

Lindbladian dissipation of strongly-correlated quantum matter

Lucas Sá^{1,*}, Pedro Ribeiro^{1,2,†} and Tomaž Prosen^{3,‡}¹*CeFEMA, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal*²*Beijing Computational Science Research Center, Beijing 100193, China*³*Department of Physics, Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia*

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We propose the Sachdev-Ye-Kitaev Lindbladian as a paradigmatic solvable model of dissipative many-body quantum chaos. It describes N strongly coupled Majorana fermions with random all-to-all interactions, with unitary evolution given by a quartic Hamiltonian and the coupling to the environment described by M quadratic jump operators, rendering the full Lindbladian quartic in the Majorana operators. Analytical progress is possible by developing a dynamical mean-field theory for the Liouvillian time evolution on the Keldysh contour. By disorder-averaging the interactions, we derive an (exact) effective action for two collective fields (Green's function and self-energy). In the large- N , large- M limit, we obtain the saddle-point equations satisfied by the collective fields, which determine the typical timescales of the dissipative evolution, particularly the spectral gap that rules the relaxation of the system to its steady state. We solve the saddle-point equations numerically and find that, for strong or intermediate dissipation, the system relaxes exponentially, with a spectral gap that can be computed analytically, while for weak dissipation, there are oscillatory corrections to the exponential relaxation. In this letter, we illustrate the feasibility of analytical calculations in strongly correlated dissipative quantum matter.

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The dynamics of complex interacting many-body open quantum systems, their timescales, and their correlations are a timely topic with major conceptual and experimental significance. In addition to dissipation and decoherence, contact with different environments may induce currents of otherwise conserved quantities, such as energy and charge, and the observables of the system typically attain a steady state. A compact form for describing the dynamics of a quantum system in the presence of an environment with a short memory time (i.e., in the Markovian approximation) is to consider the quantum master equation for the density matrix of the system $\partial_t \rho = \mathcal{L}(\rho)$, where the Liouvillian generator is of the Lindblad form [1–3]:

$$\mathcal{L}(\rho) = -i[H, \rho] + \sum_{m=1}^M (2L_m \rho L_m^\dagger - \{L_m^\dagger L_m, \rho\}). \quad (1)$$

Here, H is the Hamiltonian of the system, while the M jump operators L_m describe the effective coupling to the environment.

Even within this simplified setup, describing interacting open quantum systems is a daunting task. Considerable progress has been achieved in the past decade for integrable models. In generic (i.e., chaotic) closed many-body quantum systems, interactions may entail such a complex structure that the Hamiltonian behaves in several aspects like a large random matrix, as conjectured by Bohigas, Giannoni, and Schmit [4]. Extending this result to the dissipative realm is a fundamental problem that has attracted considerable attention recently [5–11]. Along similar lines, the past couple of years have seen the development of the (non-Hermitian) random matrix theory of Lindbladian dynamics [12–20]. By randomly sampling the Hamiltonian and jump operators, many statistical properties, including the spectral support [12] and distribution [19], the spectral gap [13–15], and the steady state [15,21] have been computed. However, physical systems have few-body interactions, rendering them very different from dense random matrices. It is natural to ask what properties are similar (i.e., universal) in both cases. Steps in this direction were taken in Refs. [16,17,20], where local operators were modeled as Pauli strings with a fixed number of nonidentity operators.

We instead propose using the Sachdev-Ye-Kitaev (SYK) model [22–26], a model of N Majorana fermions with random all-to-all couplings, to describe both the Hamiltonian and the jump operators of the dissipative system. The SYK model originated in nuclear physics 50 years ago [27–31] but has seen a recent surge of interest because of its connection to two-dimensional quantum gravity, after it was shown to be maximally chaotic, exactly solvable at strong coupling, and near conformal [23–25,32–37]. Later, it was also found that it displays an exponential growth of low-energy excitations

*lucas.seara.sa@tecnico.ulisboa.pt

†ribeiro.pedro@tecnico.ulisboa.pt

‡tomaz.prosen@fmf.uni-lj.si

typical of black holes and heavy nuclei [38–40], that it realizes the full Altland-Zirnbauer classification [38,39,41–48], and that it captures many features of non-Fermi liquids [49–53] and wormholes [54–64]. These developments placed the SYK model in a prominent position at the intersection of high-energy physics, condensed matter, and quantum chaos, as one of the few analytically tractable models of both holography and strongly interacting quantum matter. Moreover, several experimental implementations have been proposed [65–70], and its practical and technological relevance has been highlighted [71–76]. Finally, non-Hermitian SYK models have also started gaining traction, with studies focusing on thermodynamics and wormhole physics [77,78], symmetries and universality [11], entanglement dynamics [79,80], and the effect of decoherence on quantum chaos [81,82].

