Generating tensor polarization from shear stress

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We derive an expression for the tensor polarization of a system of massive spin-1 particles in a hydrodynamic framework. Starting from quantum kinetic theory based on the Wigner-function formalism, we employ a modified method of moments which also takes into account all spin degrees of freedom. It is shown that the tensor polarization of an uncharged fluid is determined by the shear-stress tensor. In order to quantify this novel polarization effect, we provide a formula which can be used for numerical calculations of vector-meson spin alignment in relativistic heavy-ion collisions.

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I. INTRODUCTION

The observation of polarization phenomena in relativistic heavy-ion collisions has opened a new direction of research in the physics of the hot and dense nuclear matter [1,2]. The STAR Collaboration showed that Λ-baryons emitted in noncentral nuclear collisions are spin polarized along the direction of the global angular momentum [3,4]. This finding provides the evidence that in the quark-gluon plasma particle spin polarization is triggered by rotation (as suggested in Refs. [5–8]) in a way which resembles the time-honored Barnett effect [9]. Despite early success in describing global polarization data [8,10–16], discrepancies between theory and experiment triggered big theoretical efforts both at the phenomenological [17–30] and more formal level with the formulation of relativistic spin hydrodynamics [31–68]. More recently, experimental studies of the so-called spin alignment of massive spin-1 particles such as ϕ and $K^{\star 0}$ mesons have been also carried out [69–71]. The data shows that the spin alignment is much larger compared to theoretical predictions given by models based on the assumption of local equilibrium [8]. This poses a new puzzle which is currently the subject of intense work [72–80] for which, however, an established solution is still missing.

In heavy-ion experiments, the spin vector polarization of Λ-baryons can be directly extracted from the angular distribution of their weak decay [3,4]. The case of massive spin-1 particles is different. First, it is important to note that the polarization state of a vector meson is fully specified by three parameters corresponding to the conventional vector polarization and by additional five parameters called tensor polarization [81]. In fact, tensor polarization is a property which characterizes only particles with spin higher than $1/2$. In general, vector and tensor polarization are independent quantities and, therefore, one can have a spin-1 particle which is tensor polarized and not vector polarized, and vice versa [81]. In experiments, since for vector mesons only parity-conserving decays are studied [69,70], the spin alignment only gives information on the tensor polarization state.

In Refs. [82,83] it was shown that vector mesons emitted from a thermalized medium are in general tensor polarized even if the system is in global equilibrium without rotation. Such tensor polarization is due to the imbalance between transverse and longitudinal spectral functions [82,83]. In this paper, we propose a different mechanism. We consider an uncharged fluid composed of massive spin-1 particles near local thermodynamic equilibrium. In our framework, tensor polarization arises due the presence of shear stress in the fluid. An intuitive explanation can be given only based on parity arguments. Since tensor polarization is a parity-even rank-2 traceless and symmetric tensor [81], in a hydrodynamic framework it can only be proportional to the shear stress tensor of the fluid at first order in deviations from equilibrium. In this work we derive the expression for the tensor polarization starting from quantum kinetic theory for massive spin-1 particles. In order to calculate the dissipative corrections, we use the method of moments. In particular, we define new rank-2 spin moments which extend the previous formulations for the spin-0 [84] and spin-1/2 cases [64].

Our notation and conventions are: $a\cdot b := a^\mu b_\mu$, $a_{[\mu}b_{\nu]} := a_\mu b_\nu - a_\nu b_\mu$, $a_{(\mu}b_{\nu)} := a_\mu b_\nu + a_\nu b_\mu$, $g_{\mu\nu} := \mathrm{diag}(+,-,-,-)$, $\epsilon^{0123} = -\epsilon_{0123} := 1$. The ℓth rank projector onto the subspace of traceless symmetric tensors orthogonal to the fluid 4-velocity u^μ [85] is denoted as $\Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell}$, and we write a projected tensor A as $A^{\langle\mu_1\cdots\mu_\ell\rangle} := \Delta^{\mu_1\cdots\mu_\ell}_{\nu_1\cdots\nu_\ell} A^{\nu_1\cdots\nu_\ell}$.

II. KINETIC THEORY FOR VECTOR PARTICLES

Let us consider the Lagrangian for a Proca field V^μ of mass m,

$$\mathcal{L} = -\hbar\left(\frac{1}{2}V^{\dagger\mu\nu}V_{\mu\nu} - \frac{m^2}{\hbar^2}V^{\dagger\mu}V_\mu\right) + \mathcal{L}_{\text{int}}, \tag{1}$$

where $\mathcal{L}_{\text{int}}$ is a general interaction Lagrangian. The fundamental object of quantum kinetic theory is the Wigner function defined as [65,86–90]

$$W^{\mu\nu}(x,k) := -\frac{2}{(2\pi\hbar)^4\hbar}\int d^4y\, e^{-ik\cdot y/\hbar} \times \langle : V^{\dagger\mu}(x+y/2)V^\nu(x-y/2):\rangle, \tag{2}$$

where $\langle:\cdots:\rangle$ denotes the normal-ordered ensemble average. This Wigner transform of the two-point function defines a quantum analog of the distribution function known from classical kinetic theory. Assuming that quantum effects are small (meaning that the Compton wavelength of the particles has to be small compared to a typical macroscopic length scale), one can perform a so-called $\hbar$ expansion, i.e., write

$$W^{\mu\nu}(x,k) = W^{(0),\mu\nu}(x,k) + \hbar W^{(1),\mu\nu}(x,k) + \cdots, \tag{3}$$

where the Planck constant acts as a bookkeeping parameter. In the following, all results are derived from employing such an expansion up to first order in $\hbar$. Note that in Eq. (2) the momentum variable k is not necessarily on the mass shell. However, one can show [44,91,92] that, to first order in the $\hbar$ expansion, the off-shell terms cancel in the evolution equation of the Wigner function, such that it is sufficient to consider the part that is on shell. Considering the fact that a charged vector field has 3 (complex) independent components [93], it is evident that the Wigner function must have nine independent degrees of freedom, while the remaining seven components can be expressed in terms of these [65]. These degrees of freedom can be shown to consist of a scalar (one component), a pseudovector (three components), and a traceless symmetric tensor (five components). As shown in Appendix A, the pseudovector degree of freedom can be related to the vector polarization of the particles, while the traceless symmetric tensor corresponds to the tensor polarization. A convenient way to treat these nine independent components in a compact fashion is to enlarge the phase space by introducing an additional "spin" variable $\mathfrak{s}^\mu$ [37], together with a respective measure

$$dS(k) := \frac{3m}{2\sigma\pi}d^4\mathfrak{s}\,\delta(\mathfrak{s}^2+\sigma^2)\delta(k\cdot\mathfrak{s}), \quad \sigma^2 := 2. \tag{4}$$

Note that we have the following identities,

$$\int dS(k) = 3, \quad \int dS(k)\mathfrak{s}^\mu\mathfrak{s}^\nu = -2K^{\mu\nu},$$
$$\int dS(k)K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta\mathfrak{s}_\rho\mathfrak{s}_\sigma = \frac{8}{5}K^{\mu\nu}_{\rho\sigma}, \tag{5}$$

while the integral over any odd number of spin vectors vanishes. Here, $K^{\mu\nu} := g^{\mu\nu} - k^\mu k^\nu/m^2$ and $K^{\mu\nu}_{\rho\sigma} := 1/2K^\mu_{(\rho}K^\nu_{\sigma)} - 1/3K^{\mu\nu}K_{\rho\sigma}$ denote the projectors onto subspaces irreducible with respect to the little group of k^μ. We can then define a scalar distribution function [65]

$$f(x,k,\mathfrak{s}) := H^{\nu\mu}(k,\mathfrak{s})W^{\text{on-shell}}_{\mu\nu}(x,k),$$
$$H^{\mu\nu}(k,\mathfrak{s}) := \frac{1}{3}K^{\mu\nu} + \frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\frac{k_\alpha}{m}\mathfrak{s}_\beta + \frac{5}{8}K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta, \tag{6}$$

where $W^{\text{on-shell}}_{\mu\nu}$ denotes the part of the Wigner function proportional to $\delta(k^2-m^2)$. It is important to note that, to first order in the $\hbar$ expansion, the distribution function $f(x,k,\mathfrak{s})$ contains the complete information necessary to reconstruct the full Wigner function. In the noninteracting case, the following inverse relation also holds:

$$W^{\text{on-shell}}_{\mu\nu}(x,k) = \int dS(k)h_{\mu\nu}(k,\mathfrak{s})f(x,k,\mathfrak{s}),$$
$$h_{\mu\nu}(k,\mathfrak{s}) := \frac{1}{3}K_{\mu\nu} + \frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\frac{k^\alpha}{m}\mathfrak{s}^\beta + K^{\alpha\beta}_{\mu\nu}\mathfrak{s}_\alpha\mathfrak{s}_\beta. \tag{7}$$

Starting from the equations of motion for the vector field that follow from the Lagrangian (1), it can be shown that the evolution equation of the phase-space distribution function reads [92]

$$k\cdot\partial f(x,k,\mathfrak{s}) = \mathfrak{C}[f], \tag{8}$$

where

$$\mathfrak{C}[f] := \frac{1}{2}\int d\Gamma_1\, d\Gamma_2\, d\Gamma'\, d\bar{S}(k)(2\pi\hbar)^4\delta^{(4)}(k+k'-k_1-k_2)$$
$$\times \mathcal{W}[f(x+\Delta_1-\Delta,k_1,\mathfrak{s}_1)f(x+\Delta_2-\Delta,k_2,\mathfrak{s}_2)$$
$$- f(x+\Delta'-\Delta,k',\mathfrak{s}')f(x,k,\bar{\mathfrak{s}})] \tag{9}$$

and we introduced the $(x,k,\mathfrak{s})$-phase-space measures

$$d\Gamma := dK dS(k), \quad dK := \frac{2}{(2\pi\hbar)^3}d^4k\,\delta(k^2-m^2). \tag{10}$$

