Perspective

Reversible to irreversible transitions in periodic driven many-body systems and future directions for classical and quantum systems

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Reversible to irreversible (R-IR) transitions arise in numerous periodically driven collectively interacting systems that, after a certain number of driving cycles, organize into a reversible state where the particle trajectories repeat during every or every few cycles. On the irreversible side of the transition, the motion is chaotic. R-IR transitions were first systematically studied for periodically sheared dilute colloids, and have now been found in a wide variety of both soft and hard matter periodically driven systems, including amorphous solids, crystals, vortices in type-II superconductors, and magnetic textures. It has been shown that in several of these systems, the transition to a reversible state is an absorbing phase transition with a critical divergence in the organization timescale at the transition. The same systems are capable of storing multiple memories and may exhibit return point memory. We give an overview of R-IR transitions including recent advances in the field and discuss how the general framework of R-IR transitions could be applied to a much broader class of nonequilibrium systems in which periodic driving occurs, including not only soft and hard condensed matter systems, but also astrophysics, biological systems, and social systems. In particular, some likely candidate systems are commensurate-incommensurate states, systems exhibiting hysteresis or avalanches, nonequilibrium pattern forming states, and other systems with absorbing phase transitions. Periodic driving could be applied to hard condensed matter systems to see if organization into reversible states occurs for metal-insulator transitions, semiconductors, electron glasses, electron nematics, cold atom systems, or Bose-Einstein condensates. R-IR transitions could also be examined in dynamical systems where synchronization or phase locking occurs. We also discuss the possibility of using complex periodic driving, such as changing drive directions or using multiple frequencies, to determine whether these systems can still organize to reversible states or retain complex multiple memories. Finally, we describe features of classical and quantum time crystals that could suggest the occurrence of R-IR transitions in these systems.

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I. INTRODUCTION

Driven many-body deterministic nonlinear systems generally exhibit disordered or chaotic dynamics, as found in turbulence [1], particle flow over disordered media [2], plasmas [3], sheared materials [4,5], granular matter [6], earthquakes [7], gravitational systems [8], and biological systems [9]. Chaotic dynamics can arise even in systems with only three degrees of freedom [10,11] so it can be expected that driven many-body disordered systems with hundreds or thousands of degrees of freedom will generally exhibit fluctuating or chaotic dynamics. Recently, a growing number of many-body systems have been shown to exhibit a transition from time periodic or reversible motion to chaotic irreversible motion under oscillatory driving [12–26]. In these studies, the particle positions are compared from one driving cycle to the next. In the chaotic phase, the particles do not return to the same positions and undergo diffusive motion away from the initial positions over many driving cycles. For certain driving amplitudes or system parameters, however, the particles can organize over many cycles into a reversible state in which they return to the same positions after every or every few cycles, and the long time diffusive behavior is lost.

Reversible behavior in viscosity-dominated flows was famously demonstrated by G. I. Taylor [27] using a two cylinder setup in which the inner cylinder is rotated multiple times and then rotated back. Pine *et al.* used the same shearing geometry and viscous fluid as Taylor but considered the case where there are additional colloidal particles in the fluid that can collide with each other, so that any irreversible behavior would be due to the particle collisions rather than the fluid itself [12]. Periodically sheared dilute colloidal particle experiments allow for the systematic study of transitions from irreversible to reversible motion in many-body systems.

Pine *et al.* showed that there is a critical strain amplitude below which the steady-state behavior is reversible and above which the behavior becomes diffusive or irreversible. The colloids are in a viscous fluid and are large enough that thermal effects are negligible; since they are also electrically neutral, the only interactions capable of producing an irreversible state are contact forces between the colloids during collisions. During the initial shear cycles, particles move by different amounts and some particles collide with one another. In a steady irreversible state, collisions occur during each cycle and cause the particles to wander away from their initial positions in a Brownian-like diffusion, where the distances traveled along the shear direction $\langle x^2\rangle$ and perpendicular to the shear direction $\langle z^2\rangle$ increase linearly with time.

In further work, Corté *et al.* [13] studied the number of drive cycles required for the system to reach a reversible state from a randomized initial state. They introduced a simple toy model for particles that either do not overlap or undergo a stochastic collision process. The stochastic rule is partially motivated by experimental studies [28] showing that even for collisions of just two colloidal particles in a shear flow, there is a stochastic element due to the roughness of the particle surfaces. In the model Corté *et al.* use, during the first cycles the motion is chaotic, but after many cycles the system may organize to a state where collisions are absent. The number of transient cycles n_τ spent reaching the reversible state diverges as a power law $n_\tau = |\gamma - \gamma_c|^{-\nu}$, where γ_c is a critical shear amplitude [13]. A similar divergence can occur for fixed γ and increasing density, where there is a critical density ϕ_c below which the system organizes to a reversible state and above which it is in an irreversible state [13]. This critical behavior suggests that the transition to the reversible state is a nonequilibrium phase transition, and the observed exponents $\beta = 0.45(2)$ and $\nu_\parallel = 1.33(2)$ are similar to the critical exponents of the two-dimensional directed percolation (DP) universality class [29], namely, $\beta = 0.584(4)$ and $\nu_\parallel = 1.295(6)$. Although extensive theoretical studies of DP transitions have been performed, clear observations of these transitions in experimental systems have only been obtained relatively recently [30–32]. A related universality class is conserved direction percolation (CDP), which has exponents very similar to those of DP [29,33], so it is an open question whether the dilute systems are in the DP or CDP universality class. Some works suggest that CDP could be more relevant for different driving protocols [34,35].

Since R-IR transitions were first observed in dilute systems, it might seem reasonable to imagine that these transitions only arise under specialized circumstances where the interactions are of sufficiently short range or the system is sufficiently dilute that it is possible to reach a state where collisions never occur. This would imply that it would be difficult to find reversible states in disordered strongly coupled systems where the particles are always in contact or where long-range interactions are relevant; however, R-IR transitions have in fact been observed for periodically driven granular systems [17] where the particles are always in contact as well as for vortices in type-II superconductors [14,24] at densities for which the vortices are strongly interacting. One of the most extensively studied strongly interacting systems exhibiting an IR-R transition is periodically sheared amorphous solids [15,16,18,19,21,23,25,36–38]. Here the particles or atoms are always in contact and long-range strain fields are present. Reversible behavior is expected to occur in solids when the strain amplitude is small so that the system behaves completely elastically; however, the R-IR transitions in the amorphous solids appear well into regimes where strong plastic deformations occur and the particles are exchanging neighbors. In general, in strongly coupled systems, the particle trajectories during reversible states show more complex loops or return to the same position not after every drive cycle but after multiple drive cycles, such as in examples where the motion repeats every six cycles [16,21,38,39]. Plasticity is thought to be an inherently irreversible process; however, these works indicate that it is possible for *plastically deforming reversible regimes* to emerge, suggesting that similar transitions to reversible states could arise in many other strongly coupled systems. One example is the work of Keim *et al.* [18] in which individual reversible plastic events were observed experimentally. There have now been several other studies of many-body interacting systems undergoing IR-R transitions including crystals [40], systems with quenched disorder [14,24,41], chiral active matter [42,43], skyrmions [44], and magnetic materials [45]. Additionally, it has been shown that in the reversible phase, the system can be trained to store multiple memories [20,37]. In this case, the system can retain a memory of the deformation amplitude applied under periodic cycling in the form of a dip that appears in the response to a dc drive that is swept up from zero. For a dilute system, such memories would be stored in particular spatial arrangements of the particle locations, whereas in dense systems, the memories could take the form of particular arrangements of plastically deforming regions.

In this paper, we give an overview of work on reversible-irreversible transitions in both dilute and strongly coupled systems, and discuss how ideas about R-IR transitions could be more broadly applied to other areas including polymeric soft matter, pattern forming systems, and frictional systems. The R-IR transition could be induced via global driving such as a shear or external field, or via local driving such as the periodic motion of a local probe. We also discuss classes of active matter systems that could exhibit R-IR transitions in the limit where thermal effects are negligible but the activity can be treated as periodic. Most existing studies involved a periodic drive applied in a single direction at one frequency; however, much more complicated periodic drives could be applied with multiple frequencies or changing directions, opening a new area of investigation. Such systems might organize to more complex reversible states and could retain a memory of past drive protocols as the driving is changed from more to less complex. We also discuss possible applications of R-IR transitions to a wider number of hard condensed matter systems under periodic driving, including metal-insulator systems, charge ordering systems, Bose-Einstein condensates, superfluids, sliding charge density waves, Wigner crystals, systems showing nonlinear transitions, classical time crystals, and numerous magnetic textures including magnetic domain walls and magnetic bubbles. These types of systems can be subjected to numerous forms of periodic driving, such as electric currents, magnetic fields, or optical excitations. The most promising quantum systems for observing R-IR transitions are the time crystals [46], which already show a number of

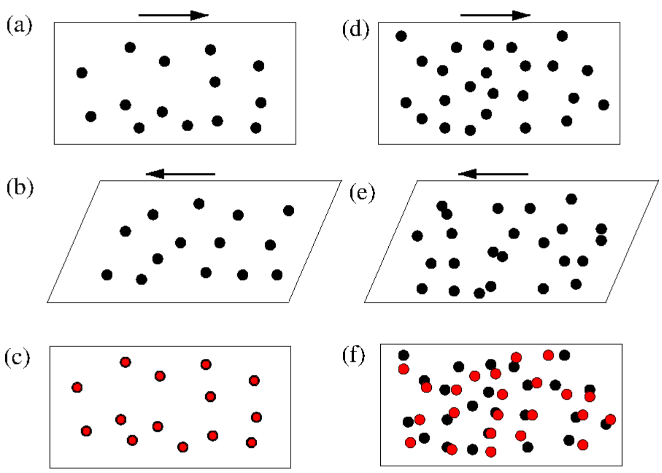

FIG. 1. A schematic of the system studied by Pine *et al.* [12], consisting of a dilute suspension of colloids (black disks) subjected to periodic shearing. Arrows indicate the shearing direction. In (a) and (b), the system is in the dilute limit, and the comparison of the starting (black) and ending (red) positions of the particles shown in (c) indicates reversible behavior. In (d) and (e), the system is in the dense limit, the particles collide repeatedly under shear, and the image of starting and ending positions in (f) indicates the occurrence of irreversible motion.

features consistent with a transition from a chaotic state to a time periodic state. There is also a broader class of nonlinear coupled systems such as coupled oscillators or networks that have been shown to exhibit synchronization and phase locking effects [47,48] and nonequilibrium pattern formation [49]. Synchronization effects can occur in many-body coupled systems when the many degrees of freedom become coupled so that the response looks like that of a few body system, and we argue that such transitions could be viewed as R-IR transitions, suggesting that these systems could also be candidates for exhibiting transitions into absorbing states.

II. SHEARED SYSTEMS

The dilute colloid experiments performed by Pine *et al.* [12] showing an irreversible to reversible transition inspired a variety of studies that tested for similar behavior in other many-body systems under periodic driving. In Fig. 1, representative configurations show the situation in the reversible versus irreversible steady states for systems with low [Fig. 1(a)] and high [Fig. 1(b)] colloid densities subject to the same amplitude of shearing. Comparison of the starting (black dots) and ending (red dots) positions of the particles, shown in Figs. 1(c) and 1(f) for the two densities, reveals that the motion is reversible in the low density system where collisions do not occur and irreversible in the higher density system where collisions are ongoing. Here, in "reversible" motion all of the particles return to their previous positions after the shearing deformation is reversed, while for "irreversible" motion this is not the case. Figure 2(a) shows the experimentally measured particle positions from Ref. [12] at the end of each cycle over multiple cycles in the irreversible regime. Here the dynamics is Brownian-like, and the particles gradually diffuse away

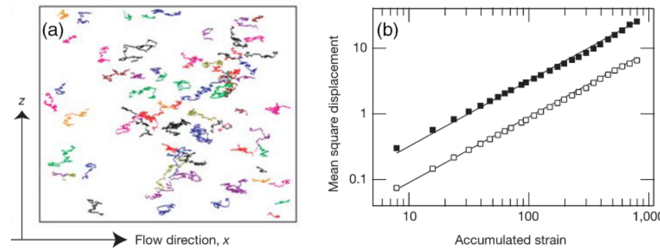

FIG. 2. (a) Particle trajectories measured experimentally for the sheared colloid system in Ref. [12] in the irreversible or chaotic regime. (b) The corresponding mean square particle displacements in the direction parallel, $\langle x^2\rangle$ (filled squares), and perpendicular, $\langle z^2\rangle$ (open squares), to the drive, showing diffusive motion. Reprinted by permission from Springer Nature, D. J. Pine *et al.* [12].

from their initial positions. In Fig. 2(b), the corresponding measured mean square displacement $\langle x^2\rangle$ in the direction of drive versus accumulated stain increases linearly as expected for Brownian motion. The displacement $\langle z^2\rangle$ in the direction perpendicular to the drive shows the same scaling but with a smaller prefactor.

