Letter

Anomalous universal adiabatic dynamics: The case of the Fredkin model

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When a system is driven across a second-order quantum phase transition, the number of defects which are produced scales with the speed of the variation of the tuning parameter according to a universal law described by the Kibble-Zurek mechanism. We study a possible breakdown of this prediction proving that the number of defects can exhibit another universal scaling law which is still related only to the critical exponents z and ν, but differs from the Kibble-Zurek result. Finally, we provide an example, the deformed Fredkin spin chain, where this violation of the standard adiabatic dynamics can occur.

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I. INTRODUCTION

Near a quantum phase transition the behavior of some physical quantities is generally described only in terms of critical exponents [1]. In particular, by driving the system across a second-order quantum phase transition by slowly changing a parameter, the density of defects that come out scales algebraically with the speed of the variation of the tuning parameter [2–4]. In detail, the exponent of the scaling law depends only on the critical exponents z and ν, as predicted by the Kibble-Zurek mechanism [5,6]. This phenomenon has been confirmed theoretically by using different approaches and observed experimentally in different physical systems (see, e.g., Refs. [7,8] and references therein).

Here, we prove that there are systems undergoing a universal dynamics different from the one described in Ref. [4]. In detail, we show that the exponent of the density of defects depends only on the critical exponents z and ν, but it is different from that predicted by the Kibble-Zurek mechanism. At the quantum phase transition, approaching the thermodynamic limit, these systems might have a finite size scaling which depends on the excited states. In this case and if this dependence is smooth, we have still a universal adiabatic dynamics, different from the Kibble-Zurek prescription. We show that our theory applies to the deformed Fredkin spin chain [9]. The Fredkin model, in principle, can be realized experimentally by cold atoms in optical lattices, and a three-spin interaction can be designed as in Ref. [10]. The system is driven through a deformation parameter g across a quantum critical point described by the Fredkin spin chain [11,12], which, however, does not have a conformal invariance, being the dynamical exponent $z \geqslant 2$.

II. UNIVERSAL ADIABATIC DYNAMICS

In order to derive our main result, we consider a one-dimensional many-body system of size N described by a Hamiltonian $H(\lambda)$ which shows a second-order quantum phase transition at $\lambda = 0$, with critical exponents z and ν. If the parameter λ changes linearly in time, like $\lambda(t) = t/\tau$, the time-evolved state can be expressed as a linear combination of the eigenstates $|E_l(\lambda)\rangle$ with eigenvalues $E_l(\lambda)$ (in a nondecreasing order), as $|\psi(t)\rangle = \sum_l c_l(t) e^{-i\int_{t_{in}}^{t} E_l(\lambda(t'))dt'} |E_l(\lambda(t))\rangle$. As shown in Ref. [4], if the initial state is the ground state $|E_0(\lambda(t_{in}))\rangle$, the relative number of adiabatic excitations $n_{ex} = \sum_{l>0} |c_l(t)|^2$ can be expressed as

$$n_{ex} = \sum_l \left| \int_{-\infty}^{\infty} d\lambda \langle E_l(\lambda)|\partial_\lambda|E_0(\lambda)\rangle e^{i\tau \int_{-\infty}^{\lambda} d\lambda' [E_l(\lambda') - E_0(\lambda')]} \right|^2. \tag{1}$$

Let us introduce the momenta k_l, such that both the scaling relations

$$E_l(\lambda) - E_0(\lambda) = \lambda^{z\nu} \tilde{F}\left(\lambda^{z\nu}/k_l^z\right), \tag{2}$$

$$\langle E_l(\lambda)|\partial_\lambda|E_0(\lambda)\rangle = \frac{\lambda^{z\nu-1}}{k_l^z} \tilde{G}\left(\frac{\lambda^{z\nu}}{k_l^z}\right) \tag{3}$$

hold, and if k_l are homogeneously distributed around zero, $k_l \sim 0$, with proper universal scaling functions $\tilde{F}$ and $\tilde{G}$, we obtain the Kibble-Zurek scaling law, i.e., $n_{ex} \sim \tau^{-\gamma_0}$ as $\tau \to \infty$, with $\gamma_0 = \nu/(z\nu + 1)$ [4]. If, instead, the distribution of k_l is $\rho(k) \sim k^\beta$ for $k \sim 0$, it is easy to see that $n_{ex} \sim \tau^{-(1+\beta)\gamma_0}$.

