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Letter

Geometrical torque on magnetic moments coupled to a correlated antiferromagnet

Nicolas Lenzing,[1] David Krüger,[1] and Michael Potthoff[1,2]

[1]*University of Hamburg, Department of Physics, Notkestraße 9-11, 22607 Hamburg, Germany*

[2]*The Hamburg Centre for Ultrafast Imaging, Luruper Chaussee 149, 22761 Hamburg, Germany*

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The geometrical spin torque mediates an indirect interaction of magnetic moments, which are weakly exchange coupled to a system of itinerant electrons. It originates from a finite spin-Berry curvature and leads to a non-Hamiltonian magnetic-moment dynamics. We demonstrate that there is an unprecedentedly strong geometrical spin torque in the case of an electron system, where correlations cause antiferromagnetic long-range order. The key observation is that the anomalous torque is strongly boosted by low-energy magnon modes emerging in the two-electron spin-excitation spectrum due to spontaneous breaking of SU(2) spin-rotation symmetry. As long as single-electron excitations are gapped out, the effect is largely universal, i.e., essentially independent of the details of the electronic structure, but decisively dependent on the lattice dimension and spatial and spin anisotropies. Analogous to the reasoning that leads to the Mermin-Wagner theorem, there is a lower critical dimension at and below which the spin-Berry curvature diverges.

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I. INTRODUCTION

A magnetic moment coupled to a system of itinerant electrons via a local exchange interaction of strength J experiences a spin torque which leads to precession dynamics. For several magnetic moments $\boldsymbol{S}_m$ (with $m = 1, \dots, M$), usually described as classical fixed-length spins, there are further torques caused by, e.g., indirect exchange interactions mediated by the electron system. These *Hamiltonian* spin torques, well known in micromagnetics [1] and in the theory of coupled spin-electron dynamics [2–8], all derive from interaction terms in the quantum-classical Hamiltonian [9] for the spin and electron degrees of freedom. In addition, there is a non-Hamiltonian spin torque that has a purely *geometric* nature. This geometrical spin torque represents the feedback of the Berry physics [10] on the classical magnetic-moment dynamics.

Generally, such feedback effects have been pointed out early [11–13] but have not been studied in spin dynamics theory until recently [14]. For weak J compared to the typical energy scales of the electron system, the classical spin dynamics is slow, such that the electron system accumulates a geometrical phase which is gauge independent in the case of a cyclic motion [10,15,16]. This Berry phase is closely related to the Berry curvature, a two-form which, when integrated in classical parameter space over a two-dimensional surface bounded by a closed path $\mathcal{C}$, yields the Berry phase associated with $\mathcal{C}$. For example, in molecular physics [17] and when treating the coordinates of the nuclei classically, the feedback of the Berry physics produces an additional geometrical force, where the Berry curvature plays the role of a magnetic field in the nuclei equations of motion. This effect is known as "geometrical magnetism" [18,19].

The geometrical spin torque resulting from the spin-Berry curvature (SBC) [14] is the analogous concept in the field of atomistic spin dynamics [4,20]. As opposed to the closely related geometrical friction term [18,19], i.e., Gilbert damping [21], it is energy conserving. But, importantly, the SBC is non-Hamiltonian and emerges for weak J, i.e., in the limit of slow classical spin dynamics. However, the effects are typically weak [22] for a solid [23], such that it appears difficult to disentangle the effect of the geometrical spin torque from other contributions [24].

In this Letter we study the geometrical spin torque for magnetic moments coupled to a magnetic solid: a correlated D-dimensional antiferromagnetic (AF) insulator. This is a generic situation realized, e.g., by magnetic impurities in the bulk or by magnetic adatoms on the surface of the antiferromagnet. We demonstrate that the magnitude of the SBC is governed by the magnon-excitation spectrum. This has very general consequences: the SBC must diverge for $D = 1$ but is regular for $D \geqslant 3$, see Table I. For $D = 2$ the SBC generically exhibits a logarithmic divergence as a function of any perturbation causing a gap in the magnon dispersion, such as magnetic anisotropies or external magnetic fields. The magnitude of the SBC and thus the impact on the magnetic-moment dynamics is studied for the Hubbard model at half-filling and zero temperature as a prototype of a correlation-induced insulator.

