Letter

Drag-induced dynamical formation of dark solitons in Bose mixture on a ring

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Andreev-Bashkin drag plays a very important role in multiple areas such as superfluid mixtures, superconductors, and dense nuclear matter. Here we point out that the drag phenomenon can be also important in the physics of solitons, ubiquitous objects arising in a wide array of fields ranging from tsunami waves and fiber-optic communication to biological systems. So far, fruitful studies have been conducted in ultracold atomic systems where nontrivial soliton dynamics occurred due to intercomponent density-density interaction. In this work we show that current-current coupling between components (Andreev-Bashkin drag) can lead to a substantially different kind of effect, unsupported by density-density interactions, such as a drag-induced dark soliton generation. This also points out that soliton dynamics can be used as a tool to experimentally study the dissipationless drag effect.

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Solitons are ubiquitous objects appearing in various physical systems, including nonlinear optics, fluid dynamics [1–9], and ultracold atomic systems [10,11]. Ultracold bosons form Bose-Einstein condensate effectively described by the Gross-Pitaevskii equation (GPE) [10–12]. The nonlinearity present in the GPE can balance dispersive effects, supporting nonuniform solutions (solitons) preserving shape in time. This, together with great progress in cold-atom experimental techniques, makes ultracold bosonic systems an excellent platform for studies on matter-wave solitons [13–27]. Solitons also occur in fermionic ultracold atomic systems [28–32].

A conventional superfluid is described by a complex field $\psi = \sqrt{n}e^{i\varphi}$. The phase gradient can be identified with the superfluid velocity $\mathbf{v} = \frac{\hbar}{m}\nabla\varphi$, where m is the particle mass [10–12,33]. Andreev and Bashkin demonstrated that in a two-component interacting superfluid mixture the relation between superfluid velocities and superflows becomes very nontrivial due to existence of a dissipationless drag transport effect [34]. Indeed, the corresponding free-energy density takes the form $f = \sum_\alpha \rho_\alpha \mathbf{v}_\alpha^2/2 + \rho_d \mathbf{v}_a \cdot \mathbf{v}_b$, where ρ_α ($\mathbf{v}_\alpha$) represents a superfluid density (superfluid velocity) of component $\alpha \in \{a, b\}$ and ρ_d is the Andreev-Bashkin (AB) drag coefficient [34]. Consequently, the superflows, i.e., $\mathbf{j}_\alpha = \partial_{\mathbf{v}_\alpha} f = \rho_\alpha \mathbf{v}_\alpha + \rho_d \mathbf{v}_{\beta\neq\alpha}$, reveal that the component possessing no superfluid velocity, e.g., $\mathbf{v}_a = \mathbf{0}$, will still exhibit a nonzero superflow, $\mathbf{j}_a \neq \mathbf{0}$, as long as $\mathbf{v}_b \neq \mathbf{0}$.

The AB effect strongly affects vortex lattices in superfluids [35,36] and can change the nature of topological solitons in superconductors [37]. It is also crucial for the understanding of properties of dense nuclear matter [38,39] and observed pulsar dynamics [40–42]. At the microscopic level the drag effect originates from intercomponent particle-particle interaction [34,43–47]. Especially interesting is the case of strongly correlated superfluids whose parameters are precisely controllable in optical lattices [48,49]. There the AB drag originates from the interplay between intercomponent particle-particle interaction and lattice effects and can be, in relative terms, arbitrarily strong and ρ_d can be also negative [43–47,50–59]. Interestingly, AB drag signatures have been found in quantum droplets collisions [60]. The drag effect can have various forms. Recently, it was demonstrated that in certain asymmetrical lattices there exists also a perpendicular entrainment referred to as vector drag [61].