Let us now consider the case where Eq. (3) is modified. Our aim is to understand how the number of excitations n_{ex} scales with τ in this case. Due to Eq. (2), we have that

$$n_{ex} = \sum_l \left| \int d\lambda \langle E_l|\partial_\lambda|E_0\rangle e^{i\tau \int^{\lambda} d\lambda \lambda^{z\nu} \tilde{F}\left(\lambda^{z\nu}/k_l^z\right)} \right|^2. \tag{4}$$

Let us suppose that the main contribution to the integral comes only from small k_l, as is usually the case, and assume that $|\langle E_l|\partial_\lambda|E_0\rangle|$ has a global maximum at $\lambda_c(N) \sim N^{-1/\nu}$,

which scales as $k_l^{-a_l}$ for $k_l \sim 0$. For $N \to \infty$, we expect that $\langle E_l|\partial_\lambda|E_0\rangle$ is well approximated by a Gaussian in a neighborhood of $\lambda_c(N)$, i.e., $\langle E_l|\partial_\lambda|E_0\rangle \sim k_l^{-a_l} e^{-[\lambda-\lambda_c(N)]2/(2\sigma_\lambda^2)}$. We make the following finite size scaling ansatz [13]:

$$\langle E_l|\partial_\lambda|E_0\rangle = N^{a_l} g_l(N^{1/\nu}[\lambda - \lambda_c(N)]). \tag{5}$$

If this formula holds, then we get $\sigma_\lambda \sim k_l^{1/\nu}$. For instance, if the scaling law in Eq. (3) is fulfilled we get a homogeneous critical exponent a_l, which is equal to $a_l = 1/\nu$. For $\tilde{F}(x) \sim 1/x$, considering the integral (up to a global phase)

$$\int d\lambda k_l^{-a_l} e^{-[\lambda-\lambda_c(N)]^2/(2\sigma_\lambda^2)+i\tau k_l^z \lambda} \sim k_l^{1/\nu-a_l} e^{-c\tau^2 k_l^{2z+2/\nu}/2}, \tag{6}$$

from Eq. (4), we get

$$n_{ex} \sim \sum_l A_l k_l^{2/\nu-2a_l} e^{-c\tau^2 k_l^{2z+2/\nu}}. \tag{7}$$

Defining $a(k_l) = a_l$, we get, in the thermodynamic limit,

$$n_{ex} \sim \int \rho(k) k^{2/\nu-2a(k)} e^{-c\tau^2 k^{2z+2/\nu}} dk. \tag{8}$$

We note that c defines the time scale, and we can consider $c = 1$, without loss of generality, in order to calculate the limit $\tau \to \infty$. We consider $a(k)$ smooth such that $a(k) \sim a(0) + uk$ as $k \sim 0$, and we assume that the scaling formula of Eq. (3) holds for the first excited state $l = 1$, so that $a(0) = 1/\nu$. If $u = 0$, the integral of Eq. (8) can be easily calculated, and, for $\beta = 0$, we recover the Kibble-Zurek scaling law $n_{ex} \sim \tau^{-\gamma_0}$. Let us consider the case $u \neq 0$. By performing the substitution $k = (\eta/\tau)^{\gamma_0}$, and extending the integral over η from zero to infinity, for $\beta = 0$ and $c = 1$, from Eq. (8) we get

$$n_{ex} \sim \int_0^\infty d\eta (\eta/\tau)^{\gamma_0} \eta^{-1} e^{f(\eta)}, \tag{9}$$

