Letter

Fast generation of high-fidelity mechanical non-Gaussian states via additional amplifier and photon subtraction

Dong-Long Hu [1], Jia-Jin Zou [1], Feng-Xiao Sun [2], Jie-Qiao Liao [3], Qiongyi He [2] and Ze-Liang Xiang [1,*]

[1] *School of Physics, Sun Yat-sen University, Guangzhou 510275, China*

[2] *State Key Laboratory for Mesoscopic Physics, School of Physics, Frontiers Science Center for Nano-Optoelectronics, and Collaborative Innovation Center of Quantum Matter, Peking University, Beijing 100871, China*

[3] *Key Laboratory of Low-Dimensional Quantum Structures and Quantum Control of Ministry of Education, Key Laboratory for Matter Microstructure and Function of Hunan Province, Department of Physics and Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha 410081, China*

(Received 12 November 2022; accepted 25 August 2023; published 6 September 2023)

Non-Gaussian states (NGSs) with higher-order correlation properties have wide-range applications in quantum information processing. However, the generation of such states with high quality still faces practical challenges. Here, we propose a protocol to faithfully generate two types of mechanical NGSs, i.e., Schrödinger cat states and Fock states, in open optomechanical systems, even when the cooperativity is smaller than one ($g^2/\kappa\gamma < 1$). In contrast to the usual scheme, a short squeezed field is pumped to rapidly entangle with a mechanical resonator via a beam-splitter-like optomechanical interaction, effectively reducing the mechanical decoherence. Furthermore, by performing an additional amplifier and a following multiphoton subtraction on the entangled optical field, one can selectively obtain the high-fidelity mechanical cat and Fock states. This protocol is robust to various imperfections, allowing it to be implemented with state-of-the-art experimental systems with close to unit fidelity. Moreover, it can be extended to generate a four-component cat state and provide possibilities for future quantum applications of NGSs.

DOI: 10.1103/PhysRevResearch.5.L032031

Non-Gaussian states (NGSs) with Wigner negativity are of paramount importance for their advantages in quantum information processing, which cannot be simulated by classical resources [1–5]. Such states, including Schrödinger cat states (CSs) and Fock states, are widely applied in quantum error correction [6,7], quantum metrology [8,9], and quantum sensing [10,11]. By now, one can prepare NGSs by using four-wave mixing with Kerr nonlinearities in superconducting circuits [6,7,12] or by performing non-Gaussian operations, such as photon subtraction or addition, on given Gaussian states in optical systems [13–19]. In addition, NGSs can be remotely generated between distant sites through the shared entanglement in optical and microwave systems, offering intrinsic security and efficiency [20–23].

Among different systems, the well-studied cavity optomechanical system is a promising candidate for studying NGSs [24,25]. This system, driven by the radiation pressure, provides an important means to manipulate and detect mechanical motion in the quantum regime using light; hence it has received substantial attention and intensive investigations. So far, many fundamental quantum phenomena of mechanical motion have been experimentally observed in this system, such as optical-mechanical [26–28] and mechanical-mechanical entanglement [29,30], and mechanical squeezing [31,32]. Fortunately, these results are incredibly favorable for preparing the mechanical NGSs. Based on optomechanical systems, various schemes have been proposed for the controllable generation of mechanical NGSs with decent quality [33–36]. However, the intrinsic mechanical dissipation and the challenging realization of strong single-photon optomechanical coupling still hinder the practical implementation of such schemes.

In this Letter, we present a protocol for faithfully preparing near-perfect CSs and Fock states of mechanical motion in an open optomechanical system, as shown in Fig. 1(a). Unlike previous works [37,38], here a short squeezed vacuum, as shown in Fig. 1(b), is pulsed to rapidly entangle with a mechanical resonator via a beam-splitter-like optomechanical interaction. This effectively reduces the mechanical decoherence and can be achieved even when cooperativity is smaller than one ($g^2/\kappa\gamma < 1$). Importantly, an engineered photon subtraction (EPS) is then performed on the entangled optical mode, which consists of a phase-sensitive amplifier followed by a multiphoton subtraction, as shown in Figs. 1(c) and 1(d). There always exists appropriate gains of the amplifier to ensure the Wigner negativity of the mechanical mode after photon subtraction and projective measurement. Interestingly, specific gains of the amplifier enable the selective preparation of high-fidelity mechanical CSs and Fock states, as shown in Figs. 1(e) and 1(f). In addition, the significant experimental

