

Linking the Langevin equation to scaling properties of space plasma turbulence at sub-ion scales

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Current understanding of the kinetic-scale turbulence in weakly collisional plasmas still remains elusive. We employ a general framework in which the turbulent energy transfer is envisioned as a scale-to-scale Langevin process. Fluctuations in the sub-ion range show a global scale invariance, thus suggesting a homogeneous energy repartition. In this Letter, we interpret such a feature by linking the drift term of the Langevin equation to scaling properties of fluctuations. Theoretical expectations are verified on solar wind observations and numerical simulations, thus giving relevance to the proposed framework for understanding kinetic-scale turbulence in space plasmas.

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Space and astrophysical plasmas, such as the solar and stellar winds, the planetary magnetospheres, and the interstellar medium, often exhibit strongly turbulent dynamics. The energy of fluctuations cross-scale transfers over a vast range of spatial and temporal scales and diverse structures, including current sheets, magnetic islands, and vortices, emerge. Since about sixty years ago, a large fleet of spacecraft has been providing *in situ* measurements of the solar wind plasma and the interplanetary magnetic field [1,2]. These observations have allowed us to investigate in great detail turbulence in space plasmas, whose study is of pivotal importance also for clarifying the dynamics of far astrophysical objects not accessible by *in situ* spacecraft.

Owing to their weak collisionality, understanding the properties of turbulence at scales similar or smaller than the inertial ion scales, henceforth referred to as sub-ion scales, is essential for clarifying the energy transfer and dissipation mechanisms in space and astrophysical plasmas. Indeed, at sub-ion scales, the solar wind dynamics are a complex puzzle composed of several intertwined plasma processes. Such processes generally manifest in the entire phase space, since interparticle collisions are infrequent and plasmas can be far from the thermal equilibrium [3].

Historically, the investigation of plasma dynamics at sub-ion scales has aimed at identifying the plasma processes successfully explaining *in situ* observations. These include kinetic-scale fluctuations (e.g., kinetic-Alfvén and whistler waves) [4–7], microinstabilities [8], magnetic reconnection [9,10], energy transfer highlighted by pressure-strain inter-

actions [11], Landau damping [12], etc. (see Ref. [13] for a review). These mechanisms often depend on different plasma parameters (e.g., the plasma β) and hence, their applicability is specialized to specific plasma conditions. Moreover, some of these mechanisms, such as kinetic-scale fluctuations, microinstabilities, and Landau damping are often introduced within a linear or quasilinear framework, thus possibly making their application less suitable in cases of strong turbulence, as observed in the solar wind [14].

A complementary and additional perspective proposes to interpret space plasma turbulence through a small set of macroscopic variables, whose fluctuations are not negligible but rather control the system dynamics. Such a definition sets the ground for a general and interdisciplinary framework whose underlying idea is that an observed coarse-grained dynamics can be modeled as a stochastic process [15]. A stunning example of this approach is represented by hydrodynamic turbulence, where longitudinal velocity fluctuations at different scales can be described by means of a stochastic approach [e.g., 16–18], mostly based on the generalized Langevin equation [19–24]. From this point of view, the scale-by-scale evolution of the probability distribution functions (PDFs) of the longitudinal increments is governed by the Fokker-Planck equation (FPE). A similar description has been also applied to turbulence in space plasmas. An earlier work by Strumik and Macek [25] showed that the inertial-range cascade of the solar wind can be properly framed in this scenario. A step forward has been made by proving that this formalism applies to sub-ion scales, thanks to unprecedented high-resolution magnetic field observations [26–28]. These studies confirm that the FPE is accurate in reproducing the observed PDFs of magnetic field fluctuations, and show that PDFs outline a globally self-similar scenario [26–30].

A common way to investigate solar wind turbulence is to analyze high-order statistics of magnetic field fluctuations,

viz., the structure functions $S_r^{(q)} = \langle b_r^q \rangle$, where

$$b_r \doteq B(x+r) - B(x) \quad (1)$$

are the increments of the magnetic field $B(x)$ at the spatial scale r . Fluctuations b_r are *globally* scale invariant if

$$r \rightarrow \lambda r \Rightarrow b_{\lambda r} = \lambda^h b_r, \quad (2)$$

where h is a scaling exponent and $\lambda \in \mathbb{R}^+$ is a dilatation/contraction parameter representing a zoom in/out procedure through magnetic field structures. The basic idea is that each structure at a scale decays into smaller-scale structures in a scale-invariant way. Indeed, Eq. (2) implies that structure functions exhibit a power-law trend as a function of r , i.e., $S_r^{(q)} \sim r^{\zeta_q}$, with the scaling exponents ζ_q . The space-time distribution of turbulent structures reflects the properties of energy dissipation rate and related underlying processes [31]. However, there are no fully deductive theories that are able to derive the scaling laws, viz., the behavior of the scaling exponents, starting from dynamic equations. For this reason, phenomenological theories, which introduce additional hypothesis based on experimental evidences, are widely used. In this context, the method described in this Letter ranks as a novel phenomenological perspective of sub-ion scale plasma turbulence.