where $f(\eta) = -2u\gamma_0(\eta/\tau)^{\gamma_0} \ln(\eta/\tau) - \eta^2$. In order to calculate the asymptotic formula for $\tau \to \infty$, we use the Laplace method. For $u > 0$, the function $f(\eta)$ has a global maximum at $\eta_0 = x_0\tau$, where

$$x_0 = \left(\frac{u\gamma_0^2}{(2-\gamma_0)\tau^2} W\left((2-\gamma_0)e^{1-2/\gamma_0}\tau^2/\left(\gamma_0^2 u\right)\right)\right)^{1/(2-\gamma_0)}, \tag{10}$$

where $W(x)$ is the Lambert W function defined such that $W(x)e^{W(x)} = x$. Then, we get

$$n_{ex} \sim \frac{1}{\sqrt{-f''(\eta_0)}} (\eta_0/\tau)^{\gamma_0} \eta_0^{-1} e^{f(\eta_0)}, \tag{11}$$

where $f(\eta_0) = 2ux_0^{\gamma_0} + (2/\gamma_0 - 1)\tau^2 x_0^2$, and $f''(\eta_0) = -2u\gamma_0^2\tau^{-2}x_0^{\gamma_0-2} + 2(\gamma_0 - 2)$. For extremely slow driving, $\tau \to \infty$, we get $n_{ex} \sim \tau^{-\gamma}(\ln \tau)^{(\gamma-1)/2}$, where

$$\gamma = -\lim_{\tau\to\infty} \frac{\tau\partial_\tau n_{ex}}{n_{ex}} = \frac{\gamma_0}{2-\gamma_0}, \tag{12}$$

which is again a universal critical exponent depending only on z and ν. However, as shown in Fig. 1, n_{ex} changes very slowly with τ, and the asymptotic value is reached at very large times τ, where, apart from slowly varying logarithmic corrections, $n_{ex} \sim \tau^{-\gamma}$, with γ given by Eq. (12), explicitly $\gamma = \nu/(2 - \nu + 2\nu z)$. In the transient regime, n_{ex} locally

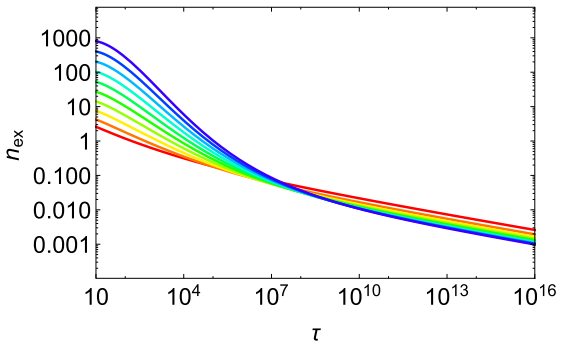

FIG. 1. Plot of n_{ex} (up to an overall prefactor) given by Eq. (11), as a function of τ, for different values of u, from 1 (red line) to 10 (purple line), increasing by one at a time. For not very large τ (in the transient regime) the exponent $\gamma(\tau)$ depends on u and increases as u increases, while for large τ it becomes universal. We used the values $z = 2.69$ and $\nu = 2/3$, valid for the Fredkin model in the zero magnetization sector.

scales approximately as $n_{ex} \sim \tau^{-\gamma(\tau)}$, where $\gamma(\tau) = -\frac{\tau\partial_\tau n_{ex}}{n_{ex}}$, which depends also on u. Finally, we also consider $\beta \neq 0$. In this case, calculating the limit of Eq. (12), we get $\gamma = \frac{\gamma_0+2\beta}{2-\gamma_0}$.