In this letter, we exploit the solvability of the SYK model and develop an analytic theory for the relaxation of generic strongly interacting dissipative quantum systems. Nonequilibrium real-time Hamiltonian dynamics of the SYK model (e.g., thermalization and transport) have been studied before by either coupling it to an external bath [83–90] or quenching its interactions [91–95] (see also Refs. [96,97] for non-Markovian entropy dynamics and Refs. [98–100] for a continuously monitored SYK model), but a fully fledged quantum-master-equation approach to strongly interacting dissipative dynamics has remained unaddressed. Here, we bridge this gap. Working on the Keldysh contour [101–104], we extend the dynamical mean-field theory for the collective degrees of freedom (mean-field Green’s function and self-energy) [26,105–107] to the Lindbladian evolution. Because the interactions are random and all to all, this mean-field theory is exact. From its saddle-point equations, we compute the retarded Green’s function and determine the approach to the nonequilibrium steady state.

To start, we consider the Hamiltonian H and the M jump operators L_m in Eq. (1) to be SYK operators:

$$H = \sum_{i<j<k<l}^N J_{ijkl} \chi_i \chi_j \chi_k \chi_l \quad \text{and} \quad L_m = i \sum_{i<j}^N \ell_{m,ij} \chi_i \chi_j. \quad (2)$$

The Majorana operators χ_i satisfy the N -dimensional Clifford algebra $\{\chi_i, \chi_j\} = \delta_{ij}$, and the totally antisymmetric couplings J_{ijkl} and $\ell_{m,ij}$ are independent Gaussian random variables with zero mean and variance:

$$\langle J_{ijkl}^2 \rangle = \frac{3!J^2}{N^3} \quad \text{and} \quad \langle |\ell_{m,ij}|^2 \rangle = \frac{\gamma^2}{N^2}, \quad (3)$$

respectively (J_{ijkl} must be real to ensure Hermiticity of the Hamiltonian, while $\ell_{m,ij}$ can generally be complex). Notice the nontrivial scaling of the quadratic SYK couplings, which is required for a nontrivial theory in the large- N limit. The scales J and γ measure the strength of the unitary and dissipative contributions to the Liouvillian, respectively. The Hamiltonian describes coherent long-range four-body interactions, while each L_m gives an independent channel for incoherent two-body interactions, in such a way that the full Liouvillian is quartic in the Majorana

operators:

$$\begin{aligned} \mathcal{L}(\rho) = & -i \sum_{i<j<k<l}^N J_{ijkl} (\chi_i \chi_j \chi_k \chi_l \rho - \rho \chi_i \chi_j \chi_k \chi_l) \\ & - \sum_{\substack{i<j \\ k<l}}^N \Gamma_{ijkl} (2\chi_i \chi_j \rho \chi_k \chi_l - \chi_k \chi_l \chi_i \chi_j \rho \\ & - \rho \chi_k \chi_l \chi_i \chi_j), \end{aligned} \quad (4)$$

where we defined the positive-definite matrix:

$$\Gamma_{ijkl} = \sum_{m=1}^M \ell_{m,ij} \ell_{m,kl}^*, \quad (5)$$

which satisfies $\Gamma_{ijkl} = -\Gamma_{jikl} = \Gamma_{jilk} = -\Gamma_{ijlk} = \Gamma_{klji}^*$. If we let $N, M \rightarrow \infty$ with $m = M/N$ fixed, Γ_{ijkl} also becomes Gaussian distributed. Then Eq. (3) implies that the mean and the variance of Γ_{ijkl} are, respectively,

$$\langle \Gamma_{ijij} \rangle = \frac{m\gamma^2}{N} \quad \text{and} \quad \langle |\Gamma_{ijkl}|^2 \rangle_{\text{con}} = \frac{m\gamma^4}{N^3}. \quad (6)$$

The system undergoes nonunitary time evolution toward a steady state ρ_∞ , satisfying $\mathcal{L}(\rho_\infty) = 0$. For simplicity, we restrict ourselves to Hermitian jump operators (i.e., real $\ell_{m,ij}$). In that case, the steady state is the infinite-temperature state $\rho_\infty = \mathbb{1}/2^{N/2}$. We are interested in the relaxation to ρ_∞ . To that end, we consider the retarded Green’s function:

$$iG^R(t) \delta_{ij} = \Theta(t) \langle \text{Tr}[\rho_\infty \{\chi_i(t), \chi_j\}] \rangle, \quad (7)$$

where $\langle \dots \rangle$ denotes the average over both J_{ijkl} and Γ_{ijkl} , and the (Heisenberg-picture) Majorana operator $\chi_i(t)$ satisfies the adjoint Lindblad equation $\partial_t \chi_i = \mathcal{L}^\dagger(\chi_i)$. The relaxation dynamics are characterized by the late-time decay of $iG^R(t)$. An exponential decay signals a well-defined spectral gap (relaxation rate).

