Letter

Hubble selection of the weak scale from QCD quantum critical point

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(Received 1 August 2021; accepted 9 May 2022; published 31 May 2022)

There is growing evidence that the small weak scale may be related to self-organized criticality. In this regard, we note that if the strange quark were lighter, the QCD phase transition could have been first order, possibly exhibiting quantum critical points at zero temperature as a function of the Higgs vacuum expectation value v_h smaller than (but near) the weak scale. We show that these quantum critical points allow a dynamical selection of the observed weak scale, via quantum-dominated stochastic evolutions of the value of v_h during eternal inflation. Although the values of v_h in different Hubble patches are described by a probability distribution in the multiverse, inflationary quantum dynamics ensures that the peak of the distribution evolves toward critical points (self-organized criticality), driven mainly by the largest Hubble expansion rate there—the Hubble selection of the universe. To this end, we first explore the quantum critical points of the three-flavor QCD linear sigma model, parametrized by v_h at zero temperature, and we present a relaxion model for the weak scale. Among the patches that have reached reheating, it results in a sharp probability distribution of v_h near the observed weak scale, which is critical not to the crossover at $v_h = 0$ but to the sharp transition at $\sim\Lambda_{\rm QCD}$.

DOI: 10.1103/PhysRevResearch.4.L022048

I. INTRODUCTION

The Planck weak-scale hierarchy may be addressed by the near criticality of the Higgs mass parameter [1,2]. In this viewpoint, the small weak scale close to zero is special because the universe transitions between broken and unbroken phases of the electroweak symmetry at zero. The transition could generate various standard model (SM) backreactions that allow dynamical selection of the weak scale [3–8]. However, this transition is a second-order crossover in the SM, providing only relatively smooth selection rules. In addition, the SM Higgs potential, once renormalization-group evolved, was found to yield another almost degenerate vacuum near the Planck scale [9,10]. This surprising coincidence provides more evidence that the particular (seemingly unnatural) value of the weak scale might be related to criticality. This motivated ideas of the multiple-point principle [11–15], classical scale invariance [16–18], as well as Higgs inflation [19–21]. Extremely small dark energy is also thought to be near the critical point. But yet, whether and how criticality plays a crucial role in naturalness remain unclear.

Recently, a cosmological selection mechanism for criticality was developed in Ref. [22], where inflationary quantum-dominated evolution of the relaxion inevitably drives a theory parameter close to a quantum critical point. In one setup, it is crucial that the critical point be the first-order separation between discrete phases with a significant energy difference, so that the Hubble rate can be sharply largest there. Then after long enough inflation (essentially eternal as will be discussed), such Hubble patches having a theory near the critical point will dominate the multiverse, as they are expanding and are reproduced most rapidly—Hubble selection of the universe. This mechanism realizes self-organized criticality [22]; see also Refs. [23,24].

Then we ask the following: Why is the selection of criticality the selection of the observed small weak scale? To what first-order critical points does the Higgs mass have relevance? Reference [22] analyzed the aforementioned renormalization-scale dependence of the SM Higgs vacuum structure [9,10], but the critical scale was found to be far above the weak scale. References [5,6] studied a prototype model with multiple axions, where a QCD barrier trapping an axion disappears when v_h turns off, so that the axion suddenly rolls down to the minimum, generating a large energy contrast necessary for Hubble selection. Critical changes of a theory could also induce the small weak scale in association with much smaller dark energy [7,8].

In this Letter, we present a cosmological account of the weak scale from possible first-order zero-temperature (hence, quantum) critical points of QCD.[1] We first point out that QCD may have built-in quantum critical points at some $v_h^* \lesssim v_{\rm EW} = 246$ GeV; this has yet to be studied, and we initiate an

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[1] The small weak scale, if not due to criticalities or symmetries, could also be a result of the cosmological selection of anthropic [7,25–29] or entropic [30–35] principles in a multiverse.

exploration using the three-flavor linear sigma model (LSM) of low-energy QCD. Then we present a relaxion model that realizes Hubble selection of the QCD criticality and self-organizes v_h close to the observed value. Then, v_h is critical to $\sim\Lambda_{\rm QCD}$ (not to the crossover at zero). An added benefit is that the weak scale and $\Lambda_{\rm QCD}$ are generically close, which otherwise is accidental. Furthermore, building upon earlier works, we elaborate Hubble selection with different semiquantitative derivations.