*xiangzliang@mail.sysu.edu.cn

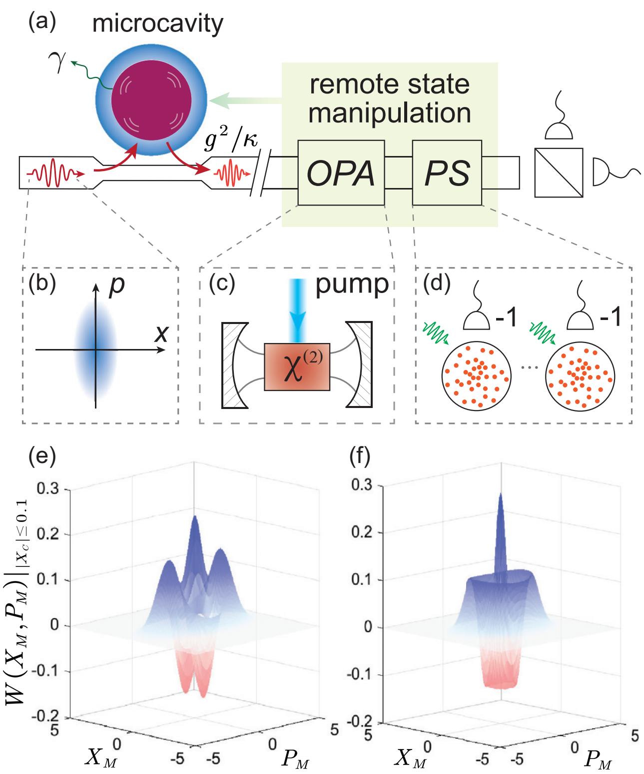

FIG. 1. (a) Sketch of the fast generation of mechanical NGSs, where a short squeezed pulse interacts with the mechanical resonator and propagates to a distant site. By performing an EPS and a further projective measurement on the entangled optical mode, the desired mechanical NGSs can be faithfully produced. (b) Schematic of a squeezed vacuum. (c) Schematic of an optical parametric amplifier (OPA) and (d) schematic of a controlled multiphoton subtraction with cascaded Rydberg atom ensembles. (e),(f) Resulting mechanical NGSs, a cat state and a Fock state (fidelities ∼0.99), with different gains of a two-photon EPS, respectively.

progress of deterministic photon subtraction utilizing the Rydberg-blockade effect [39–41] ensures that our protocol can efficiently generate large-size NGSs. Furthermore, this protocol can be extended to prepare a four-component cat state with high fidelity and provides a possibility to explore advanced NGSs. All the results here are obtained using the accessible parameters from state-of-the-art experiments.

Cavity optomechanical system. We consider a typical cavity optomechanical system, consisting of a mechanical mode of a resonator coupled to an optical mode of a cavity via the radiation pressure [24]. The mechanical mode is initially prepared in the ground state at a low environment temperature [42]. The dynamics of the system can be described by $H = \omega_m m^\dagger m + \omega_c c^\dagger c + g_0 c^\dagger c(m + m^\dagger)$, where m (c) is the annihilation operator of the mechanical (optical) mode with the resonate frequency $\omega_{m(c)}$, g_0 denotes the single-photon optomechanical coupling rate, and $\hbar = 1$. Under the red-detuned drive on the optical mode, the effective Hamiltonian reduces to (see the Supplemental Material [43])

$$H_{\text{eff}} = \omega_m m^\dagger m + \Delta c^\dagger c - ig(mc^\dagger - m^\dagger c), \tag{1}$$

where $\Delta = \omega_c - \omega_p$ is the frequency detuning with ω_p the frequency of the driving field, and $g = g_0|\beta|$ is the linearized optomechanical coupling rate with $|\beta|$ the light amplitude of the cavity mode. This interaction describes a beam-splitter-type scattering between the optical photons and the mechanical phonons. Unlike previous schemes that require a strong optomechanical coupling, i.e., $C_{\text{om}} = g^2/\kappa\gamma > 1$, where γ represents the mechanical dissipation rate and κ denotes the decay rate of the optical cavity, our protocol is also valid in the weak-coupled regime. To show this, here we consider a low optomechanical cooperativity $C_{\text{om}} = 0.8$, with the parameters $g/2\pi = 3$, $\kappa/2\pi = 7$, and $\gamma/2\pi = 1.6$ MHz, which can be easily achieved in current experiments [44–47].