In general, for any random initial condition in sheared colloidal systems, during the first few cycles the particles do not return to their original positions after each cycle. Instead, over time the particles either organize to a reversible state or remain in an irreversible state. Corté *et al.* [13] used a combination of simulation and experiments to further explore the system studied by Pine *et al.* [12] and applied different strain amplitudes to identify the manner in which the system reaches a steady irreversible or reversible state. Figures 3(a) and 3(b) show the positions of the active particles, or particles that did not return to their original positions after each cycle, for a simulation of two-dimensional (2D) sheared hard disks at two different strain amplitudes of $\gamma_0 = 3.0$ [Fig. 3(a)] and $\gamma_0 = 2.0$ [Fig. 3(b)] [13]. At the end of the first shear cycle, there are numerous active particles in each case, but over time the system reaches a steady state that is either irreversible with a finite number of active particles, as shown for $\gamma_0 = 3.0$ in Fig. 3(a), or reversible with no active particles, as shown for $\gamma_0 = 2.0$ in Fig. 3(b). A time series of the fraction of active particles as a function of shear cycle number appears in Fig. 3(c) for the same two strain amplitudes. After 2500 cycles, there are no active particles remaining in the $\gamma_0 = 2.0$ sample, but the activity in the $\gamma_0 = 3.0$ sample plateaus at a finite steady-state value where about 39% of the particles remain active. The inset in Fig. 3(c) shows the steady-state active particle fraction versus strain amplitude, indicating that there is a transition from irreversible (active) to reversible (nonactive) behavior near a critical strain of $\gamma_0^c = 2.66$.

By fitting the curves in Fig. 3(c) to a stretched exponential, Corté *et al.* [13] obtained the mean time τ required to reach a steady reversible or irreversible state, plotted in Fig. 4 as a function of strain amplitude γ_0. At the critical amplitude γ_0^c, τ diverges as a power law according to

$$\tau \propto \left|\gamma_0 - \gamma_0^c\right|^{-\nu}. \tag{1}$$

In the 2D simulations, it was found that $\nu \approx 1.33$ on both sides of the transition, while the three-dimensional (3D)

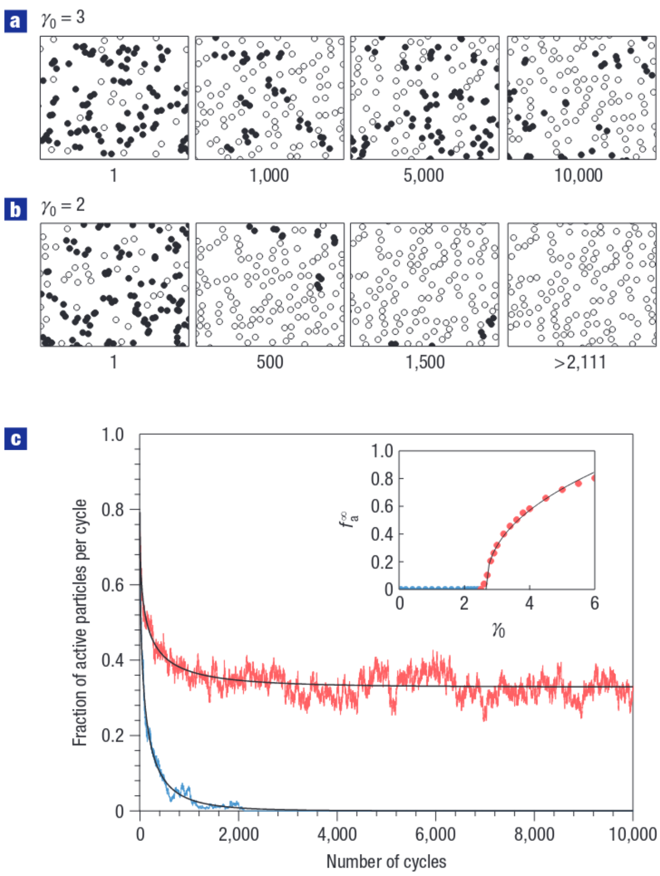

FIG. 3. [(a) and (b)] Snapshots from simulations in Ref. [13] of a hard disk model of the experiments in Fig. 2 as a function of time for different strain amplitudes γ_0. Black disks are moving irreversibly between cycles and open disks are returning to the same position between cycles. (a) $\gamma_0 = 3$ in the irreversible regime. (b) $\gamma_0 = 2$ in the reversible regime. (c) For the same system, the fraction of particles that are active (moving irreversibly) in each cycle as a function of the number of shear cycles applied at $\gamma_0 = 3$ (red) and 2 (blue). The inset shows the steady-state fraction of active particles as a function of γ_0. Reprinted by permission from Springer Nature, L. Corté *et al.* [13].

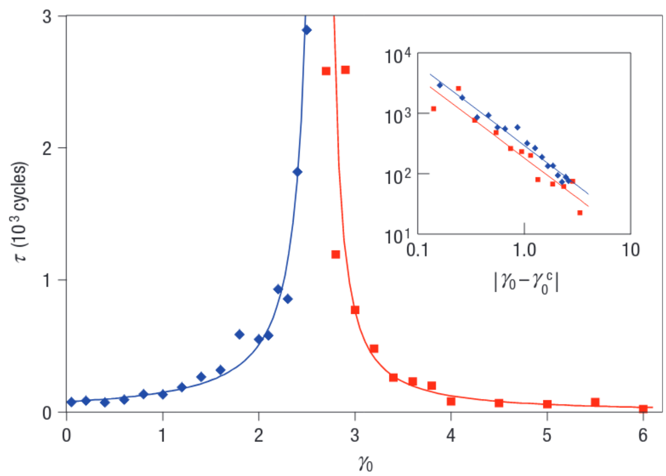

FIG. 4. For the simulation of sheared 2D disks from Fig. 3, a plot of the time τ to reach a steady state vs shear amplitude γ_0 for the reversible (blue diamonds) and irreversible (red squares) regimes. The inset shows the same data as a power-law plot of τ vs $|\gamma_0 - \gamma_0^c|$, where the critical shear amplitude is $\gamma_0^c = 2.66 \pm 0.05$. The lines indicate fits of the data to $\tau \propto |\gamma_0 - \gamma_0^c|^{-\nu}$ with $\nu = 1.33 \pm 0.02$. Reprinted by permission from: Springer Nature, L. Corté *et al.* [13].

experiments produced a similar divergence with $\nu = 1.1$. This transition has the hallmarks of what is known as an absorbing phase transition, which appears in nonequilibrium systems and often falls in the directed percolation (DP) universality class [29,50]. The irreversible state can be viewed as a dynamically fluctuating state in which the particles continue to exchange positions and long time diffusion is occurring. In contrast, the reversible state corresponds to the absorbed state where the fluctuations are lost and the behavior becomes completely time repeatable, indicating that the system is dynamically frozen or trapped in a limit cycle.

Corté *et al.* [13] called the transition into the reversible state "random organization" since the particles form a random configuration in which collisions do not occur. Since that time there have been further studies of random organization in these sheared dilute colloidal systems [51], as well as a number of studies indicating that randomly organized states near the critical threshold are hyperuniform [22,42,52–57]. Hyperuniform particle arrangements contain no large-scale deviations in the density from the average density, unlike random particle arrangements. Periodic crystals are hyperuniform by definition; however, certain random structures also have hyperuniform properties [58], and there is ongoing work to understand the conditions under which random hyperuniform states can occur. In a random organized state, the particles form configurations where collisions are absent. This increases the average distance between particles and reduces large local variations in the particle positions, thereby diminishing large density fluctuations and giving a more uniform density that extends out to long length scales. An open question is whether all systems near a R-IR transition exhibit hyperuniformity, or whether this is a property found only in systems that are dilute or that have short-range interactions.

There has also been work on ordered pattern formation in dilute systems where the R-IR transition overlaps with a disorder (irreversible) to order (reversible) transition [59] for hard disk colloidal particles interacting with a periodic array of obstacles. Figure 5(a) shows the fraction R_n of active particles at the end of each cycle for periodically driven disks moving through an ordered array of posts at a fixed drive amplitude for different disk densities ϕ. When $\phi > 0.3716$, the system remains in an irreversible state, while for $\phi \leqslant 0.3716$, the disks organize into a reversible state. Figures 5(b) and 5(c) illustrate the particle positions in the steady state, which is irreversible and disordered in Fig. 5(b) at $\phi = 0.3962$, and reversible and ordered at $\phi = 0.3716$ in Fig. 5(c). In Ref. [59], the authors also found a power law divergence of the time required to reach the reversible state. The power-law exponents are similar to those observed in 2D random organization systems, suggesting that the periodically driven disks fall in the same universality class as those systems. Another interesting

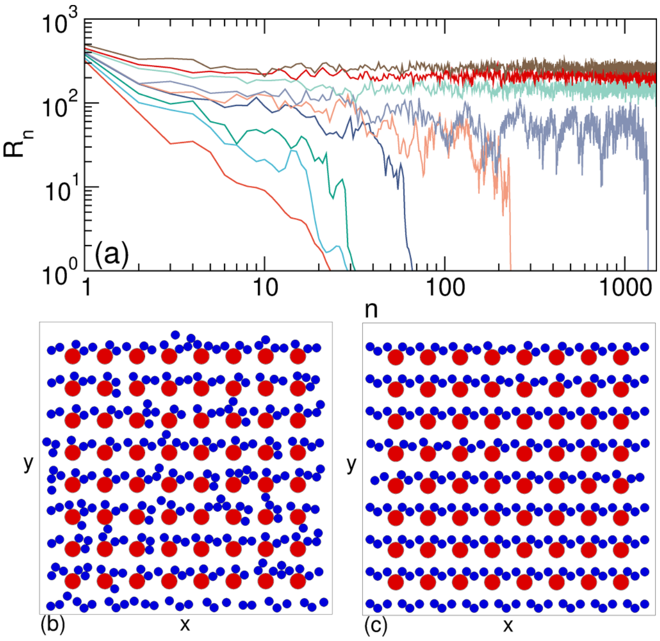

FIG. 5. Simulations of cyclically driven disks in a periodic array of obstacles from Ref. [59]. (a) The number R_n of active or irreversibly moving disks vs cycle number n for different disk densities ranging from $\phi = 0.335$ (bottom) to $\phi = 0.3962$ (top) under shearing with an amplitude of $A = 0.031623$ at an angle of $\theta = 18.435^\circ$ from the x-axis symmetry direction of the obstacle array. [(b) and (c)] Images of the disk locations (blue) and obstacles (red) in a portion of the sample for the same driving amplitude and direction as in (a). (b) An irreversible state at $\phi = 0.3962$. (c) A reversible state at $\phi = 0.3716$. Reprinted from C. Reichhardt and C. J. O. Reichhardt [59] with the permission of AIP Publishing.

feature of the reversible state in Fig. 5(c) is that collisions are not absent, but instead the particles collide with the obstacles in a repeating pattern. Pattern formation in a reversible state was also studied for bidisperse systems in which half of the particles move in circles and the other half do not, where it was shown that there is a transition from a mixed fluid to a pattern forming phase separated reversible state [43]. There could be other dilute systems on complex landscapes that could give rise to similar types of reversible pattern formation.

III. CONDENSED SYSTEMS

We next consider R-IR transitions in systems close to or just at jamming, as well as in amorphous solids and systems deep in the jammed phase. In dilute systems, the irreversible state generally has a liquid structure and the particles do not form a solid. At the transition into the absorbing state, the particles either experience a small number of repeating collisions or have no collisions at all. As a result, it might be assumed that R-IR transitions are limited only to systems that are dilute or have contact interactions, making it possible to provide the particles with enough space to rearrange and organize into a reversible state. Below we show that this is not the case.