In conclusion, we get a breakdown of the Kibble-Zurek mechanism due to the fact that the overlaps $\langle E_l|\partial_\lambda|E_0\rangle$ do not all scale with the same critical exponent. From a heuristic point of view, we have that the typical size of the defects scales as $\ell \sim \lambda^{-\nu_e}$, where ν_e is an effective critical exponent which depends on the critical exponents $a(k)$, then by separating the dynamics in impulsive and adiabatic as in the Kibble-Zurek mechanism, we get $n_{ex} \sim \tau^{-\nu_e/(z\nu+1)}$. In particular, in the limit $\tau \to \infty$ we expect $\nu_e = \nu/(2 - \gamma_0)$.

It is worth observing that the scaling formula applies to expectation values $\langle O\rangle$ of observables which are diagonal in the energy basis at the final time. To prove it, we note that at the final time we have $\langle O\rangle = \sum_l O_l|c_l|^2$, where O_l are the diagonal elements of the observable, providing that $O_0 = 0$. In the thermodynamic limit we get an integral over k, and if the main contribution comes from $k \sim 0$, and $O(k_l) = O_l$ is a smooth function, we can approximated $O(k)$ with $O(0)$ in the integral, so that we expect $\langle O\rangle \sim n_{ex}$, namely, $\langle O\rangle$ and n_{ex} have the same scaling behavior in τ.

III. THE MODEL

As a physical example, we consider a chain of $N = 2n$ spins 1/2, described by the Hamiltonian $H(g) = H_\partial + \sum_{j=1}^{N-2} H_j(g)$, where $H_\partial = |{\downarrow_1}\rangle\langle{\downarrow_1}| + |{\uparrow_N}\rangle\langle{\uparrow_N}|$ and

$$H_j(g) = |{\uparrow_j}\rangle\langle{\uparrow_j}| \otimes |s_{j+1,j+2}(g)\rangle\langle s_{j+1,j+2}(g)| + |s_{j,j+1}(g)\rangle\langle s_{j,j+1}(g)| \otimes |{\downarrow_{j+2}}\rangle\langle{\downarrow_{j+2}}|, \tag{13}$$

with $|s_{i,j}(g)\rangle = (|{\uparrow_i}{\downarrow_j}\rangle - g|{\downarrow_i}{\uparrow_j}\rangle)/\sqrt{1+g^2}$, where $|{\uparrow_j}\rangle$ and $|{\downarrow_j}\rangle$ are eigenstates of σ_j^z with eigenvalues 1 and -1, where σ_j^α with $\alpha = x, y, z$ are the Pauli matrices. As shown in Ref. [9], the unique ground state $|E_0(g)\rangle$ is frustration free and it is a weighted superposition of so-called Dyck paths. We note that for $g = 0$ the ground state is $|{\uparrow\downarrow}\rangle^{\otimes n}$; conversely, in the limit $g \to \infty$, the ground level is degenerate

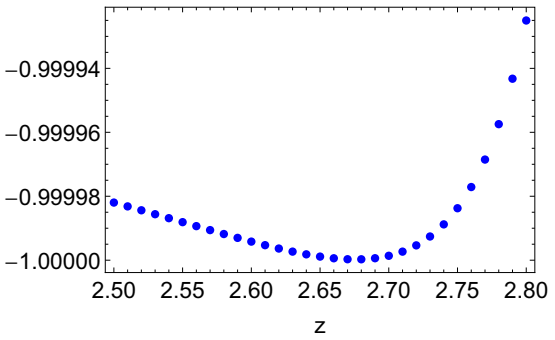

FIG. 2. Plot of the Pearson correlation coefficient, corresponding to the first excited energy level in the subspace $S_z = 0$, between $\ln \Delta_n(z)$ and $\ln n$ for $n \in [5, 8]$, by changing z by steps of 0.01. It is minimum for $z = 2.68$.