The transformation introduced in Eq. (2) is satisfied by $b_r \sim r^h$ [31]. In the multifractal theory of magnetohydrodynamic (MHD) turbulence, h is the generalized Hölder exponent $h(x, r)$ [32] and thus magnetic field fluctuations are *locally* scale-invariant $b_r \sim r^{h(x, r)}$. Here, $h(x, r)$ represents a fluctuating quantity [33], hence the global scale-invariance holds only on average [31] and can be expressed in terms of the most probable Hölder exponent h_0 . For a globally scale-invariant dynamics, a relation between h_0 and the spectral slope β of the Fourier power spectral density can be derived

$$\beta = 2h_0 + 1, \quad (3)$$

with $\beta \in (1, 3)$ for the nonstationary process $B(x)$ with stationary increments b_r [34].

In terms of FPE, the scale-evolution equation of the q th order structure function can be written as [35–37]

$$-\frac{\partial}{\partial r} S_r^{(q)} = q \langle b_r^{q-1} D^{(1)}(b_r) \rangle + q(q-1) \langle b_r^{q-2} D^{(2)}(b_r) \rangle. \quad (4)$$

This scale-evolution equation depends on the two Kramers-Moyal (KM) coefficients which define the stochastic process. For the solar wind, the following parametrization of the KM coefficients correctly reproduce the fluctuation statistics at sub-ion scales [26–28]:

$$\begin{aligned} D^{(1)}(b_r) &= -\gamma(r) b_r, \\ D^{(2)}(b_r) &= \alpha(r) + \delta(r) b_r^2. \end{aligned} \quad (5)$$

By inserting Eqs. (5) in Eq. (4), a direct relation between KM coefficient parameters and the local scaling exponent $\xi_q(r)$ can be drawn [36,38]:

$$\begin{aligned} \xi_q(r) &= \frac{r}{S_r^{(q)}} \frac{\partial S_r^{(q)}}{\partial r} \\ &= r q \left(\gamma(r) + (1-q) \left(\delta(r) + \frac{S_r^{(q-2)}}{S_r^{(q)}} \alpha(r) \right) \right). \end{aligned} \quad (6)$$

In the framework of our model, it means that the evolution in scale of the fluctuating trajectories b_r can be described according with the Langevin equation [39]

$$-\frac{\partial b_r}{\partial r} = D^{(1)}(b_r, r) + \sqrt{2D^{(2)}(b_r, r)} \eta(r), \quad (7)$$

where $\eta(r)$ is a unit-variance Gaussian noise term such that $\langle \eta(r) \eta(r') \rangle = \delta(r - r')$. Equation (7) indicates that the dynamics of magnetic field increments as a function of the scale r is the result of the superposition of two mechanisms: The *drift* which models the self-similar decay of magnetic field structures from larger to smaller ones, and the *diffusion* that accounts for the stochastic repartition of energy during the structure breakdown process.

This scenario applies in the general framework of turbulence. In the following we specialize our discussion to the sub-ion scale dynamics by considering three different data samples: Super-Alfvénic solar wind observations gathered by Parker solar probe (PSP) during its first perihelion [40], a high-speed stream observed by the Cluster mission in the near-Earth solar wind [41], and a data sample obtained from Eulerian hybrid Vlasov-Maxwell (HVM) simulations of decaying plasma turbulence [42,43]. The HVM numerical framework has been widely adopted to describe significant features of turbulence in space plasmas [44–46], including the transition to a monofractal scaling when approaching sub-ion scales [47]. The data selected for the analysis show a similar level of fluctuation $b_{\text{rms}}/B_0 \sim 0.5$, with B_0 being the mean magnetic field, and different values of the proton plasma beta (0.4 for PSP, 1.5 for Cluster, and 2 for HVM). The analysis of spacecraft measurements is carried out by assuming the Taylor's hypothesis $r = \langle V \rangle \tau$, where τ indicates the time scale and $\langle V \rangle$ the mean solar wind speed [48,49]. Further details about observations and numerical setup are respectively given in Secs. A and B of the Supplemental Material [50]. Plasma turbulence in the solar wind is known to be strongly anisotropic and the energy cascade develops mostly in the transverse directions with respect to the mean local magnetic field [51–54]. Hence, we analyze fluctuations along one of the transverse magnetic field directions for each data sample considered here. For *in situ* observations we compute increments along the maximum variance component $b_r = b_{r,\text{max}}$, while in the case of HVM simulations we consider increments along the x component $b_r = b_{r,x}$ since the mean field is directed along the z direction [42]. In the last case we also verified the validity of the Markov condition at sub-ion scales, as reported in Sec. C of the Supplemental Material [50].