In binary systems very interesting solitonic effects are driven by intercomponent density-density interaction [62–80]. In this paper we study the consequences of the AB effect (current-current interaction) on the solitonic dynamics. We consider a one-dimensional (1D) binary bosonic superfluid mixture modeled by the energy functional $\mathcal{E} = N\int(\varepsilon_0 + \varepsilon_d)dx$, with $\varepsilon_0 = \sum_\alpha(-\hbar^2\psi_\alpha^*\partial_x^2\psi_\alpha/2m + g_\alpha N|\psi_\alpha|^4/2)$ and $\varepsilon_d = g_d N\sum_\alpha J_\alpha^2/2 + g_d N J_a J_b = g_d N(J_a + J_b)^2/2$. Here ψ_α is the condensate field of component $\alpha \in \{a, b\}$ normalized to unity $|\langle\psi_\alpha|\psi_\alpha\rangle|^2 = 1$. The particles, whose numbers are equal and conserved in both components, $N_\alpha = N$, possess equal masses $m_\alpha = m$ and are confined in a ring of circumference L, i.e., we assume periodic boundary conditions (PBC) $\psi_\alpha(x + L, t) = \psi_\alpha(x, t)$. The condensates are subjected to an intracomponent contact interaction of strength governed by $g_{\alpha 0}$ and the AB intercomponent drag incorporated by scalar product of $J_\alpha = \hbar\psi_\alpha^*\partial_x\psi_\alpha/2mi + \text{c.c.}$ with strength given by $g_d > 0$. The contributions proportional to J_α^2 in ε_d are required for $\mathcal{E}$ to be bounded from below. Such a phenomenological effective model of the AB drag has been studied previously in other contexts [37,81].

Our goal is to investigate the effects of current-current interaction. Hence, in this work we specifically set the well-studied intercomponent density-density interaction to zero. However, the effect of the latter is discussed in

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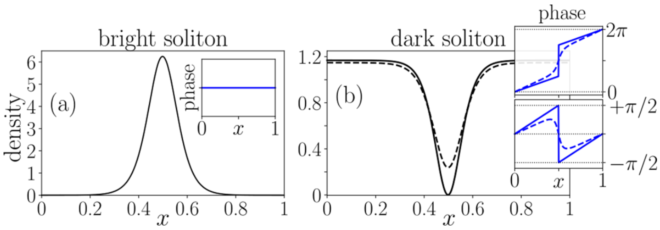

FIG. 1. Illustration of well-localized solitons confined in a ring of circumference $L = 1$: (a) lowest-energy bright soliton density and (b) densities of two types of dark solitons, black (solid line) and gray (dashed line). The corresponding phase distributions are depicted in the respective insets. While the stationary bright soliton in (a) has a uniform phase, a dark soliton notch is always accompanied by a phase slip that can be either facing down or facing up. The upper and lower insets of (b) show phase distributions characterized by $W = 1$ and 0, respectively.

the Supplemental Material [82]. The corresponding system of dimensionless time-dependent Gross-Pitaevskii-like equations reads ($\alpha \in \{a, b\}$ and $\gamma \neq \alpha$)

$$i\partial_t \psi_\alpha = -\frac{\partial_x^2 \psi_\alpha}{2} + g_\alpha |\psi_\alpha|^2 \psi_\alpha + g_d \mathcal{J}_{\alpha\alpha} + g_d \mathcal{J}_{\alpha\gamma}, \qquad (1)$$

where the length scales are measured in units of the ring circumference L, time in units of $\frac{mL^2}{\hbar}$, and energy in units of $\frac{\hbar^2}{mL^2}$. Here we also rescaled $g_\alpha \to \frac{mL}{\hbar^2} g_\alpha N$, $g_d \to \frac{N}{mL} g_d$, and defined $\mathcal{J}_{\alpha\beta} = [2(\partial_x \psi_\alpha) J_\beta + \psi_\alpha(\partial_x J_\beta)]/2i$ with the redefined dimensionless $J_\alpha = \psi_\alpha^* \partial_x \psi_\alpha / 2i + \text{c.c.}$. In the absence of drag, i.e., for $g_d = 0$, Eq. (1) becomes independent and supports both bright and dark soliton solutions that for PBC can be expressed analytically in terms of Jacobi functions [83–87]. A stationary bright soliton in ring geometry (PBC) forms spontaneously in the ground state when $g_\alpha < g_c = -\pi^2$. On the other hand, dark solitons are collective excitations characterized by density notches accompanied by phase slips in phase distribution φ and appear for any $g_\alpha > 0$ [87,88]. For finite rings, i.e., $L < \infty$, a single dark soliton always propagates with some finite velocity because the phase cyclicity condition $\varphi(L) - \varphi(0) = 2\pi W$, where the winding number $W \in \mathbb{Z}$, requires a nonzero phase gradient to be satisfied in the presence of a solitonic phase slip. In the limiting case of a totally dark, i.e., black, soliton the corresponding density vanishes in the dip where the phase reveals a single-point discontinuity by π. Therefore, to satisfy PBC the phase φ has to accumulate at least as $\pm i\pi x/L$. Solitons with a shallower density notch accompanied by a smooth $\varphi(x)$ are often called gray solitons. Note that two gray solitons revealing identical densities may possess phase distributions characterized by different W and in consequence different average momenta $\langle p \rangle = -i\hbar \int dx\, \psi^* \partial_x \psi$. In Fig. 1 we show typical density and phase distributions of the lowest-energy bright soliton and two types of dark solitons: black and gray.