We are inspired by observations that if the strange quark were slightly lighter, the (finite-temperature T) QCD chiral phase transition could have been first order. Although not yet firmly established [36–43], this possibility has been expected based on the (non)existence of infrared fixed points in the three-dimensional LSM [44,45]. In other words, QCD at $T = 0$ too (relevant during inflation) may have a rich vacuum structure, as a function of variable quark masses or v_h. Our initial phenomenological exploration of the vacuum structure shall be verified by dedicated research.

The Letter discusses the basic model ingredients (Sec. II), exploration of QCD quantum critical points (Sec. III), Hubble selection (Sec. IV), realization of the weak-scale criticality (Sec. V), and conclusions with future improvements.

II. MODEL

The model consists of the relaxion ϕ [3], the Higgs h, and the meson field Σ: $V_{\rm tot} = V_\phi + V_h + V_\Sigma$. The relaxion couples only to the Higgs sector, scanning v_h. But the change of v_h induces changes in the Σ sector, developing the desired quantum criticality at v_h^*. Then the Hubble selection (acting on ϕ) self-organizes the universe to the critical point.

The real-scalar relaxion potential is axionlike:

$$V_\phi = \Lambda_\phi^4 \cos\frac{\phi}{f_\phi}. \qquad (1)$$

For Hubble selection, its field range f_ϕ shall exceed the Planck scale (see later), which is possible with multiple axions [46–48].

The Higgs potential takes the SM form ($\lambda_h \simeq 0.13$) plus the coupling to the relaxion (see Ref. [3] for details),

$$V_h = \frac{1}{2}(M^2 - g\widetilde{\phi})h^2 + \frac{\lambda_h}{4}h^4 \to -\frac{1}{2}(g\phi)h^2 + \frac{\lambda_h}{4}h^4, \qquad (2)$$

where h is the real Higgs field in unitary gauge. We shift ϕ such that the quadratic term $\mu_h^2 = -g\phi$ vanishes at $\phi = 0$. $v_h^2 \equiv -\mu_h^2/\lambda_h = g\phi/\lambda_h$ is used to label the relaxion scanning ($v_{\rm EW} = 246$ GeV).[2] The required field range of ϕ to scan μ_h^2 up to the cutoff M^2 is $\delta\phi \sim M^2/g$, thus we set $f_\phi = M^2/g$. The dimensionful coupling g is a spurion of the relaxion shift symmetry, and thus can be small naturally.

Below $\Lambda_{\rm QCD} = 200$ MeV, meson fields $\Sigma_{ij}(x)$ are relevant degrees of freedom, whose condensates are order parameters for chiral symmetry breaking. This vacuum structure as a function of v_h is what we want to explore. It can be conveniently described by the LSM with $U(N_f)_L \times U(N_f)_R$ symmetry of QCD [49–51],

$$\begin{aligned} V_\Sigma = {} & \mu^2\,{\rm Tr}[\Sigma\Sigma^\dagger] + \lambda_1({\rm Tr}[\Sigma\Sigma^\dagger])^2 + \lambda_2\,{\rm Tr}[(\Sigma\Sigma^\dagger)^2] \\ & - c(\det\Sigma + \det\Sigma^\dagger) - {\rm Tr}[\mathcal{H}(\Sigma+\Sigma^\dagger)], \end{aligned} \qquad (3)$$

where fields and parameters are decomposed as $\Sigma = (\sigma_a + i\pi_a)T^a$, $\mathcal{H} = h_a T^a$ with generators T^a satisfying ${\rm Tr}[T^aT^b] = \delta^{ab}/2$ for $a = 0, \ldots, N_f^2 - 1$. Without losing generality, $\lambda_{1,2}, h_a$ are real, $c > 0$, and μ^2 can take either sign. Σ is bifundamental under the symmetry. The first line of Eq. (3) conserves ${\rm SU}(N_f)_L \times {\rm SU}(N_f)_R$; λ_2 is nonzero, otherwise symmetry is enhanced to $O(2N_f^2)$. One of the remaining $U(1)$'s is identified as the conserved baryon number $U(1)_V$, simply omitted in our discussion. The other $U(1)_A$ is anomalous, broken by the instanton contribution c down to $Z_A(N_f)$ [52,53]. Symmetries are further broken by $\mathcal{H}$, the leading chiral-symmetry-breaking mass term. We fix $N_f = 3$ with the isospin symmetry $m_u = m_d$, as a first exploration; only $h_0, h_8 \neq 0$.

It is worthwhile to note that the LSM indeed possesses necessary features for quantum critical points. For $N_f = 3$, the instanton term is a cubic potential, possibly creating local vacua (even with $\mu^2 > 0$). The linear term $\mathcal{H}$ can destabilize the local vacua at critical quark masses or v_h^*, just as the external magnetic field (the linear term) in ferromagnets flips higher-energy spin directions at a critical field strength.