The first step in our reasoning consists of assessing the different role played by the deterministic and the stochastic terms in Eq. (7) at different scales. To this purpose we define a set of *trajectories* b_r , see Eq. (1), for different starting points x . For a nearly constant value of the Hölder exponent, say h_0 , the amplitude of fluctuations is expected to decrease as a power law in r . If we normalize each trajectory by the standard deviation σ_{b_r} , which then scales as r^{h_0} , we expect to observe constant and scale-independent values of the normalized fluctuations. In this view, Fig. 1 reports a few normalized trajectories which allow us to identify two different dynamical regimes: (i) the sub-ion range where normalized fluctuations show a nearly constant value; (ii) the lower end of the inertial range,

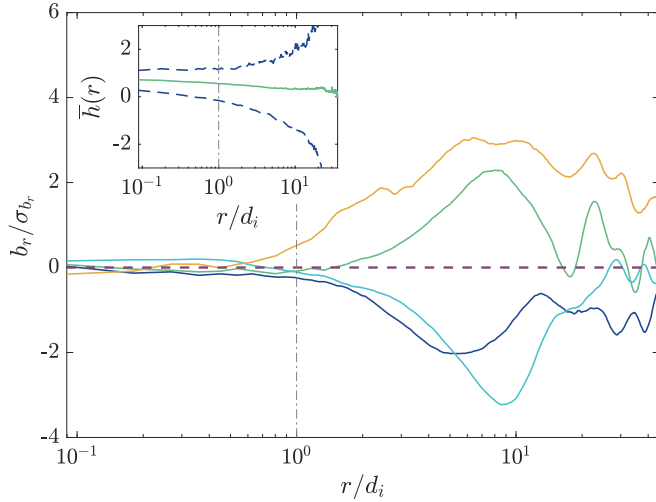


FIG. 1. Fluctuating magnetic field trajectories normalized to the standard deviation scale-by-scale from the PSP data sample (color lines). The inset shows the averaged Hölder exponent $\bar{h}(r)$ as a function of r (solid line) along with 95% confidence bounds (dashed lines) for the PSP data sample. Vertical lines indicate the ion inertial length.

e.g., at scales $\gtrsim 10d_i$, where trajectories exhibit stochastic fluctuations. The different level of stochasticity of the magnetic field fluctuations at different scales can be quantified by estimating the local Hölder exponent. The inset of Fig. 1 reports the scale-by-scale averaged Hölder exponent $\bar{h}(r)$ (solid line) along with the 95% confidence interval bounds (dashed lines). The stochastic character of turbulent fluctuations is related to the spreading of the local Hölder exponent (i.e., spreading of the confidence bounds) when approaching the inertial range ($r > 10d_i$). In the context of Langevin equation, this scenario suggests that the diffusion term does not significantly contribute to the dynamics at sub-ion scales, whereas it plays an important role in the inertial range. Results of Fig. 1 are derived from PSP data, but analogous considerations can be also extended to the other data samples (not shown). Based on this observation, we introduce the nondiffusive limit of the Langevin equation in order to model sub-ion scale fluctuations.

Hence, we assume that the small-scale dynamics is governed only by the first-order KM coefficient, viz., $D^{(1)} \gg D^{(2)}$, which in terms of the parametrization introduced in Eqs. (5) corresponds to neglecting the parameters $\alpha(r)$ and $\delta(r)$. Furthermore, $\gamma(r)$ follows a power-law behavior at sub-ion scales [26–28], i.e.,

$$\gamma(r) = h_0 r^\mu, \quad (8)$$

allowing us to derive the coefficients h_0 and the exponents μ directly from data. Their values are listed in Table I. In the nondiffusive limit, Eq. (6) becomes

$$\xi_q(r) \sim r q \gamma(r) = q h_0 r^{1+\mu}. \quad (9)$$

For both solar wind data and numerical simulations $\mu \sim -1$ (see Table I), then Eq. (9) reads

$$\zeta_q \equiv \xi_q(r) = q h_0, \quad (10)$$

TABLE I. Fitted h_0 and μ parameters appearing in Eq. (8). Errors represent the fit 95% confidence bounds.