From the many-body perspective, dark solitons are directly connected with a specific class of the so-called yrast states [89–106], i.e., lowest-energy states for a given total momentum. Similar many-body excitations correspond to dark solitons also in the presence of open boundary conditions [107] (for an overview see [87]). Here we study whether current-current drag interactions can lead to yrast excitations, inducing the formation of dark solitons.

Let us assume that in our system one of the components, say, the b component, exhibits $J_b \neq 0$ while $J_a = 0$. If the spatial translation symmetry is broken and $\mathcal{J}_{\alpha\gamma} \neq 0$, then a dynamic drag-related current generation and a momentum transfer between the components can be expected. To study this problem, we consider the case in which component a is initially prepared in the uniform ground state ψ_{a0} for repulsive interaction $g_a > 0$. At the same time the b component is prepared in the ground state ψ_{b0}, but for attractive interactions characterized by $g_b < g_c$ that is associated with a stationary bright soliton. In such a case $\langle p_a \rangle = \langle p_b \rangle = 0$, $J_a = J_b = 0$, and the drag interactions have no impact on these states. To have $J_b, \mathcal{J}_{\alpha\gamma} \neq 0$ we additionally set the bright soliton in motion such that initially $\langle p_b \rangle, J_b \neq 0$.

Basing on the relationship between yrast states and dark solitons in a single-component repulsive Bose gas with PBC, one can ask if the drag-related momentum transfer from component b to a can induce a dark soliton formation in the latter component. We argue that preparing component b in a well-localized bright soliton state may reduce excitations of kinds other than the collective solitonic ones. That is, the bright soliton would slow down its propagation when transferring the momentum from b to a, while preserving an approximately unchanged shape due to strong intracomponent attraction. In such a case, there is a chance that most of the energy gained by component a would correspond to the collective motion characterized by the transferred momentum. Thus, excluding the drag interaction energy, the resulting excited state in component a would have energy close to the one possessed by the yrast state with $\langle p_a \rangle$. If so, then one may expect an emergence of dark soliton signatures (density notch and phase slip) in the a component.

Given that the above-mentioned scenario takes place, the induced dark soliton is expected to be different depending on the amount of momentum injected into component a: The latter is likely to change over time. One may ask whether or not it is possible for a specific dark soliton to form in component a that would coexist with the bright soliton in the other component for timescales longer than the period of a single revolution of the anticipated dark soliton along the ring. We suppose that this can happen when both the target dark soliton and the bright soliton propagate with comparable velocities.

The well-localized (narrow in comparison to L) bright soliton can be approximately described by the famous sech-shaped soliton wave function [11] which reveals its particlelike behavior. Note that $\langle p \rangle = \hbar \int dx |\psi|^2 \partial_x \varphi$ and $\varphi(x) = \varphi(0) + mvx/\hbar + \mathcal{S}(x)$, where $\mathcal{S}(x)$ encodes other phase features like phase slips. For well-localized bright solitons $\partial_x \mathcal{S} \approx 0$ in the vicinity of the soliton clump and thus such states propagate with the velocity $v \approx \langle p \rangle / m$. Generally, $\int dx |\psi|^2 \partial_x \mathcal{S}$ is non-negligible for dark solitons, making the relationship between v and $\langle p \rangle$ more complicated. The special case is a black soliton, for which $\partial_x \mathcal{S} \neq 0$ only at the soliton dip where $|\psi_{\rm bs}|^2 = 0$. Thus, for the black soliton $v_{\rm bs} = \langle p_{\rm bs} \rangle / m$, where $\langle p_{\rm bs} \rangle / \hbar = \pi/L + 2\pi n/L$, with $n \in \mathbb{Z}$.

