD of the *Sokoban*. This equation explained the observed density dependence of the mean exploration radius. A prime corollary coming from Eq. (4) is that the exploration radius is finite for any positive obstacle density $\rho > 0$, suggesting that the *Sokoban* undergoes dynamical caging at all obstacle densities. Consequently, the percolation transition is lost. However, Eq. (4) is not exact, and numerical determination of the mean exploration radius at extremely low obstacle densities is very challenging computationally and beyond our reach. Thus the existence of a critical density in the *Sokoban* model, or lack of it thereof, remains to be proven rigorously.

Despite their superficial similarity, the *Sokoban* random walk and AIL exhibit qualitatively different behaviors. While the AIL becomes unbounded below a critical obstacle density, the *Sokoban* random walk undergoes dynamical caging well beyond this density. From this we learn that the ability to push away obstacles is not always beneficial for a random walker seeking to explore its surroundings. Indeed, depending on the obstacle density, the asymptotic MSDs of the *Sokoban* random walk and AIL may differ by orders of magnitude. However, for a very narrow range of densities near the percolation threshold the asymptotic MSDs are similar, thus making it difficult to discriminate the two walks based solely on this static measure. To this end, we recall that near criticality an AIL on a percolation cluster displays subdiffusive behavior [58]. In contrast, we find that the *Sokoban* displays regular diffusion, i.e., MSDs that (prior to saturation) grow linearly with time (Fig. S3 [56]).

Short-ranged tracer-media interactions are often neglected as they are not believed to significantly impact transport properties at the macroscale. However, the findings presented herein and in [51] vividly demonstrate that even a limited ability of a random walker to dynamically modify its local environment could drastically alter its long-ranged transport behavior and first-passage properties. In such cases, i.e., where strong deviations from the inert "ant in a labyrinth" paradigm occur, the *Sokoban* provides an alternative null model, which can be further adapted and refined according to need.

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