	PSP	Cluster	HVM
h_0	0.87 ± 0.06	0.87 ± 0.03	0.88 ± 0.52
μ	-1.02 ± 0.01	-1.04 ± 0.01	-1.04 ± 0.22

where h_0 , in this fashion, corresponds to the most probable Hölder exponent. This means that the condition $D^{(1)} \gg D^{(2)}$ implies a *global scale invariance*. This represents the central result of this work since it establishes a direct link between the drift term at sub-ion scales and the most probable Hölder exponent of the multifractal theory. Such a connection will be now verified by exploiting both *in situ* observations and numerical simulations in a twofold sense: On a statistical level, e.g., through structure functions, and on an individual level, e.g., by considering single fluctuating trajectories.

Scaling exponents can be derived from solar wind observations through the structure function analysis within the sub-ion range, as described in Sec. D of the Supplemental Material [50]. The scaling exponents obtained from the three data samples are here compared with those obtained through Eq. (10), Fig. 2. An excellent agreement between observations and model predictions is found, strongly supporting the established link between the first-order KM coefficient and the Hölder exponent h_0 . Concerning the spectral analysis, we can use Eq. (3) to estimate the spectral slope corresponding to the observed most probable Hölder exponent h_0 from the three data samples. By using the values estimated from the drift coefficient, reported in Table I, we obtain $\beta \sim [2.7, 2.8]$, which is in agreement with typical findings in the sub-ion range [55,56].

The linear trend of the scaling exponents ζ_q as a function of the order of the structure function q observed from data suggests that magnetic field fluctuations should exhibit a weak

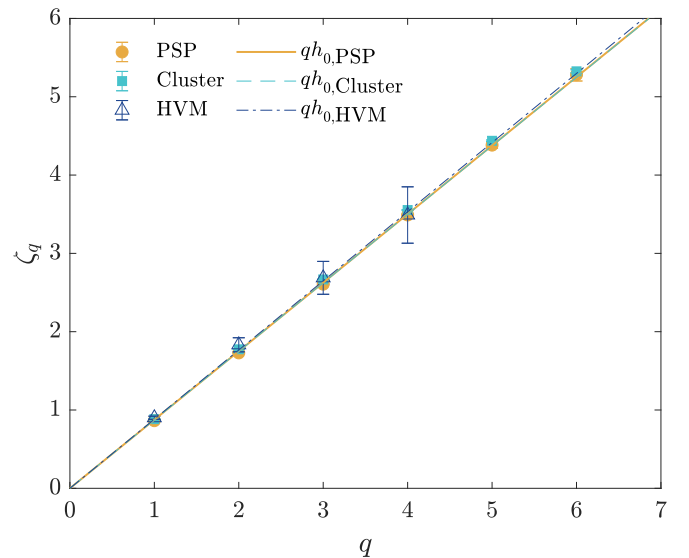


FIG. 2. Scaling exponents ζ_q as a function of the order q of the structure functions (markers). Straight lines represent the estimation provided by Eq. (10) by using the values of Table I.