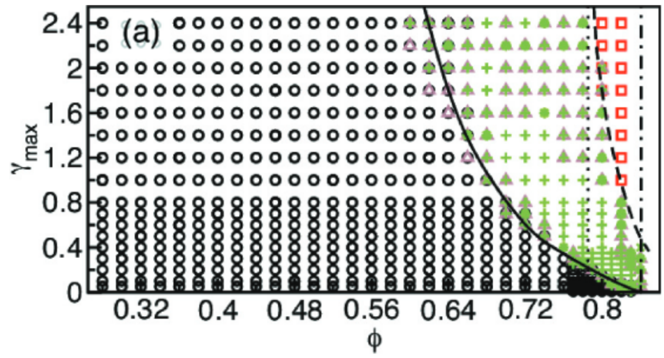

FIG. 6. R-IR transition in simulations of a 2D bidisperse assembly of hard disks from Ref. [60]. The dynamic phases are plotted as a function of the strain amplitude $\gamma_{\max}$ vs the disk density ϕ. Black symbols indicate reversible states where the disks return to their original locations after a single cycle. For the green symbols, the states are loop reversible with complicated disk trajectories that repeat after one or more periods. In the region with red symbols, the steady state is irreversible. Reprinted with permission from C. F. Schreck *et al.* [60]. Copyright by the American Physical Society.

A. Reversibility near jamming and packing

Schreck *et al.* [60] studied a granular matter version of R-IR transitions for a periodically driven 2D bidisperse disk assembly, and focused on the formation of a reversible state below the jamming transition, $\phi < \phi_J = 0.84$. Well below jamming, the system organizes into a reversible state where there are no collisions. Schreck *et al.* term this a "point reversible" state, and it is the same as a random organization state. For higher densities, the system can still organize to a reversible state, but the disk trajectories become much more complex and involve some collisions. Schreck *et al.* named these "loop reversible" states since the reversible orbits form loop structures instead of straight lines. At densities near jamming, an irreversible steady state emerges. Figure 6 shows the phase diagram as a function of strain amplitude $\gamma_{\max}$ and disk density ϕ from Ref. [60], where the black region is point reversible, the green region is loop reversible, and the irreversible states are colored in red. Schreck *et al.* also obtained similar results in 3D systems. Recent studies by Gosh *et al.* on disk packings showed that the transition to a reversible state coincides with a transition to a crystalline state, which is interesting because it was not expected *a priori* that these transitions would occur at the same point [61].

Both experiments and simulations have been performed for R-IR transitions in granular matter as a function of varied shear amplitude [62] and friction [17]. Other works address transitions to reversible states or random organization at the approach to jamming or random close packing [63], as well as possible ways to connect jamming and yielding in a unified framework [64–66]. Open questions for systems transitioning between jammed and unjammed states include how the nature of the trajectories changes across jamming and whether there could be different types of absorbing transitions. Other effects to consider would be adding quenched disorder sites to a jamming system [67,68] in order to determine how the R-IR transition is affected, or to study whether R-IR

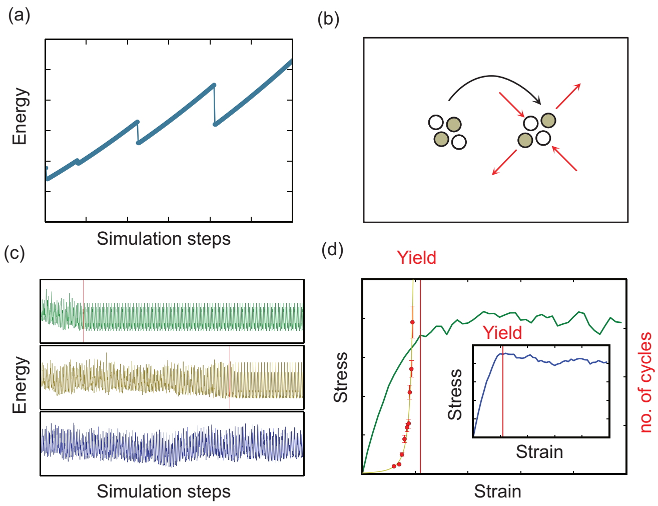

FIG. 7. (a) The energy as a function of simulation steps for an athermal quasistatic simulation of an amorphous solid. The discontinuous drops in the energy occur due to plastic rearrangements. Reprinted under CC license from I. Regev *et al.* [36]. (b) An illustration of particle motion in the most fundamental plastic rearrangement event, the soft-spot. (c) Three different potential energy time series for three different maximal strain amplitudes, which increase from top to bottom. The red lines mark the onset of repetitive behavior or the formation of a limit cycle. (d) Stress-strain curve under linear shear (green line). The red vertical line marks the R-IR transition, while the red data points indicate the number of cycles required to reach a limit cycle under oscillatory shear. The inset shows a similar stress-strain curve obtained for different initial conditions. Reprinted with permission from I. Regev *et al.* [16]. Copyright by the American Physical Society.

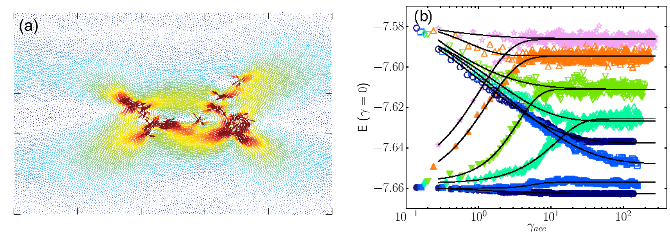

FIG. 8. (a) A repeating avalanche in the reversible state of the system from Fig. 7. Arrows and colors indicate the direction and magnitude, respectively, of the displacements during the avalanche motion. Reprinted under CC license from I. Regev *et al.* [36]. (b) The potential energy E per particle in the steady state under zero strain ($\gamma = 0$) vs accumulated strain γ_{acc} for different values of maximum shear strain $\gamma_{\max}$ increasing from bottom to top under temperatures $T = 1.0$ (open symbols) and $T = 0.466$ (closed symbols). Reprinted with permission from D. Fiocco *et al.* [15]. Copyright by the American Physical Society.

transitions change in the presence of Griffiths [69] or Gardner transitions [70].

B. Amorphous solids

For systems such as solids or glasses well above the jamming density subjected to shear, it is known that at small strains, the response is elastic and plastic rearrangements do not occur, while for intermediate strains, plastic events start to appear, and for even higher strains the system exhibits plastic yielding. In plastic events, particles can exchange neighbors, and continuous plasticity is often associated with mixing and irreversible deformations. It is possible for reversible plasticity to occur in which the particles can exchange one or more neighbors during the first portion of the drive cycle, but then these exchanges are inverted during the second portion of the drive cycle, bringing the system back to its original configuration. In this way, a jammed system could have a reversible elastic response, a reversible plastic response, and irreversible plastic events, as well as mixtures of reversible and irreversible plastic events.

Currently there is a considerable amount of work on elucidating the nature of the yielding transition in sheared solids and glasses, understanding the shape of the stress-strain curve, and determining the way in which shear response and the yielding transition depend on how the system is prepared. Figure 7(a) shows the potential energy of an amorphous solid subject to an athermal strain increase every simulation step. The energy increases parabolically, as is expected from a rigid elastic material, and decreases discontinuously when a plastic rearrangement of the particles occurs. The most basic plastic rearrangement involves a change of nearest neighbors called a "soft-spot" [71–74] that is illustrated in Fig. 7(b). Most plastic events involve several soft-spots, as will be discussed below. Amorphous and crystalline systems can also be subjected to periodic shearing, and for strains where the response is plastic under unidirectional shear, it could be assumed that the system would only exhibit irreversible states; however, Fiocco *et al.* [15], Regev *et al.* [16] and Priezjev [75] studied 3D and 2D model glasses subject to cyclic shear and showed the existence of a transition from reversible to irreversible dynamics at a critical strain amplitude. Priezjev [75] demonstrated that, at a finite temperature, there is a transition from an almost periodic, subdiffusive regime, to a diffusive regime. Fiocco *et al.* [15] and Regev *et al.* [16] used athermal quasistatic simulations to show that below the critical point the dynamics is exactly periodic and particles repeat the same positions after each cycle. Regev *et al.* [16] found that the number of cycles needed to reach a limit-cycle diverges at this point. In their study, Fiocco *et al.* [15] also showed that the postyield dynamics involves a loss of memory of the initial configuration.

Figure 7(c) shows the potential energy as a function of strain cycles for increasing strain amplitudes. At the lowest amplitude, on the top portion of the panel, the system is initially in an irreversible state and settles after a short transient into a reversible state, as indicated by the transitions from a fluctuating non-repeating signal to a periodic signal. As the strain amplitude increases, it takes longer for the system to reach a reversible state. Remarkably, even in the reversible states the system shows large scale *reversible* plastic deformations. Figure 8(a) shows an example of a reversible plastic avalanche event for the system from Fig. 7 [36]. Figure 7(d) illustrates the number of cycles required to reach a reversible state as a function of strain amplitude along with the stress-strain curve. At yielding, marked by the red vertical line, there is a divergence in the time needed to reach the reversible state. This work also showed that there is a power-law divergence

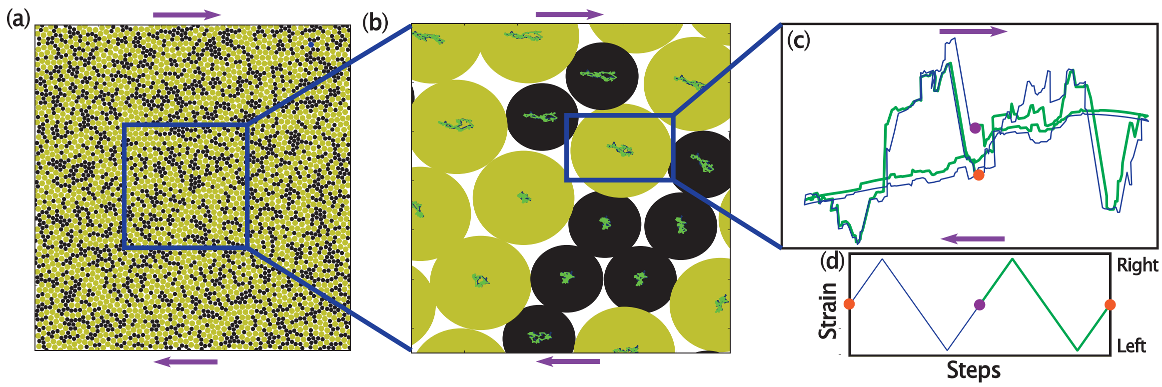

FIG. 9. Particle trajectories in an amorphous solid following a multiperiodic limit cycle. (a) A system of 4096 particles subject to periodic shear. (b) A blow-up showing individual particles and the trajectories performed by their centers marked in blue and green, where blue represents the first cycle and green represents the second cycle. (c) A blow-up showing the trajectory of a single particle. During the first cycle the particle performs the blue trajectory, followed by the green trajectory during the second cycle. (d) The strain as a function of simulation steps (quasistatic equivalent of time) in the cycle. Reprinted under CC license from I. Regev *et al.* [36].

in the timescale to reach the reversible state as a function of strain amplitude; however, the observed critical exponent $\nu \approx 2.6$ differs from the value $\nu \approx 1.33$ obtained by Corté *et al.* [13], suggesting that the transition is in a different universality class. There are several possible reasons why this might be the case. Fiocco *et al.* [15] studied the potential energy per particle in zero strain configurations for different numbers of cycles at varied strain amplitude under two different temperatures, as shown in Fig. 8(b). For low values of the maximum shear strain $\gamma_{\max}$, the final steady state is affected by how the system was prepared, while at higher $\gamma_{\max}$, the system always converges to the same energy regardless of how the system was prepared. The time required to reach a steady state is largest near a critical value of $\gamma_{\max}$ separating these two regimes.