We now switch to the Keldysh path-integral representation of the Majorana Liouvillian (see the Supplemental Material (SM) [108] for a derivation and Ref. [109] for the bosonic version). We introduce real Grassmann fields $a_i(z)$ living on the closed-time contour $z \in \mathcal{C} = \mathcal{C}^+ \cup \mathcal{C}^-$, where real time runs from $-\infty$ to $+\infty$ (branch \mathcal{C}^+) and then back again to $-\infty$ (branch \mathcal{C}^-). The Grassmann field $a_i(t^+)$ [$a_i(t^-)$], with $t^+ \in \mathcal{C}^+$ ($t^- \in \mathcal{C}^-$), propagates forward (backward) in time and is the path-integral representation of a Majorana operator acting on the density matrix from the left (right). Using Eq. (4), we can immediately write down the partition function:

$$Z = \int \prod_{i=1}^N \mathcal{D}a_i \exp(iS[a_i]), \quad (8)$$

where we omitted an initial-state contribution that is irrelevant for the long-time dissipative dynamics, and the Lindblad-Keldysh action is

$$\begin{aligned} iS[a_i] = & i \int_{\mathcal{C}} dz \frac{1}{2} \sum_{i=1}^N a_i(z) i\partial_z a_i(z) \\ & - i \int_{\mathcal{C}} dz \sum_{i<j<k<l}^N J_{ijkl} a_i(z) a_j(z) a_k(z) a_l(z) \end{aligned}$$

$$+ \int_{\mathcal{C}} dz dz' K(z, z') \sum_{\substack{i < j \\ k < l}}^N \Gamma_{ijkl} a_i(z) a_j(z) a_k(z') a_l(z'). \quad (9)$$

The memory kernel $K(z, z')$ allows for both Markovian and non-Markovian dissipative dynamics (see the SM [108] for a discussion on how a non-Markovian thermal bath fits into our framework). Comparing Eqs. (4) and (9), we can read off the lesser and greater components of the Markovian kernel:

$$K^<(t_1, t_2) = K(t_1^+, t_2^-) = 2 \delta(t_1 - t_2), \quad (10)$$

$$K^>(t_1, t_2) = K(t_1^-, t_2^+) = 0, \quad (11)$$

respectively. Equations (9)–(11) can also be derived microscopically by tracing out the environment in a unitary system-plus-environment theory [108]. We further define the (mean-field) Green's function:

$$G(z, z') = -\frac{i}{N} \sum_{i=1}^N a_i(z) a_i(z'), \quad (12)$$

and the self-energy $\Sigma(z, z')$ as a Lagrange multiplier enforcing the definition of Eq. (12) in the path integral.

To proceed, we average over the random couplings J_{ijkl} and Γ_{ijkl} , in the limit $N, M \rightarrow \infty$ with $m = M/N$ fixed. The averaging procedure is straightforward and is presented in the SM [108]. The resulting averaged partition function is

$$\langle Z \rangle = \int \mathcal{D}G \mathcal{D}\Sigma e^{iS_{\text{eff}}[G, \Sigma]}, \quad (13)$$

with mean-field action:

$$\begin{aligned} iS_{\text{eff}}[G, \Sigma] = & \frac{N}{2} \left\{ \text{Tr} \log(i\partial - \Sigma) - \int_{\mathcal{C}} dz dz' \Sigma(z, z') G(z, z') - \frac{J^2}{4} \int_{\mathcal{C}} dz dz' [G(z, z')]^4 \right. \\ & + \frac{m\gamma^4}{4} \int_{\mathcal{C}} dz dz' dw dw' K(z, z') K(w, w') [G(z, w')]^2 [G(z', w)]^2 \\ & \left. + m\gamma^2 \int_{\mathcal{C}} dz dz' K(z, z') [G(z, z')]^2 \right\}. \quad (14) \end{aligned}$$