The transition rate is given by

$$\mathcal{W} := \frac{(2\pi\hbar)^3}{32}M^{\gamma_1\gamma_2\delta_1\delta_2}M^{\zeta_1\zeta_2\eta_1\eta_2}h_{1,\gamma_1\eta_1}h_{2,\gamma_2\eta_2}h'_{\zeta_2\delta_2}$$
$$\times(H_{\zeta_1}{}^{\alpha}\bar{h}_{\alpha\delta_1} + \bar{h}_{\zeta_1}{}^{\alpha}H_{\alpha\delta_1}), \tag{11}$$

while the vectors Δ_1, Δ_2, Δ', and Δ read

$$\Delta_1^\mu := \frac{2}{3}\frac{1}{\mathcal{W}}\frac{(2\pi\hbar)^3}{64}\frac{i\hbar}{2m^2}M^{\gamma_1\gamma_2\delta_1\delta_2}M^{\zeta_1\zeta_2\eta_1\eta_2}\left(h^\mu_{1\ \eta_1}k_{1,\gamma_1} - k_{1,\eta_1}h_{1,\gamma_1}{}^\mu\right)h_{2,\gamma_2\eta_2}h'_{\zeta_2\delta_2}H_{\zeta_1\delta_1}, \tag{12a}$$

$$\Delta_2^\mu := \frac{2}{3}\frac{1}{\mathcal{W}}\frac{(2\pi\hbar)^3}{64}\frac{i\hbar}{2m^2}M^{\gamma_1\gamma_2\delta_1\delta_2}M^{\zeta_1\zeta_2\eta_1\eta_2}h_{1,\gamma_1\eta_1}\left(h^\mu_{2\ \eta_2}k_{2,\gamma_2} - k_{2,\eta_2}h_{2,\gamma_2}{}^\mu\right)h'_{\zeta_2\delta_2}H_{\zeta_1\delta_1}, \tag{12b}$$

$$\Delta'^\mu := \frac{2}{3}\frac{1}{\mathcal{W}}\frac{(2\pi\hbar)^3}{64}\frac{i\hbar}{2m^2}M^{\gamma_1\gamma_2\delta_1\delta_2}M^{\zeta_1\zeta_2\eta_1\eta_2}h_{1,\gamma_1\eta_1}h_{2,\gamma_2\eta_2}\left(h'^\mu{}_{\delta_2}k'_{\zeta_2} - k'_{\delta_2}h'_{\zeta_2}{}^\mu\right)H_{\zeta_1\delta_1}, \tag{12c}$$

$$\Delta^\mu := \frac{2}{3}\frac{1}{\mathcal{W}}\frac{(2\pi\hbar)^3}{64}\frac{i\hbar}{2m^2}M^{\gamma_1\gamma_2\delta_1\delta_2}M^{\zeta_1\zeta_2\eta_1\eta_2}h_{1,\gamma_1\eta_1}h_{2,\gamma_2\eta_2}h'_{\zeta_2\delta_2}\left(H^\mu{}_{\delta_1}k_{\zeta_1} - k_{\delta_1}H_{\zeta_1}{}^\mu\right), \tag{12d}$$

where we abbreviated $h_1 := h(k_1, \mathfrak{s}_1)$ (and analogously for h_2, h', $\bar{h}$, and H). The vectors (12) denote shifts in the particle position from the point x, characterizing the nonlocality of the collision. It has been shown in Refs. [37,91] that this nonlocality is essential to explain the spin polarization of particles, as it introduces a nonvanishing orbital angular momentum into the collision that can then be converted into spin, since the total angular momentum is conserved. However, it will become clear in Sec. IV that the tensor polarization of vector particles does not depend on these nonlocalities, but arises from purely local effects. In Eqs. (11) and (12), M is the tree-level vertex of the theory, and is related to the transfer matrix elements via

$$\begin{aligned}&\langle k, k'; \lambda, \lambda'|\,\hat{t}\,|k_1, k_2; \lambda_1, \lambda_2\rangle \\ &\quad = \epsilon^{*(\lambda)}_\mu(k)\epsilon^{*(\lambda')}_\nu(k')\epsilon^{(\lambda_1)}_\alpha(k_1)\epsilon^{(\lambda_2)}_\beta(k_2)M^{\mu\nu\alpha\beta},\end{aligned} \tag{13}$$

where, e.g., $|k_1, k_2; \lambda_1, \lambda_2\rangle$ denotes a two-particle state with momenta (k_1, k_2) and spins (λ_1, λ_2), while $\epsilon^{(\lambda)}_\mu(k)$ is the polarization vector of a vector particle with momentum k and spin λ. Note that the form of the Boltzmann equation [Eqs. (8) and (9)] for binary elastic collisions closely resembles the formulation presented in Refs. [37,91,94].

III. RELATIVISTIC HYDRODYNAMICS AND TENSOR POLARIZATION

We consider an uncharged fluid with spin degrees of freedom and tensor polarization governed by the conservation equations

$$\partial_\mu T^{\mu\nu} = 0, \quad \hbar\partial_\lambda S^{\lambda,\mu\nu} = T^{[\nu\mu]}, \tag{14}$$

where $T^{\mu\nu}$ is the energy-momentum tensor and $S^{\lambda,\mu\nu}$ is the spin tensor. In this work we choose the Hilgevoord-Wouthuysen (HW) pseudogauge up to first order in $\hbar$ [39,65],

$$\begin{aligned}T^{\mu\nu} &:= \int d\Gamma\, k^\mu k^\nu f(x, k, \mathfrak{s}),\\ S^{\lambda,\mu\nu} &:= \int d\Gamma\, k^\lambda\left(\Sigma^{\mu\nu}_{\mathfrak{s}} - \frac{\hbar}{3m^2}k^{[\mu}\partial^{\nu]}\right)f(x, k, \mathfrak{s}),\end{aligned} \tag{15}$$

where we defined $\Sigma^{\mu\nu}_{\mathfrak{s}} := -(1/m)\epsilon^{\mu\nu\alpha\beta}k_\alpha\mathfrak{s}_\beta$. The (momentum-dependent) tensor polarization is given by

$$\Theta^{\mu\nu}(k) = \frac{1}{2}\sqrt{\frac{3}{2}}\frac{1}{N(k)}\int d\Sigma_\lambda k^\lambda \int dS(k)K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta f(x, k, \mathfrak{s}), \tag{16}$$

where the prefactor is defined in accordance with Ref. [81], $N(k) := \int d\Sigma_\lambda k^\lambda K_{\alpha\beta}W^{\alpha\beta}$ and $d\Sigma_\lambda$ denotes integration over a spacelike hypersurface, which, for example, can be taken to be the freeze-out hypersurface. As will be shown later, this quantity is related to the spin alignment measured in experiments [69–71]. A derivation of Eq. (16) is provided in Appendix A.

A. Moment expansion

In order to determine the dissipative corrections to the tensor polarization, we extend the formalism developed in Ref. [84] for spin-0 particles and in Ref. [64] for spin-1/2 particles to the case of spin 1. We split the distribution function $f(x, k, \mathfrak{s})$ into a local-equilibrium and a dissipative contribution

$$f(x, k, \mathfrak{s}) = f_{\text{eq}}(x, k, \mathfrak{s}) + \delta f_{\mathbf{k}\mathfrak{s}}, \tag{17}$$

with the local-equilibrium part [91]

$$f_{\text{eq}}(x, k, \mathfrak{s}) := \exp\left(-\beta_0 E_{\mathbf{k}} - \frac{\hbar}{2m}\epsilon^{\mu\nu\alpha\beta}\Omega_{\mu\nu}k_\alpha\mathfrak{s}_\beta\right), \tag{18}$$

where $E_{\mathbf{k}} := k\cdot u$. Note again that Eq. (18) as well as all calculations in this paper are valid up to first order in $\hbar$. The Lagrange multipliers for the four-momentum and total angular momentum are given by $\beta_0 u^\mu$ and $\Omega_{\mu\nu}$, respectively, with β_0 being the inverse temperature, u^μ the fluid four-velocity and $\Omega_{\mu\nu}$ the spin potential. Since the tensor polarization is not related to any conserved quantity, it does not appear in the local-equilibrium distribution function. The deviation from local equilibrium $\delta f_{\mathbf{k}\mathfrak{s}}$ is first expanded in the spin variable $\mathfrak{s}^\mu$, where it is at most bilinear, cf. Eq. (6). Thus we can write

$$\delta f_{\mathbf{k}\mathfrak{s}} = f_{0\mathbf{k}}\left(\phi_{\mathbf{k}} - \mathfrak{s}_\mu\zeta^\mu_{\mathbf{k}} + \mathfrak{s}_\alpha\mathfrak{s}_\beta K^{\alpha\beta}_{\mu\nu}\xi^{\mu\nu}_{\mathbf{k}}\right), \tag{19}$$

where $f_{0\mathbf{k}} := \exp(-\beta_0 E_{\mathbf{k}})$ is the zeroth-order equilibrium distribution function. Here we assumed $\zeta^\mu_{\mathbf{k}}$ and $\xi^{\mu\nu}_{\mathbf{k}}$ to be orthogonal to the four-momentum and (in the case of $\xi^{\mu\nu}_{\mathbf{k}}$) traceless, which can be done without loss of generality due to the symmetries of $\mathfrak{s}^\mu$ and $K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta$ [64]. Then, it is possible to explicitly use these properties to eliminate the components of $\zeta^\mu_{\mathbf{k}}$ and $\xi^{\mu\nu}_{\mathbf{k}}$ that are parallel to the fluid four-velocity u^μ, obtaining