For a reversible state in dilute systems, there are no collective effects since the particle-particle contacts vanish, whereas in the amorphous system, the reversible plastic events indicate that there is strong coupling among the particles, implying that there may be a dynamical length scale present and that the microscopics of the reversible state are different in the amorphous and dilute systems. Another feature in the work of Regev *et al.* [16] is that the divergence in the reorganization timescale correlates well with the point at which a yielding transition appears, so that below yielding, the oscillating drive creates a reversible plastic state, while above yielding, the system can never reach a reversible state. As shown in Fig. 9 and Ref. [16], the reversible state does not have to recur during each drive cycle, but may instead recur after multiple driving cycles. For example, the pattern might repeat after two cycles, and in fact limit cycles of up to seven or more cycles have been observed [16,38,39]. Multiple other studies found that periodically sheared amorphous systems can organize to reversible states with a varied number of limit cycles. This behavior is reminiscent of the routes into chaotic states that arise in low dimensional systems [10,11,76]. Keim *et al.* [38] showed that the multiperiodicity can be explained as resulting from interactions between soft-spots such as the ones shown in Fig. 10, while Szulc *et al.* [77] explained how oscillations in the activation thresholds of the soft-spots cause multiperiodicity. There have also been a number of experiments on transitions into reversible plastic states in amorphous jammed systems. Keim *et al.* found that jammed systems can organize into reversible plastic states [18], as highlighted in Fig. 11. In this case, under a cyclic drive the plastic deformations were either irreversible (red) or reversible (blue), with numerous irreversible events appearing during the first few cycles but a larger number of reversible events arising over time. The reversible events often consisted of groups of four particles switching positions in a periodic fashion. Nagamanasa *et al.* [78] also found a power-law diverging timescale for the transition to an irreversible state in a binary colloidal glass.

In several works [79–86], the effect of sample preparation on the potential energy in the steady-state was considered. For subcritical amplitudes, the steady-state potential energy

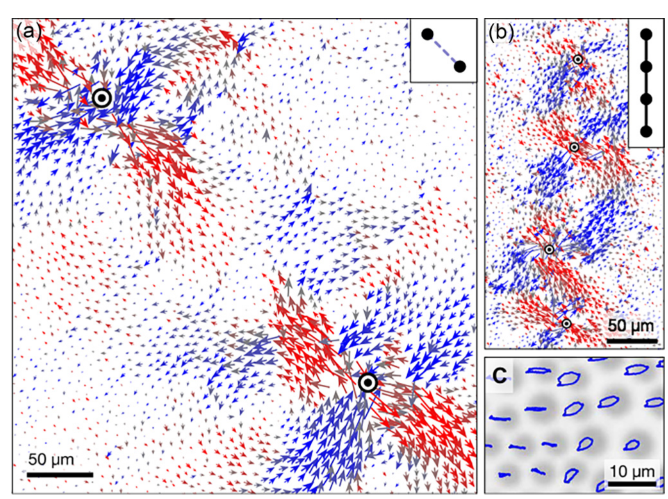

FIG. 10. Experimental observation of interacting plastically deforming regions or "soft spots" in the reversible state of a jammed solid. (a) Arrows indicate the magnitude of the particle displacements and colors indicate the two principal shear axes. (b) Cooperative interactions among multiple soft spots. (c) Trajectories (blue) of individual particles in a reversible state where the particles return to their initial positions after each cycle. Reprinted under CC license from N. C. Keim and J. D. Paulsen [38].

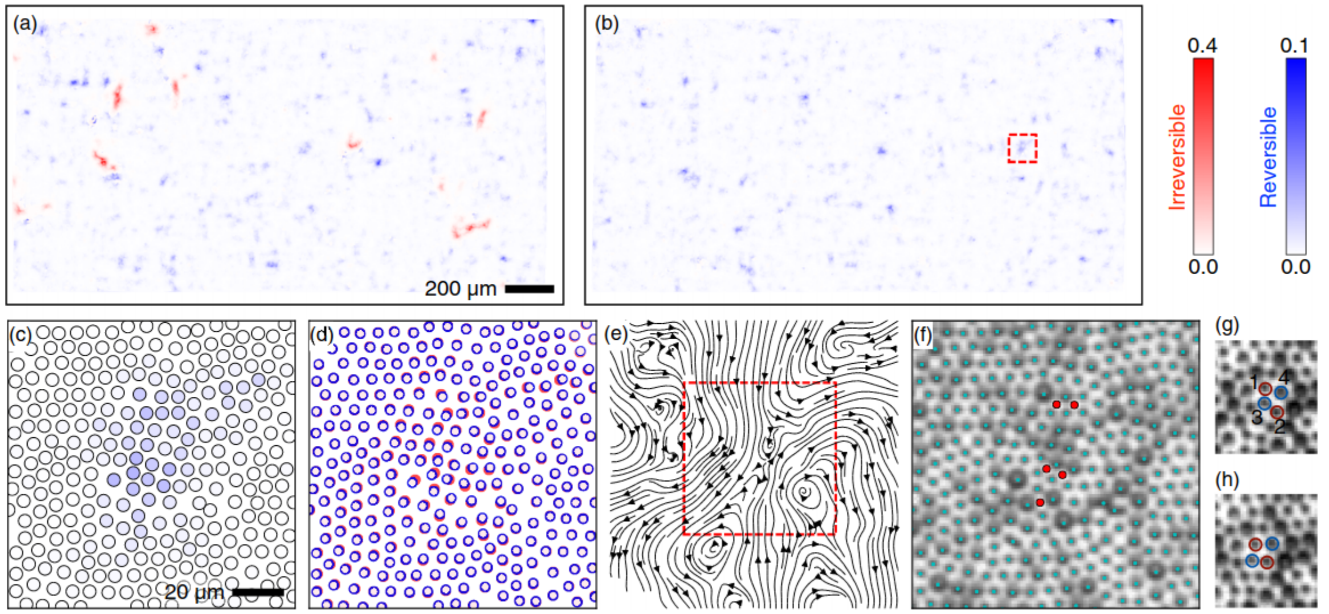

FIG. 11. An experimental measure of plastic events for a periodically sheared 2D jammed configuration showing that motion occurs in irreversible plastic regions (red) and reversible but plastic regions (blue), with the amount of reversible plasticity increasing over time. The plastic regions are often composed of groups of four particles. [(a) and (b)] Reversible (blue) and irreversible (red) motion after (a) 8 shear cycles and (b) 20 shear cycles showing that the system settles into a reversible state. (c) Detailed view of the residual displacement for the reversible cluster outlined with a red box in panel (b). (d) Relative maximum displacements of the particles in this cluster at the point of minimum shear (red) and at maximum shear (open blue circles). (e) The corresponding flow streamlines. [(f)–(h)] Images showing the details of a plastic rearrangement event in the same region. Reprinted with permission from N. C. Keim and P. E. Arratia [18]. Copyright by the American Physical Society.

of samples with different initial mean energies depends on the initial conditions, but for postcritical initial conditions, the steady-state potential energy is independent of the preparation protocol. These studies also showed that for postcritical amplitudes, the potential energy increases up to saturation. These observations were explained using models of dynamics on random energy landscapes [82,87,88].

Other works have shown that in certain cases, the yielding transition in amorphous systems can be discontinuous or first order [84–86]. This opens up the possibility for studying further R-IR effects such as hysteresis, where a system that has reached a reversible state could remain stuck in that state even for drives at which the steady state should be irreversible. Studies that used special quenching protocols allowing the system to be equilibrated at very low initial temperatures T_{init} prior to being quenched to zero temperature have shown that the transition from reversible to irreversible dynamics can be either abrupt or continuous, depending on the preparation protocol. Soft samples that were prepared by a fast quench from the liquid exhibited a smooth transition while samples that were prepared using a slow quench (sometimes using special preparation protocols) exhibited a sharp transition and hysteresis [83,89]. Other studies have shown that within the reversible state, although the system remains solid it becomes a softer solid, particularly when the reversible trajectories form loops [90]. It would be interesting to understand the circumstances under which first or second order R-IR transitions occur, and whether it is possible that adding some critical amount of disorder to a system could change the transition from first to second order.

Efforts to model the R-IR transition in amorphous solids subject to cyclic shear have so far focused on using integer automata models that represent an amorphous solid as a lattice of soft-spots interacting by elastic interactions. Using such a model, Khirallah *et al.* [23] found that such systems indeed exhibit a transition at a critical amplitude from asymptotically periodic dynamics, where the system repeats after n forcing cycles, to asymptotically diffusive dynamics. They also found that, similar to what is observed in particle simulations, the transition between the reversible and irreversible phases is marked by a power-law divergence in the number of cycles required to reach a reversible state. The exponent observed in this case was $\nu \approx 2.7$, consistent with the exponents found by Regev *et al.* [16] and lending further support to the idea that the universality class of R-IR transitions in amorphous solids differs from that of the dilute systems. Khirallah *et al.* also observed that similar to the observations from particle simulations [15,91], the diffusion coefficient on the irreversible side of the transition increases as a power law in the magnitude of the strain amplitude, and showed that there are still a large number of reversible plastic events that occur within the irreversible state. This work suggests that cellular automata models, which are computationally faster than particle simulations and are easier to model theoretically, can capture many of the relevant behaviors at R-IR transitions, and that similar reversible plastic to irreversible diffusive phases could arise in other types of cellular automata models.

There are several possible avenues for continued research in the study of amorphous solids under cyclic shear. First, there are a variety of different amorphous systems in which R-IR transitions have not yet been studied such as polymer glasses, metallic glasses, gels, and emulsions. It is not clear if such systems will show the same transition and, if so, whether the transition would be of the same character. Second, the theoretical understanding of the transition in amorphous solids remains undeveloped. Although it is clear that in dilute systems, reversible dynamics arises due to the emergence of states where the particles are spaced in such a way that they are not interacting, it is not clear why in some amorphous configurations the interactions between soft-spots lead to irreversible dynamics whereas in others the same interactions allow for periodic states. Third, it would be interesting to study how the R-IR transition varies for different kinds of particles. For example, droplets in densely packed emulsions can undergo a variety of shape changes under compression or have very different types of elastic properties compared to hard particles [51,53,92]. It may be that the ability of individual particles to distort would introduce another form of dissipation that could increase the range of reversible behavior; however, such distortions could also inject additional degrees of freedom, giving more possible ways for the packing to change and promoting irreversible motion.

Beyond disordered solids, R-IR transitions have also been studied in polycrystals [19,40] and point dislocation models [90]. There have been only a few studies of R-IR transitions in crystalline systems [40], but there are a variety of effects that could be studied in ordered states, such as the motion of grain boundaries or of isolated defects such as dislocation lines and disclinations. In crystalline systems, application of periodic driving could generate defects, leading over time to work hardening and eventual failure; however, there could be regimes in which the system reaches a steady reversible state under cyclic driving. This could be tested for crystalline systems found in soft matter, materials science, and certain hard condensed matter systems such as superconducting vortices or Wigner crystals. In Fig. 12, dislocations in a 2D colloidal assembly are manipulated using local stresses, shears, and dilatations [93]. One could consider applying local or global periodic driving to such a system in order to determine whether the motion of the individual defects illustrated in Figs. 12(a)–12(d) or of the grain boundary illustrated in Fig. 12(e) is reversible.

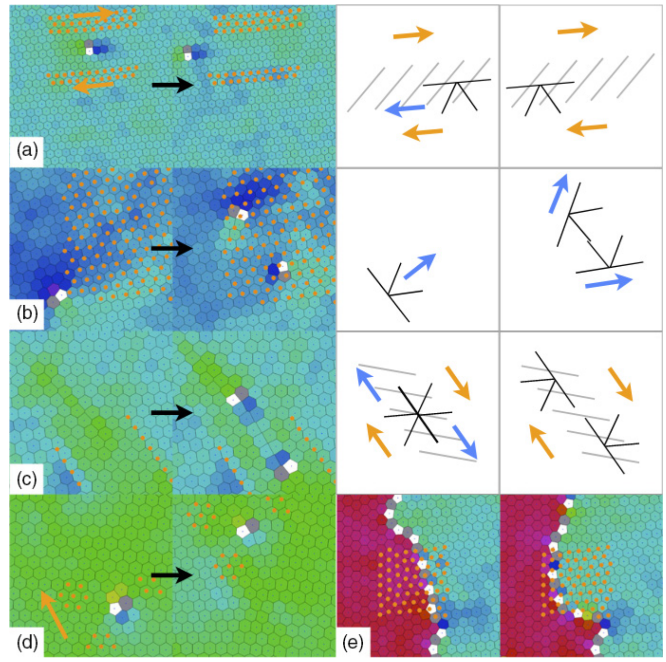

FIG. 12. Motion of dislocations and grain boundaries being controlled with optically induced "topological" tweezers in experiments on a colloidal assembly. (a) Inducing glide with localized shearing. (b) Inducing climb by dilatation. (c) Dislocation fissioning through applied shear. (d) Glide of a dislocation that has been trapped by opposing shear stresses. (e) Moving a grain boundary by applying a potential that is commensurate with the lattice on one side of the boundary. From W. T. M. Irvine *et al.* [93].