II. TIME-REVERSAL SYMMETRY

Within adiabatic spin-dynamics theory [14,22], geometrical spin torque is obtained from the SBC of the electron

TABLE I. Spin-Berry curvature of a spontaneously symmetry-broken antiferromagnetic state with gapped single-particle excitations. $\boldsymbol{k}$: wave vector. See text for discussion.

Lattice dimension	SBC	Distance dependence	Magnetic ground state
1	Divergent	–	–
2	Log. divergent	–	Stable
3	Regular	$1/R$	Stable
$D \geqslant 4$	$\sim \int_0^{\Lambda_{\text{cutoff}}} dk\, k^{D-3}$	$1/R^{D-2}$	Stable

system, see Eq. (2) below. Importantly, a finite SBC generally requires time-reversal symmetry (TRS) breaking in the electron system [22]. If J is strong, as assumed in Ref. [14], TRS is broken by the classical spin moment itself, as this acts like a local symmetry-breaking field. TRS breaking can be waived only at the cost of working with a non-Abelian extension of the theory well beyond the adiabatic limit [25], where the dynamics is governed by the generically finite non-Abelian spin-Berry curvature. Another approach is to replace the electron system with an entirely classical model composed of "slow" and "fast" spin moments [26,27]. This circumvents the necessity of TRS breaking altogether but still exhibits the feedback of holonomy effects in purely classical systems [28]. For magnetic moments coupled to *quantum* systems and in the physically relevant weak-J regime, a finite SBC can be achieved with an external magnetic field, or with a (staggered) orbital field as considered recently [22] with the Haldane model [29] as a prototype of a TRS-breaking Chern insulator [30]. However, fine tuning of the parameters is required to achieve considerable effects [22]. Here we consider an electron system in which correlations induce a TRS-breaking AF state. The AF order not only enables a finite SBC but also strongly boosts its magnitude due to magnon modes in the spin-excitation spectrum.

III. DYNAMICS OF MAGNETIC MOMENTS

We are interested in the slow dynamics of M magnetic moments, described as classical spins $\boldsymbol{S}_m$ of unit length, which are coupled to a correlated electron system with Hamiltonian H_{el} via a local exchange interaction $H_{\text{int}} = J \sum_{m=1}^{M} \boldsymbol{s}_{i_m} \boldsymbol{S}_m$. Here, i_m is the site the mth moment is coupled to, and $\boldsymbol{s}_i = 1/2 \sum_{\sigma\sigma'} c^\dagger_{i\sigma} \boldsymbol{\tau}_{\sigma\sigma'} c_{i\sigma'}$, where $\boldsymbol{\tau}$ is the vector of Pauli matrices, is the local spin moment at site i of the electron system. The total Hamiltonian is $H = H(\boldsymbol{S}) = H_{\text{el}} + H_{\text{int}}(\boldsymbol{S})$ and depends on the configuration $\boldsymbol{S} = (\boldsymbol{S}_1, \ldots, \boldsymbol{S}_M)$ of the magnetic moments.

Assuming that the electron system at any instant of time t is in its instantaneous ground state for the spin configuration $\boldsymbol{S}(t)$, i.e., $|\Psi(t)\rangle = |\Psi_0(\boldsymbol{S}(t))\rangle$, the equation of motion of adiabatic spin dynamics is given by [14,22]

$$\dot{\boldsymbol{S}}_m = \left(\boldsymbol{T}^{(\text{H})}_m + \boldsymbol{T}^{(\text{geo})}_m\right) \times \boldsymbol{S}_m \, . \tag{1}$$

Here $\boldsymbol{T}^{(\text{H})}_m \times \boldsymbol{S}_m$ with $\boldsymbol{T}^{(\text{H})}_m = \partial \langle H(\boldsymbol{S})\rangle / \partial \boldsymbol{S}_m = J \langle \boldsymbol{s}_{i_m}\rangle$ is the conventional (Hamiltonian) spin torque, where $\langle \cdots \rangle$ is the instantaneous ground-state expectation value.

