Letter

# Robust transitionless quantum driving: Concatenated approach

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We propose a concatenated approach for implementing transitionless quantum driving regardless of adiabatic conditions while being robustness with respect to all kinds of systematic errors induced by pulse duration, pulse amplitude, detunings, and Stark shift, etc. The current approach is particularly efficient for all time-dependent pulses with arbitrary shape, and only the phase differences between pulses is required to properly modulate. The simple physical implementation without the help of pulse shaping techniques or extra pulses makes this approach quite universal and provides a different avenue for robust quantum control by the time-dependent Hamiltonian.

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*Introduction.* The adiabatic passage (AP) [1–3] plays an important role in quantum optics [4] and quantum computation [5–7], due to the insensitivity to various errors in physical parameters. In the adiabatic process, the system always evolves along an eigenstate connecting the initial state with the target state. One typical example of AP is the stimulated Raman adiabatic passage (STIRAP) [8–13], a popular way for population inversion in three-level systems. Regardless of its robustness, the common characteristic of AP are incompleteness of population transfer and slow change of parameters over time.

To overcome these shortcomings, a technique called shortcuts to adiabaticity (STA) [14–24] has emerged that can directly cancel nonadiabatic transitions by introducing extra counterdiabatic fields. This technique allows the system to perfectly evolve along the eigenstate of the original Hamiltonian at a very fast rate but suffers a great loss on the robustness of AP since it requires exactly knowing the system parameters in advance. Recently, several works are devoted to the robustness of STA [25–28]. Another essential requirement in STA is the pulse shaping technique to tailor the original waveform, and some counterdiabatic fields sometimes are prohibited from a physical point of view so its realization may be challenging.

As another alternative, the composite adiabatic passage (CAP) technique [29], a combination of the best of both composite pulses [30–44] and adiabatic passage [45,46], has been proposed to achieve complete population inversion in two-level systems, while its extension version named composite STIRAP [47,48] achieves the same objective in three-level systems. The main advantage of CAP over STA is the retained robustness of AP and the simplicity of implementation, because one just properly modulates the relative phases between different pulses. Nevertheless, some fundamental issues still need to be addressed in CAP, e.g., the unaccessible of obtaining universal rotation operations (or arbitrary superposition states) and the requirement of longer duration compared with traditional adiabatic passages.

In this work, through carefully designing the phase differences, we propose a different dynamical mechanism for achieving perfect transitionless quantum driving in a robust manner, while simultaneously performing high precision quantum operations even *without* knowing the magnitudes of various systematic errors. The current approach is constructed by multiply concatenating the Hamiltonians with the same arbitrary pulse shape but different well-designed constant phases. In particular, it is unnecessary to satisfy the adiabatic condition, and thus the total duration does not have to be long.

*Gauge invariance.* Consider a quantum system with near-neighbor interactions, and the general form of the time-dependent Hamiltonian is ($\hbar = 1$)

$$H(t) = \sum \lambda_{j,j}(t)|j\rangle\langle j| + \lambda_{j,j+1}(t)|j\rangle\langle j+1| + \text{H.c.}, \quad (1)$$

where $\lambda_{j,j}(t)$ denote level energies, and $\lambda_{j,j+1}(t)$ are the coupling strengths of near-neighbor levels. When introducing extra constant phases to the coupling strengths, the Hamiltonian becomes

$$\begin{aligned} H(t,\boldsymbol{\theta}) &= \sum \lambda_{j,j}(t)|j\rangle\langle j| + \lambda_{j,j+1}(t)e^{i\theta_j}|j\rangle\langle j+1| + \text{H.c.} \\ &= \sum \lambda_{j,j}(t)|\tilde{j}\rangle\langle\tilde{j}| + \lambda_{j,j+1}(t)|\tilde{j}\rangle\langle\widetilde{j+1}| + \text{H.c.}, \quad (2) \end{aligned}$$

where $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots)$ represents a vector parametrizing different constant phases in external fields. Through making appropriate transformations to the original basis:

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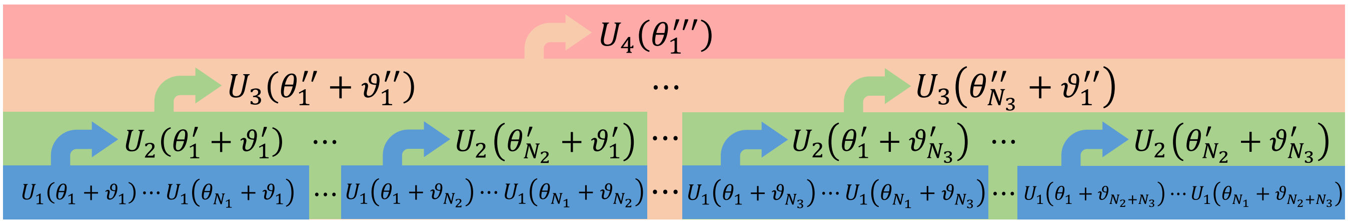


FIG. 1. Sketch diagram of the concatenated approach. Different colors represent different uses, and $\boldsymbol{\vartheta}_1=\boldsymbol{\vartheta}_1'=\boldsymbol{\vartheta}_1''=0$. In the blue box (the first hierarchy), the phase differences $\boldsymbol{\theta}_{mn}$ in the propagator $U_2(\boldsymbol{\theta}_1')=U_1(\boldsymbol{\theta}_{N_1})\cdots U_1(\boldsymbol{\theta}_1)$ are applied to achieve perfect transitionless quantum driving. The only difference between the different blue boxes is the phase shift $\boldsymbol{\vartheta}_n$. By continuing to concatenate the propagators $U_2(\boldsymbol{\theta}_n')$ in the green box (the second hierarchy), i.e., $U_3(\boldsymbol{\theta}_1'')=U_2(\boldsymbol{\theta}_{N_2}')\cdots U_2(\boldsymbol{\theta}_1')$, we design the phase differences $\boldsymbol{\theta}_{mn}'$ for the robustness against the nonadiabatic transition induced by various errors. Similarly, the phase differences $\boldsymbol{\theta}_{mn}''$ in the orange box (the third hierarchy) are devoted to performing high precision quantum operations, where $U_4(\boldsymbol{\theta}_1''')=U_3(\boldsymbol{\theta}_{N_3}'')\cdots U_3(\boldsymbol{\theta}_1'')$. Certainly, the propagators $U_4(\boldsymbol{\theta}_n''')$ can be further concatenated to realize other uses. In this way, we achieve transitionless quantum driving in a robust manner even in the presence of various systematic errors.

$|\tilde{j}\rangle=\exp(-i\sum_{k=1}^{k=j-1}\theta_k)|j\rangle$, the form of the Hamiltonian with different constant phases is the same as the previous one. Namely, the addition of constant phases simply means that we are choosing a set of new basis and the expression of the propagator remains unchanged, i.e., $U_1(t)=\tilde{U}_1(t,\boldsymbol{\theta})$, where $U_1(t)$ and $\tilde{U}_1(t,\boldsymbol{\theta})$ represent the propagators in the basis $|j\rangle$ and $|\tilde{j}\rangle$, respectively. Therefore, the physical property of the Hamiltonian (2) is still trivial after introducing arbitrary constant phases. Actually, this trivial property originates from the gauge invariance of the system [49].