C. Magnetic systems

Hysteresis is frequently observed in condensed matter and materials science, and the best known example is in magnetism where cycling an applied field generates a hysteretic magnetic response in the material [94]. Hysteresis in magnetic systems is a result of disorder and exchange interaction between the spins of different atoms. In the case of ferromagnetic interactions, disorder can take the form of a local random field, as in the random field Ising model (RFIM), which has the Hamiltonian [95]:

$$H = -J\sum_{\langle i,j\rangle} s_i s_j - \sum_i (h_i s_i - h s_i), \tag{2}$$

where $s_i = \{-1, 1\}$ is the direction of the ith spin, $J > 0$ is a constant ferromagnetic coupling constant, h_i is the random field, and h is an externally applied field. Alternatively, there can be randomness in the effective spin-spin interactions, as captured by the Edwards-Anderson spin-glass model [96]:

$$H = -\sum_{\langle i,j\rangle} (J_{ij} s_i s_j - h s_i). \tag{3}$$

In this case, the coupling constant J_{ij} is a random variable that can be both positive and negative, leading to geometric frustration. Models with ferromagnetic interactions where $J > 0$ always reach a limit cycle after a transient of two cycles or less due to the "no-passing" property first proved by Middleton [95,97]. For this reason, such systems cannot have a R-IR transition. Models with couplings that can be both positive or negative, such as a spin-glass, can have long transients and thus, in principle, can have both reversible and irreversible behavior. In the case of the Edwards-Anderson spin-glass and related systems, each spin has only two states, which may hinder the emergence of completely irreversible dynamics. At the thermodynamic limit, however, the transients may become infinitely long, and the system can then become effectively irreversible.

Basak *et al.* [45] considered cyclic driving of uniaxial random field XY models with disorder. They found that for increasing field amplitude, the system goes from an Ising ferromagnetic state to a paramagnetic state that does not repeat, as shown in Fig. 13. In the reversible case, the spin patterns

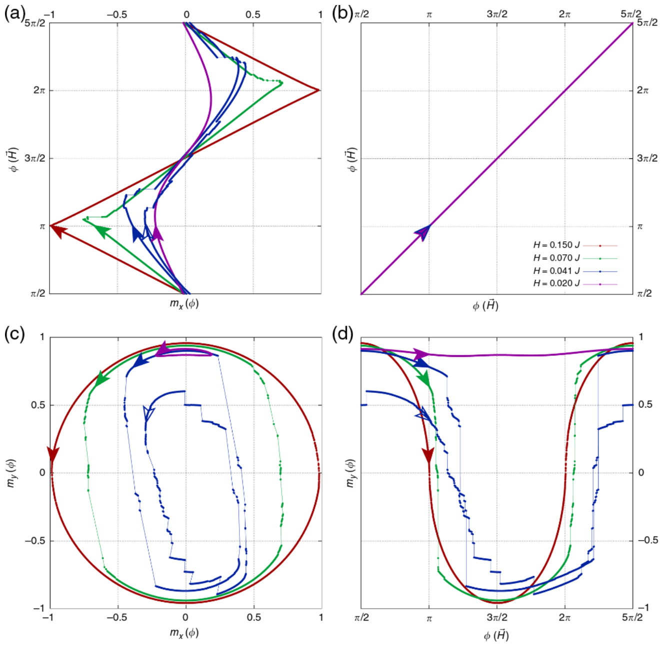

FIG. 13. Cyclic driving of uniaxial random field XY models with disorder. (a) The driving field angle ϕ vs the magnetic response in the x direction m_x for increasing driving strengths, where purple is the lowest drive and red is the highest drive. (b) A plot of ϕ vs ϕ indicating that the applied field is rotated counterclockwise. (c) The y direction magnetic response m_y plotted vs m_x. (d) m_y vs ϕ. Reprinted under CC license from S. Basak *et al.* [45].

repeat after n cycles, and the number of cycles to reach a reversible state increases as the critical point is approached. The plot of the y and x magnetizations m_y versus m_x in Fig. 13(c) indicates that there is an initial transient before the system settles into a period-2 limit cycle. This implies that the magnetic system can organize into a repeatable pattern spanning one or multiple periods, similar to what was found in the amorphous solids discussed previously.

Uniaxial random field XY models can be applied to many systems, including Josephson junctions [98], superfluids in a uniaxially stressed aerogel [99], uniaxially stressed 2D Wigner crystals [100], the half-integer quantum Hall effect, and the graphene quantum Hall ferromagnet [101]. Electron nematics [102,103] are also promising. Each of these are candidate systems in which to look for R-IR transitions under periodic driving.

IV. FUTURE DIRECTIONS IN CONDENSED SYSTEMS

A. Commensurate-incommensurate systems

Another class of systems that are good candidates for examining R-IR transitions is that in which commensurate-incommensurate transitions can occur [104–107]. These systems can be described in terms of interacting particles on a periodic substrate, where the ratio of the number of particles N to the number of substrate minima N_s is given by the filling ratio $f = N/N_s$. For example, Fig. 14(a) shows a schematic of charged colloidal particles interacting with a 2D periodic egg-carton substrate at a filling close to $f = 1$ [108].

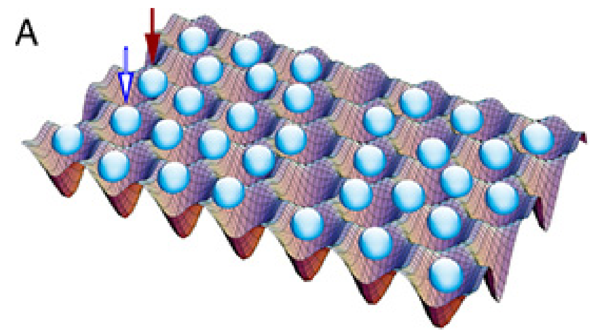

FIG. 14. An illustration of a 2D version of a Frenkel-Kontorova model consisting of colloidal particles interacting with a periodic substrate. From A. Vanossi *et al.* [108].

Commensurate conditions can arise for fractional matching when $f = n/m$ with integer n and m [107]. The most celebrated example of this is the Frenkel-Kontorova model for one-dimensional (1D) chains of elastically coupled particles on a substrate [106]; however, commensurate-incommensurate transitions arise across a remarkable variety of hard and soft condensed matter systems in both one and two dimensions. Specific systems include atomic ordering on surfaces [104,105], cold atoms in optical traps [109], vortices in nanostructured superconductors [110,111], vortices in Bose-Einstein condensates [112], and colloidal systems [108,113–115]. In quantum systems, there can be transitions from commensurate Mott phases to incommensurate superfluids [116,117]. For most of the systems listed above, the particles are not strictly elastically coupled but can undergo exchanges with one another, permitting plastic deformations, defect generation, and phase slips to occur.

In typical commensurate-incommensurate systems, the particles are largely localized under commensurate conditions, while for slightly incommensurate states there are kinks or antikinks that are more mobile than the particles and that depin first under an external drive [107]. At more strongly incommensurate states, the system becomes increasingly disordered and can exhibit glasslike behavior. Since the amount of order can be tuned by changing the filling, it would be possible to examine R-IR transitions for varied fillings. For example, under commensurate conditions the system may be strongly reversible for a range of drives, whereas incommensurate fillings might either organize to a reversible state similar to what is found in the sheared systems or remain in an irreversible state. There could be a critical drive amplitude marking the R-IR transition, or there could be fillings for which the system becomes fluid-like and is always irreversible. Whether the response of a commensurate-incommensurate system is reversible or irreversible may also depend on the type of drive applied. In addition to their relevance to a wide class of systems, another advantage of studying commensurate-incommensurate systems is that a variety of distinct types of defect structures can be realized, ranging from individual solitons or kinks and domain walls to strongly amorphous phases. To illustrate why commensurate-incommensurate systems could show different reversible or irreversible behaviors, Fig. 15 shows that a wide range of defect morphologies emerge for colloids on periodic substrates

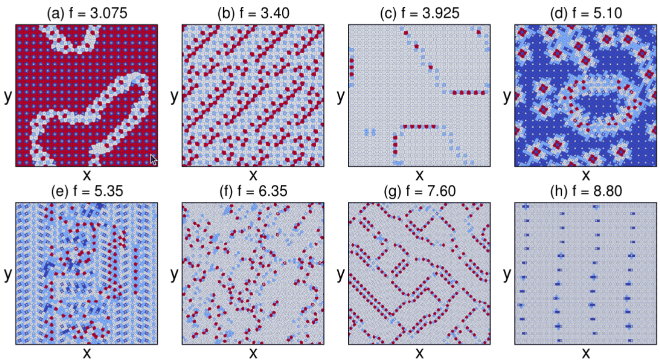

FIG. 15. Voronoi tessellations of colloidal particles interacting with a periodic substrate for different nearly commensurate fillings ranging from $f = 3.075$ to 8.80. Dark blue polygons are fourfold coordinated, light blue polygons are fivefold coordinated, gray polygons are sixfold coordinated, and red polygons are sevenfold coordinated. A variety of grain boundary and dislocation structures appear. Reproduced from Ref. [118] with permission from the Royal Society of Chemistry.

at different incommensurate fillings [118]. It would be interesting to understand whether the grain boundaries and isolated defects that emerge in a system such as this one would exhibit reversible or irreversible dynamics under periodic driving, and whether a possible R-IR transition would resemble that found in dilute or amorphous systems, or whether it would fall into an entirely new universality class that has not yet been observed for R-IR transitions. Such a study would relate to the broader question of the nature of R-IR transitions in interacting soliton systems.

B. Local driving

In the systems discussed up to this point, the driving is applied globally via shearing, a current, or a field; however, it is also possible to subject a system to local driving. In soft matter, such driving is referred to as active rheology, where a single particle is dragged through a background medium in order to create measurable perturbations [119–123]. Studies of this type have been used to examine changes in the drag [121,123] and fluctuations [119,122,124], as well as to determine whether there is a threshold force needed for motion of the probe particle [119,120,125] as the system passes through different types of glassy or jamming transitions. Figure 16(a) shows an example of a single probe particle driven through a glassy background of bidisperse colloidal particles, and the corresponding velocity time series contains a series of jumps that indicate the occurrence of plastic events in the medium surrounding the probe particle [119]. Similar active rheology techniques have also been used to study plastic rearrangements in crystalline systems [126,127].

In hard condensed matter systems, local driving can be achieved using a magnetic force tip or scanning tunneling microscope for superconducting vortices [128,129] or an optical probe for both superconducting vortices [130] and magnetic skyrmions [131]. Other possible methods include injecting a current at a single localized spot or applying time dependent inhomogeneous fields. Most studies of active rheology have

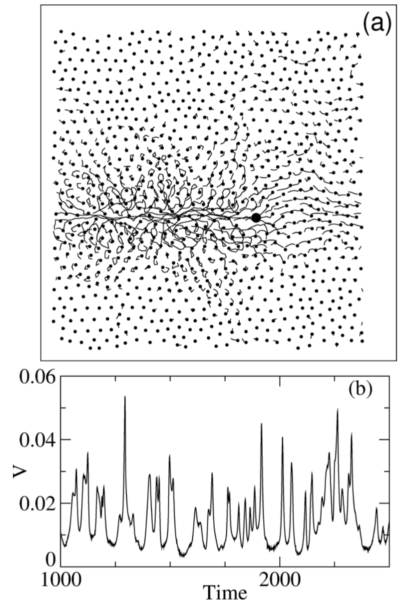

FIG. 16. (a) Particle locations (small black dots) and trajectories (lines) in a simulation of a single probe particle (large black dot) being dragged through a glassy background of bidisperse colloidal particles. (b) Time series of the velocity of the probe particle in panel (a) shows jumps corresponding to plastic rearrangement events. Reprinted with permission from M. B. Hastings *et al.* [119]. Copyright by the American Physical Society.

used dc driving, but ac driving could also be applied. Local driving has been applied to plastic to elastic transitions as well as situations where the probe particle particle creates local plastic deformations in the system. Similar techniques could be used to explore local R-IR. For example, if the probe particle is periodically driven at different driving amplitudes, as illustrated schematically in Fig. 17, the system could organize to an elastic reversible state with no plastic deformation, to a state with reversible plastic events or loop reversibility, or to a state that is continuously fluid-like or irreversible. It would be possible to measure whether a reversible or irreversible interface extends out from the region containing the driven particle, or if there is a fluctuating or continuous border surrounding the local probe region. To test for memory effects, the ac drive could be applied in one direction until the system reaches a reversible state, and could then be rotated into a new direction to see whether the system still remains reversible.