Because the optical cavity decay rate $\kappa \gg g$, the cavity field is adiabatically eliminated, resulting in a direct interaction between the mechanical mode and the propagating squeezed vacuum field in a waveguide, as described by the input-output relation $c_{\text{out}} = c_{\text{in}} + \sqrt{2\kappa}c$ [48,49]. This interaction gives rise to an effective decay rate of the mechanical mode $G = g^2/\kappa + \gamma$, simultaneously, to the propagating pulse (g^2/κ) and the environment (γ), as shown in Fig. 1(a). Therefore, the quantum coherence feature of the mechanical mode can be protected by pumping the squeezed vacuum with a short duration,

$$\tau \lesssim \frac{1}{2G} \quad (R = e^{-2G\tau} > e^{-1}), \tag{2}$$

where R is the reflectivity of the effective beam splitter that plays the same role of the duration τ (see Supplemental Material [43]). In this fast entanglement preparation regime, the remote generation of mechanical NGSs is insensitive to the decoherence of the mechanical mode, which we will show in the following.

Squeezing-induced entanglement and EPR steering. Along the squeezed vacuum propagates through and interacts with the mechanical resonator, the excepted nonlocal correlations between the cavity output mode (C) and the mechanical (M) mode can deterministically produce. We assume the quadrature $X_{C_{\text{in}}}$ of the input pulse is squeezed with a strength S_{in}, where the quadratures are $X_a = (a + a^\dagger)/\sqrt{2}$, $P_a = (a - a^\dagger)/i\sqrt{2}$, and a is a generic bosonic annihilation operator. The entanglement can be qualified by the logarithmic negativity E_N [50] and the Einstein-Podolsky-Rosen (EPR) steering from the mechanical mode to the cavity output mode, $\mathcal{G}^{M\to C}$ [51] (see the Supplemental Material [43]). The EPR steering is a quantum phenomenon that one party can remotely influence the wave function of the other distant party by performing suitable measurements [52]. Note that the EPR steering is a necessary resource for the remote generation of Wigner negativity in the mechanical mode via the photon subtraction [53,54].

In Fig. 2(a), we illustrate the quantum correlations as functions of the reflectivity R with different squeezing levels, where the photon-phonon entanglement can effectively produce with low cooperativity ($C_{\text{om}} = 0.8$). For a good initialization of the mechanical mode with negligible thermal occupation, arbitrary $S_{\text{in}} \neq 0$ dB ($x \to 10\log_{10} x$ dB) is feasible for preparing effective entanglement. For example, we choose $S_{\text{in}} = -6$ dB in the following discussions for generating high-fidelity mechanical NGSs. In experiments, the