*Nontriviality of phase differences*. Even though the trivial property do not change in any way for arbitrary constant phases, *phase differences* are of nontrivial physical significance and can be used for implementing perfect transitionless quantum driving in a robust manner. To make it more clear, we demonstrate the detailed design workflow in Fig. 1, where the propagator corresponding to the Hamiltonian $H(t,\boldsymbol{\theta}_n)$ reads $U_1(\boldsymbol{\theta}_n)=\mathbf{T}\exp[-i\int H(t,\boldsymbol{\theta}_n)t]$ with $\mathbf{T}$ being a time-ordering operator. As shown in the blue box, we initially concatenate $N_1$ Hamiltonians $H(t,\boldsymbol{\theta}_n)$ with the same time-varying shape but different $\boldsymbol{\theta}_n$ to create a composite propagator $U_2(\boldsymbol{\theta}_1')=U_1(\boldsymbol{\theta}_{N_1})\cdots U_1(\boldsymbol{\theta}_1)$, where the phase differences $\boldsymbol{\theta}_{mn}=\boldsymbol{\theta}_m-\boldsymbol{\theta}_n$ are properly modulated to achieve perfect transitionless quantum driving.

When the quantum system exhibits various errors, transitionless quantum driving becomes imperfect and thus produces nonadiabatic transitions. To solve it, we continue to concatenating $N_2$ propagators $U_2(\boldsymbol{\theta}_n')$ to produce a new one $U_3(\boldsymbol{\theta}_1'')=U_2(\boldsymbol{\theta}_{N_2}')\cdots U_2(\boldsymbol{\theta}_1')$, where $U_2(\boldsymbol{\theta}_n')$ are generated by adding different constant phase shifts $\boldsymbol{\vartheta}_n$ to $N_1$ Hamiltonians $H(t,\boldsymbol{\theta}_n)$. Through altering the phase differences $\boldsymbol{\theta}_{mn}'=\boldsymbol{\theta}_m'-\boldsymbol{\theta}_n'$, the propagator $U_3(\boldsymbol{\theta}_1'')$ would sharply suppress the nonadiabatic transition induced by various errors.

Note that the propagator $U_3(\boldsymbol{\theta}_1'')$ becomes approximately diagonalized after the first two concatenations, but there still exists phase errors on diagonal elements and those errors would reduce the precision of quantum operations. Thus, we require to further concatenate $N_3$ propagators $U_3(\boldsymbol{\theta}_n'')$ to obtain the new one $U_4(\boldsymbol{\theta}_1''')=U_3(\boldsymbol{\theta}_{N_3}'')\cdots U_3(\boldsymbol{\theta}_1'')$. Similarly, the phase differences $\boldsymbol{\theta}_{mn}''=\boldsymbol{\theta}_m''-\boldsymbol{\theta}_n''$ are finely tuned to promote the accuracy of quantum operations. Indeed, the propagators $U_4(\boldsymbol{\theta}_n''')$ can be also concatenated for more other uses if necessary. Through this concatenated approach, we obtain a sequence for implementing robust quantum operations under transitionless quantum driving.

*Transitionless quantum driving*. To illustrate this, let us consider a three-level $\Lambda$ system driven by a Stokes and pump (SP) pulse pair, while two-photon resonance is kept. The form of the Hamiltonian reads

$$H(t)=\Delta(t)\sigma_{ee}+[\Omega_p(t)e^{i\theta_p}\sigma_{ge}+\Omega_s(t)e^{i\theta_s}\sigma_{fe}+\mathrm{H.c.}],\quad(3)$$

where $\sigma_{kl}=|k\rangle\langle l|$ $(k,l=g,f,e)$ and the $|g\rangle(|f\rangle)\leftrightarrow|e\rangle$ transition is driven by the pump (Stokes) pulse with the coupling strength $\Omega_p(t)$ $[\Omega_s(t)]$, the phase $\theta_p$ $(\theta_s)$, and the detuning $\Delta(t)$. The instantaneous eigenstates of $H(t)$ are $|d(t)\rangle=\cos\phi(t)\exp(-i\theta_{sp})|g\rangle-\sin\phi(t)|f\rangle$, $|E_+(t)\rangle=\sin\varphi(t)\exp(i\theta_s)|b(t)\rangle+\cos\varphi(t)|e\rangle$, and $|E_-(t)\rangle=\cos\varphi(t)\exp(i\theta_s)|b(t)\rangle-\sin\varphi(t)|e\rangle$, with $\theta_{sp}=\theta_s-\theta_p$, $\Omega(t)=\sqrt{\Omega_p(t)^2+\Omega_s(t)^2}$, $\tan 2\varphi(t)=2\Omega(t)/\Delta(t)$, $\tan\phi(t)=\Omega_p(t)/\Omega_s(t)$, and the bright state $|b(t)\rangle=\sin\phi(t)|g\rangle+\cos\phi(t)\exp(i\theta_{sp})|f\rangle$.