IV. GEOMETRICAL SPIN TORQUE

The second term, the geometrical spin torque $\boldsymbol{T}^{(\text{geo})}_m \times \boldsymbol{S}_m$, is necessary to enforce the constraint $|\Psi(t)\rangle = |\Psi_0(\boldsymbol{S}(t))\rangle$ and has been derived within a quantum-classical Lagrange formalism in Refs. [14] and [22]. This assumes that the ground state is nondegenerate (otherwise non-Abelian spin-dynamics theory [25] must be used) and that J is sufficiently weak so that the classical spin dynamics is much slower than typical relaxation time scales of the quantum system H_{el}. Alternatively, the term may be derived within adiabatic response theory [18,19,31] as the first nontrivial correction in a systematic expansion of the response of a driven system with respect to the driving speed, when applied to spin dynamics [32]. It is given by

$$\boldsymbol{T}^{(\text{geo})}_m = \sum_\alpha \sum_{m'\alpha'} \Omega_{m'm,\alpha'\alpha}(\boldsymbol{S}) \dot{S}_{m'\alpha'} \boldsymbol{e}_\alpha , \tag{2}$$

with $\alpha = x, y, z$ and the αth unit vector $\boldsymbol{e}_\alpha$, and where

$$\Omega_{mm',\alpha\alpha'}(\boldsymbol{S}) = \frac{\partial}{\partial S_{m\alpha}} A_{m'\alpha'}(\boldsymbol{S}) - \frac{\partial}{\partial S_{m'\alpha'}} A_{m\alpha}(\boldsymbol{S}) \tag{3}$$

is the spin-Berry curvature. At each spin configuration $\boldsymbol{S}$, this is a real antisymmetric tensor ($\Omega_{m'm,\alpha'\alpha} = -\Omega_{mm',\alpha\alpha'}$), which is invariant under local gauge transformations of the ground states $|\Psi_0(\boldsymbol{S})\rangle \mapsto e^{i\phi(\boldsymbol{S})}|\Psi_0(\boldsymbol{S})\rangle$. It is the exterior derivative of the spin-Berry connection $\boldsymbol{A}_m = i\langle \Psi_0 | \frac{\partial}{\partial \boldsymbol{S}_m} | \Psi_0\rangle$, which describes parallel transport of the ground state $|\Psi_0(\boldsymbol{S})\rangle$ on the manifold of spin configurations $\mathcal{M}$. For M classical spins $\boldsymbol{S}_m \in S^2$, this is given by the M-fold Cartesian product of 2-spheres $\mathcal{M} \equiv S^2 \times \cdots \times S^2$.

V. SPONTANEOUS ANTIFERROMAGNETIC ORDER

We consider a coupling of the magnetic spin moments to the single-band Hubbard model [33,34] on a D-dimensional hypercubic lattice as a prototypical model for itinerant magnetic order. Its Hamiltonian is $H_{\text{el}} = -t \sum_{ij}^{\text{n.n.}} \sum_{\sigma=\uparrow,\downarrow} c^\dagger_{i\sigma} c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow}$, where the nearest-neighbor hopping $t = 1$ fixes the energy and (with $\hbar \equiv 1$) the time scales. $c_{i\sigma}$ annihilates an electron at site i with spin projection σ, and $n_{i\sigma} = c^\dagger_{i\sigma} c_{i\sigma}$. The sums over i, j are restricted to nearest neighbors, and L is the total number of sites. It is well known [35–39] that at half-filling, repulsive Hubbard-U and for $D \geqslant 2$, the ground state of the system in the thermodynamical limit $L \to \infty$ develops long-range AF correlations. SU(2) spin-rotation symmetry and therewith TRS are spontaneously broken, and the ordered state is characterized by a finite staggered magnetization $\boldsymbol{m} = m\boldsymbol{e}_z$ with $m = L^{-1} \sum_i z_i \langle n_{i\uparrow} - n_{i\downarrow}\rangle$ and $z_i = \pm 1$ for i in sublattice A or B, respectively. We assume $m > 0$ for sublattice A.

At weak U, AF order is driven by the Slater mechanism and perturbatively accessible [36,38]. Within self-consistent Hartree-Fock theory [40], the one-electron excitation spectrum displays a gap $\Delta = Um$ at wave vector $Q = (\pi, \pi, \ldots)$ in the conventional Brillouin zone. The two-electron spin-excitation spectrum is well described by standard random-phase approximation (RPA) but for the symmetry-broken AF state [41–44].