Variation of Eq. (14) with respect to $\Sigma(z, z')$ and $G(z, z')$ (recall that both are antisymmetric in their contour indices) leads to the Schwinger-Dyson equations on \mathcal{C} :

$$(i\partial - \Sigma) \cdot G = \mathbb{1}_{\mathcal{C}}, \quad (15)$$

$$\begin{aligned} \Sigma(z, z') = & -J^2 [G(z, z')]^3 + \frac{m\gamma^4}{2} G(z, z') \int_{\mathcal{C}} dw dw' [K(z, w)K(w', z') + K(w, z)K(z', w')] [G(w, w')]^2 \\ & + m\gamma^2 [K(z, z') + K(z', z)] G(z, z'), \quad (16) \end{aligned}$$

where Eq. (15) is to be understood as a matrix equation, while Eq. (16) acts on each matrix element individually. These equations are exact for the SYK Lindbladian in the large- N , large- M limit.

We now move back from contour times (z, z') to real times (t_1, t_2) . For Majorana fermions, there is a single independent Green's function [91, 110], say, the greater component $G^>(t_1, t_2) = G(t_1^-, t_2^+)$, while the lesser component $G^<(t_1, t_2) = G(t_1^+, t_2^-)$ satisfies $G^<(t_1, t_2) = -G^>(t_2, t_1)$. Restricting Eq. (15) to $(z, z') = (t_1^-, t_2^+)$ and using Eqs. (10) and (11), Eq. (16) reads as [108]

$$\begin{aligned} \Sigma^>(t_1, t_2) = & G^>(t_1, t_2) \{-J^2 [G^>(t_1, t_2)]^2 \\ & - m\gamma^4 [G^<(t_1, t_2)]^2 \\ & + 2m\gamma^2 \delta(t_1 - t_2)\}. \quad (17) \end{aligned}$$

Next, we change variables to $t = t_1 - t_2$ and $\mathcal{T} = (t_1 + t_2)/2$. For long times $\mathcal{T} \rightarrow \infty$, the system loses any information about its initial state and relaxes to the steady state. The Green's function $G^>$ depends now only on t , and we move

to Fourier space with continuous frequencies ω . We further perform a Keldysh rotation by defining the real quantities [111]:

$$\rho^{\pm}(\omega) = -\frac{1}{2\pi i} [G^>(\omega) \mp G^<(-\omega)], \quad (18)$$

$$\rho^{\text{H}}(\omega) = -\frac{1}{\pi} \mathcal{P} \int dv \frac{\rho^-(v)}{\omega - v}, \quad (19)$$

and analogously for $\Sigma^>(\omega)$. Here, $\rho^+(\omega)$ is proportional to the Keldysh component of the Green's function $G^{\text{K}}(\omega) = -2\pi i \rho^+(\omega)$, while the spectral function $\rho^-(\omega)$ is normalized $\int d\omega \rho^-(\omega) = 1$ and, together with its Hilbert transform $\rho^{\text{H}}(\omega)$, completely determines the retarded Green's function $G^{\text{R}}(\omega) = -\pi [\rho^{\text{H}}(\omega) + i\rho^-(\omega)]$. Because the steady state is the infinite-temperature state, we have $\rho^+(\omega) = 0$, and we can write Eq. (7) as

$$iG^{\text{R}}(t) = \Theta(t) \int d\omega \rho^-(\omega) \cos \omega t. \quad (20)$$

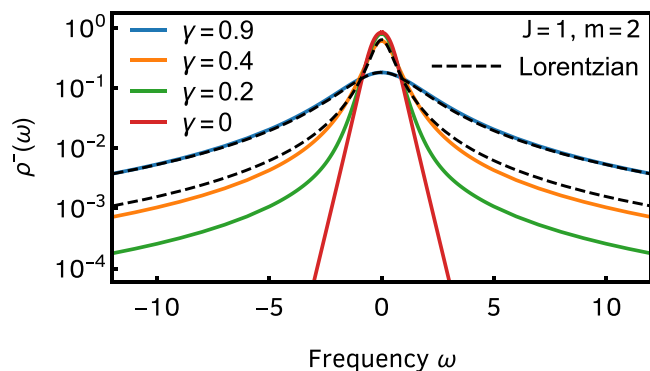


FIG. 1. Spectral function $\rho^-(\omega)$ obtained from the numerical solution of the Schwinger-Dyson equations, Eqs. (21) and (22), for $J = 1$, $m = 2$, and different γ . For large γ , the solution is well described by a Lorentzian (dashed lines) with the width computed analytically, Eq. (25). For intermediate γ (e.g., $\gamma = 0.4$), the Lorentzian ansatz still gives a reasonable description of the result (especially for low frequencies), but it fails for low dissipation.