In STIRAP, there is a time delay in the SP pulse pair. Because of this asynchrony, $\phi(t)$ is a variable quantity over time, inevitably leading to nonadiabatic transitions between distinct eigenstates. Here we demand that the SP pulse pair must be synchronized [i.e., $\phi(t)$ is constant] to make the dark state $|d(t)\rangle$ completely decouple to other adiabatic states [50]. To extract a freely adjustable phase (e.g., $\theta_s$), the eigenstates $|E_+(t)\rangle$ and $|E_-(t)\rangle$ are reassociated to form a set of new basis $\{|d(t)\rangle,|b(t)\rangle,|e\rangle\}$, and we adopt this set of basis to reveal the dynamical mechanism of transitionless quantum driving.

In the synchronization of the SP pulse pair, the general expression for the system propagator in the basis $\{|d(t)\rangle,|b(t)\rangle,|e\rangle\}$ is given by

$$U_1(\boldsymbol{\theta})=\begin{bmatrix}1&0&0\\0&se^{i\alpha}&re^{-i\theta_s}\\0&-r^*e^{i\theta_s}&se^{-i\alpha}\end{bmatrix},\quad(4)$$

where $r$ and $\alpha$ are determined by the parameters of the SP pulse pair and $s=\sqrt{1-|r|^2}$. Definitely, the appearance of $r$ in Eq. (4) is the result of the coupling between the states $|b(t)\rangle$ and $|e\rangle$ in this system. To completely nullify this nonadiabatic transition, we just have to concatenate two SP pulse pairs with well-designed phase differences, and the propagator reads

$U_2(\boldsymbol{\theta}'_1) = U_1(\boldsymbol{\theta}_2)U_1(\boldsymbol{\theta}_1)$, where $U_1(\boldsymbol{\theta}_{n_1})$ is the propagator for the $n_1^{\rm th}$ SP pulse pair with $\boldsymbol{\theta}_{n_1} = (\theta_{s,n_1}, \theta_{p,n_1})$ and $\theta_{s(p),n_1}$ refers to the phase of the $n_1^{\rm th}$ Stokes (pump) pulse, $n_1 = 1, 2$. When the phase differences satisfy [50]

$$\theta_{21} = \theta_{s,2} - \theta_{s,1} = \pi - 2\alpha, \quad \theta_{s,n_1} - \theta_{p,n_1} = \theta_{sp}, \tag{5}$$

the propagator becomes diagonalizable, i.e.,

$$U_2(\boldsymbol{\theta}'_1) = |d(t)\rangle\langle d(t)| + e^{i2\alpha}|b(t)\rangle\langle b(t)| + e^{-i2\alpha}|e\rangle\langle e|. \tag{6}$$

According to this propagator, we can perfectly steer the system evolution along the dark/bright state without any transitions to the excited state $|e\rangle$. Therefore, two SP pulse pairs are sufficient to achieve perfect transitionless quantum driving, where the shape of the Stokes (pump) pulse can be arbitrary and does not require to satisfy the adiabatic condition.