$$\delta f_{\mathbf{k}\mathfrak{s}} = f_{0\mathbf{k}}\left(\phi_{\mathbf{k}} - \mathfrak{s}^\nu\Xi_{\nu\mu}\zeta^\mu_{\mathbf{k}} + \mathfrak{s}_\alpha\mathfrak{s}_\beta K^{\alpha\beta}_{\mu\nu}\Xi^{\mu\nu}_{\rho\sigma}\xi^{\rho\sigma}_{\mathbf{k}}\right), \tag{20}$$

where we defined the tensors

$$\begin{aligned}\Xi_{\mu\nu} &:= \Delta_{\mu\nu} + \frac{k_{\langle\mu\rangle}k_{\langle\nu\rangle}}{E^2_{\mathbf{k}}},\\ \Xi_{\mu\nu,\alpha\beta} &:= \frac{1}{2}(\Xi_{\mu\alpha}\Xi_{\nu\beta} + \Xi_{\mu\beta}\Xi_{\nu\alpha}) - \frac{1}{\Xi^2}\Xi_{\mu\gamma}\Xi_\nu{}^\gamma\Xi_{\delta\alpha}\Xi^\delta{}_\beta\end{aligned} \tag{21}$$

with $\Xi^2 := \Xi^{\mu\nu}\Xi_{\mu\nu} = 2 + m^4/E^4_{\mathbf{k}}$. Expanding $\phi_{\mathbf{k}}$, $\zeta^\mu_{\mathbf{k}}$ and $\xi^{\mu\nu}_{\mathbf{k}}$ terms of irreducible moments, we find

$$\delta f_{\mathbf{k}\mathfrak{s}} = f_{0\mathbf{k}}\sum_{\ell=0}^\infty k_{\langle\mu_1}\cdots k_{\mu_\ell\rangle}\left(\sum_{n\in\mathbb{S}^{(0)}_\ell}\mathcal{H}^{(0,\ell)}_{\mathbf{k}n}\rho^{\mu_1\cdots\mu_\ell}_n - \mathfrak{s}^\nu\Xi_{\nu\mu}\sum_{n\in\mathbb{S}^{(1)}_\ell}\mathcal{H}^{(1,\ell)}_{\mathbf{k}n}\tau^{\langle\mu\rangle,\mu_1\cdots\mu_\ell}_n + \mathfrak{s}_\alpha\mathfrak{s}_\beta K^{\alpha\beta}_{\mu\nu}\Xi^{\mu\nu}_{\rho\sigma}\sum_{n\in\mathbb{S}^{(2)}_\ell}\mathcal{H}^{(2,\ell)}_{\mathbf{k}n}\psi^{\langle\rho\sigma\rangle,\mu_1\cdots\mu_\ell}_n\right). \tag{22}$$

Here $\mathbb{S}_\ell^{(n)}$ denotes the set of moments of tensor-rank ℓ in momentum and n in spin that are included in the theory, and the irreducible moments are given by

$$\rho_r^{\mu_1\cdots\mu_\ell} := \int d\Gamma E_{\mathbf{k}}^r k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle}\delta f_{\mathbf{k}\mathfrak{s}}, \tag{23a}$$

$$\tau_r^{\mu,\mu_1\cdots\mu_\ell} := \int d\Gamma E_{\mathbf{k}}^r \mathfrak{s}^\mu k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle}\delta f_{\mathbf{k}\mathfrak{s}}, \tag{23b}$$

$$\psi_r^{\mu\nu,\mu_1\cdots\mu_\ell} := \int d\Gamma E_{\mathbf{k}}^r K_{\alpha\beta}^{\mu\nu}\mathfrak{s}^\alpha\mathfrak{s}^\beta k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle}\delta f_{\mathbf{k}\mathfrak{s}}. \tag{23c}$$

Note that, as was the case in Ref. [64], due to the explicit removal of redundant degrees of freedom [cf. Eq. (20)] only moments orthogonal to the four-velocity in all indices enter the expansion (22). Furthermore, we introduced the polynomials

$$\mathcal{H}_{\mathbf{k}n}^{(j,\ell)} := \frac{(2j+1)!!}{2^j j!}\frac{W^{(\ell)}}{\ell!}\sum_{m\in\mathbb{S}_\ell^{(j)}}\sum_{q=0}^{m} a_{mn}^{(\ell)}a_{mq}^{(\ell)}E_{\mathbf{k}}^q, \tag{24}$$

where the coefficients $a_{mn}^{(\ell)}$ are constructed via Gram-Schmidt orthogonalization, cf. Ref. [84]. The normalization reads $W^{(\ell)} := (-1)^\ell/I_{2\ell,\ell}$, where we defined the standard thermodynamic integrals

$$I_{nq} := \frac{1}{(2q+1)!!}\int d\Gamma E_{\mathbf{k}}^{n-2q}\left(E_{\mathbf{k}}^2-m^2\right)^q f_{0\mathbf{k}}. \tag{25}$$

The rank-($2+\ell$) tensors in Eq. (23c) are new compared to the previously developed hydrodynamic framework for spin-0 and spin-1/2 particles [64,84] and correspond to dissipative degrees of freedom associated with tensor polarization, cf. Sec. IV. Inserting Eq. (17) into Eq. (8), the Boltzmann equation takes the form

$$\delta\dot{f}_{\mathbf{k}\mathfrak{s}} + \dot{f}_{0\mathbf{k}} + E_{\mathbf{k}}^{-1}k\cdot\nabla f_{0\mathbf{k}} + E_{\mathbf{k}}^{-1}k\cdot\nabla\delta f_{\mathbf{k}\mathfrak{s}} = E_{\mathbf{k}}^{-1}\mathfrak{C}[f], \tag{26}$$

which is the starting point for the derivation of the equations of motion for the irreducible moments. For the purpose of this paper the full set of coupled equations of motion is not needed and we will only focus on the tensor-polarization moments $\psi_r^{\mu\nu,\mu_1\cdots\mu_\ell}$. Integrating Eq. (26) over $\int dS(k)K_{\alpha\beta}^{\mu\nu}\mathfrak{s}^\alpha\mathfrak{s}^\beta$, we obtain equations of motion of the form

$$\dot{\psi}_r^{\langle\mu\nu\rangle,\langle\mu_1\cdots\mu_\ell\rangle} - C_{r-1}^{\langle\mu\nu\rangle,\langle\mu_1\cdots\mu_\ell\rangle} = \mathcal{O}(\mathrm{Re}^{-1}\partial)^{\langle\mu\nu\rangle,\mu_1\cdots\mu_\ell}. \tag{27}$$

Here we used that $\int dS(k)K_{\alpha\beta}^{\mu\nu}\mathfrak{s}^\alpha\mathfrak{s}^\beta f_{\mathrm{eq}}(x,k,\mathfrak{s}) = 0$, which follows from Eq. (18). This implies that tensor polarization vanishes in equilibrium up to first order in $\hbar$. The contributions from the last term on the left-hand side of Eq. (26) to Eq. (27), denoted by $\mathcal{O}(\mathrm{Re}^{-1}\partial)$, correspond to quantities linear in gradients of dissipative quantities, i.e., of first order in the so-called inverse Reynolds numbers Re^{-1}. Note that the first term on the left-hand side of Eq. (27) is also of order $\mathcal{O}(\mathrm{Re}^{-1}\partial)$. Furthermore, we defined the generalized collision integrals

$$C_r^{\mu\nu,\langle\mu_1\cdots\mu_\ell\rangle} := \int d\Gamma E_{\mathbf{k}}^r K_{\alpha\beta}^{\mu\nu}\mathfrak{s}^\alpha\mathfrak{s}^\beta k^{\langle\mu_1}\cdots k^{\mu_\ell\rangle}\mathfrak{C}[f]. \tag{28}$$

The explicit form of the right-hand side of Eq. (27) is of no importance for the following discussion, since we will assume that the tensor-polarization moments are given by their Navier-Stokes values, which are determined by neglecting contributions of order $\mathcal{O}(\mathrm{Re}^{-1}\partial)$ in Eq. (27). This is justified since, in contrast to the components of the energy-momentum tensor or spin tensor, the tensor-polarization moments are not part of the conserved quantities (15). Therefore, it is not necessary to treat them dynamically in second-order hydrodynamics, and it is reasonable to expect that the Navier-Stokes values will constitute the leading-order contribution, while possible second-order terms would lead to small corrections.

B. Truncation

Since we expect the conserved quantities (15) to dominate the evolution of the system on long time scales, it is reasonable to take the irreducible moments appearing there as the dynamical degrees of freedom of our theory. Decomposing the energy-momentum tensor with respect to the fluid velocity u^μ as

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - \Delta^{\mu\nu}(P_0+\Pi) + \pi^{\mu\nu}, \tag{29}$$

where ϵ is the energy density, P_0 is the isotropic pressure, Π is the bulk-viscous pressure, $\pi^{\mu\nu}$ denotes the shear-stress tensor, and imposing the Landau frame condition $T^{\mu\nu}u_\nu = \epsilon u^\mu$ as well as the matching condition $u_\mu u_\nu T^{\mu\nu} = u_\mu u_\nu T_{\mathrm{eq}}^{\mu\nu}$, we identify the dynamical moments $\rho_0 \equiv -(3/m^2)\Pi$ and $\rho_0^{\mu\nu} \equiv \pi^{\mu\nu}$, while $\rho_1 = \rho_2 = 0$, $\rho_1^\mu = 0$ [84]. Therefore, we have $\mathbb{S}_0^{(0)} = \mathbb{S}_2^{(0)} = \{0\}$ and $\mathbb{S}_1^{(0)} = \emptyset$[1], while $\mathbb{S}_\ell^{(n)} = \emptyset$ for $n > 2$. In principle, the transport coefficients in the equations of motion for ρ_0 and $\rho_0^{\mu\nu}$ are modified through the coupling to the tensor-polarization moments, known in the nonrelativistic case as the Senftleben effect [95]. However, it is expected that the modifications of both the conventional transport coefficients and the tensor polarization due to this effect are small [95]. Furthermore, although the components of the spin tensor should also be treated dynamically [64], we will not consider them in this work since they do not couple to the tensor-polarization moments.