In the strong-U limit, the one-electron spectrum is dominated by a large Hubbard gap $\Delta \sim U$ and well-developed local spin moments, coupled via Anderson's superexchange [35,38]. Here, the model maps onto the Heisenberg spin-1/2 Hamiltonian with AF exchange $J_{\rm H} = 4t^2/U$ and AF long-range order, see Refs. [37], [45], and [46], for example. To compute the low-energy magnon dispersion and states, we can apply spin-wave theory (SWT) [47] to the AF Heisenberg model and use the Holstein-Primakoff transformation [48] at linear order. Linear SWT is motivated by the fact that single-magnon decay requires overlap with the two-magnon continuum, so that the picture of a stable magnon gas is protected by kinematic restrictions at low energies [49–52].

VI. SPIN-BERRY CURVATURE OF AN ANTIFERROMAGNET

To compute the geometrical spin torque, we make use of a Lehmann-type representation of the SBC starting from Eq. (3). This is straightforwardly derived [22] using a resolution of the unity, $\mathbf{1} = \sum_n |\Psi_n(\boldsymbol{S})\rangle\langle\Psi_n(\boldsymbol{S})|$, with an orthonormal basis of instantaneous eigenstates of $H_{\rm el} + H_{\rm int}(\boldsymbol{S})$:

$$\Omega_{mm',\alpha\alpha'} = -2J^2 {\rm Im} \sum_{n\neq 0} \frac{\langle\Psi_0|s^{\alpha}_{i_m}|\Psi_n\rangle\,\langle\Psi_n|s^{\alpha'}_{i_{m'}}|\Psi_0\rangle}{(E_n - E_0)^2}\,. \tag{4}$$

Note that, due to the J^2 prefactor, the $\boldsymbol{S}$ dependence of the eigenenergies and eigenstates will provide corrections to Eq. (4) only at order J^3. As we refer to the weak-J limit, these will be neglected in the following.

In the AF phase and assuming that the order parameter is aligned to the z axis, $\langle \boldsymbol{s}_i\rangle = (-1)^i m \boldsymbol{e}_z$, there is a remaining SO(2) symmetry of the energy eigenstates under spin rotations around $\boldsymbol{e}_z$. This unbroken spin-rotation symmetry, together with the spatial inversion and translation symmetries of $H_{\rm el}$, and the antisymmetry $\Omega_{mm',\alpha\alpha'} = -\Omega_{m'm,\alpha'\alpha}$ [see Eq. (3)] imply that the spin-Berry curvature tensor is entirely fixed by a single real number $\Omega \equiv \Omega_{mm',xy} = -\Omega_{mm',yx}$ for each fixed pair of sites i_m, $i_{m'}$. All other elements must vanish, as is detailed by the symmetry analysis in Sections A and B of the Supplemental Material (SM) [53].

In a first step, for weak U, we compute the SBC via

$$\Omega_{mm'} = -iJ^2 \frac{\partial}{\partial\omega} \chi_{i_m i_{m'},xy}(\omega)\Big|_{\omega=0} + \mathcal{O}(J^3)\,, \tag{5}$$

where $\chi_{ii',\alpha\alpha'}(\omega) = L^{-1}\sum_{\boldsymbol{k}} e^{i\boldsymbol{k}(\boldsymbol{R}_i - \boldsymbol{R}_{i'})}\chi_{\alpha\alpha'}(\boldsymbol{k},\omega)$ is the real-space retarded susceptibility, obtained by the RPA (see SM, Sec. C [53]). The relation Eq. (5) is easily derived by comparing the representation Eq. (4) of the SBC with the Lehmann representation of the susceptibility (SM, Secs. A and B [53]). Therewith, the susceptibility in the symmetry-broken AF state is seen to play a dual role for the spin dynamics: (i) via Eq. (5) and Eq. (2) its frequency derivative at $\omega = 0$ yields the geometrical spin torque $\boldsymbol{T}^{\rm (geo)}_m \times \boldsymbol{S}_m$, and (ii) the static susceptibility yields, in the weak-J regime, the conventional RKKY spin torque $\boldsymbol{T}^{\rm (H)}_m \times \boldsymbol{S}_m$ with $\boldsymbol{T}^{\rm (H)}_m = \partial H_{\rm RKKY}/\partial \boldsymbol{S}_m$, where $H_{\rm RKKY} = J^2\sum \chi_{i_m i_{m'},\alpha\alpha'}(\omega=0) S_{m\alpha} S_{m'\alpha'}$ is the perturbative RKKY Hamiltonian of the AF state.