After the Fourier transformation and Keldysh rotation, the self-energy, Eq. (17), is given by

$$\sigma^-(\omega) = \frac{m\gamma^2}{\pi} + \frac{J^2 + m\gamma^4}{4} \times \int d\mu dv \rho^-(\omega - \mu - v) \rho^-(\mu) \rho^-(v), \quad (21)$$

while the Dyson equation, Eq. (15), reads as

$$\rho^-(\omega) = \frac{\sigma^-(\omega)}{[\omega + \pi\sigma^H(\omega)]^2 + [\pi\sigma^-(\omega)]^2}. \quad (22)$$

The Schwinger-Dyson equations can be solved numerically in a self-consistent manner by proposing an ansatz for $\rho^-(\omega)$ and $\sigma^-(\omega)$ and then iterating Eqs. (21) and (22) until convergence is achieved [111]. Details on our numerical procedure are given in the SM [108]. The results for $J = 1$, $m = 2$, and different values of γ are plotted in Fig. 1. For large-enough γ , the spectral function is well approximated by a Lorentzian. Fourier transforming back to the time domain [Eq. (20)], see Figs. 2(a) and 2(b), this implies a well-defined spectral gap Δ (i.e., relaxation rate), as the retarded Green's function decays exponentially $iG^R(t) = \Theta(t) \exp\{-\Delta t\}$. The spectral gap can be determined analytically as follows. We propose the Lorentzian ansatz:

$$\rho^-(\omega) = \frac{1}{\pi} \frac{\Delta}{\omega^2 + \Delta^2}, \quad (23)$$

for the spectral function, and because the Lorentzian is stable under convolution, Eq. (21) leads to the self-energy:

$$\sigma^-(\omega) = \frac{m\gamma^2}{\pi} + \frac{J^2 + m\gamma^4}{4\pi} \frac{3\Delta}{\omega^2 + (3\Delta)^2}. \quad (24)$$

Since we are interested in the low-frequency response, we set $\omega = 0$. The regime of validity of this approximation can be determined self-consistently and is presented in the SM [108]. Plugging Eqs. (23) and (24) back into the Dyson equation,

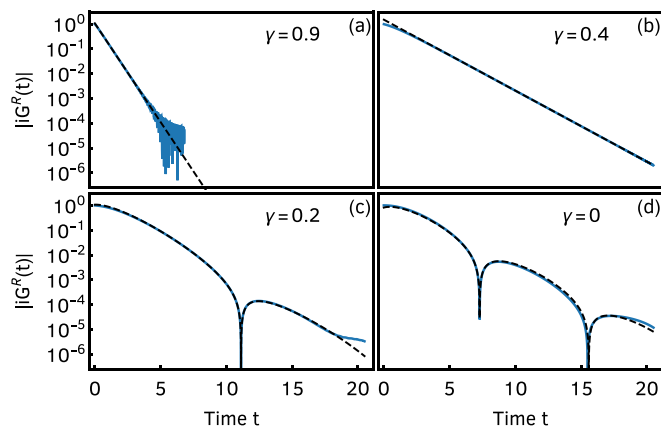


FIG. 2. Retarded Green's function as a function of time for $J = 1$, $m = 2$, and different γ . The full blue curves are obtained by Fourier transforming the numerical solution for $\rho^-(\omega)$, as prescribed in Eq. (20). The dashed black lines give the best asymptotic fit to Eq. (26).

Eq. (22), we find

$$\Delta = \frac{m\gamma^2}{2} \left(1 + \sqrt{\frac{3m+1}{3m} + \frac{J^2}{3m^2\gamma^4}} \right). \quad (25)$$