The tensor polarization in Eq. (16), when integrated over momentum space, can be expressed in terms of the irreducible moments as

$$\bar{\Theta}^{\mu\nu} := \int dK N(k)\Theta^{\mu\nu}(k) = \frac{1}{2}\sqrt{\frac{3}{2}}\int d\Sigma_\lambda\left(u^\lambda\psi_1^{\mu\nu} + \psi_0^{\mu\nu,\lambda}\right). \tag{30}$$

In order to keep the degrees of freedom which enter the expression for the tensor polarization (30), we choose $\mathbb{S}_0^{(2)} = \{1\}$ and $\mathbb{S}_1^{(2)} = \{0\}$ in the moment expansion.

IV. TENSOR POLARIZATION FROM SHEAR STRESS

Using the truncation procedure outlined in the previous section, the Navier-Stokes limits of Eq. (27) for $r \in \mathbb{S}_\ell^{(2)}$ simply become

$$C_0^{\langle\mu\nu\rangle} = 0, \qquad C_{-1}^{\langle\mu\nu\rangle,\langle\lambda\rangle} = 0. \tag{31}$$

[1]Due to the restriction to an uncharged fluid, we do not need to consider the moment ρ_0^μ related to charge diffusion.

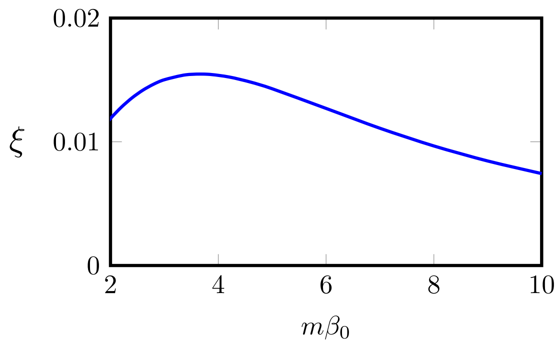

FIG. 1. The coefficient ξ for the case of a four-point interaction, $\mathcal{L}_{\rm int} \sim (V^\dagger \cdot V)^2$.

When expressing these collision terms through the irreducible moments, we note that, since in this work we only consider parity-conserving interactions, all integrals over $\mathcal{W}$ containing an odd number of spin vectors vanish [64]. This implies that there is no coupling between the moments $\tau_r^{\mu,\mu_1\cdots\mu_\ell}$ and $\psi_r^{\mu\nu,\mu_1\cdots\mu_\ell}$. The second equation in (31) immediately implies $\psi_0^{\langle\mu\nu\rangle,\lambda} = 0$ since there are no tensor structures with the appropriate symmetries. On the other hand, the first equation in (31) yields

$$\sum_{n\in\mathbb{S}_0^{(2)}} \mathcal{C}_{1n}^{(0)}\psi_n^{\langle\mu\nu\rangle} + \sum_{n\in\mathbb{S}_2^{(0)}} \mathcal{D}_{1n}^{(2)}\rho_n^{\mu\nu} = 0, \tag{32}$$

where we linearized the collision term (9), plugged it into (28) and used the expansion for the distribution function (22). Furthermore, we introduced the collision integrals

$$\mathcal{C}_{1n}^{(0)} := \frac{1}{5}\int [dK] f_{0\mathbf{k}} f_{0\mathbf{k}'}\Delta_{\mu\nu,\alpha\beta}\big(\mathcal{M}^{\mu\nu,\alpha\beta}_{(k\mathfrak{s})(k_1\mathfrak{s}_1)}\mathcal{H}^{(2,0)}_{k_1n} + \mathcal{M}^{\mu\nu,\alpha\beta}_{(k\mathfrak{s})(k_2\mathfrak{s}_2)}\mathcal{H}^{(2,0)}_{k_2n} - \mathcal{M}^{\mu\nu,\alpha\beta}_{(k\mathfrak{s})(k'\mathfrak{s}')}\mathcal{H}^{(2,0)}_{k'n} - \mathcal{M}^{\mu\nu,\alpha\beta}_{(k\mathfrak{s})(k\bar{\mathfrak{s}})}\mathcal{H}^{(2,0)}_{kn}\big), \tag{33a}$$

$$\mathcal{D}_{1n}^{(2)} := \frac{1}{5}\int [dK] f_{0\mathbf{k}} f_{0\mathbf{k}'}\mathcal{M}^{\mu\nu}_{(k\mathfrak{s})}\big(\mathcal{H}^{(0,2)}_{k_1n}k_{1,\langle\mu}k_{1,\nu\rangle} + \mathcal{H}^{(0,2)}_{k_2n}k_{2,\langle\mu}k_{2,\nu\rangle} - \mathcal{H}^{(0,2)}_{k'n}k'_{\langle\mu}k'_{\nu\rangle} - \mathcal{H}^{(0,2)}_{kn}k_{\langle\mu}k_{\nu\rangle}\big), \tag{33b}$$

with $[dK] := dK_1 dK_2 dK' dK$ and

$$\mathcal{M}^{\mu\nu}_{(k\mathfrak{s})} := \frac{1}{2}(2\pi\hbar)^4\delta^{(4)}(k+k'-k_1-k_2)\int [dS]d\bar{S}(k)\,\mathcal{W}K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta, \tag{34a}$$

$$\mathcal{M}^{\mu\nu,\alpha\beta}_{(k_i\mathfrak{s}_i)(k_j\mathfrak{s}_j)} := \frac{1}{2}(2\pi\hbar)^4\delta^{(4)}(k+k'-k_1-k_2)\Xi_j^{\gamma\delta,\alpha\beta}\int [dS]d\bar{S}(k)\,\mathcal{W}K^{\mu\nu}_{i,\rho\sigma}\mathfrak{s}_i^\rho\mathfrak{s}_i^\sigma\, K^{\zeta\eta}_{j,\gamma\delta}\mathfrak{s}_{j,\zeta}\mathfrak{s}_{j,\eta}. \tag{34b}$$

Here, we defined $[dS] := dS_1(k_1)dS_2(k_2)dS'(k')dS(k)$, and $K^{\mu\nu}_{i,\alpha\beta}$ denotes the symmetric traceless projector onto the subspace orthogonal to $k_i \in \{k_1, k_2, k', k\}$. Similarly, $\Xi^{\alpha\beta}_{j,\gamma\delta}$ is the tensor introduced in Eq. (21) with the momentum k replaced by k_i. A more detailed derivation of Eq. (32) is provided in Appendix B.

Employing the truncation introduced in Sec III B in Eq. (32) and using that $\rho_0^{\mu\nu} = \pi^{\mu\nu}$ yields

$$\psi_1^{\langle\mu\nu\rangle} = \xi\,\beta_0\pi^{\mu\nu}, \tag{35}$$

where

$$\xi := -\frac{1}{\beta_0}\frac{\mathcal{D}_{10}^{(2)}}{\mathcal{C}_{11}^{(0)}} \tag{36}$$

denotes a coefficient that can only depend on the ratio of mass over temperature $m\beta_0$. With details relegated to Appendix C, we plot the value of ξ in Fig. 1 for the case of a simple four-point interaction.

Equation (35) is one of the main results of this work, showing that the Navier-Stokes values of the moments related to the tensor polarization are determined from collisions. Furthermore, the value of the coefficient ξ is determined solely by local collisions, i.e., the nonlocality of the collision term (9) has no influence on the tensor polarization, provided that the interactions conserve parity. Note that, neglecting the moments of first order in spin, the deviation of the single-particle distribution function from local equilibrium reads at this point

$$\delta f_{\mathbf{k}\mathfrak{s}} = f_{0\mathbf{k}}\bigg(-\frac{3}{m^2}\mathcal{H}^{(0,0)}_{\mathbf{k}0}\Pi + \mathcal{H}^{(0,2)}_{\mathbf{k}0}k_{\langle\mu}k_{\nu\rangle}\pi^{\mu\nu} + \xi\,\beta_0\mathcal{H}^{(2,0)}_{\mathbf{k}1}\mathfrak{s}_\alpha\mathfrak{s}_\beta K^{\alpha\beta}_{\mu\nu}\,\Xi^{\mu\nu}_{\rho\sigma}\pi^{\rho\sigma}\bigg). \tag{37}$$

V. SPIN ALIGNMENT IN HEAVY-ION COLLISIONS

We now connect these results to the spin alignment measured in experiments which, in turn, is related to the 00-element of the spin-density matrix $\rho_{\lambda\lambda'}$ [71]. In analogy with Ref. [96], one obtains

$$\rho_{\lambda\lambda'}(k) = \frac{\int d\Sigma_\alpha k^\alpha \epsilon^{(\lambda)\mu}W_{\mu\nu}\epsilon^{*(\lambda')\nu}}{\sum_{\sigma=1}^3\int d\Sigma_\alpha k^\alpha\epsilon^{(\sigma)\mu}W_{\mu\nu}\epsilon^{*(\sigma)\nu}}. \tag{38}$$