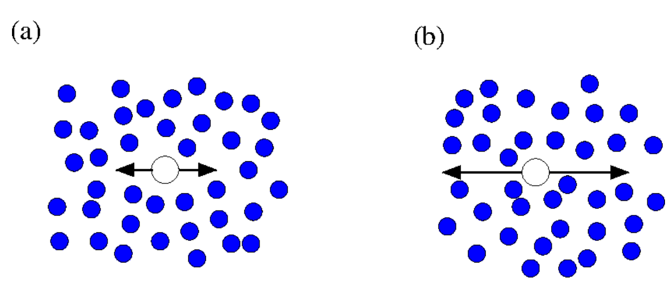

FIG. 17. Schematic of how local periodic driving could be used to detect an R-IR transition with a local probe by oscillating the probe at low (left) and high (right) amplitudes.

Such an approach would also be useful for understanding how the dissipation or drag is affected when the system transitions from an irreversible to a reversible state.

C. Systems with density gradients and confined geometries

In many of the systems examined so far, the density is roughly uniform across the sample; however, there can be a variety of cases in which density gradients arise, such as for particles subjected to gravity or being pushed against a wall by a driving field. It could also be possible to construct systems in which half of the sample contains quenched disorder and the other half is free of quenched disorder, or to introduce spatial gradients in the quenched disorder itself. Guasto *et al.* [132] studied R-IR transitions for oscillatory shear in pipe flow, and found an onset of irreversibility throughout the system even in regimes where the stain is very small. This suggests that gradients can enhance irreversible behavior, and opens the question of whether irreversible spatial regions can coexist with reversible regions, or whether the boundaries between such regions always generate enough randomness to allow the irreversibility to propagate throughout the system. In work examining the sedimentation of colloidal particles under a periodic shear [35], the presence of a density gradient made it possible to determine that the denser regions are irreversible while the less dense regions are reversible, and that the reversible and irreversible regions are separated by a well defined coexistence front. Another area in which to explore R-IR transitions is in confined geometries such as pipes, channels, or labyrinths. Here, although the introduction of spatial gradients may enhance the irreversible behavior, the reduction of phase space may enhance the reversible behavior, leading to a competition. It would be interesting to explore whether such systems could have regions that are almost completely reversible as well as other regions that are strongly irreversible. Due to the geometric constraints, these regions could be sufficiently isolated from each other that the R and IR phases could exhibit dynamical coexistence over long times.

D. Complex interactions

Up until this point, R-IR transitions have been studied only for systems with relatively simple short-range interactions. There are, however, numerous systems in which more complex interactions appear, such as long-range repulsion and short-range attraction, frustrated interactions, and interactions with multiple length scales. When these types of complex interactions are present, pattern formation or phase separation often occurs, as found in many soft matter systems [133–136] as well as in hard condensed matter systems such as electron liquid crystals [137–139], doped Mott insulators [140], charge ordering in Jahn-Teller systems [141] and magnetic domains [142]. In Fig. 18, we show some examples of pattern formation in a system with multiple length scale interactions, which cause both short-range crystalline order and larger scale ordering to occur. It would be interesting to test whether systems of this type could exhibit R-IR transitions under periodic driving. For soft matter systems, this could be achieved through shearing, while for hard condensed matter systems such as electron liquid crystals, the driving could take the form

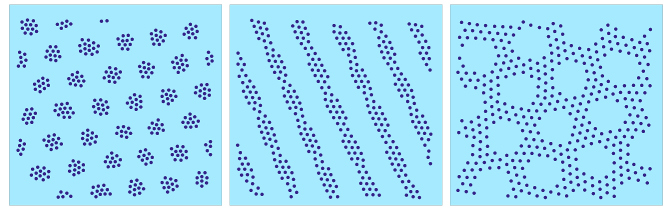

FIG. 18. Example of clump (left), stripe (center), and void (right) patterns that form for particles with competing long-range repulsion and short-range attraction from a system such as that described in Ref. [133].

of an oscillating current [138,139]. Other systems with long range interactions where some form of periodic driving can be imposed include dusty plasmas or charged dust particles that form microscale crystals [143,144]. Such systems could be periodically sheared or driven with other methods [145].

Due to the multiple length scales that are present in the interaction, it is possible that individual particles might undergo irreversible deformations even as the larger scale pattern remains reversible, producing a situation with microscopic irreversibility but macroscopic reversibility. If this were the case, there might be multiple R-IR transitions as well as the possibility for complex memory formation.

E. Quasireversibility and universality classes

The systems described so far are in either a reversible or an irreversible state during one or multiple drive cycles. Another possibility is that even in the absence of long time diffusion, the system could follow one of several available loop paths, where on any given cycle the particular path that is chosen is selected randomly. This could occur if the system shows only a local ergodicity or if there are regions where the particles can locally explore many different possible states but local fluctuation paths remain confined, so that the particles are unable to diffuse over long times. There could also be glassy reversibility. In many of the systems described so far, the irreversible states were characterized by observing diffusive behavior of the particles; however, there could be regimes of very long time diffusion in which the system is trapped in one cyclic orbit for long times before making a rare jump to a new orbit. Glassy behaviors of this type could arise when thermal fluctuations become important. For example, the system could be in a reversible regime for $T = 0$, but become irreversible at finite T when it can occasionally thermally hop to different periodic orbits. Finally, there could be orbits that are quasiperiodic in time, in analogy to orbits that are quasiperiodic in space.

Another possibility is smectic-like irreversibility in which there is long time diffusion in one direction but not in the perpendicular direction. In driven systems with quenched disorder, moving smectic states have been observed in which the diffusion is finite in only one direction [107]. For colloidal particles or amorphous solids under a periodic drive, 1D reversibility of this type could occur for anisotropic particles or layered systems. There could then be two irreversible transitions that occur separately, with one for each direction. It would be interesting to understand whether there are only a

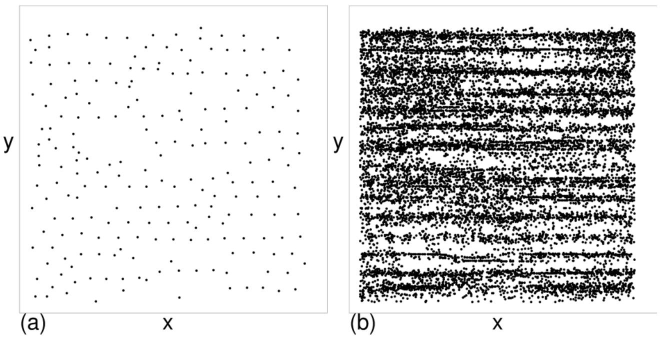

FIG. 19. Stroboscopic snapshots of the positions during 25 cycles of superconducting vortices cyclically driven over a random landscape. (a) The reversible regime for small ac drive amplitudes. (a) The irreversible regime for large ac drive amplitudes. Reprinted with permission from N. Mangan *et al.* [14]. Copyright by the American Physical Society.

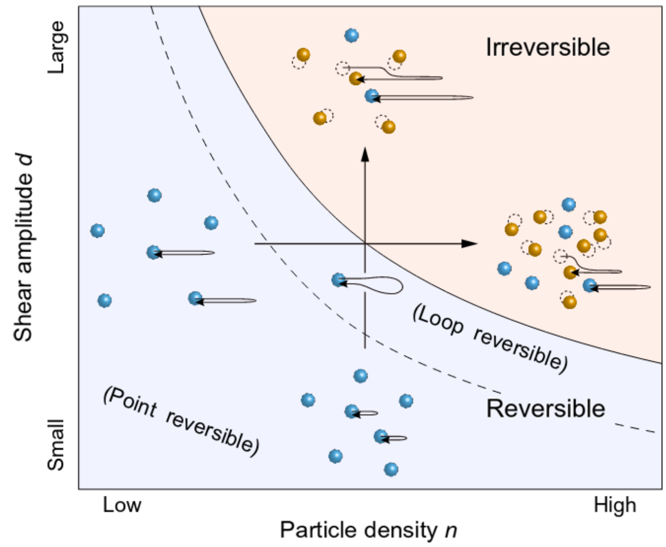

FIG. 20. Schematic illustration of R-IR phases in a superconducting vortex system as a function of shear amplitude d vs vortex or particle density n. Point reversible states appear for low densities or small shears, while a region of loop reversibility emerges prior to the transition to irreversible behavior at high densities and large shears. Reprinted under CC license from S. Maegochi *et al.* [24].

few types of distinct universality classes for R-IR transitions, or whether multiple universality classes could be observed by changing the nature of the particle-particle interactions or the driving.

V. VORTICES, SKYRMIONS, AND OTHER HARD CONDENSED MATTER SYSTEMS

There are many other systems containing assemblies of particles that can be periodically driven. These include cases in which the particles are coupled to random or periodic substrates [2,107], as found for sliding friction [146], vortices in type-II superconductors [147,148], sliding charge density waves [149,150], colloids [151], pattern forming systems [136], active matter [152], election liquid crystals [153], and Wigner solids [154]. The driving can be applied uniformly using a current, an electric field, or a magnetic field. If there is no quenched disorder present and the drive is uniform, the particles simply translate back and forth; however, when disorder is present, certain particles can become trapped while other particles move past them, leading to plastic events. Under an increasing dc drive, these systems typically exhibit a pinned phase, a plastically deforming phase, and a moving crystal or moving smectic state. If the disorder is weak, the system depins elastically without the creation of topological defects.

Mangan *et al.* [14] performed numerical simulations of superconducting vortices in a 2D system with random disorder and ac driving where the driving period, vortex density, and pinning strength are varied. Figure 19(a) shows the vortex positions at the end of each cycle for 25 cycles under a drive at which each vortex moves a distance of 45λ during a single cycle, where λ is the London penetration depth. Here the system is in a reversible regime. In Fig. 19(b), when the driving amplitude is increased so that each vortex moves a distance of 160λ during every cycle, the vortices do not return to the same positions and the system is irreversible. For the reversible state, the diffusion is zero and the time series of the average vortex velocity repeats the same pattern during every drive cycle, which would produce a periodic voltage signal if measured experimentally. In the irreversible regime, the vortices exhibit a finite diffusion and the voltage signal is chaotic. Okuma *et al.* [41] studied R-IR transitions experimentally by shearing superconducting vortices in a Corbino geometry, and mapped out the phase diagram for the transition from reversible to irreversible flow as a function of vortex displacement per cycle.

Additional work has since been performed [24] to more clearly show that superconducting vortices can also exhibit point reversible states similar to the random organization found for dilute colloids, loop reversible states similar to those observed in amorphous systems, and irreversible states, as illustrated in Fig. 20. At low densities where the vortices are far apart, the behavior resembles that of a dilute colloidal system, and point reversible states emerge, while at higher density where interactions between the vortices become important, the system shows loop reversibility. For large densities and large driving amplitudes, the system is irreversible. In Ref. [24], there is also a diverging timescale for the system to settle into a reversible or irreversible state. The power-law divergence reported in Fig. 4 of Ref. [24] is similar to that found in the random organization system and has an exponent $\nu = 1.33$ that is close to the value expected for directed percolation [30]. Even though the vortices can exhibit collective effects due to their longer range interactions, the exponents observed are consistent with the dilute colloidal system rather than with amorphous solids, where exponents closer to $\nu = 2.6$ are found. This could be due to the nature of the plastic events that occur in the different systems. There has also been work showing evidence for R-IR transitions in ac driven superconducting vortices in linear geometries [155].