III. QCD QUANTUM CRITICAL POINTS

To explore the vacuum structure as a function of v_h, we first fix the benchmark "SM point" parameters of V_Σ, reproducing a measured meson spectrum, and then we deduce how these parameters change with v_h.

The masses of pions and kaons, being pseudo-Goldstones, are given by symmetry-breaking terms $\mathcal{H}$, related by partially conserved axial-vector currents,

$$\partial_\mu j_a^{\mu 5} = m_{\pi_a}^2 f_{\pi_a}\pi_a = \pi_b h_c d_{abc}, \qquad (4)$$

where the last equality is obtained by the variation of V_Σ under chiral transformations. For pions $\pi^0 = \pi_3$ with $d_{3b0} = \sqrt{2/3}\delta_{b3}$, $d_{3b8} = 1/\sqrt{3}\delta_{b3}$, and for kaons $K^0 = (\pi_6 + i\pi_7)/\sqrt{2}$ with $d_{Kb0} = \sqrt{2/3}\delta_{bK}$, $d_{Kb8} = -1/\sqrt{12}\delta_{bK}$, we have

$$m_\pi^2 f_\pi = \sqrt{\frac{2}{3}}h_0 + \frac{h_8}{\sqrt{3}}, \quad m_K^2 f_K = \sqrt{\frac{2}{3}}h_0 - \frac{h_8}{2\sqrt{3}}. \qquad (5)$$

Using measured values of $m_{\pi,K}$ (Table I), we fix the SM-point value of $h_{0,8}$ [54],

$$h_0(v_{\rm EW}) = (287\ {\rm MeV})^3, \quad h_8(v_{\rm EW}) = -(312\ {\rm MeV})^3. \qquad (6)$$

We proceed to fit masses of other pseudoscalar and scalar mesons to data. The minima of V_Σ are numerically found by considering the stability along all 18 field directions. The $N_f = 3$ LSM is known to have three types of vacua at $\mathcal{H} = 0$ [56,57]: ${\rm SU}(3)_L \times {\rm SU}(3)_R$ ($s_1 = s_3 = 0$), ${\rm SU}(3)_V$ ($s_1 = s_3 \neq 0$), and ${\rm SU}(2)_L \times {\rm SU}(2)_R \times U(1)_V$ ($s_1 = 0, s_3 \neq 0$), where $\langle\Sigma\rangle = \sigma_0 T^0 + \sigma_8 T^8 = {\rm diag}(s_1, s_1, s_3)$. In particular, the global ${\rm SU}(3)_V$ vacuum (that we live in today) and the local ${\rm SU}(3)_L \times {\rm SU}(3)_R$ vacuum *coexist* if $\mu^2 > 0$ and

[2] QCD backreaction $V_h \ni y_q h\langle\bar{q}q\rangle/\sqrt{2}$ is ignored, as it is sizable only for the ${\rm SU}(3)_V$ vacuum which is not Hubble selected for $v_h \lesssim v_h^*$

TABLE I. Predictions of the benchmark SM point [Eqs. (6) and (7)], compared with data from the Particle Data Group [55]. In units of MeV.

Parameter	f_π	f_K	m_π	m_K	m_η	$m_{\eta'}$	m_{a_0}	$m_{f_0(500)}$	$m_{f_0(1370)}$	$m_{K^*(1430)}$
Measured	92.4	113	$139.57^{\pm 0.005}$	$497.61^{\pm 0.013}$	$547.86^{\pm 0.017}$	$957.78^{\pm 0.06}$	$980^{\pm 20}$	$500^{\pm 150}$	$1370^{\pm 150}$	$1425^{\pm 50}$
Benchmark	92.4	113	137	491	534	973	1050	731	1260	1140

$K \equiv \frac{c^2}{2\mu^2(3\lambda_1+\lambda_2)} > 4.5$ with $3\lambda_1+\lambda_2>0$ [57]. Thus, this parameter space is our focus, that potentially exhibits first-order quantum critical points.