When returning to the original representation, the propagator (6) in the subspace spanned by $\{|g\rangle, |f\rangle\}$ becomes $\mathcal{U} = \exp(-i\alpha \boldsymbol{n}\cdot\boldsymbol{\sigma})$ with $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ being the Pauli operators. It actually represents a rotation operation around the axis $\boldsymbol{n} = (-\sin\phi(t)\cos\theta_{sp}, \sin\phi(t)\sin\theta_{sp}, \cos\phi(t))$ by the angle $\alpha$ in the Bloch sphere. As a result, we obtain a universal quantum gate, its evolution along the dark/bright state without any transitions. In particular, different rotation operations [i.e., reflecting in different $\alpha$, $\phi(t)$, and $\theta_{sp}$] can be freely modulated by the coupling strength $\Omega_s(t)$, the detuning $\Delta(t)$, or the phase $\theta_s$ in this system.

*Robustness.* In reality, there are many factors that prevent us from exactly acquiring the full information of a quantum system. For example, in the context of atoms driven by external fields, the external fields are usually assumed to be monochromatic while they actually have a certain linewidth. Due to their interactions, the energy level of atoms may also shift, i.e., the so-called Stark shift [4]. Furthermore, the inhomogeneous distribution of external fields as well as tiny vibrations of atoms at equilibrium, can create small uncertainties in the interactions of these systems. All these uncertainties can be regarded as errors in various physical parameters, such as pulse duration, pulse amplitude, and detuning. These errors mainly lead to two unfavorable effects: the generation of nonadiabatic transitions and the imprecision of quantum control. Actually, both effects can be largely avoided by further concatenating the propagators with well-designed constant phases. Next, we elaborate on this point.

We can see in Eq. (4) that systematic errors mainly cause the deviation of two quantities: $r$ and $\alpha$. Fortunately, the concatenated dynamical mechanism for transitionless quantum driving is inherently immune to the deviation in the quantity $r$, because the design of phase differences is independent of $r$ [50]. As a result, the system evolution is completely unaffected by the deviation in the quantity $r$. For the deviation in the quantity $\alpha$, it makes the phase difference satisfying Eq. (5) slightly different, resulting in the generation of the nonadiabatic transition. To reduce this transition, we can concatenate $N_2$ propagators with different constant phases: $U_3(\theta''_1) = U_2(\boldsymbol{\theta}'_{N_2})\cdots U_2(\boldsymbol{\theta}'_1)$, where $\boldsymbol{\theta}'_{n_2} = (\theta_{s,n_2}, \theta_{p,n_2})$ and $\theta_{s(p),n_2}$ refers to the phase of the $(2n_2-1)$th Stokes (pump) pulse in this situation, $n_2 = 1, \ldots, N_2$. For $N_2 = 2^M$, $M = 1, 2, \ldots$, when the phase differences satisfy [50]

$$\theta'_{2^M n_2 - 2^{M-1}+1, 2^M n_2 - 2^M + 1} = \pi - 2^{M+1}\alpha, \tag{7}$$

where $\theta'_{m,n} = \theta_{s,m} - \theta_{s,n}$, the sequence is accurate to the $(M+1)$th order deviation. For other pulse numbers, i.e., $N_2 \neq 2^M$, we can adopt the concatenated method or numerical method to obtain the phase differences [50].

On the nonadiabatic transition induced by the deviation in the quantity $\alpha$ being dramatically suppressed, the propagator $U_3(\theta''_1)$ can be approximately written as a diagonalizable form: $U_3(\theta''_1) \approx |d(t)\rangle\langle d(t)| + \exp[i2\beta(1+\delta_\beta)]|b(t)\rangle\langle b(t)| + \exp[-i2\beta'(1+\delta_{\beta'})]|e\rangle\langle e|$. It is worth mentioning that the deviation $\delta_\beta$ cannot be completely eliminated in the second concatenation [51] and thus introduces the fault of quantum operations. To decline its influence, we need to return back to the original basis $\{|g\rangle, |f\rangle\}$ and then further concatenate $N_3$ propagators with distinct phase differences: $U_4(\theta'''_1) = U_3(\theta''_{N_3})\cdots U_3(\theta''_1)$, where $U_3(\theta''_{n_3}) \approx \exp[-i\beta(1+\delta_\beta)\boldsymbol{n}\cdot\boldsymbol{\sigma}]$ and $\theta''_{n_3}$ refers to the phase differences $\theta_{sp,n_3}$ between the $(4n_3-1)$th Stokes pulse and pump pulse, $n_3 = 1, \ldots, N_3$. Here the pulse number $N_3 (N_3 \geqslant 3)$ can be arbitrarily selected. When $N_3$ is small, e.g., $N_3 = 3$, the analytical expression of phase differences is given by [50]