Magnetic skyrmions are another particle-like magnetic texture that have many similarities to superconducting vortices in that they can be driven by an applied current [156,157]. An important distinctive feature is that skyrmions have a strong

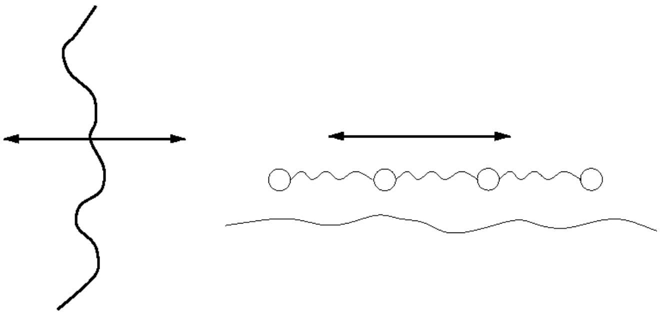

FIG. 21. Schematic of other elastic systems that could exhibit R-IR transitions under ac driving (arrows), such as a moving contact line (left) or a sliding charge density wave (right).

gyrotropic or Magnus force. Simulations of skyrmions under ac drives with quenched disorder [44] showed an R-IR transition associated with a diverging timescale where the exponent is close to $\nu = 1.29$. In this system, the Magnus term enhances the irreversible behavior by increasing the number of dynamically accessible orbits. This is in contrast to the behavior of an overdamped system, such as strongly damped skyrmions and vortices in type-II superconductors. It has been shown that in the overdamped limit, there appear to be two different R-IR transitions, since the diffusion constant first drops to zero in the direction perpendicular to the drive, followed by a regime in which the behavior is reversible both parallel and perpendicular to the drive. This suggests that the number of possible R-IR transitions may depend on the effective dimensionality of the system. Experimental results that seem consistent with the predictions of Ref. [44] were found in the superconducting vortex system [158].

VI. FUTURE DIRECTIONS FOR SYSTEMS WITH QUENCHED DISORDER

R-IR transitions under periodic driving could be explored in charge density wave systems [149,150], which often exhibit narrow band noise or temporal ordering that may be the hallmarks of reversible behavior. It would be interesting to understand if there is also an irreversible regime and if there is a diverging timescale for reaching the reversible regime. This is of particular relevance since recent work on classical time crystals has suggested that sliding charge density waves under ac driving are examples of discrete time crystals [159], which we discuss further in Sec. VIII. Other elastic systems that can be driven periodically include magnetic domain walls [160], moving contact lines [161], depinning interfaces [162], and slider block models [2]. Examples of possible materials science systems are ac driven grain boundaries [163], twin planes [164], and dislocation patterns [165,166], as illustrated in the schematic of Fig. 21. It is possible that systems of this type would always organize to a reversible state; however, there could be different kinds of reversible states separated by transitions with diverging timescales. For example, there could be a critical point separating point reversible states and loop reversible states.

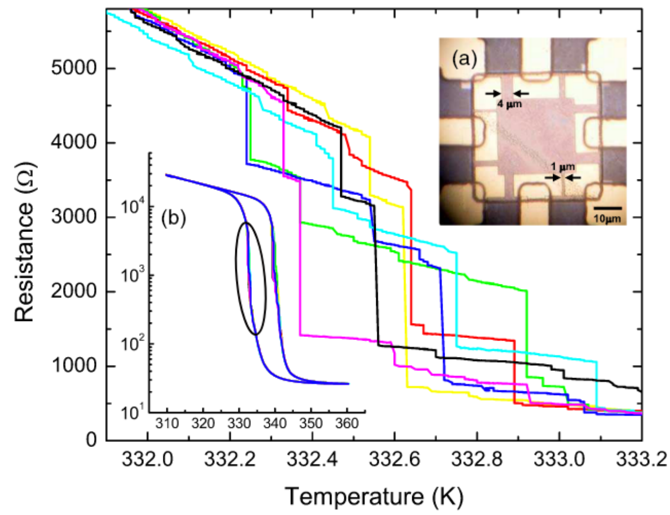

FIG. 22. Experimentally measured hysteresis loops in a VO_2 sample at the metal-insulator transition. (a) Illustration of the sample geometry showing that eight devices are present on a single sample; the widths of two of the devices are marked with arrows. (b) Full hysteresis loop as a function of resistance vs temperature. The main panel shows consecutive resistance vs temperature cycles zoomed in on the portion of the full hysteresis loop circled in (b). Jumps occur that are not repeatable from cycle to cycle. Reprinted with permission from A. Sharoni *et al.* [167]. Copyright by the American Physical Society.

A. Systems with avalanches and noise

Further directions include studying systems with hysteresis that exhibit repeating avalanches to seek diverging time and/or length scales under repeating hysteretic cycles. Figure 22 illustrates experimentally measured avalanches across the metal-insulator transition of VO_2 [167]. In systems of this type, it would be possible to perform repeated cycles to determine whether the same resistance values recur, and whether there is a finite number of cycles that must be performed before the system reaches a repeating pattern.

In many condensed matter systems, noise is generated under the application of a current or field. This noise can take the form of avalanches or crackling noise [168,169], telegraph noise [170], switching [171], narrow band noise [172,173], or broad band noise [173,174], and can be characterized using the power spectrum or second spectrum [169]. It would be interesting to explore whether such a system exhibits a perfectly repeatable noise pattern under applied periodic driving, and if so, whether a finite number of cycles must be performed before the system reaches this type of reversible state.

B. Astrophysical systems

Another set of systems that can be considered to have periodic driving and that also exhibits avalanches or bursts are pulsar glitches in neutron stars [175–178] and brightness variability for certain stars [179]. Here the periodic driving arises from the rotation of the stars. When a pulsar glitch occurs, there is a shift $\delta\nu$ in the rotation frequency ν of the pulsar. In general, glitching pulsars fall into two classes: those with Poisson-like waiting times [175–177], and

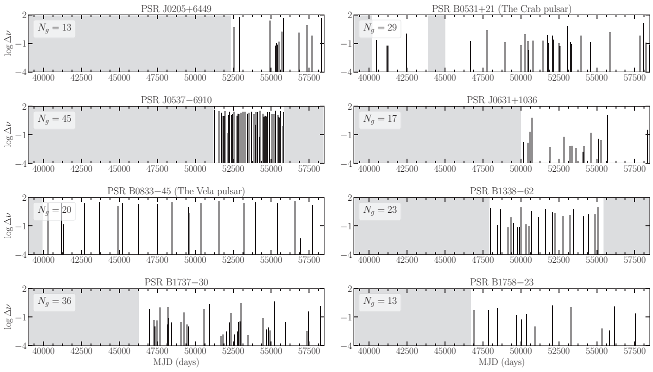

FIG. 23. Observations of glitches $\delta\nu$ (in μHz) of the rotation frequencies ν for different pulsars as a function of time in units of the modified Julian date (MJD). The total number of glitches observed for each pulsar is listed as N_g, and windows of time in which no observations were made for at least 3 months are shaded in gray. Reprinted under CC license from J. R. Fuentes *et al.* [180].

those with unimodal or quasiperiodic waiting times [177,178]. Figure 23 shows some example time series of pulsar glitches exhibiting different levels of periodicity [180]. It is possible that the glitches could be a sign that over time the star is organizing to a more periodic state, and if so, a reversible state could emerge in which there is a power-law distribution of periodic waiting times. It would be interesting to understand whether there could be a transition from chaotic glitch intervals to periodic or quasiperiodic glitches, and whether a diverging timescale appears in these systems. Sheikh *et al.* [179] measured avalanches in the brightness variability of stars and found scaling exponents that suggest the system may be near a nonequilibrium critical point. A system of this type might be irreversible on one side of the nonequilibrium point and reversible on the other side.

VII. MEMORY EFFECTS

Sheared colloidal systems can organize to a reversible state when the applied strain γ_i satisfies $\gamma_i < \gamma_c$, where γ_c is the critical stain below which the system is always reversible. If the strain amplitude is changed so that $\gamma_i > \gamma_c$, reversible behavior cannot appear. In Fig. 24(a), a numerical study [181] of a system with a critical strain level of $\gamma_c = 4.0$. showed that when a training shear pulse amplitude of $\gamma_1 = 3.0$ is applied, a "reading" of the system with a trial strain level of γ produces a signature or memory of γ_1 in the form of a cusp. It is also possible to store the values of two distinct strain levels, as shown in Fig. 24(b) for $\gamma_2 = 2$ and $\gamma_2 = 3.0$, where there are now two kinks that appear as the trial strain level is varied. This implies that multiple memories can be storied in these systems as long as the strain amplitude remains below the critical stain where the system becomes irreversible and loses all memory. Paulsen *et al.* [182] considered an experimental dilute colloidal system and observed that a memory effect emerges for strains smaller than γ_c.

Fiocco *et al.* [37] studied a protocol similar to that used by Keim *et al.* [181] but employed an amorphous solid in which the particles always remain in contact. They find that

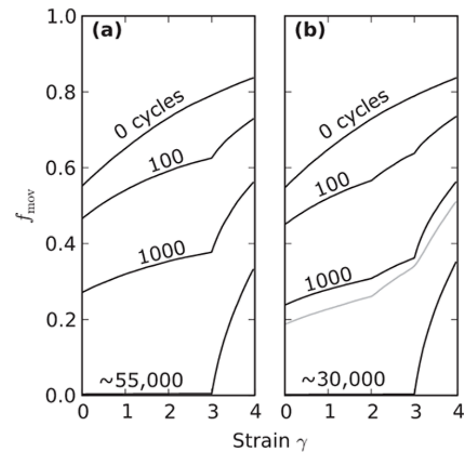

FIG. 24. The fraction of moving or irreversible particles $f_{\rm mov}$ vs the trial strain level γ for different numbers of cycles in a numerical model of sheared colloidal particles, demonstrating the emergence of memory. (a) Application of a single shear level γ_1 gives a kink in the trial strain γ at $\gamma = \gamma_1$; reversible behavior eventually emerges for $\gamma < \gamma_1$ but the system remains irreversible for $\gamma > \gamma_1$. (b) Application of a smaller shear level γ_2 for five cycles followed by application of a larger shear level γ_3 for one cycle gives the trial strain level γ a memory of the value of both γ_2 and γ_3. When the system organizes to a reversible state, only memory of the larger strain level γ_3 is retained. Reprinted with permission from N. C. Keim and S. R. Nagel [181]. Copyright by the American Physical Society.

a similar memory effect, shown in Fig. 24(a), can be achieved by using a training shear amplitude. Since this is a jammed solid, the memory is likely stored in a different manner than for the dilute systems. For example, the memory may reside in the plastic events or long-range strain fields, rather than in the spatial configurations as found in the dilute systems.

Mungan *et al.* [183] showed that the configurations of an amorphous solid sheared along a fixed plane can be represented by a directed graph where nodes represent stable particle configurations and arrows represent transitions between them due to plastic events. Figure 25 shows an example of such a network where all the transitions are reversible. In each configuration, a soft spot is switched on or off. The soft spot consists of a localized region in which the particles can undergo a plastic event or the elasticity is small. Regev et al. [184] further showed that the strongly connected components of the graphs, representing clusters of configurations where every two configurations (nodes) are reachable by a path of plastic deformation, include all the possible limit-cycles of the system. Since transitions between different strongly connected components are irreversible, as the strain magnitude increases to a level close to the R-IR transition, irreversible transitions become more frequent and the strongly connected components become smaller and can contain only small limit cycles, such as the yellow arrows in Fig. 25. Some open questions include whether introducing a change in the direction of shearing or driving in this regime could reduce the persistence of the memory. Although memory has been studied in sheared colloidal particles and sheared amorphous solids, less is known about whether similar memory effects can occur in other systems. For example, Dobroka *et al.* [185] found that

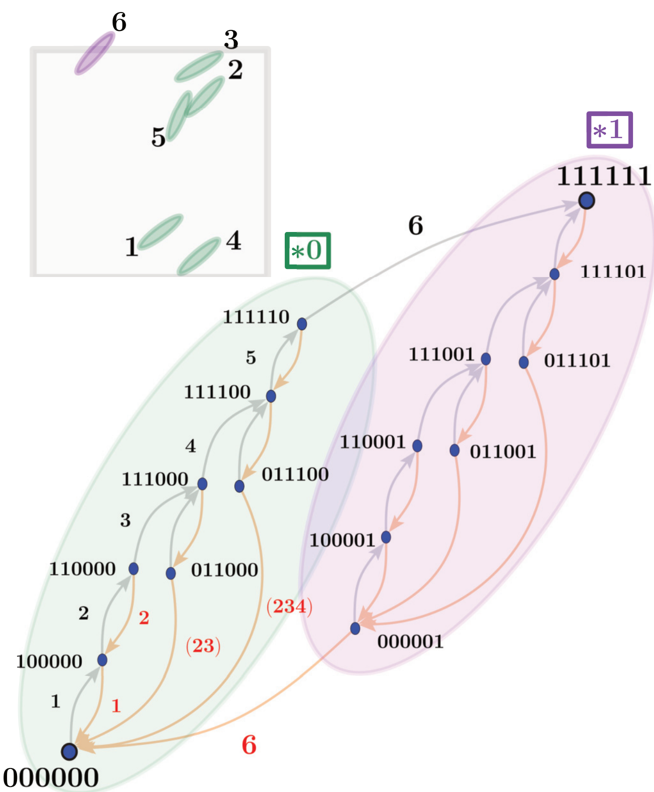

FIG. 25. A network representation of the transitions between different states of a model of an amorphous solid subjected to cyclic shear deformations. The inset shows the soft-spot locations and numbers. Each state is represented by a binary number where 1 or 0 in the second digit represents a state where soft spot number 2 is "switched on" or "switched off," respectively. Arrows represent transitions due to an increase (black) or decrease (red) in the strain causing soft-spot switching. The green and purple shaded regions are limit-cycles with soft-spot number 6 switched off or on, respectively. Reprinted with permission from M. Mungan *et al.* [183]. Copyright by the American Physical Society.

kinks in the response similar to those observed by Paulsen *et al.* [182] can occur for periodically driven superconducting vortices.