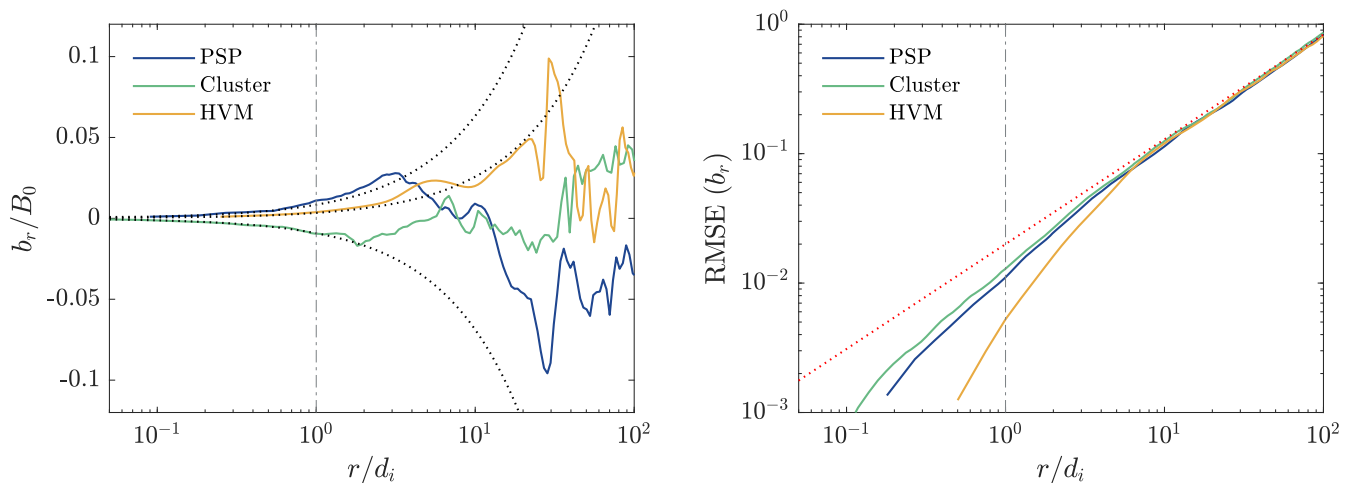


FIG. 3. Comparison between three trajectories selected from different data samples (left panel) and the corresponding nondiffusive trajectories b_r^* (black dotted lines). Root-mean-square error of the fluctuations b_r with respect to b_r^* as a function of the scale (right panel). The red dotted line indicates a reference power-law trend. The scales are reported in units of d_i and the magnetic field is normalized by the mean field value B_0 for clarity of presentation.

stochastic behavior at sub-ion scales. In fact, the nondiffusive limit of Eq. (7) corresponds to a damping equation with a scale dependent drift term

$$\frac{\partial b_r^*}{\partial r} = \gamma(r)b_r^* = \frac{h_0 b_r^*}{r}, \quad (11)$$

whose solution is

$$b_r^* = b_0 \left(\frac{r}{r_0} \right)^{h_0}, \quad (12)$$

where b_r^* indicates the deterministic solution and b_0 is the initial condition, i.e., a value of the fluctuation at the scale r_0 . The left panel of Fig. 3 shows a comparison of the power-law damping predicted by Eq. (12) with a set of fluctuating trajectories from three different data samples. This figure points out that the nondiffusive approximation is quite satisfactory for $r \ll d_i$, whereas the diffusion starts to significantly affect the dynamics of the magnetic field fluctuations at $r \gtrsim d_i$. The root-mean-square error between the whole set of measured trajectories and the corresponding model predictions is reported in Fig. 3 (right panel). This figure shows that errors decrease more rapidly when approaching sub-ion scales for all three data samples.

The proposed approach allows us to unveil connections between the global statistical features of the plasma dynamics and the peculiar characteristics of each realization of the cascading process. For instance, the Hölder exponent h_0 quantifies the regularity (i.e., the roughness) of the topology of the fluctuations associated with a dynamical process [57]: Irregular bursts are characterized by $h_0 \rightarrow 0$, whereas a field of increasingly regular fluctuations is associated with larger h_0 and leans towards a homogeneous fractal structure when $h_0 \rightarrow 1$. By looking at Fig. 3 we can observe that the deterministic damping shown by empirical fluctuating trajectories is characterized by a weakly fluctuating trend that corresponds to a well-defined sign of the fluctuation amplitude among different scales. The high values obtained for $h_0 \sim [0.87, 0.88]$ represent a natural consequence of this intuitive framework.

In the case of the Langevin equation, the nondiffusive limit is achieved by simply neglecting the noise term and then reducing the stochastic equation to a deterministic one. The same procedure cannot be applied when dealing with the FPE, for which the nondiffusive limit is singular [58,59]. In this case the correct way to perform this limit is to introduce a new variable y such that $b_r = b_r^* + \epsilon y$, where ϵ is an arbitrarily small parameter. Hence, the nondiffusive limit corresponds to $\epsilon \rightarrow 0$ in which case the stochastic fluctuation vanishes and b_r approaches the deterministic solution b_r^* . This approximation represents an ideal limit in which the FPE predicts weak fluctuations around the deterministic damping [59]. Such scenario seems to be fairly accurate by inspecting Fig. 3, where the trajectories exhibit weak fluctuations around the asymptotic solution at small scales.

All the evidence reported in this work is consistent with a homogeneous and globally self-similar repartition of energy through the scales in the sub-ion range. As a consequence, the typical multifractal character of the nonhomogeneous energy repartition observed in the inertial range, here enclosed in the diffusion term of the Langevin equation, does not represent a major contribution at these scales. The result is an unstructured fluctuation field suggesting that typical space-time inhomogeneities associated with turbulent structures have sizes larger than d_i . This is supported by the linear scaling law of ζ_q , here derived in the nondiffusive limit, and by the fact that trajectories at sub-ion scales are captured with a single exponent h_0 . In this context, our newly introduced framework constitutes a complementary view with respect to the existing phenomenological models, allowing us to recover important results of solar wind turbulence at sub-ion scales and to provide new constraints for future model developments.

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