$$\theta''_{21} = 2\arctan \pm\Big(\sqrt{1-P_f^2} \pm \sqrt{2P_f - P_f^2}\Big), \tag{8a}$$

$$\theta''_{32} = 2\arctan \pm\Big(\sqrt{1-P_f^2} \mp \sqrt{2P_f - P_f^2}\Big), \tag{8b}$$

where $\theta''_{mn} = \theta''_m - \theta''_n$ and $P_f$ represents the population of the state $|f\rangle$. As for a longer sequence (i.e., $N_3 > 3$), the performance of the robustness against the deviation $\delta_\beta$ becomes much better since more adjustable phases are contained, and it is instructive to adopt numerical calculations to obtain the solutions.

Note that the phase differences given by Eq. (8) are used to compensate for the population deviation of the target state $|\psi_T\rangle$. To be able to simultaneously compensate for the phase deviation of the target state, we require to perform a Taylor expansion of the fidelity $F = |\langle\Psi_T|\Psi(N_3T)\rangle|^2$ instead, where $|\Psi(N_3T)\rangle$ represents the final state after concatenating $N_3$ pulses. Definitely, the process of resolving phase differences is similar to that of using population formulas (see also in Ref. [50]), and such a design actually helps to suppress the phase sensitivity of the target state. Furthermore, with the same number of pulses the system robustness designed by fidelity formulas may be marginally inferior to that of using population formulas. The reason is obvious, i.e., more phases are involved to compensate for the deviation $\delta_\beta$. Therefore, a longer sequence may be necessary to achieve the same robust effect in this situation.

In Fig. 2, we demonstrate the robust performance of the quantum operation in the presence of two systematic errors by different $\mathcal{R}_{\frac{\pi}{4}}(N_1 \times N_2, N_3)$ sequences, where $\mathcal{R}_{\frac{\pi}{4}}(N_1 \times N_2, N_3)$ represents the rotation around the axis $\boldsymbol{n}$ with the angle $\pi/4$ and the total pulse number $N = N_1 \times N_2 \times N_3$ with $N_1 = 2$. For simplicity, the shape of each Stokes pulse is chosen as Guassian here, while other shapes can still work well [50].