The derivation of Eq. (38) is provided in Appendix A. Since we are interested in a diagonal element of the spin-density matrix, with the corresponding polarization vector $\epsilon^{(0)\mu} := (0, 0, 0, 1)$ being real, the antisymmetric part of the Wigner function does not contribute. One can verify with the aid of Eq. (16) that the 00-element of Eq. (38) is given by

$$\rho_{00}(k) = \frac{1}{3} - \sqrt{\frac{2}{3}}\epsilon^{(0)}_\mu\epsilon^{(0)}_\nu\Theta^{\mu\nu}(k). \tag{39}$$

Using Eq. (37), we arrive at the final expression

$$\rho_{00}(k) = \frac{1}{3} - \frac{4}{15}\frac{\int d\Sigma_\alpha k^\alpha \xi\, \beta_0 f_{0\mathbf{k}} \mathcal{H}^{(2,0)}_{\mathbf{k}1} \epsilon^{(0)}_\alpha \epsilon^{(0)}_\beta K^{\alpha\beta}_{\mu\nu}\, \Xi^{\mu\nu}_{\rho\sigma} \pi^{\rho\sigma}}{\int d\Sigma_\alpha k^\alpha f_{0\mathbf{k}} \big(1 - 3\mathcal{H}^{(0,0)}_{\mathbf{k}0}\Pi/m^2 + \mathcal{H}^{(0,2)}_{\mathbf{k}0} \pi^{\mu\nu} k_{\langle\mu} k_{\nu\rangle}\big)}, \tag{40}$$

where we used that there is no tensor polarization in local equilibrium which follows from Eq. (18). The polynomials $\mathcal{H}$ appearing in Eq. (40) read [84]

$$\mathcal{H}^{(0,0)}_{\mathbf{k}0} = \frac{1}{I_{00}}, \quad \mathcal{H}^{(0,2)}_{\mathbf{k}0} = \frac{1}{2I_{42}}, \quad \mathcal{H}^{(2,0)}_{\mathbf{k}1} = \frac{15}{8}\frac{I_{00}E_{\mathbf{k}} - I_{10}}{I_{20}I_{00} - I_{10}^2}. \tag{41}$$

Equation (40) is the main result of our work which shows how vector particles can become tensor polarized due to the presence of shear stress. Since this effect is independent of vorticity, one may choose a quantization axis different from the global angular-momentum direction [69–71], where the strength might be larger.

It is important to note that the expression (40) depends on the details of the interaction between particles only through the coefficient ξ.

VI. CONCLUSIONS

In this work, starting from quantum kinetic theory and using the method of moments, we have shown that shear stress can induce tensor polarization in an uncharged fluid. This novel polarization mechanism is purely related to out-of-equilibrium properties of the system and it is independent of fluid rotation. Thus, one does not need to include nonlocal collisions [37] since such an effect is not determined by the conservation of total angular momentum. Our main result is a formula which can be used for quantitative predictions for vector-meson spin alignment in heavy-ion collisions using hydrodynamic simulations. The present work can be extended by relaxing the assumption of charge neutrality of the fluid. In fact, particle diffusion will also contribute to the tensor polarization. Furthermore, the method of moments discussed here can be used to derive relativistic dissipative spin-1 hydrodynamics with dynamical spin degrees of freedom.

Note added. Recently, we became aware of a related study [97].

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APPENDIX A: RELATIONS BETWEEN THE WIGNER FUNCTION AND POLARIZATION OBSERVABLES

In this Appendix, we prove the relation of the spin-density matrix to the Wigner function, following the same steps as outlined in Ref. [96] for spin-1/2 particles. Furthermore, we prove the relation between tensor polarization and the Wigner function reported in the main text.

The spin-density matrix is defined as

$$\rho_{\lambda\lambda'}(k) := \frac{\langle \hat{a}^\dagger_\lambda(k)\hat{a}_{\lambda'}(k)\rangle}{\sum_\sigma \langle \hat{a}^\dagger_\sigma(k)\hat{a}_\sigma(k)\rangle}. \tag{A1}$$

The goal is to relate the Wigner function

$$W^{\mu\nu}(x,k) := -\frac{2}{(2\pi\hbar)^4\hbar}\int d^4y e^{-ik\cdot y/\hbar} \times \langle : V^{\dagger\mu}(x+y/2)V^\nu(x-y/2):\rangle \tag{A2}$$

to the averages over creation and annihilation operators appearing in Eq. (A1). Expressing the fields in terms of creation and annihilation operators

$$V^\mu(x) := \sqrt{\hbar}\sum_\sigma \int \frac{d^3k}{(2\pi\hbar)^3 2k^0} \times [e^{-\frac{i}{\hbar}k\cdot x}\epsilon^{(\sigma)\mu}(k)\hat{a}_\sigma(k) + e^{\frac{i}{\hbar}k\cdot x}\epsilon^{*(\sigma)\mu}(k)\hat{b}^\dagger_\sigma(k)] \tag{A3}$$

and inserting them into the Wigner function, we obtain $W^{\mu\nu} = W^{\mu\nu}_+ + W^{\mu\nu}_- + W^{\mu\nu}_S$, where $W^{\mu\nu}_\pm$ denote the particle and antiparticle contributions, respectively (i.e., their associated momenta are timelike with $k^0 > 0$ or $k^0 < 0$), while $W^{\mu\nu}_S$ denotes the Wigner function whose momentum is spacelike. These three quantities read explicitly

$$W^{\mu\nu}_+(x,k) = -2\sum_{\sigma,\sigma'}\int \frac{d^3p}{(2\pi\hbar)^3 2p^0}\int \frac{d^3p'}{(2\pi\hbar)^3 2p'^0} \times \delta^{(4)}[k-(p+p')/2]e^{i(p-p')\cdot x/\hbar}\epsilon^{*(\sigma)\mu}(p) \times \epsilon^{(\sigma')\nu}(p')\langle \hat{a}^\dagger_\sigma(p)\hat{a}_{\sigma'}(p')\rangle, \tag{A4a}$$

$$W^{\mu\nu}_-(x,k) = -2\sum_{\sigma,\sigma'}\int \frac{d^3p}{(2\pi\hbar)^3 2p^0}\int \frac{d^3p'}{(2\pi\hbar)^3 2p'^0} \times \delta^{(4)}[k+(p+p')/2]e^{i(p-p')\cdot x/\hbar} \times \epsilon^{(\sigma)\mu}(p)\epsilon^{*(\sigma')\nu}(p')\langle \hat{b}^\dagger_\sigma(p')\hat{b}^\dagger_{\sigma'}(p)\rangle, \tag{A4b}$$

$$W^{\mu\nu}_S(x,k) = -2\sum_{\sigma,\sigma'}\int \frac{d^3p}{(2\pi\hbar)^3 2p^0}\int \frac{d^3p'}{(2\pi\hbar)^3 2p'^0} \times \delta^{(4)}[k-(p-p')/2][e^{i(p+p')\cdot x/\hbar}\epsilon^{*(\sigma)\mu}(p) \times \epsilon^{*(\sigma')\nu}(p')\langle \hat{a}^\dagger_\sigma(p)\hat{b}^\dagger_{\sigma'}(p')\rangle + e^{-i(p+p')\cdot x/\hbar}\epsilon^{(\sigma)\mu}(p') \times \epsilon^{(\sigma')\nu}(p)\langle \hat{a}_\sigma(p')\hat{b}_{\sigma'}(p)\rangle], \tag{A4c}$$

where we employed that $\langle : \hat{a}_\sigma(p)\hat{a}^\dagger_{\sigma'}(p') : \rangle = \langle \hat{a}^\dagger_{\sigma'}(p')\hat{a}_\sigma(p)\rangle$ due to the bosonic nature of the particles. Using now that

$$\int d\Sigma_\alpha k^\alpha W_+^{\mu\nu}(x,k) \equiv k^0 \int d^3x W_+^{\mu\nu}(x,k) = \sum_{\sigma,\sigma'} \delta(k^2-m^2)\Theta(k^0)\epsilon^{*(\sigma)\mu}(p)\epsilon^{(\sigma')\nu}(p')\langle \hat{a}^\dagger_\sigma(p)\hat{a}_{\sigma'}(p')\rangle \tag{A5}$$

as well as the completeness and orthogonality relations of the polarization vectors

$$\epsilon^{*(\lambda)\mu}(k)\epsilon^{(\lambda')}_\mu(k) = -\delta_{\lambda\lambda'}, \quad \sum_\lambda \epsilon^{*(\lambda)\mu}(k)\epsilon^{(\lambda)\nu}(k) = -K^{\mu\nu}, \tag{A6}$$

we find the sought-after relation

$$\langle \hat{a}^\dagger_\lambda(k)\hat{a}_{\lambda'}(k)\rangle = \int d\Sigma_\alpha k^\alpha \epsilon^{(\lambda)}_\mu(k) W_+^{\mu\nu}(x,k)\epsilon^{*(\lambda')}_\nu(k), \tag{A7}$$

which lets us express the spin-density matrix of the particles as

$$\rho_{\lambda\lambda'}(k) = \frac{\int d\Sigma_\alpha k^\alpha \epsilon^{(\lambda)}_\mu(k) W_+^{\mu\nu}(x,k)\epsilon^{*(\lambda')}_\nu(k)}{\sum_\sigma \int d\Sigma_\alpha k^\alpha \epsilon^{(\sigma)}_\mu(k) W_+^{\mu\nu}(x,k)\epsilon^{*(\sigma)}_\nu(k)}. \tag{A8}$$

Note that a similar relation holds also for the antiparticles.