Let us operate with the dimensionless units and restrict our considerations to states ψ_α possessing $0 \leqslant \langle p_\alpha \rangle \leqslant 2\pi$

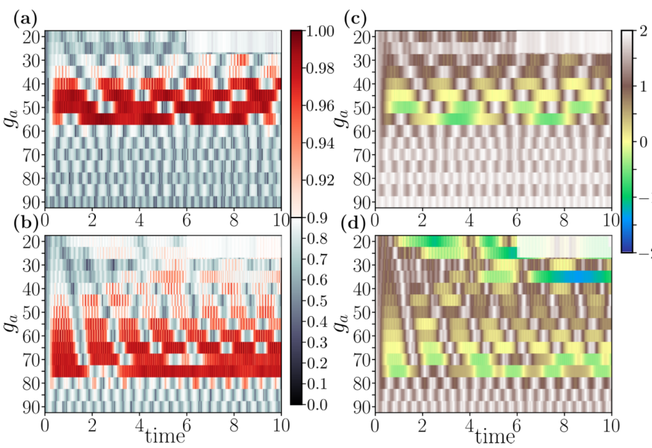

FIG. 2. Time evolution of (a) and (b) the overlap $|\langle\psi_a|\psi_{\rm bs}\rangle|^2$ and (c) and (d) the momentum difference $(\langle p_b\rangle - \langle p_a\rangle)/\pi$ obtained for $g_d = 0.1$, with (a) and (c) $g_b = -20$ and (b) and (d) $g_b = -30$ characterizing the bright soliton state. The regions where $|\langle\psi_a|\psi_{\rm bs}\rangle|^2 > 0.9$ coincide with small values of $|\langle p_b\rangle - \langle p_a\rangle|$ indicating that a (nearly) black soliton forms when approximately half of $\langle p_b\rangle|_{t=0} = 2\pi$ is transferred to component a.

measured in $\hbar/L$ units. We are going to analyze the possibility of a drag-induced formation of the most distinct of dark solitons, namely, the black soliton. We suppose that a long-living coexistence of black and bright solitons may be possible when both objects propagate with comparable velocities. Therefore, at $t = 0$, we set the initial ground-state bright soliton (b component) in motion with $\langle p_b\rangle = 2\langle p_{\rm bs}\rangle = 2\pi$. This is done by multiplying $\psi_{b0}(x)$ by $e^{i2\pi x}$, i.e., $\psi_b(x, t = 0) = \psi_{b0}(x)e^{i2\pi x}$. Since $\langle p_b\rangle + \langle p_a\rangle = 2\pi$ is a conserved quantity in our system, we expect that if the momentum is transferred from component b to a, the above-mentioned coexistence my appear when $\langle p_b\rangle - \langle p_a\rangle \approx 0$. In such a case $\langle p_b\rangle \approx \langle p_a\rangle \approx \pi$ and the corresponding solitons should propagate with comparable velocities.

We prepare the initial bright soliton state $\psi_{b0}(x)$ by means of an imaginary time evolution of (1) with $\alpha = b$, $g_d = 0$, and four different $g_b = -20, -25, -30, -35$ separately. These values of g_b are all substantially below the critical value $g_c = -\pi^2$, which guarantees that the resulting bright soliton density is well localized. This state is then set in motion with $\langle p_b\rangle|_{t=0} = 2\pi$ by incorporating a phase factor as previously described. Component a is prepared in a similar way but with $g_a \in \{20, 25, \dots, 90\}$ resulting in the lowest-energy state $\psi_{a0} = \psi_a(x, t = 0) = 1$ (up to a global phase). After the state preparation we switch on the AB drag by setting $g_d = 0.1$ while keeping g_a and g_b fixed. We then numerically evolve Eq. (1) in real time up to $t = 10$, a time more than 30 times longer than the characteristic period of the black soliton revolution around the ring $T = 1/\pi \approx 0.32$.