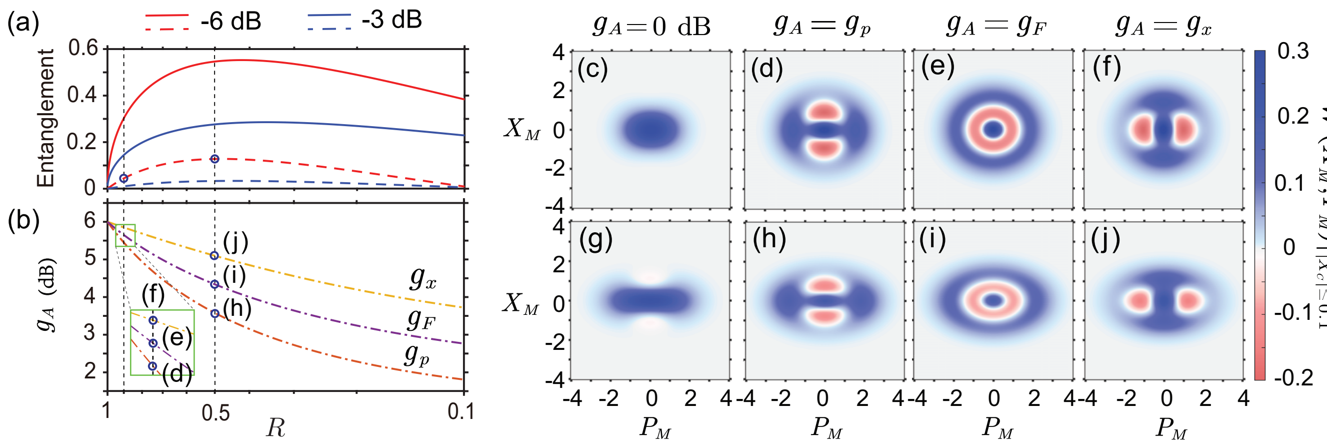

FIG. 2. (a) The logarithmic negativity E_N (solid line) and EPR steering $\mathcal{G}^{M\to C}$ (dashed line) vs the reflectivity $R = e^{-2G\tau}$. The red (blue) line corresponds to -6 dB (-3 dB) of the input squeezing. (b) The required gains g_A of the amplifier for preparing a high-fidelity P_M-direction cat state (g_p), Fock state (g_F), and X_M-direction cat state (g_x) vs the reflectivity $R = e^{-2G\tau}$. (c)–(j) The resulting states by choosing the gains of the amplifier from $g_A = 0$ to $g_A = g_x$ with $R = 0.9$ and $R = 0.5$, respectively. Here, a two-photon subtraction is adopted, and the effective mechanical decay rate $G = g^2/\kappa + \gamma = 2.9$ MHz with $C_{\rm OM} = 0.8$.

squeezing strength $S_{\rm in} \approx -15$ dB for optical modes has been achieved [55].

Remote state generation and manipulation. After performing an EPS and a further projective measurement on the entangled optical mode, as shown in Fig. 1(a), the mechanical mode will immediately collapse to an NGS, ρ_M. For investigating the precise relations between the resulting state ρ_M and the EPS, we describe the n-photon EPS as a combined operator, $\mathbf{E}(g_A, n) := c^n\mathcal{U}(g_A)$. Here the phase of the amplifier $\mathcal{U}$ matches the input squeezing, which offers a direct amplification for X_C with gain g_A, and n is the subtracted photon number from the optical mode. With the outcome $X_C = 0$, the relation between ρ_M and the entangled state $\rho_{\rm out}$ is $\rho_M = \langle \mathbf{E}(g_A, n)\rho_{\rm out}\mathbf{E}^\dagger(g_A, n)\rangle_{X_C=0}$. Ideally, with $\gamma = 0$, we can analytically derive the wave function $\psi(X_M)$ of ρ_M in the representation of X_M, as

$$\psi(X_M) \propto \phi_{n,\xi}(X_M)\exp\left(-\frac{X_M^2}{2\sigma_{11}^{-1}}\right), \tag{3}$$

$$\phi_{n,\xi}(X_M) \propto \sigma_{13}^n\Sigma_{k=0}^{[n/2]}\frac{(-1)^k n!2^{n-2k}}{k!(n-2k)!}\left(\frac{X_M}{\sqrt{2\sigma_{11}^{-1}}}\right)^{n-2k}\xi^k. \tag{4}$$

Here the non-Gaussian features of $\psi(X_M)$ are exhibited by $\phi_{n,\xi}(X_M)$, which relates to the nth-order Hermite polynomial $\phi_{n,\xi}(X_M) \propto H_n(X_M/\sqrt{2\xi\sigma_{11}^{-1}})$ when $\xi \neq 0$. Surprisingly, we find that ξ intimately relates to the non-Gaussianity of ρ_M and can be remotely controlled by the applied amplifier simultaneously, i.e., $\xi = (\sigma_{33} - g_A)/(\sigma_{13}^2\sigma_{11}^{-1})$, where $\sigma = 1/2V^{-1}$, V is the covariance matrix of the entangled state $\rho_{\rm out}$, and σ_{ij} is the matrix element of σ (see Supplemental Material [43]).

From Eqs. (3) and (4), we find the following: (i) when $\xi = 0$ or $\xi = 1$, $\psi(X_M)$ corresponds to a near-perfect squeezed CS $\sim\hat{S}\mathcal{N}_n(|\alpha\rangle + (-1)^n|-\alpha\rangle)$, with a coherent amplitude $\alpha = \sqrt{n}e^{i\frac{\pi}{2}\xi}$ squeezed by $s = \sigma_{11}^{-e^{-i\pi\xi}}/2$ and a high-fidelity $\mathcal{F} \approx 1 - 0.03/n$ (i.e., the overlap between actual and target states, defined as $\mathcal{F} = \langle\psi_t|\rho_M|\psi_t\rangle$) [16]; (ii) when $\xi = 0.5$, a mechanical Fock state $\hat{S}|n\rangle$ squeezed by $s = \sigma_{11}^{-1}$ can be strictly generated. Here, $\hat{S}$ is a formalistic squeezing operator, $\mathcal{N}_n$ is the normalization coefficient, and $\sigma_{11} \approx R + TS_{\rm in}^{-1}$ indicates the mechanical squeezing effect coherently transferred from the optical squeezing, which increases with the duration τ (see Supplemental Material [43]). This precise mapping allows the faithful preparation of two typical mechanical NGSs, i.e., the CS and Fock state, by simply adjusting the gain g_A of the amplifier. The required gain can be inversely solved by