and the ground states are of the kind $\otimes_i (|\uparrow\rangle^{\otimes n_i^{\uparrow}} \otimes |\downarrow\rangle^{\otimes n_i^{\downarrow}})$, with $n_i^{\uparrow} \geqslant 2$ and $n_i^{\downarrow} \geqslant 2$, so that $\sum_i n_i^{\uparrow} = \sum_i n_i^{\downarrow} = n$, i.e., we get magnetic domains not smaller than two sites. For the special value $g = 1$ we get the Fredkin spin chain [11,12]. In this case, the ground state is a homogeneous superposition of Dyck paths which displays an entanglement entropy that grows logarithmically with the size of the block. It is shown that for the Fredkin chain, i.e., for $g = 1$, the gap closes in the thermodynamic limit as $E_1 \sim N^{-z}$, and there are different dynamical critical exponents z depending on the subspace defined by a given value of the magnetization $S_z = \sum_{j=1}^{N} \sigma_j^z$ (see Ref. [14]). For instance, in the sector with total spin $S_z = 0$, by performing a Density Matrix Renormalization Group analysis for large sizes, one gets $z \approx 2.69$ [11,14]. Let us derive z by means of an alternative approach which allow us to get a good result already for not too large system sizes. Let us consider the ratio $E_1(n)/E_1(n+1)$. For large n we expect that $E_1(n)/E_1(n+1) \sim (1 + 1/n)^z$. To estimate the value of z, we consider the difference $\Delta_n(z) = |E_1(n)/E_1(n+1) - (1 + 1/n)^z|$. The correct value of z is the one for which $\Delta_n(z) \to 0$ as $n \to \infty$. We assume that $\Delta_n(z) \sim n^{-\alpha}$ for the true value of z. Actually, we have numerical evidences that this is the case in the neighbourhood of $z = 2.69$. By calculating the Pearson correlation coefficient between $\ln \Delta_n(z)$ and $\ln n$ for $n \in [5, 8]$, we find that it reaches the value -1 for $z \approx 2.68$ (see Fig. 2). With this approach, therefore, we obtain a value of z very close to the known result, $z = 2.69$, already considering small systems sizes. For $g \sim 1$, in the thermodynamic limit, the gap closes as $E_1 \sim |g - 1|^{z\nu}$, so that for $g = g_c = 1$ we have a second-order quantum phase transition. Thus, we perform the finite size scaling ansatz [13]

$$E_1(N, g) = N^{-z(N)} f(N^{\frac{1}{\nu}}[g - g_c(N)]) \tag{14}$$

where $z(\infty) = z$ and $g_c(\infty) = g_c = 1$. The pseudo-critical value $g_c(N)$, which is expected to be $g_c(N) \sim g_c + bN^{-1/\nu}$ for large N, can be calculated as the fixed point of the scale transformation [13,15]

$$N^z E_1(N, g) = N'^z E_1(N', g'). \tag{15}$$

Thus, by considering $N' = N - 2$, $g_c(N)$ is such that $N^z E_1(N, g_c(N)) = (N-2)^z E_1(N-2, g_c(N))$, i.e., corresponds to the value g where the curves $N^z E_1(N, g)$ and $(N-2)^z E_1(N-2, g)$ (as functions of g) cross. The value of the exponent ν is such that the points $N^z E_1(N, g)$ versus $N^{\frac{1}{\nu}}[g - g_c(N)]$ collapse into a single curve $f(v)$ as N goes to infinity. In particular, if the ansatz in Eq. (14) is true, for finite N the points collapse to a curve proportional to $f(v)$. By performing this analysis, we get that the gap closes with $z \approx 2.69$ and $\nu = 2/3$ (see Fig. 3). After having determined the critical exponents, we proceed by considering a time evolution generated by changing the parameter g linearly in time, as $g(t) = g_{\text{fin}} t/\tau$, in the time interval $t \in [0, \tau]$. The initial state is the ground state $|E_0(0)\rangle = |\uparrow\downarrow\rangle^{\otimes n}$. The time evolution occurs in the invariant subspace of the Dyck words, having a dimension $C_N = \binom{2n}{n}/(n+1)$ and characterized by a magnetization S_z equal to zero. Since for very large g_{fin} in the ground state there are not magnetic domains smaller than two sites, we consider as defects the domains of only one site. For very large g we get $H_j(g) \sim w_j = |\uparrow\downarrow\uparrow\rangle\langle\uparrow\downarrow\uparrow| + |\downarrow\uparrow\downarrow\rangle\langle\downarrow\uparrow\downarrow|$, so, the average number of these defects tends to be equal to the average value $\langle\psi(\tau)|H(g_{\text{fin}})|\psi(\tau)\rangle$, i.e., the irreversible work produced. Based on the observation clarified previously, since the irreversible work scales in the same way as the density of defects, we can consider $w = \sum_j \langle\psi(\tau)|w_j|\psi(\tau)\rangle/N$ to determine the scaling of defects, which is easier to calculate, resorting to the matrix product state and second-order Trotter decomposition [16] (see Fig. 4). If the conventional theory of the Kibble-Zurek mechanism could be applied, we would expect $w \sim \tau^{-\gamma_0}$, with $\gamma_0 = \nu/(z\nu + 1) < 1$, if $z > 1$. However,