Our results indicate that the bright soliton in the b component survives the evolution for all the parameters considered. For each of $g_b = -20, -25, -30, -35$ we find a region in the g_a parameter where clear dark soliton signatures (density notch and phase slip) emerge in $\psi_a(x, t)$ (see Ref. [82] for snapshots of typical system dynamics). Figure 2 shows the temporal behavior of the overlap $|\langle\psi_a|\psi_{\rm bs}\rangle|^2$ and the momentum difference $(\langle p_b\rangle - \langle p_a\rangle)/\pi$ for different g_a and $g_b = -20, -30$. The overlaps $|\langle\psi_a|\psi_{\rm bs}\rangle|^2$ are calculated with the analytical black soliton solution $\psi_{\rm bs}$ characterized by the corresponding g_a and located at a position of the phase slip recognized in $\psi_a(x, t)$. By choosing a specific color code in the overlap plots we discriminate the regions where $|\langle\psi_a|\psi_{\rm bs}\rangle|^2 > 0.9$ (red intensity) from those where $|\langle\psi_a|\psi_{\rm bs}\rangle|^2 < 0.9$ (gray intensity). Note that overlaps above 0.9 appear when the momenta $\langle p_b\rangle$ and $\langle p_a\rangle$ are similar and are maintained for timescales significantly longer than T. We observe that the critical g_a above which a dark soliton appears depends on the value of g_b. That is, for stronger attraction, i.e., a narrower bright soliton in the b component, the regime of the (nearly) black soliton formation shifts to larger g_a corresponding to the narrower dark solitons. In Ref. [83] we also analyze how drag-induced states ψ_a would evolve if drag is quenched to zero (drag-free dynamics) at a time when $|\langle\psi_a|\psi_{\rm bs}\rangle|^2 \approx 1$. It turns out that such generated states reveal a genuine dark soliton drag-free evolution.

To better understand the system dynamics, in Fig. 3 we study more closely cases with $g_b = -30$ and $g_a = 65, 70, 75$. As before, we analyze the time dependence of the overlap $|\langle\psi_a|\psi_{\rm bs}\rangle|^2$ and momentum $\langle p_a\rangle/\pi$. Additionally, we monitor the minimum Euclidean distance Δ along the ring between the bright soliton and the drag-induced dark soliton, the minimum reached by an anticipated density notch $\min[|\psi_a(t)|^2]$, and the ratio of the bright soliton height to its initial value $\max[|\psi_b(t)|^2]/\max[|\psi_b(0)|^2]$. In all the cases an initial momentum transfer leads to the formation of a (nearly) black soliton. Indeed, the overlap $|\langle\psi_a|\psi_{\rm bs}\rangle|^2$ increases together with $\langle p_a\rangle$, and the density notch is simultaneously being carved as indicated by the decreasing value of $\min[|\psi_a(t)|^2]$. At the same time the distance Δ reveals an increasing separation between solitons in the two components reaching a maximum $\Delta \approx 0.5$ at a time in the middle of the plateau of $|\langle\psi_a|\psi_{\rm bs}\rangle|^2 \approx 1$. The seemingly linear trend in Δ for $\Delta \gtrsim 0.1$ reveals a constant relative motion between the spatially separated solitons $|v_b - v_a| \approx 1$ three times slower than the single-component black soliton velocity $v_{\rm bs} = \pi$. This behavior of Δ repeats multiple times during the evolution.

Due to different velocities and assumed ring system geometry, the solitons collide multiple times during the course of evolution. It turns out that the induced (nearly) black soliton state often is substantially disturbed or even completely destroyed when both solitons meet, i.e., when $\Delta \to 0$, which results in an abrupt drop of the overlap value $|\langle\psi_a|\psi_{\rm bs}\rangle|^2$. The dark soliton relocalizes again when Δ increases. Such a mechanism is the origin of quasiperiodic patterns visible in Figs. 2 and 3. However, as indicated by the behavior of $\max[|\psi_b(t)|^2]/\max[|\psi_b(0)|^2]$, the bright soliton remains almost unaffected when passing through the dark one. On the other hand, as shown in Fig. 3(b) for $t > 7$ and Fig. 3(c) for $t > 3$, the drag-induced dark soliton can also survive an encounter with the bright soliton. Additionally, in Fig. 3(c) for $t \in (6.3, 7)$ and $t \in (8, 9)$, one can observe signatures of the existence of long-living dark-bright soliton composites characterized by $\Delta \approx 0$. (For more intuition, see snapshots of the system evolution in Ref. [82].)

In summary, we have studied the dynamics of a bosonic binary mixture confined in a 1D ring geometry with