$$g_A = \sigma_{33} - \xi\sigma_{13}^2\sigma_{11}^{-1}. \tag{5}$$

To be more intuitive, we rewrite the state preparation conditions $\xi = (1, 1/2, 0)$ into the gain domain as $g_A = (g_p, g_F, g_x)$, and plot the curves of g_p, g_F, and g_x versus effective reflectivity R in Fig. 2(b).

By using the above precise mapping in a concrete example, we illustrate the Wigner functions of the resulting mechanical NGSs caused by a two-photon EPS [43], as shown in Figs. 2(c)–2(j). Figures 2(c)–2(f) are the resulting mechanical states generated with $R = 0.9$ ($\tau \approx 3$ ns). Instead of directly performing photon subtraction [Fig. 2(c)], a prior amplifier allows one to selectively obtain the P_M-direction squeezed CS [Fig. 2(d)], the two-phonon state [Fig. 2(e)], and the X_M-direction squeezed CS [Fig. 2(f)] with fidelity $\mathcal{F} > 0.98$, by choosing the corresponding gains g_A that are shown in the inset of Fig. 2(b). Furthermore, we show the resulting states generated with $R = 0.5$ ($\tau \approx 19$ ns) in Figs. 2(g)–2(j), where the EPR steering reaches its maximal value. It shows that a longer duration will lead to an increased squeezing and decoherence effect of our desired mechanical NGSs. Here, the two more-separated "cats" [Fig. 2(h)] on P_M and the two closer "cats" [Fig. 2(j)] on X_M are obtained with fidelity $\mathcal{F} > 0.88$.

Feasibility and imperfections. In experiments, the amplifier $\mathcal{U}$ can be implemented by a cavity (with frequency at ω_c) that contains a $\chi^{(2)}$ gain medium [56], as shown in Fig. 1(a). By pumping the gain medium with driving frequency $\omega_d = 2\omega_c$, amplitude Λ and phase Φ_d. The Hamiltonian of the cavity is $H_A = i\chi^{(2)}\Lambda(a^2e^{i\Phi_d} - a^{\dagger 2}e^{-i\Phi_d})$ in a frame rotating with ω_c. The phase $\Phi_d = \omega_c l/c$ is used to match the phase of the propagated optical field. From the input-output theory, the amplitude quadrature X_C of the optical field passing through the cavity will obtain a gain,

$$g_A = (1 + \chi^{(2)}\Lambda/\kappa_A)^2/(1 - \chi^{(2)}\Lambda/\kappa_A)^2, \quad (6)$$

where κ_A is the decay rate of the cavity mode, and the gain g_A can be modulated via the external driving for a desired ξ that determines the mechanical NGSs. However, the noise of the practical amplification process and the photon loss during the optical pulse transits to the amplifier both lead to a reduction of the photon-phonon entanglement. Here we quantify these two imperfections by optical transmission efficiency η and amplification noise n_A that describes an incoherent squeezing $s = (1 + n_A)/g_A$ of phase quadrature P_C when the amplitude quadrature X_C obtains a gain g_A.

In addition, the usual photon subtraction event is implemented by a beam splitter with high transmittance which has a low probability of success [13,17–19]. To address this issue, we introduce the deterministic photon subtraction implemented using the Rydberg-blockade effect [39], and the corresponding experiment of three-photon subtraction has recently been achieved by cascaded cold atomic ensembles [41]. The dark counts of the photon subtraction are considered here, which leads to a negligible imperfection.