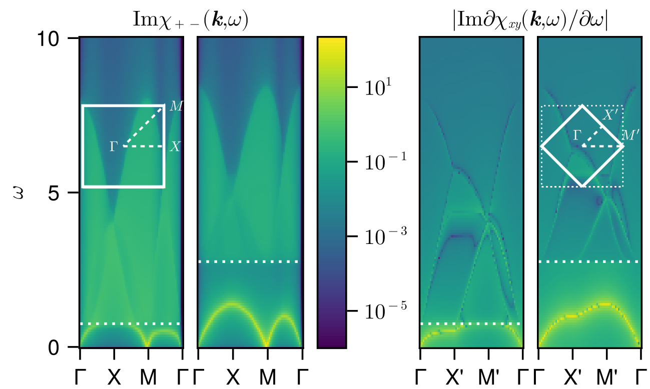

FIG. 1. Left: Transversal retarded ground-state spin susceptibility ${\rm Im}\,\chi_{+-}(\boldsymbol{k},\omega)$ for $U = 2$ and $U = 4$ along high-symmetry directions in the conventional $D = 2$ Brillouin zone, as obtained by RPA. Right: Frequency derivative ${\rm Im}\,\partial_\omega\chi_{xy}(\boldsymbol{k},\omega)$ (absolute values) in the mBz, related to the SBC at $\omega = 0$. White dotted lines: Slater gap $\Delta = Um$ (onset of the continuum). Lorentzian broadening $\omega \to \omega + i\eta$ with $\eta = 0.045$. Energy scale: $t = 1$.

For the Hubbard model on the $D = 2$ square lattice the spin-excitation spectrum $\chi_{+-}(\boldsymbol{k},\omega)$, see Fig. 1 (left) for $U = 2$ and $U = 4$, consists of a continuum at high frequencies $\omega > \Delta = Um$ ($\Delta \approx 0.75$ for $U = 2$, $\Delta \approx 2.76$ for $U = 4$) and, furthermore, within the gap an undamped transversal and doubly degenerate magnon mode. This mode takes most of the spectral weight. The magnon contribution to the derivative $\partial_\omega\chi_{xy}(\boldsymbol{k},\omega)$ on sublattice A (Fig. 1, right) is even more pronounced, especially for $\omega = 0$, where it is related to the SBC by Eq. (5).

VII. GOLDSTONE THEOREM, IMPLICATIONS

In our second step, we exploit the fact that the spin-excitation spectrum of an AF insulator has a universal structure at low frequencies. This is due to Goldstone's theorem, which enforces the presence of gapless magnon modes [54–56]. In the collinear AF state and corresponding to the two broken generators of the spin SU(2) symmetry, there are two degenerate modes with a linear and isotropic dispersion in the vicinity of the Γ point in the magnetic Brillouin zone (mBz). Linear SWT applied to the Heisenberg model that emerges in the strong-U limit captures this physics, i.e., the dispersion close to Γ is given by $\frac{1}{2}J_{\rm H}\omega(\boldsymbol{k}) = c_{\rm s}k + \mathcal{O}(k^2)$, where $c_{\rm s}$ is the spin-wave velocity. Using the magnon energies and eigenstates, we can compute the SBC in this limit from Eq. (4) directly (SM, Secs. D and E [53]), ending up with

$$\Omega_{mm'} = \mp\frac{2J^2}{J_{\rm H}^2}\frac{1}{(2\pi)^D}\int_{\rm mBz} d^Dk \frac{\cos\left[\boldsymbol{k}\left(\boldsymbol{R}_{i_m} - \boldsymbol{R}_{i_{m'}}\right)\right]}{\omega(\boldsymbol{k})^2}, \tag{6}$$

if both i_m, $i_{m'}$ belong to sublattice A ($-$ sign) or B ($+$ sign), and $\Omega_{mm'} = 0$ else.