By scanning with these constraints, we found a range of good parameter space (see Appendix A in Supplemental Material [58]). The benchmark SM point is [with Eq. (6)]

$$\mu^2 = (60\ \mathrm{MeV})^2, \quad c = 4800\ \mathrm{MeV}, \quad \lambda_1 = 7, \quad \lambda_2 = 46, \tag{7}$$

yielding $K = 47.8$. Its goodness of fit to the meson spectrum (Table I) is χ^2/degrees of freedom $= 0.44$ with the first seven observables and 3.11 including all. The first seven are the most reliable, while the last three are less precisely measured with unclear identities [55]. Here, 5% theoretical uncertainties are added as typical sizes of the perturbative corrections. Our benchmark is as good as existing benchmarks in the literature: Ref. [56] (with $\mu^2>0$) yielded 0.20 and 2.84, respectively, and Ref. [59] ($\mu^2<0$) yielded 1.22 and 4.64.[3] But ours differs in that quantum critical points v_h^* may exist.

We turn to discuss the vacuum structure away from the SM point for $v_h < v_{\mathrm{EW}}$. How does V_Σ, in particular $\mathcal{H}$, depend on v_h? Since this is not known, we deduce it as follows. The current divergence [Eq. (4)] calculated from QCD or chiral Lagrangian yields $m_\pi^2 \propto m_q$, which is also $\propto h_{0,8}$ from Eq. (5). It suggests $h_{0,8} \propto v_h$. Indeed, identifying the $\mathcal{H}$ term and the current mass term, $\mathcal{L} \ni -m_q(\bar{u}u + \bar{d}d) - m_s\bar{s}s$, yields the ratio $h_8/h_0 \simeq (\frac{m_q-m_s}{\sqrt{3}})/(\frac{2m_q+m_s}{\sqrt{6}}) \simeq -1.3$ using $m_q = \frac{(m_u+m_d)}{2} = 3.4$ MeV and $m_s = 93$ MeV [55], the same as the ratio from Eq. (6). Thus, we assume that $\mathcal{H}$ is linear to v_h as

$$h_{0,8}(v_h) = h_{0,8}(v_{\mathrm{EW}})\frac{v_h}{v_{\mathrm{EW}}}. \tag{8}$$

Other LSM parameters and dimensionful factors could also depend on v_h, either directly or indirectly, e.g., via condensation or Λ_{QCD}. Λ_{QCD} depends on quark masses via renormalization running but only logarithmically, and instanton contributions on masses and condensates but complicated and nonperturbative [65]. In this initial exploration, Eq. (8) is assumed to be the only change of V_Σ induced by the scanning of v_h.

The vacuum structure as a function of v_h (other parameters fixed to the benchmark) is shown in Fig. 1. As $K > 4.5$ dictates, there are coexisting vacua at $v_h = 0$. As v_h (hence, $\mathcal{H}$) increases with Eq. (8), and the metastable $SU(3)_L \times SU(3)_R$ vacuum becomes shallower and unstable at the critical point, which is found at $v_h^* \simeq 20$ MeV; in fact, a wider range of $v_h^* = \mathcal{O}(1\text{–}100)$ MeV is consistent with the meson data (see Appendix A in Supplemental Material [58]). The energy difference of the coexisting vacua at the critical point is 93 MeV, comparable to Λ_{QCD}; the potential energies are parametrized in Eqs. (14) and (15).

In all, we have shown that QCD may possess quantum critical points at $v_h^* < v_{\mathrm{EW}}$, which needs dedicated verification.

IV. HUBBLE SELECTION

Inflationary quantum fluctuations on the scalar field allow access to a higher potential regime, which is forbidden classically. Although the field in each Hubble patch always rolls down in average, larger Hubble rates at higher potentials can make a difference in the global field-value distribution among patches, culminating in Hubble selection. This section reviews and supplements it [22].

The volume-weighted (global) distribution $\rho(\phi,t)$ of the field value ϕ obeys the modified Fokker-Planck equation (FPV) [66–69]

$$\frac{\partial\rho(\phi,t)}{\partial t} = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H}\rho\right) + \frac{1}{8\pi^2}\frac{\partial^2(H^3\rho)}{\partial\phi^2} + 3\Delta H\rho. \tag{9}$$

The first two terms represent the flow and diffusion, just as in the original Fokker-Planck equation which averages over Langevin motions. The variation of the Hubble rate $3\Delta H(\phi) = \frac{V(\phi)}{2M_{\mathrm{Pl}}^2 H_0} \ll 3H_0$ accounts for volume weights within a distribution: $M_{\mathrm{Pl}} = 2.4\times10^{18}$ GeV. The meanings become clearer if we look at a solution (for a linear potential without boundary conditions),

$$\rho(\phi,t) \propto \exp\left\{\frac{-1}{2\sigma_\phi^2(t)}\left[\phi - \left(\phi_0 + \dot{\phi}_c t + \tfrac{3}{2}(\Delta H)'\sigma_\phi^2 t\right)\right]^2\right\}, \tag{10}$$

where the exponent describes the motion of a peak. $\dot{\phi}_c = -V'/3H$ is classical rolling. Remarkably, an additional