In the next step we will relate the traceless symmetric components of the Wigner function to the tensor polarization, which is defined as [81]

$$\Theta^{\mu\nu}(k) := \frac{1}{2}\sqrt{\frac{3}{2}}\mathrm{Tr}\left[\left(\hat{S}^{(\mu}\hat{S}^{\nu)} + \frac{4}{3}K^{\mu\nu}\right)\hat{\rho}(k)\right], \tag{A9}$$

where $\hat{\rho}(k)$ is the spin-density operator restricted to the four-momentum k^μ, and

$$\hat{S}^\mu := -\frac{1}{2m}\epsilon^{\mu\nu\alpha\beta}\hat{J}_{\nu\alpha}\hat{P}_\beta \tag{A10}$$

denotes the Pauli-Lubanski operator divided by the particle mass [39,96]. Here, $\hat{J}^{\mu\nu}$ is the generator of Lorentz transformations, while $\hat{P}^\mu$ generates space-time translations. From, e.g., Eq. (14) in Ref. [96] we know that we can represent the matrix elements of the operator $\hat{S}^\mu$ as

$$\langle k,\lambda|\,\hat{S}^\mu\,|k,\lambda'\rangle = -\frac{1}{2m}\epsilon^{\mu\nu\alpha\beta}k_\nu D^S([k])^{-1}D^S(J_{\alpha\beta})D^S([k]), \tag{A11}$$

where $D^S(J^{\mu\nu})$ and $D^S([k])$ are the spin-S representation of the total angular-momentum operator and the standard Lorentz boost to the four-momentum k^μ, respectively. From this relation we can infer

$$\Theta^{\mu\nu}(k) = \frac{1}{2}\sqrt{\frac{3}{2}}\left\{\frac{1}{2}\epsilon^{\mu\alpha\beta\gamma}\epsilon^{\nu\rho\sigma\lambda}\frac{k_\alpha k_\lambda}{m^2}\mathrm{Tr}\left[D^S([k])^{-1}D^S(J_{\beta\gamma})D^S(J_{\rho\sigma})D^S([k])\rho(k)\right] + \frac{4}{3}K^{\mu\nu}\right\}. \tag{A12}$$

For massive spin-1 particles, we work in the (1/2,1/2) representation of the Lorentz group, where

$$D^S(J_{\beta\gamma})^{\mu\nu} = i(g^\mu_\beta g^\nu_\gamma - g^\mu_\gamma g^\nu_\beta), \quad (D^S(J_{\beta\gamma})D^S(J_{\rho\sigma}))^{\mu\nu} = g^\mu_\beta g^\nu_\rho g_{\gamma\sigma} + g^\mu_\gamma g^\nu_\sigma g_{\beta\rho} - g^\mu_\beta g^\nu_\sigma g_{\gamma\rho} - g^\mu_\gamma g^\nu_\rho g_{\beta\sigma}. \tag{A13}$$

In a basis where the polarization vectors in the particle rest frame [i.e., the frame where $k^{\star\mu} = (m,0,0,0)$] coincide with the cartesian axes $\epsilon^{(\lambda)\mu}(k^\star) = -g^{\lambda\mu}$, we can express the standard Lorentz transformation as

$$D^S([k])^{\mu\lambda} = \epsilon^{(\lambda)\mu}(k). \tag{A14}$$

Inserting this into Eq. (A12) and using the spin-density matrix (A8) as well as the completeness relation (A6), we find

$$\begin{aligned}\Theta^{\mu\nu}(k) &= \frac{1}{2}\sqrt{\frac{3}{2}}\left[2\epsilon^{\mu\alpha\beta\gamma}\epsilon^{\nu\rho\sigma\lambda}\frac{k_\alpha k_\lambda}{m^2}g_{\gamma\sigma}K_{\beta\eta}K_{\rho\zeta}\frac{\int d\Sigma_\epsilon k^\epsilon W_+^{\eta\zeta}(x,k)}{\int d\Sigma_\epsilon k^\epsilon K_{\phi\psi}W_+^{\phi\psi}(x,k)} + \frac{4}{3}K^{\mu\nu}\right] \\ &= \sqrt{\frac{3}{2}}\left[(K^\mu_\alpha K^\nu_\beta - K^{\mu\nu}K_{\alpha\beta})\frac{\int d\Sigma_\gamma k^\gamma W_+^{\alpha\beta}(x,k)}{\int d\Sigma_\gamma k^\gamma K_{\rho\sigma}W_+^{\rho\sigma}(x,k)} + \frac{2}{3}K^{\mu\nu}\right] \\ &= \sqrt{\frac{3}{2}}K^{\mu\nu}_{\alpha\beta}\frac{\int d\Sigma_\gamma k^\gamma W_+^{\alpha\beta}(x,k)}{\int d\Sigma_\gamma k^\gamma K_{\rho\sigma}W_+^{\rho\sigma}(x,k)}.\end{aligned} \tag{A15}$$

Translating this expression into integrals over spin space and abbreviating

$$\int d\Sigma_\gamma k^\gamma K_{\rho\sigma}W_+^{\rho\sigma}(x,k) = \int d\Sigma_\gamma k^\gamma \int dS(k) f(x,k,\mathfrak{s}) =: N(k), \tag{A16}$$

we have

$$\Theta^{\mu\nu}(k) = \frac{1}{2}\sqrt{\frac{3}{2}}\frac{1}{N(k)}\int d\Sigma_\gamma k^\gamma \int dS(k) K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta f(x,k,\mathfrak{s}). \tag{A17}$$

For completeness, we furthermore list the expression for the vector polarization of spin-1 particles, which is defined as

$$S^\mu(k) := \mathrm{Tr}[\hat{S}^\mu\hat{\rho}(k)]. \tag{A18}$$

Inserting the representation of the total angular momentum operator (A13), we obtain

$$S^\mu(k) = \frac{i}{2}\epsilon^{\mu\nu\alpha\beta}\frac{k_\nu}{m}\frac{\int d\Sigma_\gamma k^\gamma W_{+,\alpha\beta}(x,k)}{\int d\Sigma_\gamma k^\gamma K_{\rho\sigma} W_+^{\rho\sigma}(x,k)}, \tag{A19}$$

which in extended phase space becomes

$$S^\mu(k) = \frac{1}{N(k)}\int d\Sigma_\gamma k^\gamma \int dS(k)\mathfrak{s}^\mu f(x,k,\mathfrak{s}). \tag{A20}$$

APPENDIX B: DERIVATION OF EQ. (32)

Considering the definition of the irreducible moments of the collision integrals (28), Eq. (31) reads explicitly

$$\begin{aligned}
0 = C_0^{\langle\mu\nu\rangle} &= \int d\Gamma\, K_{\alpha\beta}^{\langle\mu\nu\rangle}\mathfrak{s}^\alpha\mathfrak{s}^\beta\mathfrak{C}\\
&= \frac{1}{2}\int [d\Gamma] d\bar{S}(k)(2\pi\hbar)^4\delta^{(4)}(k+k'-k_1-k_2)\\
&\quad\times \mathcal{W} K_{\alpha\beta}^{\langle\mu\nu\rangle}\mathfrak{s}^\alpha\mathfrak{s}^\beta[f(x+\Delta_1-\Delta,k_1,\mathfrak{s}_1)\\
&\quad\times f(x+\Delta_2-\Delta,k_2,\mathfrak{s}_2)\\
&\quad- f(x+\Delta'-\Delta,k',\mathfrak{s}')f(x,k,\bar{\mathfrak{s}})],
\end{aligned} \tag{B1}$$

where we abbreviated $[d\Gamma] := d\Gamma_1 d\Gamma_2 d\Gamma' d\Gamma$. Due to our assumption that the interaction conserves parity, all integrals over $\mathcal{W}$ weighted with an odd number of spin vectors vanish [64], i.e.,

$$\int [dS] d\bar{S}(k)\mathcal{W}\,\mathfrak{s}_i^\mu = 0, \tag{B2a}$$

$$\int [dS] d\bar{S}(k)\mathcal{W}\, K_{i,\alpha\beta}^{\mu\nu}\mathfrak{s}_i^\alpha\mathfrak{s}_i^\beta\mathfrak{s}_j^\lambda = 0. \tag{B2b}$$

From these identities we see that only the components of the distribution functions which are proportional to either zero or two spin vectors contribute to Eq. (B1). The nonlocal shifts Δ_1, Δ_2, Δ', and Δ however are linear in the spin vector $\mathfrak{s}^\mu$ [94], which follows from Eq. (12) by considering the symmetries of M together with the assumption that spin effects are at least of order $\mathcal{O}(\hbar)$. This implies that neither the nonlocal part of the collision term nor the spin-dependent part of the local-equilibrium distribution function (18) give a nonvanishing contribution to Eq. (B1). Linearizing the collision term in the deviations from equilibrium, inserting the moment expansion (22), and using the conservation of linear momentum, Eq. (B1) becomes