Memory can also be associated with the hysteresis that appears in magnets, semiconductors [186], and metal-insulator systems [187] under oscillating voltages, and at dynamic transitions in magnetic systems subjected to fast oscillations where the hysteresis occurs in the dynamic variables [188]. For example, in the phenomenon of return point memory [189,190], spin configurations are examined at the end of each minor loop cycle to see if the configurations fully overlap from cycle to cycle according to an overlap function. In periodically driven colloidal spin ice systems [191], the spin overlap function q was examined for repeated minor loops. A value $q = 1.0$ means that exactly the same effective spin configuration appears during each cycle, indicating reversible behavior. In Ref. [191], the system did not adopt the same spin configuration for every cycle, but as shown in Fig. 26, q increases with time until saturating at a reversible state after a fixed number of cycles, similar to the random organization

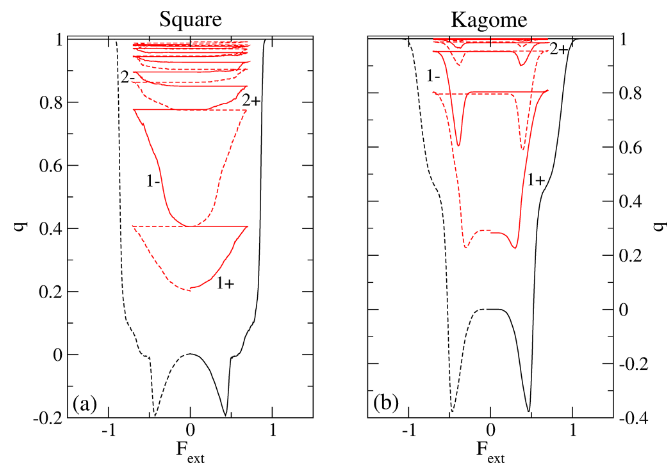

FIG. 26. Return point memory for minor loops in an artificial spin ice system constructed from colloidal particles. When $q = 1$, the system reaches a reversible state. For both the (a) square ice and (b) kagome ice geometries, q increases as the number of minor loop cycles performed increases (numbers), indicating the emergence over time of reversible behavior. Reprinted with permission from A. Libál *et al.* [191]. Copyright by the American Physical Society.

observed for dilute colloids. In addition, introducing quenched disorder increased the memory of the system.

VIII. FUTURE DIRECTIONS FOR TIME CRYSTALS AND QUANTUM SYSTEMS

Another promising area in which to seek R-IR transitions is time crystals, which have been proposed for both quantum [192] and classical systems [193]. In a time crystal, the lowest energy state of the system is periodic not only in space but also in time. Shortly after the initial proposals, it was recognized that true time crystals as originally envisioned cannot occur under conditions of strict equilibrium [194,195]; however, the time crystal concept has now generated a wealth of ideas for creating time periodic systems that could arise under nonequilibrium conditions [46,196], such as periodic driving. A key feature of a time crystal is that the system can exhibit subharmonics of the oscillatory driving [196–199] (multiperiodic response). Yao *et al.* [159] have noted that there are a number of classical systems that also show subharmonic entrainment, including Faraday waves [49] and predator-prey models [200], phase locking in driven charge density wave systems [201] or Josephson junctions [202], and superconducting vortex [203] or magnetic skyrmion [204] motion on periodic pinning arrays, meaning that these systems could be examples of Classical Discrete Time Crystals (CDTCs). In quantum systems, time crystals have been studied by measuring subharmonic entrainment in unitary many-body systems or Floquet systems [199]. In some cases, time crystals can arise in quantum systems where many-body localization can prevent thermalization [46,196]. Up until now, work on time crystals has focused on identifying systems that support time crystals; however, the dynamics of how a system can organize into a time crystal state or the general phase diagram near the boundary from chaotic to time crystal behavior has many similarities to the R-IR transitions discussed above.

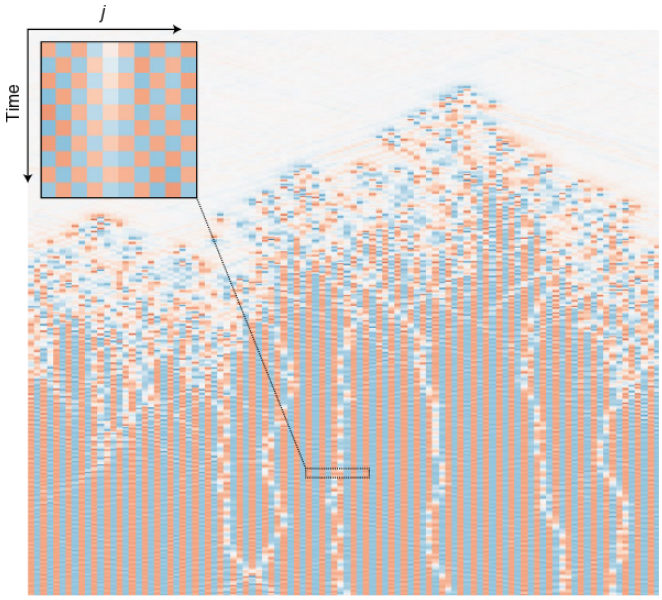

FIG. 27. A stroboscopic image as a function of time vs position j of a one-dimensional system of coupled oscillators. Oscillators that have a position coordinate q_i less than zero are red, those with $q_i > 0$ are blue, and those with $q_i \approx 0$ are white. Period-doubled oscillations emerge from the uniform state over time, forming a Classical Discrete Time Crystal (CDTC). The behavior is similar to the organization of a fluctuating system into a reversible state as shown earlier. Reprinted by permission from: Springer Nature, N. Y. Yao *et al.* [159].

An example of a classical time crystal that appears close to systems in which R-IR transitions can happen appears in the work of Yao *et al.* [159], where a one-dimensional array of coupled oscillators was studied at finite temperature. Figure 27 shows a stroboscopic view of the position coordinate q_j of each oscillator under periodic driving as a function of time. In the initial state, the system is disordered or fluctuating, but over time it organizes into an antiferromagnetic repeating state. This is reminiscent of an initially irreversible state organizing over time into a reversible state. The reversibility does not have to appear after a single driving cycle; instead, the system can repeat after two or more driving cycles. It would be interesting to see whether there is a timescale for the organization into the time periodic or CDTC state, and if so, whether this time diverges as a function of the driving parameters, similar to what is found for the R-IR transition in colloidal systems. The work of Yao *et al.* [159] can also be applied to the much broader class of systems in higher dimensions and at zero temperature, such as nonlinear systems that exhibit higher order harmonic phase locking effects. There could be a variety of other systems on periodic flashing substrates that could organize to reversible states over time, such as choreographic colloidal time crystals that have a liquid-like state and a time ordered state [205].

As discussed in Sec. VII, the reversible states can exhibit memory and training effects, so similar phenomena along with memory encoding studies could be explored in time crystal systems, where much more complicated periodic driving protocols can be employed. For example, the system might organize to a CDTC for one set of driving frequencies, but if additional frequencies were added, the system could organize into a new time crystal state. The question would be whether, if the system is first trained with the initial set of frequencies, these frequencies could be retained in the time crystal state formed with additional driving frequencies. Although we have focused on classical time crystals, similar R-IR transitions could arise in periodically driven quantum time crystals, such as periodically driven trapped atomic ions [206], driven spins in diamond [207], and arrays of superconducting qubits [208].

IX. COMPLEX NETWORKS AND DYNAMICAL SYSTEMS

In general, there are many other coupled systems that can organize into synchronized states. Systems that exhibit ergodicity breaking would also be candidates for study in the framework of R-IR transitions. For example, dynamical many-body coupled oscillators are known to show transitions to synchronized states [209]. Other systems include particles coupled to or flowing on a complex network [210] under periodic driving. In this case, the particles could form repeating loop paths on the network in the reversible state, such as those illustrated in Fig. 10(c). Such systems could include those for which particles or carriers of information cannot pass though each other and the network is rigid. Other systems such as those with nonreciprocal interactions [211] or odd viscosity [212] show transitions from chaotic to ordered edge states, and could organize to patterns containing ordered loop currents or flocking.

Complex networks and dynamical systems exhibiting synchronization arise across many biological [213,214], robotic [215], social [216], and economic systems [217]. It is known that these systems can all enter chaotic or strongly fluctuating states; however, it would be interesting to study whether under certain limited conditions there could be situations in which, when some type of periodic driving is applied, time repeating or nearly time repeating states could emerge. This could occur if the stochastic terms in the models are small. Future studies could focus on mathematical models of such systems to determine whether partial reversibility can occur or whether there is a diverging timescale near a critical point.

X. SUMMARY

We have given an overview of the recent work examining transitions from chaotic irreversible states to time periodic or reversible states for systems under periodic driving. In these systems, the locations of the particles are compared to their positions on the previous driving cycle. For irreversible states, the particles do not return to the same positions, and over multiple cycles they exhibit diffusive motion away from their initial positions. In a reversible state, the particles return to the same positions after one or an integer number of driving cycles, and there is no long time diffusion. Reversible-irreversible (R-IR) transitions were initially studied for periodically sheared dilute colloids, in which a process termed random organization produces reversible states where particle-particle collisions no longer occur, while the irreversible states have continuous collisions. R-IR transitions have also been studied in strongly interacting systems such as amorphous solids and jammed systems. Another hallmark of the R-IR transition is that there is a critical drive or density above which the system remains in an irreversible state, and there is a power-law divergence on either side of the transition for the time required for the system to settle into a steady irreversible state or organize into reversible motion. In dilute systems, the R-IR transition is consistent with directed percolation but could also fall in the class of conserved directed percolation. For amorphous solids, similar R-IR behavior is found but the transition appears to fall into a different universality class, and for dense systems, the critical amplitude coincides with the yielding transition. On the reversible side of the R-IR transition, it is possible to store a series of memories by applying training pulses. Such memory effects have been observed in both dilute and dense systems. R-IR transitions have also been studied in solid state systems such as periodically driven superconducting vortices and magnetic skyrmions, both of which show similar behavior to that found in dilute sheared colloidal systems.

We highlight how the general features of the R-IR transition could be applied to much broader classes of soft matter, hard matter, and dynamical systems. For example, many solid state systems show hysteretic behavior, and systems of this type would be fertile ground for studying transitions to reversible states under repeated cycling. The impact of repeatable noise, avalanches, or return point memory could be studied, as well as the number of cycles required to reach the reversible state, which could show critical behavior similar to that found in colloidal systems. Such studies could be performed for magnetic systems, metal-insulator transitions, charge ordering systems, and semiconductors. Other classes of systems in which to look for R-IR transitions include commensurate-incommensurate systems, frustrated systems, crumpled and corrugated sheets [218,219], and even active matter systems. These systems may also be able to store more complex memories. For example, amorphous solids or colloids could be sheared with a periodic but increasingly complex protocol to see if the system can still reach a reversible state. We also discuss some other possible states that are not time periodic but that show no long time diffusion, enabling quasiperiodic dynamics to be explored, as well as systems that do not follow the same path on each cycle but only trace out a limited number of cycles, which could arise for frustrated states.

We discuss how the R-IR framework could be applied to classical and quantum discrete time crystals where the transitions to time periodic or harmonically entrapped states could be an example of an R-IR transition. Power-law divergences in time could appear for the formation of time crystal states. Other systems to consider include more general cyclic systems such as pulsars, coupled oscillators, social systems, economics, biological systems, and particle flow dynamics on complex networks where the reversible state could arise through the formation of loop currents.

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