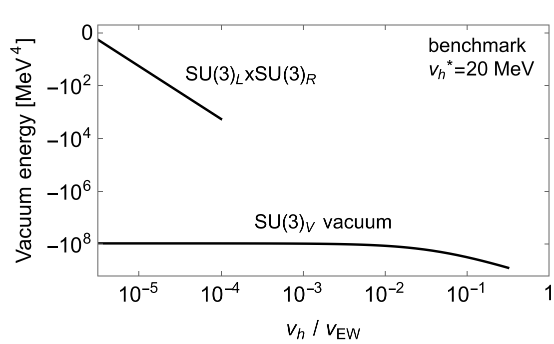

FIG. 1. Vacuum energies of benchmark coexisting QCD vacua at $T = 0$, as functions of v_h. The critical point is at $v_h^* \simeq 20$ MeV, with a Λ_{QCD}-scale energy difference.

[3]Large uncertainties of the last three observables allow various good fits; otherwise, the LSM would have been overconstrained at the tree level. Higher-order, nonperturbative, and other effects may be calculated by lattice simulations [60–64].

velocity $\dot{\phi}_H = 3(\Delta H)'\sigma_\phi(t)^2$ with opposite sign arises from volume weights within the width σ_ϕ, which grows in the beginning of FPV evolutions due to quantum diffusion $\sigma_\phi^2(t) = (\frac{H}{2\pi})^2 Ht$ from the de Sitter temperature $H/2\pi$ [70,71].

"Hubble selection" starts to operate when the peak of a distribution starts to *climb up* the potential: $\sigma_\phi^2 \simeq \frac{2}{3}M_{\rm Pl}^2$. The width at this moment is always Planckian, reflecting its quantum nature. The field excursion by this moment is non-negligible, $\Delta\phi \sim \frac{4\pi^2 M_{\rm Pl}}{9}\frac{\Lambda_\phi^4}{H^4}\frac{M_{\rm Pl}}{M^2/g}$. Thus, for a peak to climb, the field range $\delta\phi \sim M^2/g$ (needed to scan μ_h^2 up to M^2) must accommodate both the field excursion $\Delta\phi$ (stronger condition) and width σ_ϕ, yielding respectively

$$g \lesssim H\frac{H}{M_{\rm Pl}}\frac{M^2}{\Lambda_\phi^2} \lesssim \frac{M^2}{M_{\rm Pl}}. \tag{11}$$

We call this condition *global* quantum beats classical (QBC). It is stronger than the usual local QBC, $V' \lesssim H^3$, requiring $g \lesssim H\frac{H^2}{\Lambda_\phi^2}\frac{M^2}{\Lambda_\phi^2}$, because $\Lambda_\phi^2 \lesssim HM_{\rm Pl}$ from condition 1 later. It also has different meanings as it involves the field range while the local one depends only on the potential slope. It turns out to be equivalent to the Quantum+Volume (QV) condition in Ref. [22] (see Appendix B in Supplemental Material [58]) which also accounted for volume effects. If it is not satisfied, ρ makes an equilibrium at the bottom of a potential, but with the sub-Planckian width consistently $\sigma_\phi \sim H^2/m_\phi \sim H^2M^2/\Lambda_\phi^2 g \lesssim M_{\rm Pl}$ [72,73]. Thus, we require the global QBC Eq. (11) for Hubble selection.

The e-folding until this moment $\Delta N \simeq \frac{8\pi^2 M_{\rm Pl}^2}{3H^2}$ already saturates the upper bound for finite inflation, $\frac{2\pi^2}{3}\frac{M_{\rm Pl}^2}{H^2}$, given by the de Sitter entropy [74–76]. Thus, Hubble selection needs eternal inflation, and the universe eventually reaches a stationary state [77–80]. Probability distributions are to be defined within an ensemble of Hubble patches that have reached reheating [81–83]. As the latest patches dominate the ensemble with an exponentially larger number, only stationary or equilibrium distributions matter; for landscapes, this can be different [84–87].

$\rho(\phi, t)$ makes an equilibrium somewhere near the top of a potential, which is the critical point ϕ_* in this work. The distribution can be especially narrow [Planckian in the global QBC regime; see Eq. (B6) in Supplemental Material [58]] if energy drops sharply after ϕ_*. This is how Hubble selection self-organizes the universe toward critical points [22].