$$\begin{aligned}
0 &= \frac{1}{2}\int [d\Gamma] d\bar{S}(k)(2\pi\hbar)^4\delta^{(4)}(k+k'-k_1-k_2)\mathcal{W} K_{\alpha\beta}^{\langle\mu\nu\rangle}\mathfrak{s}^\alpha\mathfrak{s}^\beta f_{0\mathbf{k}}f_{0\mathbf{k}'}\\
&\quad\times\sum_{\ell=0}^\infty\Bigg[\sum_{n\in\mathbb{S}_\ell^{(0)}}\big(\mathcal{H}_{\mathbf{k}_1 n}^{(0,\ell)}k_{\langle 1,\mu_1}\cdots k_{1,\mu_\ell\rangle}+\mathcal{H}_{\mathbf{k}_1 n}^{(0,\ell)}k_{\langle 2,\mu_1}\cdots k_{2,\mu_\ell\rangle}-\mathcal{H}_{\mathbf{k}' n}^{(0,\ell)}k'_{\langle\mu_1}\cdots k'_{\mu_\ell\rangle}-\mathcal{H}_{\mathbf{k}n}^{(0,\ell)}k_{\langle\mu_1}\cdots k_{\mu_\ell\rangle}\big)\rho_n^{\mu_1\cdots\mu_\ell}\\
&\quad+\sum_{n\in\mathbb{S}_\ell^{(2)}}\big(\mathfrak{s}_{1,\gamma}\mathfrak{s}_{1,\delta}K_{1,\zeta\eta}^{\gamma\delta}\,\Xi_{1,\rho\sigma}^{\zeta\eta}\,\mathcal{H}_{\mathbf{k}_1 n}^{(2,\ell)}k_{\langle 1,\mu_1}\cdots k_{1,\mu_\ell\rangle}+\mathfrak{s}_{2,\gamma}\mathfrak{s}_{2,\delta}K_{2,\zeta\eta}^{\gamma\delta}\,\Xi_{2,\rho\sigma}^{\zeta\eta}\,\mathcal{H}_{\mathbf{k}_2 n}^{(2,\ell)}k_{\langle 2,\mu_1}\cdots k_{2,\mu_\ell\rangle}\\
&\quad-\mathfrak{s}'_\gamma\mathfrak{s}'_\delta K'^{\gamma\delta}_{\zeta\eta}\,\Xi'^{\zeta\eta}_{\rho\sigma}\mathcal{H}_{\mathbf{k}'n}^{(2,\ell)}k'_{\langle\mu_1}\cdots k'_{\mu_\ell\rangle}-\bar{\mathfrak{s}}_\gamma\bar{\mathfrak{s}}_\delta K_{\zeta\eta}^{\gamma\delta}\,\Xi_{\rho\sigma}^{\zeta\eta}\,\mathcal{H}_{\mathbf{k}n}^{(2,\ell)}k_{\langle\mu_1}\cdots k_{\mu_\ell\rangle}\big)\psi_n^{\langle\rho\sigma\rangle,\mu_1\cdots\mu_\ell}\Bigg]\\
&\equiv\sum_{\ell=0}^\infty\Bigg[\sum_{n\in\mathbb{S}_\ell^{(0)}}\big(D_{1n}^{(\ell)}\big)^{\mu\nu}_{\mu_1\cdots\mu_\ell}\rho_n^{\mu_1\cdots\mu_\ell}+\sum_{n\in\mathbb{S}_\ell^{(2)}}\big(C_{1n}^{(\ell)}\big)^{\mu\nu}_{\rho\sigma,\mu_1\cdots\mu_\ell}\psi_n^{\langle\rho\sigma\rangle,\langle\mu_1\cdots\mu_\ell\rangle}\Bigg].
\end{aligned} \tag{B3}$$

Here we defined

$$\begin{aligned}
\big(D_{1n}^{(\ell)}\big)^{\mu\nu}_{\mu_1\cdots\mu_\ell} &:= \frac{1}{2}\int [d\Gamma] d\bar{S}(k)(2\pi\hbar)^4\delta^{(4)}(k+k'-k_1-k_2)\mathcal{W} K_{\alpha\beta}^{\langle\mu\nu\rangle}\mathfrak{s}^\alpha\mathfrak{s}^\beta f_{0\mathbf{k}}f_{0\mathbf{k}'}\\
&\quad\times\big(\mathcal{H}_{\mathbf{k}_1 n}^{(0,\ell)}k_{\langle 1,\mu_1}\cdots k_{1,\mu_\ell\rangle}+\mathcal{H}_{\mathbf{k}_1 n}^{(0,\ell)}k_{\langle 2,\mu_1}\cdots k_{2,\mu_\ell\rangle}-\mathcal{H}_{\mathbf{k}'n}^{(0,\ell)}k'_{\langle\mu_1}\cdots k'_{\mu_\ell\rangle}-\mathcal{H}_{\mathbf{k}n}^{(0,\ell)}k_{\langle\mu_1}\cdots k_{\mu_\ell\rangle}\big),
\end{aligned} \tag{B4a}$$

$$\begin{aligned}
\big(C_{1n}^{(\ell)}\big)^{\mu\nu}_{\rho\sigma,\mu_1\cdots\mu_\ell} &:= \frac{1}{2}\int [d\Gamma] d\bar{S}(k)(2\pi\hbar)^4\delta^{(4)}(k+k'-k_1-k_2)\mathcal{W} K_{\alpha\beta}^{\langle\mu\nu\rangle}\mathfrak{s}^\alpha\mathfrak{s}^\beta f_{0\mathbf{k}}f_{0\mathbf{k}'}\\
&\quad\times\big(\mathfrak{s}_{1,\gamma}\mathfrak{s}_{1,\delta}K_{1,\zeta\eta}^{\gamma\delta}\,\Xi_{1,\rho\sigma}^{\zeta\eta}\,\mathcal{H}_{\mathbf{k}_1 n}^{(2,\ell)}k_{\langle 1,\mu_1}\cdots k_{1,\mu_\ell\rangle}+\mathfrak{s}_{2,\gamma}\mathfrak{s}_{2,\delta}K_{2,\zeta\eta}^{\gamma\delta}\,\Xi_{2,\rho\sigma}^{\zeta\eta}\,\mathcal{H}_{\mathbf{k}_2 n}^{(2,\ell)}k_{\langle 2,\mu_1}\cdots k_{2,\mu_\ell\rangle}\\
&\quad-\mathfrak{s}'_\gamma\mathfrak{s}'_\delta K'^{\gamma\delta}_{\zeta\eta}\,\Xi'^{\zeta\eta}_{\rho\sigma}\mathcal{H}_{\mathbf{k}'n}^{(2,\ell)}k'_{\langle\mu_1}\cdots k'_{\mu_\ell\rangle}-\bar{\mathfrak{s}}_\gamma\bar{\mathfrak{s}}_\delta K_{\zeta\eta}^{\gamma\delta}\,\Xi_{\rho\sigma}^{\zeta\eta}\,\mathcal{H}_{\mathbf{k}n}^{(2,\ell)}k_{\langle\mu_1}\cdots k_{\mu_\ell\rangle}\big).
\end{aligned} \tag{B4b}$$

Taking into account that in our truncation $\mathbb{S}_\ell^{(2)} = \emptyset$ for $\ell \geqslant 2$, it follows that the tensors defined above must take the following form,

$$\big(D_{1n}^{(\ell)}\big)^{\mu\nu}_{\mu_1\cdots\mu_\ell} \equiv \mathcal{D}_{1n}^{(2)}\Delta^{\mu\nu}_{\mu_1\mu_2}\delta_{\ell 2}, \qquad \big(C_{1n}^{(\ell)}\big)^{\mu\nu}_{\rho\sigma\mu_1\cdots\mu_\ell} \equiv \mathcal{C}_{1n}^{(0)}\Delta^{\mu\nu}_{\rho\sigma}\delta_{\ell 0}, \tag{B5}$$

where we introduced the scalar coefficients

$$\mathcal{D}_{1n}^{(2)} := \frac{1}{5}\Delta^{\mu_1\mu_2}_{\mu\nu}\left(D_{1n}^{(2)}\right)^{\mu\nu}_{\mu_1\mu_2}, \qquad \mathcal{C}_{1n}^{(0)} := \frac{1}{5}\Delta^{\rho\sigma}_{\mu\nu}\left(C_{1n}^{(0)}\right)^{\mu\nu}_{\rho\sigma}. \tag{B6}$$

The form of the coefficients in Eq. (B5) follows from the fact that the tensors $(D_{1n}^{(\ell)})^{\mu\nu}_{\mu_1\cdots\mu_\ell}$ and $(C_{1n}^{(\ell)})^{\mu\nu}_{\rho\sigma,\mu_1\cdots\mu_\ell}$ have to be orthogonal to u^μ, symmetric and traceless in the indices $(\mu\nu)$, $(\mu_1\cdots\mu_\ell)$, and (in the latter case) $(\rho\sigma)$. The only tensor structures made from $g^{\mu\nu}$ and u^μ that fulfill these requirements are given by the irreducible projectors of second rank as shown in Eq. (B5). Inserting Eqs. (B5) and (B6) into Eq. (B3), we arrive at Eq. (32) in the main text.