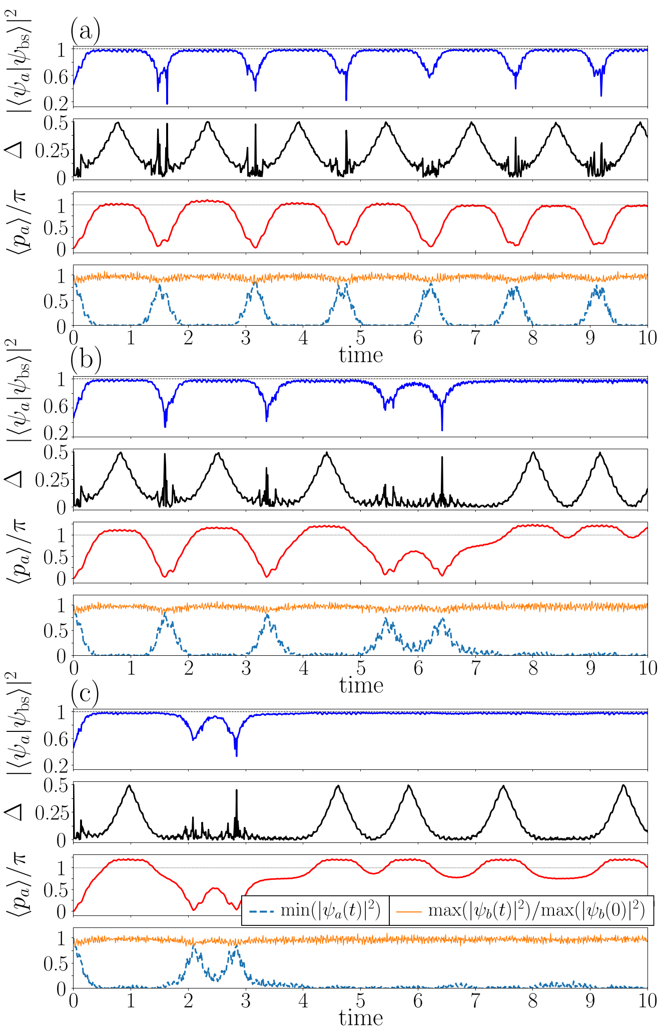

FIG. 3. Each set of plots shows, from top to bottom, the dynamics of the overlap $|\langle\psi_a|\psi_{\rm bs}\rangle|^2$, the relative distance Δ along the ring between the bright soliton (b component) and the phase slip position in $\psi_a(x,t)$, the average momentum $\langle p_a\rangle/\pi$, and the values $\min[|\psi_a(t)|^2]$ and $\max[|\psi_b(t)|^2]/\max[|\psi_b(0)|^2]$, for $g_b = -30$ and (a) $g_a = 65$, (b) $g_a = 70$, and (c) $g_a = 75$. The drag-induced dark (nearly black) soliton often is significantly disturbed, or even completely destroyed, when passing through the bright soliton, i.e., when $\Delta \to 0$. In such a case the phase slip in $\psi_a(x,t)$ is rather tiny or even unrelated to any soliton structure. This is the origin of the narrow spikes observed in the Δ plots when $\Delta \to 0$ and $\min[|\psi_a(t)|^2] \approx 1$. Nevertheless, as shown in (b) for $t > 7$ and in (c) for $t > 3$, the (nearly) black soliton can survive the encounter with the bright soliton.

intracomponent contact interactions and intercomponent Andreev-Bashkin drag. Based on the relationship between dark solitons and yrast states characterized by the lowest energy for a given momentum, we formulated and verified the hypothesis concerning a drag-induced dark soliton formation process. By numerically computing the system dynamics we tested the scenario where a propagating bright soliton interacts with the other component, prepared in the repulsively interacting uniform ground state. We demonstrated that there exist parameter regimes for which the drag interaction leads to the formation of a long-living genuine, nearly black, soliton state in the initially uniform component. While we focused on the most distinct black soliton case, the general idea provided here should also allow for generation of gray solitons. Our goal here was to study the effects of current-current interaction on soliton dynamics. An interesting question that warrants further studies is how these effects combine with intercomponent density-density interactions. This question is beyond the scope of this paper, but in [82] we show that the drag phenomenon is crucial for the dynamical formation of long-living dark solitons, while density-density intercomponent coupling does not support this effect in the setup considered. Additionally, we showed that the effect at least survives inclusion of not too strong density-density interactions. The discussed phenomenon could guide experiments for a detection of the AB drag effect in binary superfluids. This presents the possibility of studying the drag effect directly in a laboratory, shedding light on the drag effect in other systems ranging from multicomponent superconductors to superfluids in neutron stars.

In conclusion, soliton physics in binary systems was previously restricted to the role of density-density interaction. In this paper we report that a different kind of soliton dynamics arises in binary system due to current-current coupling. The results indicate that the mixed gradient coupling plays an important role in soliton physics in multicomponent systems, which warrants further investigation. We expect that competition between the drag effect and density-density intercomponent interactions will lead to even richer dynamics of multicomponent systems.

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