The flatter the potential is (with stronger quantum effects), the closer to ϕ_* is the equilibrium. The closest possible field distance is Planckian, again reflecting the uncertainty principle. For even flatter potentials, the equilibrium distribution rather spreads away from ϕ_*, because distributions will be flat in the limit $V' \to 0$. The equilibrium near ϕ_* is estimated as follows. The boundary condition $\rho(\phi \geqslant \phi_*) = 0$ (discarding Hubble patches with $\phi \geqslant \phi_*$) induces repulsive motion $\dot{\phi}_b \sim -H^3/(8\pi^2\sigma_\phi)$ [58], so that the balance requires $\dot{\phi}_c + \dot{\phi}_H + \dot{\phi}_b \simeq 0$. When $\dot{\phi}_b \gg \dot{\phi}_c$, an equilibrium is reached with

$$\sigma_\phi \simeq \left(\frac{M_{\rm Pl}^2 H^4}{4\pi^2 V'}\right)^{1/3} \simeq \left(\frac{M_{\rm Pl}^2 M^2 H^4}{4\pi^2 g\Lambda_\phi^4}\right)^{1/3}, \tag{12}$$

which is the width in the Quantum^2 + Volume (Q^{2}V) regime [22]. The width indeed increases as V flattens; nevertheless, the v_h distribution can be arbitrarily narrowed, as will be discussed. One also expects $|\phi_{\rm peak} - \phi_*| \sim \sigma_\phi$ from dimensional ground. These heuristic discussions on Q^{2}V are demonstrated with the method of images in Appendix B of Supplemental Material [58].

A theory enters the Q^{2}V regime when the balance width becomes larger than $M_{\rm Pl}$ (the width in the global QBC):

$$g \lesssim H\frac{H}{M_{\rm Pl}}\frac{H^2M^2}{\Lambda_\phi^4}. \tag{13}$$

This is equivalent to $V' \lesssim H^3 H/M_{\rm Pl}$ [22], which is also derived from the local balance near ϕ_*. Q^{2}V is typically stronger than the global QBC and not absolutely needed for Hubble selection, but later will be useful for efficient localization of v_h.

V. THE WEAK-SCALE CRITICALITY

Finally, we come to calculate the equilibrium distribution of $\rho(v_h)$ in our model. We first discuss conditions for the successful Hubble selection of v_h^*, and then present benchmark results.

The scanning of v_h starts by ϕ rolling up its potential from $\phi < 0$ to > 0. When $\phi < 0$ ($\mu_h^2 > 0$), $v_h = 0$ and $V_h = V_\Sigma = 0$ remain unchanged with ϕ. Thus ϕ simply keeps growing, driven by quantum effects. The only constraint is that V_ϕ must not affect the inflation dynamics (condition 1): $\Lambda_\phi^4 \lesssim H^2M_{\rm Pl}^2$.

As soon as $\phi > 0$ ($\mu_h^2 \leqslant 0$), the Higgs gets the vacuum expectation value $v_h > 0$, and V_h, V_Σ minima now evolve with ϕ. $V_h = -\frac{\lambda_h}{4}v_h^4$, and coexisting vacua of V_Σ are, at leading orders in v_h,

$$V_{\Sigma(L\times R)} \simeq -a_1\Lambda_{\rm QCD}^4\frac{v_h^2}{v_{\rm EW}^2}, \tag{14}$$

$$V_{\Sigma(V)} \simeq -\Lambda_{\rm QCD}^4\left(a_2 + a_3\frac{v_h}{v_{\rm EW}}\right), \tag{15}$$

where $a_1 \simeq 114$, $a_2 \simeq 0.059$, $a_3 \simeq 1.38$ for the benchmark (Fig. 1). $\langle\sigma_0\rangle_{(L\times R)} \simeq -\langle\sigma_8\rangle \simeq a_4 v_h$ with $a_4 \simeq 0.025$. Note that $V_{h,\Sigma}$ decrease with ϕ, which must be slower than the increase of $V_\phi \simeq \frac{g\Lambda_\phi^4}{M^2}\phi$, for Hubble selection.