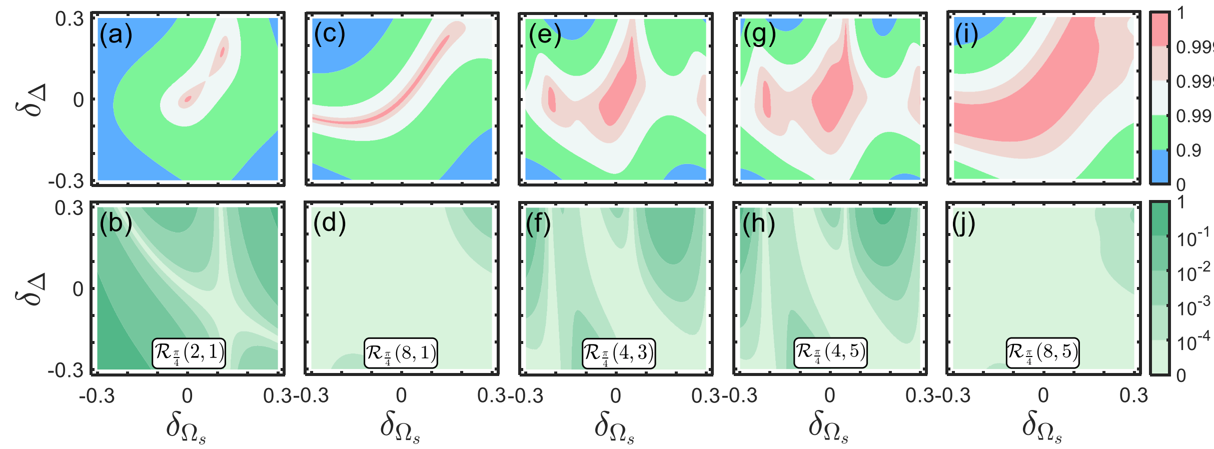


FIG. 2. Fidelity $F$ of the target state $|\Psi_T\rangle$ (top panels) and population $P_e$ of the state $|e\rangle$ (bottom panels) vs pulse amplitude error $\delta_{\Omega_s}$ and detuning error $\delta_\Delta$ for different sequences. The fidelity is defined by $F = |\langle\Psi_T|\Psi(t)\rangle|^2$ and $\mathcal{R}_\alpha(N_1 \times N_2, N_3)$ refers to performing a rotation operation around the axis $\boldsymbol{n}$ by the angle $\alpha$ in the Bloch sphere to obtain the desired state $\cos\alpha|g\rangle + \sin\alpha|f\rangle$, where $|\Psi_T\rangle = 1/\sqrt{2}(|g\rangle + |f\rangle)$, the initial state is $|g\rangle$, and the total number of SP pulse pairs is $N_1 \times N_2 \times N_3$ with $N_1 = 2$. The $n$th Stokes pulse has a Gaussian shape $\Omega_s(t) = A\Omega_0 \exp[(t - 3n\tau)^2/\tau^2]$ with the duration $T = 6\tau$.

As shown in Figs. 2(a) and 2(b), a concatenation of two SP pulse pairs to achieve perfect transitionless quantum driving possesses preliminary robustness against the pulse amplitude and detuning errors, whereas it does not work very well on large systematic errors. When the SP pulse pairs are continuously recombined for a second time [cf. $N_2 = 4$ in Figs. 2(c) and 2(d)], the robust performance of transitionless quantum driving is dramatically enhanced so as to obtain an extremely low leakage population of the state $|e\rangle$. Note that this recombination makes little contribution to promoting the precision of rotation operations, since the fidelity is sensitive to errors yet. Therefore, we need to execute the third concatenation of SP pulse pairs. Figures 2(e)–(j) shows that the high-fidelity region gradually enlarges as $N_3$ increases, while the ability of leakage suppression is still reserved, implying that the deviation $\delta_\beta$ can be favorably compensated by properly designing the phase difference of each pulse pair. Certainly, when concurrently increasing $N_2$ and $N_3$, both population leakage and operation precision are efficiently improved.

To see more clearly, we demonstrate in Fig. 3 the population evolution of two states $|f\rangle$ and $|e\rangle$ and the phase waveform of the Stokes pulse by the $\mathcal{R}_{\frac{\pi}{4}}(4, 3)$ sequence. For a single SP pulse pair, the system exhibits nonadiabatic transitions, i.e., the leakage to the excited state $|e\rangle$ (see the blue curve at $t = T$), because the adiabatic condition is broken. By concatenating two SP pulse pairs and properly adjusting the phase difference $\theta_{21}$, the transition to the excited state is completely eliminated, as shown by the blue curve at $t = 2T$. When the systematic errors are large, the evolution path seriously deviates from the original one, as shown by the dashed and dotted curves. We then concatenate four SP pulse pairs and modulate the phase difference $\theta'_{21}$ to reduce the unfavorable influence on the transition of the excited state induced by two systematic errors. In this circumstance, the nonadiabatic transition is sharply suppressed, but the fidelity of the rotation operation is not much improved; see the curves at $t = 4T$. Through further concatenating three groups of four SP pulse pairs (12 in total) and controlling the phase difference $\theta''_{21}$ and $\theta''_{32}$, the system evolution is strictly restricted in the subspace $\{|g\rangle, |f\rangle\}$, and the rotation operation becomes remarkable error tolerant.