APPENDIX C: CALCULATIONS FOR A FOUR-POINT INTERACTION

Considering a simple four-point interaction characterized by a dimensionless coupling strength G,

$$\mathcal{L}_{\text{int}} := \hbar G(V^\dagger\cdot V)^2, \tag{C1}$$

we compute the transfer-matrix elements at leading order [85,94]

$$\begin{aligned}\langle k,k';\lambda,\lambda'|\,\hat{t}\,|k_1,k_2;\lambda_1,\lambda_2\rangle &= \frac{1}{\hbar}\langle k,k';\lambda,\lambda'|:\mathcal{L}_{\text{int}}(0):|k_1,k_2;\lambda_1,\lambda_2\rangle\\ &= 2\hbar^2G\big\{\big[\epsilon^{*(\lambda')}_\alpha(k')\epsilon^{(\lambda_1)\alpha}(k_1)\big]\big[\epsilon^{*(\lambda)}_\beta(k)\epsilon^{(\lambda_2)\beta}(k_2)\big]+\big[\epsilon^{*(\lambda')}_\alpha(k')\epsilon^{(\lambda_2)\alpha}(k_2)\big]\big[\epsilon^{*(\lambda)}_\beta(k)\epsilon^{(\lambda_1)\beta}(k_1)\big]\big\},\end{aligned} \tag{C2}$$

where we used the free-field representation of the vector fields

$$V^\mu(0) = \sqrt{\hbar}\sum_{\sigma'}\int\frac{d^3\mathbf{k}'}{(2\pi\hbar)^3 2k'^0}\hat{a}(k',\sigma')\epsilon^{(\sigma')\mu}(k'). \tag{C3}$$

Recalling the relationship (13) between the vertices M and the transfer-matrix elements, we find

$$M^{\mu\nu\alpha\beta} = 2\hbar^2G(g^{\mu\alpha}g^{\nu\beta}+g^{\mu\beta}g^{\nu\alpha}). \tag{C4}$$

Using the identities

$$\int dS(k)h^{\mu\nu}(k,\mathfrak{s}) = K^{\mu\nu}, \tag{C5a}$$

$$\int dS(k)H^{\mu\nu}(k,\mathfrak{s}) = K^{\mu\nu}, \tag{C5b}$$

$$\int dS(k)K^{\rho\sigma}_{\alpha\beta}\mathfrak{s}_\rho\mathfrak{s}_\sigma h^{\mu\nu}(k,\mathfrak{s}) = \frac{8}{5}K^{\mu\nu}_{\alpha\beta}, \tag{C5c}$$

$$\int dS(k)K^{\rho\sigma}_{\alpha\beta}\mathfrak{s}_\rho\mathfrak{s}_\sigma H^{\mu\nu}(k,\mathfrak{s}) = K^{\mu\nu}_{\alpha\beta}, \tag{C5d}$$

we are able to perform the integrals over spin space in Eq. (34), obtaining

$$\int[dS]d\bar{S}(k)\mathcal{W}K^{\mu\nu}_{\alpha\beta}\mathfrak{s}^\alpha\mathfrak{s}^\beta = \frac{(2\pi\hbar)^3}{16}M^{\gamma_1\gamma_2\delta_1\delta_2}M^{\zeta_1\zeta_2\eta_1\eta_2}K_{1,\gamma_1\eta_1}K_{2,\gamma_2\eta_2}K'_{\zeta_2\delta_2}K^{\mu\nu}_{\zeta_1\delta_1}, \tag{C6a}$$

$$\int[dS]d\bar{S}(k)\mathcal{W}K^{\mu\nu}_{\rho\sigma}\mathfrak{s}^\rho\mathfrak{s}^\sigma K^{\gamma\delta}_{1,\zeta\eta}\mathfrak{s}_1^\zeta\mathfrak{s}_1^\eta = \frac{8}{5}\frac{(2\pi\hbar)^3}{16}M^{\gamma_1\gamma_2\delta_1\delta_2}M^{\zeta_1\zeta_2\eta_1\eta_2}K^{\gamma\delta}_{1,\gamma_1\eta_1}K_{2,\gamma_2\eta_2}K'_{\zeta_2\delta_2}K^{\mu\nu}_{\zeta_1\delta_1}, \tag{C6b}$$

$$\int[dS]d\bar{S}(k)\mathcal{W}K^{\mu\nu}_{\rho\sigma}\mathfrak{s}^\rho\mathfrak{s}^\sigma K^{\gamma\delta}_{2,\zeta\eta}\mathfrak{s}_2^\zeta\mathfrak{s}_2^\eta = \frac{8}{5}\frac{(2\pi\hbar)^3}{16}M^{\gamma_1\gamma_2\delta_1\delta_2}M^{\zeta_1\zeta_2\eta_1\eta_2}K_{1,\gamma_1\eta_1}K^{\gamma\delta}_{2,\gamma_2\eta_2}K'_{\zeta_2\delta_2}K^{\mu\nu}_{\zeta_1\delta_1}, \tag{C6c}$$

$$\int[dS]d\bar{S}(k)\mathcal{W}K^{\mu\nu}_{\rho\sigma}\mathfrak{s}^\rho\mathfrak{s}^\sigma K'^{\gamma\delta}_{\zeta\eta}\mathfrak{s}'^\zeta\mathfrak{s}'^\eta = \frac{8}{5}\frac{(2\pi\hbar)^3}{16}M^{\gamma_1\gamma_2\delta_1\delta_2}M^{\zeta_1\zeta_2\eta_1\eta_2}K_{1,\gamma_1\eta_1}K_{2,\gamma_2\eta_2}K'^{\gamma\delta}_{\zeta_2\delta_2}K^{\mu\nu}_{\zeta_1\delta_1}, \tag{C6d}$$

$$\int[dS]d\bar{S}(k)\mathcal{W}K^{\mu\nu}_{\rho\sigma}\mathfrak{s}^\rho\mathfrak{s}^\sigma K^{\gamma\delta}_{\zeta\eta}\bar{\mathfrak{s}}^\zeta\bar{\mathfrak{s}}^\eta = \frac{8}{5}\frac{(2\pi\hbar)^3}{16}M^{\gamma_1\gamma_2\delta_1\delta_2}M^{\zeta_1\zeta_2\eta_1\eta_2}K_{1,\gamma_1\eta_1}K_{2,\gamma_2\eta_2}K'_{\zeta_2\delta_2}K^{\mu\nu}_{\zeta_1\rho}g^{\rho\sigma}K^{\gamma\delta}_{\sigma\delta_1}. \tag{C6e}$$

Inserting the vertices given in Eq. (C4) into Eq. (C6), we perform the remaining momentum integrals (33) via slightly modifying a method outlined in Chapter XIII of Ref. [85], which we now briefly outline.

The basic idea consists in separating the integrals in Eq. (33) into a sum of elementary collision integrals

$$J^{(a,b,d,e,f)} := \int[dK]e^{-\beta P_T\cdot u}(P_T^2)^a(P_T\cdot u)^b(Q\cdot u)^d(Q'\cdot u)^e(-Q\cdot Q')^f\delta^{(4)}(k+k'-k_1-k_2), \tag{C7}$$

where the momenta k, k', k_1, and k_2 can be expressed in terms of the total momentum P_T and the relative momenta Q, Q' via

$$k^\mu = \frac{1}{2}\big(P_T^\mu + Q^\mu\big), \tag{C8a}$$

$$k'^\mu = \frac{1}{2}\big(P_T^\mu - Q^\mu\big), \tag{C8b}$$

$$k_1^\mu = \frac{1}{2}\big(P_T^\mu + Q'^\mu\big), \tag{C8c}$$

$$k_2^\mu = \frac{1}{2}\big(P_T^\mu - Q'^\mu\big). \tag{C8d}$$

Next we follow the steps in Ref. [85] and make use of the integral

$$\int_z^\infty dy\,\big(y^2 - z^2\big)^{b-1/2} y^a e^{-y} = z^{a+2b} \sum_{j=0}^{b} (-1)^j \binom{b}{j} \mathrm{Ki}_{2j-2b-a}(z), \tag{C9}$$

where $\mathrm{Ki}_r(z)$ denotes the Bickley-Naylor function of order r [98]. The result for the basic integral (C7) then reads

$$J^{(a,b,d,e,f)} = \beta^{-4-2a-b-d-e-2f} \frac{16\pi^3}{(2\pi\hbar)^{12}} \sum_{g=0}^{\min(d,e)} K(d,e,g)\sigma^{(f,g)} \sum_{h=0}^{\frac{d+e}{2}+1} \binom{\frac{d+e}{2}+1}{h} (-1)^h$$
$$\times \int_{2z}^\infty dv[v^2 - (2z)^2]^{(d+e)/2+f+1} v^{2(a-1)+b+3} \mathrm{Ki}_{-b-d-e-2+2h}(v), \tag{C10}$$

where we introduced the following factors:

$$K(d,e,g) := \begin{cases} \frac{d!e!}{(d-g)!!(d+g+1)!!(e-g)!!(e+g+1)!!} & \text{if } (d-g),\ (e-g) \text{ even,} \\ 0, & \text{otherwise} \end{cases} \tag{C11a}$$

$$\sigma^{(f,g)} := \begin{cases} (2g+1)\frac{f!\,2^g}{(f+g+1)!}\frac{\left(\frac{f+g}{2}\right)!}{\left(\frac{f-g}{2}\right)!} & \text{if } (f-g) \text{ even,} \\ 0, & \text{otherwise.} \end{cases} \tag{C11b}$$

The remaining task then consists in expanding the integrals (33) as sums of the basic integrals (C10). Note that the tensors $\Xi^{\mu\nu}$, $\Xi^{\mu\nu}_{\alpha\beta}$ do not allow for a straightforward expression in terms of polynomials of P_T, Q, and Q'. This is the case because of the factors of energy appearing in the denominator, leading to

$$\Xi^{\mu\nu} = \Delta^{\mu\nu} + \frac{\big(P_T^{\langle\mu\rangle} + Q^{\langle\mu\rangle}\big)\big(P_T^{\langle\nu\rangle} + Q^{\langle\nu\rangle}\big)}{(P_T\cdot u + Q\cdot u)^2}, \tag{C12}$$

and similarly for $\Xi^{\mu\nu}_{\alpha\beta}$. In order to bring these terms into the form required by Eq. (C7) as well, we expand them around the nonrelativistic limit (formally equivalent to taking the limit $k^\mu \simeq (m, \mathbf{0})^\mu$), leading to

$$\Xi^{\mu\nu} \simeq \Delta^{\mu\nu}, \qquad \Xi^{\mu\nu}_{\alpha\beta} \simeq \Delta^{\mu\nu}_{\alpha\beta}. \tag{C13}$$

The plot 1 is generated with this leading-order approximation, which our tests suggest is reasonable for the covered values of z, with accuracy increasing towards larger values of z.

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