Which potential dominates the ϕ dynamics? Figure 2 shows individual potential with ϕ, whose slope is

$$\frac{\delta V_\phi}{\delta\phi} \sim g\frac{\Lambda_\phi^4}{M^2}, \quad \frac{\delta V_{\Sigma(L\times R)}}{\delta\phi} \sim -a_1\frac{g}{\lambda_h}\frac{\Lambda_{\rm QCD}^4}{v_{\rm EW}^2},$$
$$\frac{\delta V_h}{\delta\phi} \sim -\frac{g}{2}v_h^2, \quad \frac{\delta V_{\Sigma(V)}}{\delta\phi} \sim -a_3\frac{g}{\lambda_h}\frac{\Lambda_{\rm QCD}^4}{v_h v_{\rm EW}}. \tag{16}$$

The dominance of growing $\delta V_\phi/\delta\phi$ up to $v_h \leqslant v_h^*$ requires $\Lambda_\phi^2/M \gtrsim v_h^*$, unless v_h is too small. After $\Lambda_{\rm QCD}$-scale energy drops in V_Σ at v_h^*, dominant V_ϕ keeps growing. For large enough $v_h \gtrsim \Lambda_{\rm QCD}(\gtrsim v_h^*)$, decreasing V_h begins to dominate and is prohibited from being Hubble selected again. So we need to make sure that V_ϕ never compensates the energy drop in the intermediate region $v_h^* \lesssim v_h \lesssim \Lambda_{\rm QCD}$: $\Delta V_\phi \simeq \frac{g\Lambda_\phi^4}{M^2}\frac{\lambda_h\Lambda_{\rm QCD}^2}{g} \lesssim \Lambda_{\rm QCD}^4$. In all, V_ϕ cannot be too flat or too steep

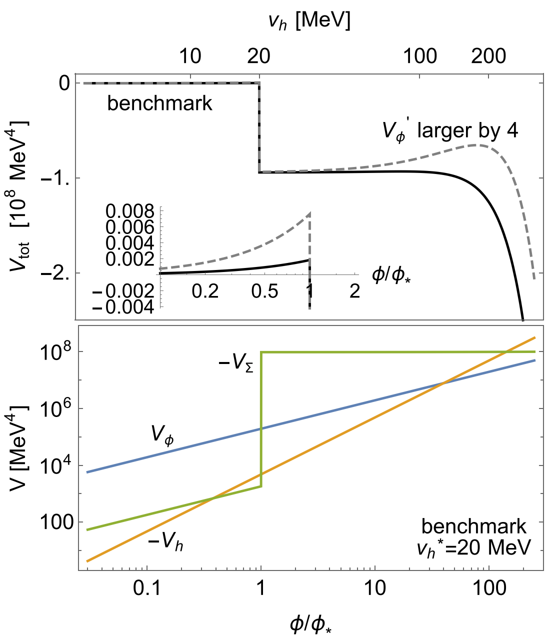

FIG. 2. Top: Total potential energy near the critical point as a function of ϕ, for the benchmark Eq. (18); dashed line for comparison. Inset: Zoom-in near ϕ_*. Bottom: Individual contribution from V_ϕ, $|V_\Sigma|$, and $|V_h|$.

(condition 2):

$$v_h^* \lesssim \Lambda_\phi^2/M \lesssim \Lambda_{\rm QCD}. \tag{17}$$

In addition, h and Σ are required to sit in their respective minima, not quantum driven to overflow their potentials. Their equilibrium widths must be small enough: $H^2/m_{h,\Sigma} \lesssim \Lambda_{\rm QCD}$, with $m_h \sim v_h$ and $m_\Sigma \sim \Lambda_{\rm QCD}$ in the higher-energy $SU(3)_L \times SU(3)_R$ vacuum. Since $v_h^* \lesssim \Lambda_{\rm QCD}$ from condition 2, we obtain (condition 3) $H \lesssim v_h^*$.

For numerical studies, we use the following benchmark (v_h^* from Sec. III),

$$\begin{aligned} &v_h^* \simeq 20\ \text{MeV}, \quad H = v_h^*, \quad M = 3\times 10^{-3} M_{\rm Pl}, \\ &\Lambda_\phi^2 = 10^{-2} H M_{\rm Pl}, \quad g = 10^{-3} H^2/M_{\rm Pl}, \end{aligned} \tag{18}$$

satisfying the global QBC [Eq. (11)](and Q^{2}V [Eq. (13)] marginally) and conditions 1–3. Potential energies near v_h^* are shown in Fig. 2. As desired, the total energy peaks sharply at v_h^*, drops significantly, and is never compensated afterwards; for much smaller or larger V_ϕ', energy would not sharply peak.