Finally, an imperfect homodyne detection with an efficiency μ is considered. By taking into account all of the above imperfections for the generation of mechanical NGSs with two-photon EPS, we analytically obtain the final Wigner function of the mechanical state,

$$W_{\rho_M} \approx \mathcal{N} \exp\left(-aX_M^2 - bP_M^2\right) \times [(2F_+ - 2e - 2f)^2 - 4(e-f)F_- - \lambda], \quad (7)$$

where $F_\pm = (cX_M)^2 \pm (dP_M)^2$ and $\lambda = e^2 + f^2 + 6ef$. The Wigner function is described by the six parameters $a - f$ that relate to the total optical detection efficiency $\Gamma = \mu + \eta$ and the amplification noise n_A, and their analytical expressions can be found in the Supplemental Material [43].

In order to demonstrate the performance of the protocol with the finite cooperativity and the above imperfections, we numerically evaluate the values of the quality of two resulting NGSs versus the reciprocal of the optomechanical cooperativity $C_{\rm OM}^{-1} = \gamma\kappa/g^2$ in Figs. 3(a) and 3(b), with a fixed reflectivity $R = 0.5$. Additionally, the impacts of the total optical detection efficiency Γ and the amplification noise n_A on the quality are illustrated in Figs. 3(c) and 3(d). The quality of the resulting states is qualified by the following: the fidelity $\mathcal{F}$, the size of mechanical CSs $|\alpha|^2$, and the Wigner negativity δ that is defined as [37]

$$\delta = \iint_{-\infty}^{\infty} [|W(X_M, P_M)| - W(X_M, P_M)] dX_M dP_M. \quad (8)$$

Here a larger value of δ and $|\alpha|^2$ indicate a better quantum feature and more distant two coherent states, respectively, and $\delta = 0$ means that the nonclassicality has vanished.

This result shows that our protocol is surprisingly robust to mechanical decay. Note that since a larger mechanical decay corresponds to a shorter interaction duration τ for a fixed reflectivity R, this can reduce mechanical decoherence but also decrease the size of the CSs. In addition, the quality

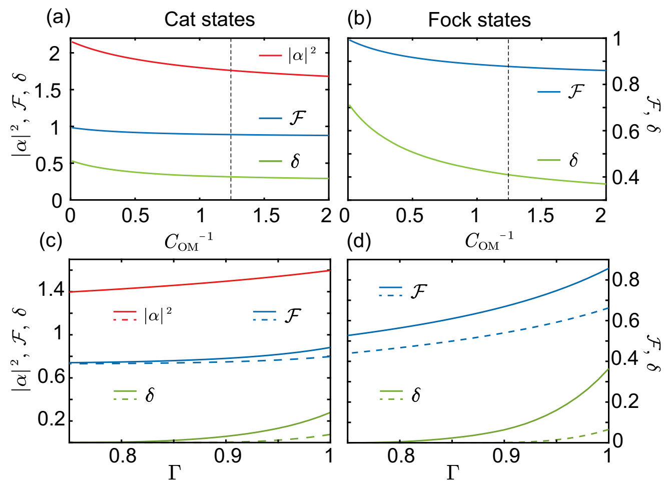

FIG. 3. (a),(b) The fidelity ($\mathcal{F}$) and Wigner negativity (δ) of two resulting NGSs vs $C_{\rm OM}^{-1} = \gamma\kappa/g^2$. The black dashed line indicates $C_{\rm OM} = 0.8$. (c),(d) The two resulting NGSs' qualities vs the total optical detection efficiency $\Gamma = \eta + \mu$. Solid (dashed) lines correspond to the amplification noise $n_A = 0$ ($n_A = 0.1$), and $|\alpha|^2$ is the size of the resulting CSs.

of the resulting states, not surprisingly, decreases with the increase of various imperfections. However, the quality of the resulting states can be further improved by decreasing the propagation distance of the cavity output light and the input squeezing level. Furthermore, the impact of imperfection in the homodyne is minor here since the homodyne detection efficiency for the optical field is usually close to unit, and we find that high-fidelity CSs can be well prepared with an arbitrary outcome, once the direction of the measurement is fixed to X_C (see Supplemental Material [43]).