Equation (25), the analytical relaxation rate of a strongly correlated dissipative quantum system, is the main result of this letter. The comparison with the numerical solution is given in Fig. 1. For $J = 1$ and $m = 2$, already for $\gamma = 0.9$, there is excellent agreement. Accordingly, we see a clear exponential decay of $iG^R(t)$ in Fig. 2(a). For intermediate γ , say, $\gamma = 0.4$, there are noticeable deviations in the tails, but the low-frequency part of ρ^- is still perfectly described by Eqs. (23) and (25), and $iG^R(t)$ still decays exponentially, see Fig. 2(b). For small γ , the tails of the spectral function are very far from Lorentzian. This signals possible power-law or oscillatory corrections to the asymptotic decay of $G^R(t)$ [depending on the precise form of $\rho^-(\omega)$, which cannot be determined analytically], which we confirm numerically, see Figs. 2(c) and 2(d). We can extract the spectral gap from $iG^R(t)$ by fitting the numerical results to an exponential function with power-law and oscillatory corrections. We found the former to be negligible in general but the latter to be relevant for small γ , i.e.,

$$iG^R(t) = A e^{-\Delta t} \cos(\Omega t + \phi) \quad (26)$$

gives an excellent fit for $t \gg 1$ with fitting parameters A , Δ , Ω , and ϕ . The resulting spectral gap is plotted in Fig. 3 as a function of γ . We conclude that, for large γ , Δ grows quadratically, in agreement with Eq. (25), while it starts to deviate from the Lorentzian ansatz at intermediate values $\gamma \approx 0.5$. As γ further decreases, our results are consistent (within the numerically accessible time window) with a bifurcation of the real gap Δ to a pair of complex-conjugated gaps $\Delta \pm i\Omega$ at $\gamma \approx 0.28$, see inset of Fig. 3. Remarkably, as $\gamma \rightarrow 0$, Δ saturates to a finite value, indicating that even an infinitesimally small amount of dissipation leads to relaxation at a finite rate. This is admissible given that we took the thermodynamic limit first. Notice that the strict limit $\gamma = 0$

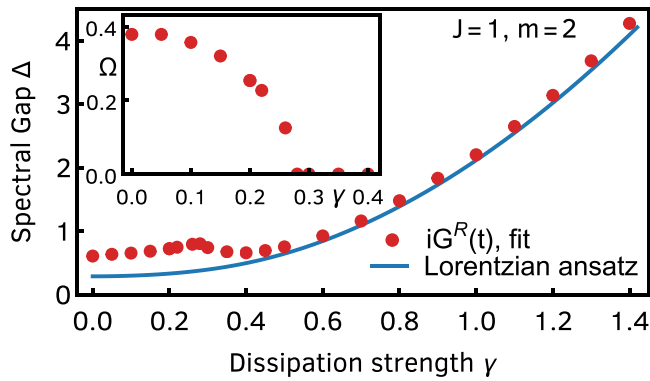


FIG. 3. Spectral gap Δ as a function of dissipation strength γ for $J = 1$ and $m = 2$. The red dots are obtained from the fit to Eq. (26), while the blue line is the analytical result from the Lorentzian ansatz, Eq. (25). The two agree for large γ but saturate to different values as $\gamma \rightarrow 0$. Inset: Frequency Ω of the oscillatory correction as a function of γ . For $\gamma \gtrsim 0.28$, the period of oscillations either diverges ($\Omega = 0$) or becomes longer than the numerically accessible time window.

is singular, as no steady state exists, and the solution of the Schwinger-Dyson equations depends on the initial state (here, the infinite-temperature equilibrium state) [108]. Although the Lorentzian ansatz and the numerical solution saturate to different values when $\gamma \rightarrow 0$, the former still gives a qualitatively correct picture for the relaxation rate of the SYK model across all dissipation scales.

In summary, we studied the dissipative dynamics of the SYK model in the framework of the Lindbladian quantum master equation. We found exponential relaxation to the infinite-temperature steady state (with possible oscillatory corrections) and analytically computed the spectral gap in the limit of strong dissipation. Our work paves the way for

further analytical investigations of dissipative strongly correlated quantum matter, as many interesting questions remain unanswered. First, our method can be straightforwardly generalized for arbitrary q -body interactions [108] (here, $q = 4$). The question arises whether the physics is qualitatively the same for all q , and, particularly, what happens in the large- q limit, where there are simplifications in the standard SYK model [34]? Second, our work can be used to study more general setups with non-Markovian dissipation by tuning the kernel $K(z, z')$. Third, going away from the scalings of Eq. (3) and considering $1/N$ corrections and non-Hermitian jump operators allows for nontrivial steady states. An analysis of the spectral and steady-state properties of general SYK Lindbladians, based on exact diagonalization along the lines of Refs. [15,21], is a natural next step. Finally, we mention the possibility of studying the symmetries of fermionic open quantum matter [11,112,113] in the context of the SYK model, for which a rich classification exists in the closed case [41–43].

Note added. After our manuscript appeared on the arXiv, we were made aware of subsequent related work [114] which independently addresses the same problem through slightly different techniques. Their findings corroborate the results of Fig. 3.

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