*Discussion.* The concatenated approach may be applicable to various kinds of quantum systems as long as phase modulation is accessible, although we just take the typical

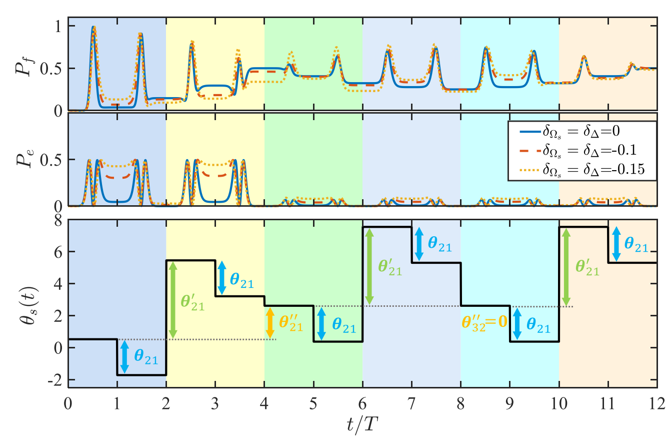


FIG. 3. Robust generation of the state $|\Psi_T\rangle$ and phase waveform of the Stokes pulse for the $\mathcal{R}_{\frac{\pi}{4}}(4, 3)$ sequence. The top (middle) panel represents the population evolution of the state $|f\rangle$ ($|e\rangle$) in the absence/presence of the errors $\delta_{\Omega_s}$ and $\delta_\Delta$. After 12 SP pulse pairs with well-designed phase differences, the system is primely driven to the state $|\Psi_T\rangle$ even though it exhibits significant pulse amplitude and detuning errors; see the dashed and dotted curves. In the bottom panel, the phase differences $\theta_{21}$ between the $2n$th and $(2n-1)$th SP pulse pairs are used for perfect transitionless quantum driving. The phase differences $\theta'_{21}$ between the $(4n-1)$th and $(4n-3)$th SP pulse pairs are devoted to suppress the nonadiabatic transition induced by the pulse amplitude and detuning errors. The phase differences $\theta''_{(n+1)n}$ between the $(4n+1)$th and $(4n-3)$th SP pulse pairs are employed for improving the fidelity of the Hadamard gate with a specific phase. All phase differences $\theta_{mn}$, $\theta'_{mn}$, and $\theta''_{mn}$ are accessible by properly modulating the phases $\theta_s$ and $\theta_p$ of SP pulse pairs.

three-level system to illustrate this issue. Actually, the phases only have to satisfy some constraints rather than arbitrary values when the Hamiltonian has long-ranged interactions [50]. On the other hand, there are several basic requirements in the current STA (e.g, see a recent review [52] and the references therein), such as multiparameter regulation, pulse shaping, and specific operation time. Also, the STA technique makes it difficult to make the system simultaneously inhibit multiple systematic errors. Nevertheless, none of them are required in the concatenated approach, since the dynamical procedure is completely out of line with the original framework of STA. In particular, the design of phase differences does not rely on the certain type of systematic errors, because they are derived from the propagator instead of the Hamiltonian. Therefore, this approach is quite universal for resisting any errors.

*Conclusion.* We have developed a concatenated approach of constructing a dynamical mechanism for both achieving perfect transitionless quantum driving and improving the robustness with respect to various systematic errors. It is particularly significant that the pulse shape can be arbitrary while the quantum system does not have to satisfy the adiabatic condition. By properly designing the phase differences between different Hamiltonians, the unwanted nonadiabatic transitions are sharply suppressed even in the presence of all kinds of errors. Meanwhile, quantum operations with very high fidelity are naturally obtained. Of course, this concatenated procedure can still be carried on for other uses. The simplicity, flexibility, and versatility of the concatenated approach opens a promising avenue for high precision control in quantum information processing.

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