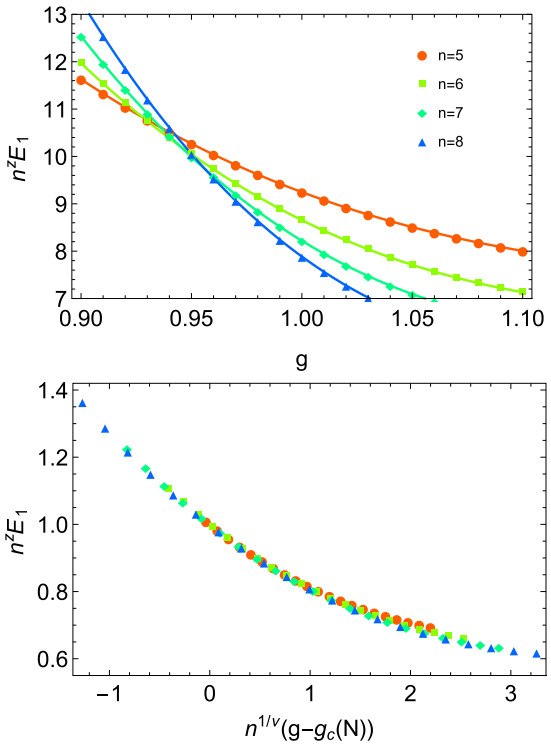

FIG. 3. In the top panel, finite size scaling plot of $n^z E_1$ versus g for different sizes $N = 2n$, for $z = 2.69$. We note that there is a crossing point at $g_c(N)$, smaller than $g_c = 1$. In the bottom panel, finite size scaling plot of $n^z E_1$ versus $n^{1/\nu}[g - g_c(N)]$, for $z = 2.69$ and $\nu = 2/3$. The values $n^z E_1$ are normalized such that $n^z E_1 = 1$ for $g = g_c(N)$. We note that all the points collapse into the same curve.

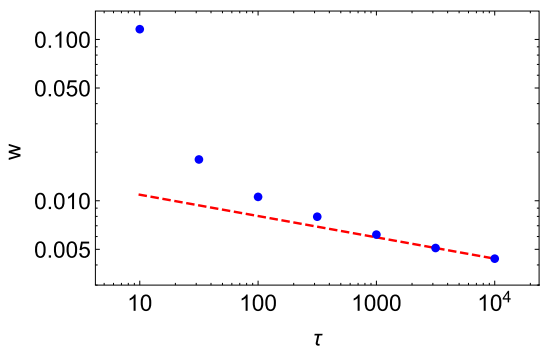

FIG. 4. Plot of w as a function of τ for a Fredkin chain with $N = 200$ subjected to an adiabatic quench from $g_{in} = 0$ to $g_{\rm fin} = 5$. We used matrix product states truncated at the value $D_{\rm max} = 10$. The time evolution is calculated by performing a second-order Trotter decomposition with time step 0.1. The dashed line is a function proportional to $\tau^{-\gamma}$, where $\gamma \approx 0.132$, calculated by fitting the last two data points. This numerical result is in agreement with the theoretical prediction for $\nu = 2/3$ and $z = 2.69$, which gives $\gamma_0 \approx 0.24$ and $\gamma = \gamma_0/(2 - \gamma_0) \approx 0.136$.