For $D = 2$, the linear dispersion close to Γ then implies a $1/k^2$ singularity of the integrand and thus a logarithmic infrared divergence. For $D \geqslant 3$, the local ($m = m'$) SBC is finite. We note that the same arguments as invoked for the Mermin-Wagner theorem [47,57], i.e., a divergence due to the low-energy spin excitations, here lead to a lower

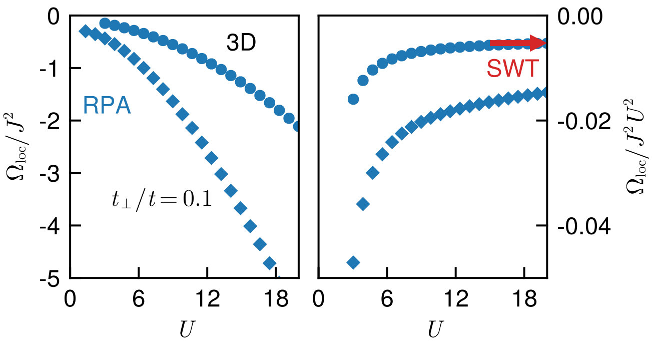

FIG. 2. Local SBC as function of U for $D = 3$, as obtained from RPA. Left: Ω_{loc}/J^2. Right: Ω_{loc}/J^2U^2. Diamonds: $t_\perp/t = 0.1$, see also Fig. 3 (left). Red arrow: $D = 3$ SWT ($U \to \infty$) result. $\eta = 0.035$.

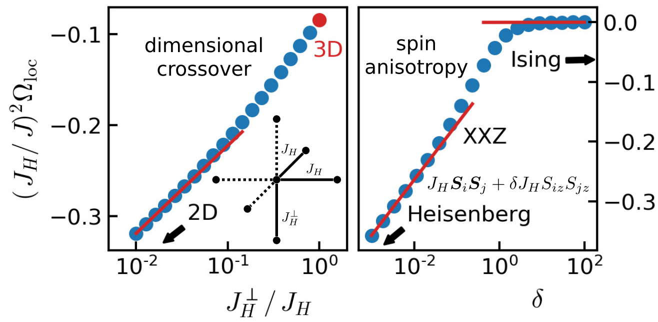

FIG. 3. SWT results (dots) for anisotropic systems. Left, dimensional crossover: local SBC for $D = 3$ but with a spatially anisotropic nearest-neighbor Heisenberg exchange $J_H^\perp \leqslant J_H$. Right, spin anisotropy: SBC as function of the coupling anisotropy parameter δ.

critical dimension ($D_c = 3$) that is shifted by one, see Table I. The numerical value for the $D = 3$ local SBC is $\Omega_{loc} \approx -0.084\,J^2/J_H^2 = -0.084\,J^2U^2/16t^4$. When scaling the hopping as $t = t^*/\sqrt{D}$ with $t^* = \text{const}$ [58,59], the modulus of the SBC decreases monotonically with D, and the SBC approaches a finite mean-field value $|\Omega_{loc}| \to J^2U^2/32t^{*4}$ for $D \to \infty$ (SM, Sec. F [53]).

VIII. MAGNITUDE OF THE SBC

SWT predicts a U^2 dependence of the SBC in the Heisenberg limit for strong U. For $U = 0$, on the other hand, TRS of the resulting paramagnetic state implies that it must vanish. For $U \to 0$, there is an intricate competition between the exponential suppression of the order parameter $m \propto e^{-1/U}$, i.e., of the "strength" of TRS breaking and thus of the SBC and, on the other hand, the exponential closure of the single-electron Slater gap $\Delta = Um$ and thus of the onset of the continuum in the spin-excitation spectrum resulting in continuum contributions that favor a large SBC. Our numerical results for the local SBC in $D = 3$, as obtained from weak-coupling RPA and strong-coupling SWT, are displayed in Fig. 2. With increasing U we find a smooth crossover from the Slater to the Heisenberg limit with a monotonically increasing $|\Omega_{loc}|$.

The nonlocal SBC at large distances $R \equiv \|\boldsymbol{R}_{i_m} - \boldsymbol{R}_{i_{m'}}\|$ is again governed by the linear dispersion at low frequencies. Carrying out the integration in Eq. (6) for $R \to \infty$ we find $\Omega(R) \propto 1/R^{D-2}$ (see Table I and SM, Sec. F [53]). For $D = 3$ this implies that the geometrical spin torque mediates a long-range coupling in the spin dynamics.

Compared to previous studies [14,22,24–27] the $D = 3$ value of the local SBC $|\Omega_{loc}| \approx 0.084\,J^2/J_H^2$ is several orders of magnitude larger for realistic parameters $J, J_H \ll t, U$. Renormalization of $c_s \to c_s' \approx 1.1c_s$ due to magnon interaction [60] leads to a slightly smaller SBC, $|\Omega_{loc}| \to (c_s/c_s')^2|\Omega_{loc}|$.