The large-time equilibrium distribution of v_h is shown in Fig. 3; see Appendix C in Supplemental Material [58] for details. The width σ_{v_h} is translated from σ_ϕ via $v_h^2 = g\phi/\lambda_h$ as

$$\sigma_{v_h} \simeq \frac{g\sigma_\phi}{2\lambda_h v_h^*}, \tag{19}$$

where $\sigma_\phi \simeq 1.3 M_{\rm Pl}$ from Eq. (12) for the benchmark with marginal Q^{2}V.[4] Thus we have $\sigma_{v_h} \simeq 0.1$ MeV $\ll v_h^*$, which is

[4]The Planckian width is a generic result of the global QBC, $\sigma_\phi \simeq \phi_*(\frac{3\phi_*^2}{2M_{\rm Pl}^2})^{-1/2} \sim M_{\rm Pl}$; see Eq. (B6) in Supplemental Material [58].

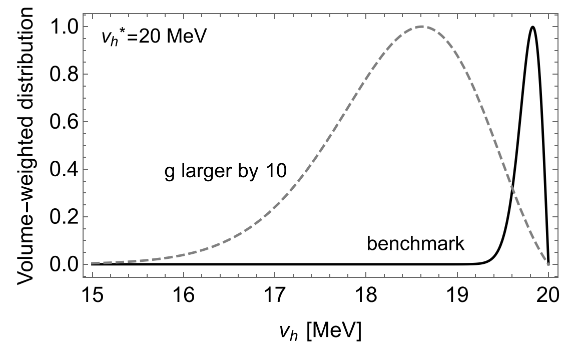

FIG. 3. The probability distribution of v_h among Hubble patches that have reached reheating. $\sigma_{v_h} \simeq 0.1$ MeV $\ll v_h^*$ for the benchmark Eq. (18); dashed line for comparison.

narrow enough so that most Hubble patches self-organize to have $v_h \approx v_h^*$ ($\sim v_{\rm EW}$). Note that it can be arbitrarily narrower at the price of arbitrarily smaller g or a larger ϕ range, moving into a deeper Q^{2}V regime; only the resulting hierarchy $f_\phi = M^2/g \gg M_{\rm Pl}$ needs to be generated consistently in field theory [46–48]. On the other limit, unwanted $\sigma_{v_h} \gtrsim v_h^*$ is resulted for 10–100 times larger g, where Q^{2}V is not even marginally satisfied.

Postinflationary dynamics is model dependent but such that ϕ slow rolls to today's $v_{\rm EW}$. Today, ϕ could be still safely slow rolling or trapped by SM backreactions. Signals from time-dependent v_h or phase transitions could be produced.

VI. DISCUSSION

In this Letter, we have discussed the self-organized criticality of the weak scale, by exploiting possible first-order quantum critical points of QCD. Although we saw some success, our exploration of critical points is much simplified and far from conclusive. We have used only LSM with $N_f = 3$ at the tree level with a simplified dependence on $v_h \leqslant v_{\rm EW}$ in Eq. (8). They shall be verified and generalized by lattice calculations [36–43], incorporating higher-order and nonperturbative effects [60–64], not only for the SM point but also away from it with $v_h \leqslant v_{\rm EW}$. $N_f > 3$ likely yields a richer vacuum structure but needs a dedicated calculation. Theoretical intuitions from confining gauge theories might also be useful. If such a critical point is indeed built in QCD, it would shed significant light on the role of near criticality of the SM.

The proposed scenario makes an advancement on the hierarchy problem, albeit not yet completely solving it. It is not complete because Hubble selection requires a mild separation of scales $\Lambda_\phi \ll M$ from Eq. (17) (if $M \sim \Lambda_\phi$ strictly, $M \lesssim \Lambda_{\rm QCD}$ is too low) but this is not quantum stable (Higgs loop diagrams with external relaxion legs yield $\Lambda_\phi \sim$ cutoff M [4,47]). Thus, a little hierarchy remains; with the fine-tuning $\epsilon \equiv \Lambda_\phi/M < 1$, the cutoff can be as high as $M \lesssim \Lambda_{\rm QCD}/\epsilon^2$. Another advancement is that choosing Λ_ϕ can be translated to a dynamical problem of choosing dimensionless parameters of the extended relaxion sector, such as in Ref. [4]. Further explorations will be enlightening.

The near criticality and naturalness of nature may be intimately connected by quantum cosmology, with necessary criticality perhaps built in just around the SM. Further theoretical and experimental studies are encouraged to unveil this connection.

ACKNOWLEDGMENTS

We thank Sang Hui Im, Hyung Do Kim, Choonkyu Lee, and Ke-Pan Xie for valuable conversations. We are supported by National Research Foundation of Korea under Grant No. NRF-2019R1C1C1010050, and S.J. also by a POSCO Science Fellowship.

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