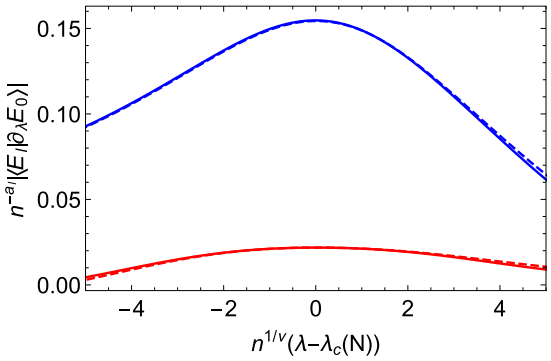

FIG. 5. Plots of the finite size scaling for the first two overlaps $\langle E_l|\partial_\lambda|E_0\rangle$. The dashed lines correspond to $n = 6$, the solid ones to $n = 7$, the blue ones to $l = 1$ (first excited level in the zero-spin sector), and the red ones to $l = 2$ (second excited level in the zero-spin sector). The value of $\lambda_c(N)$ is calculated as the maximum point of the correspondent overlap. We used $a_1 = 1.65 \approx 1/\nu$ and $a_2 = 2.18$, such that the data points for both $n = 6$ and $n = 7$ collapse in good approximation in the same curves.

as shown in Fig. 4, we find that, for not very large τ, in the transient regime, w scales with a nonuniversal exponent γ larger than one. Conversely, $w \sim \tau^{-\gamma}$ scales with an exponent γ (apart from a slow varying logarithmic function) which approaches the universal value $\gamma = \gamma_0/(2 - \gamma_0)$ for large τ, as shown in Fig. 4. This anomalous dynamical behavior for the deformed Fredkin model can be explained by requiring that the overlaps $\langle E_l|\partial_\lambda|E_0\rangle$ scale with level-dependent critical exponents $a(k_l) \sim 1/\nu + uk_l$, as shown in Fig. 5.

IV. CONCLUSIONS

The characterization of quantum phase transitions is of particular importance in condensed matter physics. In particular, the Kibble-Zurek mechanism has been thoroughly investigated over the years. Here, we considered the possibility that one of the two scaling formulas required to get the universal dynamics of the Kibble-Zurek mechanism can be generalized, so that we can have a violation of this mechanism. Nevertheless, we have shown that in this case we can also get a universal dynamics, although different from the Kibble-Zurek one. We provided an example where this anomalous universal behavior can occur. We showed that the Fredkin spin chain has the required features. By performing an explicit numerical calculation for the irreversible work, related to the density of defects, we have found a scaling law in perfect agreement with what was expected by our theory, namely, a universal adiabatic dynamics different from the Kibble-Zurek prediction.

Recently, several exceptions to the Kibble-Zurek mechanism have been found, e.g., in the presence of long-range interactions [17,18] or across a localization-delocalization transition [19], and, in general, an explanation of an anti-Kibble-Zurek behavior [20] and a different scaling in fast quenches [21] have also been investigated. The origin of the Kibble-Zurek violation, reported in our work, relies on the properties of the spectrum; however, how to detect this peculiar behavior from the Hamiltonian is still an open question. We hope that our work can give a further boost to current research in this direction, and can be useful for a deeper understanding of a class of still unexplored critical systems, inspiring further investigations and applications in the fields of quantum phase transitions and out-of-equilibrium phenomena.

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