Extension and discussion. This protocol can also be extended to rapidly prepare a four-component CS, which is widely used as cat codes in quantum error correction, by simply performing the two-photon EPS twice on the optical mode (C), as shown in Fig. 4(a). This construction allows the resulting mechanical NGSs to be manipulated by both $\mathcal{U}(g_{A1})$ and $\mathcal{U}(g_{A2})$. With the conditions $\xi_1 = 1$ and $\xi_2 = -2$, or $\xi_1 = 0$ and $\xi_2 = 3$, the mechanical mode will collapse into a four-component state, $\Sigma_{n=1}^4 |1.6e^{i(2n-1)\pi/4}\rangle$, after the projective measurement, as shown in Fig. 4(b) (see Supplemental Material [43]). The fidelity of this state reaches a value of ~0.98, which far exceeds the corresponding fidelity in the achieved experiments [6]. This extension implies that the process of EPS can be repeated and designed for preparing advanced NGSs, which provides an attractive strategy for implementing complex operations on NGSs.

Since the squeezing technology, photon subtraction, and homodyne measurement employed here are well developed in various quantum systems, the current protocol is also promising to be promoted to other physical platforms, including superconducting circuits [57–60], spin ensembles [61,62], etc. For example, in superconducting systems working in the microwave domain, field squeezing (the strength can reach ~ − 10 dB) [63], single-photon subtraction, and microwave homodyne measurement (with efficiency ~0.5) have been achieved [64,65]. Note that in addition to optical systems,

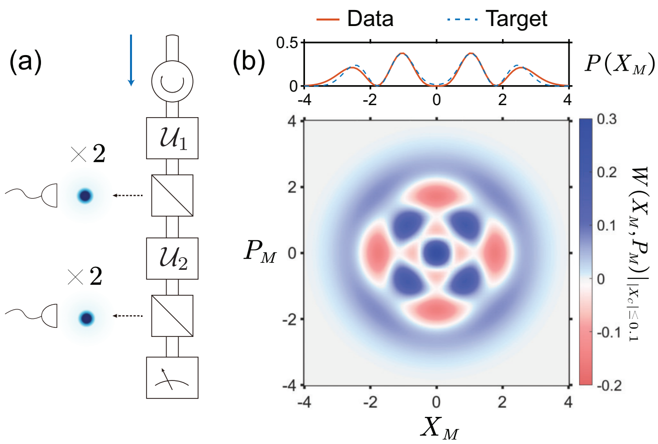

FIG. 4. (a) Sketch for preparing a four-component CS at the mechanical mode. (b) The resulting state with a high fidelity, ∼0.98, where the four cats locate at the positions with $\alpha = 1.6e^{in\pi/4}$, $n = 1, 3, 5, 7$. $P(X_M)$ is the probability distribution of the quadrature X_M. Here, $S_{\rm in} = -6$ dB, $R = 0.9$, and $C_{\rm om} = 0.8$.

superconducting circuits require a low environment temperature to get a negligible thermal occupation.

Conclusion. We have shown how mechanical non-Gaussian states can be faithfully generated and manipulated in weak-coupled cavity optomechanical systems ($C_{\rm om} = g^2/\kappa\gamma < 1$). The key ingredient is to use a squeezed pulse with a short duration to rapidly generate the EPR steering and an additional amplifier on the entangled optical mode to remotely control the mechanical states. Our protocol is robust to the mechanical dissipation and promises to deterministically generate mechanical cat states with the photon subtraction implemented by Rydberg atomic ensembles. With the parameters in state-of-the-art experiments, we simulated the preparation of various large-size mechanical non-Gaussian states, all of which maintain a high fidelity. Furthermore, the successful preparation of the four-component cat state convinces us that the cascaded construct of multiple EPSs may be used to generate high-quality advanced NGSs. Such capabilities can be promoted to various quantum systems and may have a wide-ranging impact on future quantum information processing strategies.

We thank Z. Peng for stimulating discussions. This work is supported by the National Key R&D Program of China (Grant No. 2019YFA0308200) and the National Natural Science Foundation of China (Grant No. 11874432). Q.H. thanks the National Natural Science Foundation of China (Grants No. 11975026 and No. 12125402), Beijing Natural Science Foundation (Grant No. Z190005) and the Key R&D Program of Guangdong Province (Grant No. 2018B030329001) for financial support. F.-X.S. acknowledges the National Natural Science Foundation of China (Grant No. 12147148) and the China Postdoctoral Science Foundation (Grant No. 2020M680186). J.-Q.L. is supported in part by the National Natural Science Foundation of China (Grants No. 12175061, No. 12247105, and No. 11935006) and the Science and Technology Innovation Program of Hunan Province (Grants No. 2021RC4029 and No. 2020RC4047).

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