There are at least two routes that lead to an even larger $|\Omega_{loc}|$: namely, we can take advantage of the formally infinite SBC in $D = 2$ and regularize the theory (i) by dimensional crossover to $D = 3$ [61–63], i.e., by switching on a small hopping $t_\perp$ in the third dimension (Fig. 2), implying $J_H^\perp \ll J_{H,x} = J_{H,y} = J_H$, see Fig. 3 (left), or (ii) by switching on a magnetic anisotropy to open a small gap in the magnon spectrum (Fig. 3, right), i.e., by adding an Ising term $\delta\, J_H S_{iz} S_{jz}$ to the standard Heisenberg coupling $J_H \boldsymbol{S}_i \boldsymbol{S}_j$. A moderate $J_H^\perp/J_H = 0.1$ yields a SBC $|\Omega_{loc}| \approx 0.22J^2/J_H^2$. About the same enhancement is obtained for an anisotropy parameter $\delta \sim 10^{-2}$.

IX. GEOMETRICAL SPIN DYNAMICS

For the AF ordered phase, Eq. (1) tells us that the dominating effect in the magnetic-moment dynamics is a precession around the staggered magnetization $\boldsymbol{m}$ on a time scale $1/J$. This effect dominates the weaker (and slower) anisotropic RKKY-type exchange on the scale J^2. Importantly, the SBC $\Omega \sim J^2$ enters the equations of motion as a renormalization factor (for $M > 1$ classical spins as a matrix factor) rather than a summand and thus does not compete with the stronger direct exchange of order J (SM, Sec. G [53]). For $M = 1$ this factor amounts to $1/(1 - \Omega_{loc}S_z)$, such that the most pronounced effects are found for a SBC of intermediate strength, $\Omega_{loc} = \mathcal{O}(1)$. This holds true for $M = 2$ as well, as is detailed in the SM, Sec. G [53]. Note that a singular renormalization indicates a breakdown of the theory as this is the point where the condition for nearly adiabatic spin dynamics is invalidated. Note further that the precession comes with an inverted orientation beyond the singular point.

X. CONCLUSIONS AND OUTLOOK

A hitherto unknown but generic interplay of electron correlations, spontaneous symmetry breaking, gapless Goldstone bosons, and a holonomy on the configuration space of classical spin degrees of freedom leads to non-Hamiltonian effects, such as renormalization of precession frequencies, inverted orientation of the precessional motion, or long-range interactions, in the spin dynamics. This is due to a geometrical spin torque which is finite for correlated AF ground states in lattice models with dimension $D \geqslant 3$ and diverges for $D \leqslant 2$, caused by the same mechanism that leads to the Mermin-Wagner theorem, however, shifted by one dimension. With a SBC $\Omega_{loc} = \mathcal{O}(1)$ for typical parameters, the effect is unexpectedly large. It is boosted by electron correlations and further enhanced by spatial and spin anisotropies.

We expect a strong overall impact on the phenomenology of atomistic spin dynamics, in particular on the field of antiferromagnetic spintronics [64–66], e.g., on spin-transfer torques in antiferromagnets (see Ref. [67], for example). An according concretization of the theory, however, has yet to be worked out. Anisotropic one- and two-dimensional magnetic-moment arrays, engineered atom by atom [68], or two-dimensional (anti)ferromagnetic materials [69] represent promising platforms for applications and comparison with experiments.

Treating the magnetic moments $\boldsymbol{S}_m$ as classical vectors, especially in the antiferromagnetic case [70], must be seen as an approximation that avoids a full quantum many-body setup but disregards correlation effects such as Kondo screening or heavy-fermion behavior. The approximation may be justified for high spin quantum numbers, see, e.g., Refs. [70,71], or generally in cases where there are well-formed spin moments that remain unscreened on timescales exceeding the remaining timescales of the problem. The very presence of the geometrical spin torque for the quantum-spin case, however, has been demonstrated using time-dependent density-matrix renormalization [14]. While this method and also exact time propagation (TDSE) are limited to one-dimensional or small systems and to very short femtosecond timescales, insightful results for nonclassical spin-transfer effects [72] and quantum spin transfer torque [73] were obtained recently. A consistent effective low-energy theory for a system that is entirely quantum mechanical with at least two largely different timescales has yet to be developed.

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