Letter

Universal dephasing mechanism of many-body quantum chaos

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Ergodicity is a fundamental principle of statistical mechanics underlying the behavior of generic quantum many-body systems. However, how this universal many-body quantum chaotic regime emerges due to interactions remains largely a puzzle. This paper demonstrates using both heuristic arguments and a microscopic calculation that a dephasing mechanism, similar to Altshuler-Aronov-Khmelnitskii dephasing in the theory of localization, underlies this transition to chaos. We focus on the behavior of the spectral form factor (SFF) as a function of "time" t, which characterizes level correlations in the many-body spectrum. The SFF can be expressed as a sum over periodic classical orbits and its behavior hinges on the interference of trajectories related to each other by a time translation. In the absence of interactions, time-translation symmetry is present for each individual particle, which leads to a fast exponential growth of the SFF and correspondingly loss of correlations between many-body levels. Interactions lead to dephasing, which disrupts interference, and breaks the massive time-translation symmetry down to a global time-translation/energy conservation. This in turn gives rise to the hallmark linear-in-t ramp in the SFF reflecting Wigner-Dyson level repulsion. This general picture is supported by a microscopic analysis of an interacting many-body model. Specifically, we study the complex $\mathrm{SYK}_2 + \mathrm{SYK}_2^2$ model, which allows to tune between an integrable and chaotic regime. It is shown that the dephasing mass vanishes in the former case, which maps to the noninteracting complex SYK_2 model via a time reparameterization. In contrast, the chaotic regime gives rise to dephasing, which suppresses the exponential ramp of the noninteracting theory and induces correlations between many-body levels.

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There is a widely held quantum chaos conjecture [1,2], which states that the spectral statistics of quantum chaotic systems can be universally described by random matrix theory (RMT) [3–7]. This conjecture is supported by extensive experimental [8–14] and numerical studies [15–17], and has been used as one of the diagnostics of quantum chaos. However, despite numerous efforts, the theoretical understanding of the underlying general mechanism behind this conjecture is far from complete. There have been several studies, which use the semiclassical periodic orbit theory [18] to prove that the spectral form factor—a probe of two-level statistics—indeed follows the RMT prediction for single-particle quantum chaotic systems [19–24], including disordered metals [25,26]. This approach has also been generalized to certain many-body systems with a well defined semiclassical limit [27–30]. In the context of the period-orbit theory, the Wigner-Dyson level statistics stems from constructive interference between periodic paths related by a time translation. This time-translation argument has also been employed in studies of Floquet quantum circuits [31–37], periodically kicked interacting spin and fermionic chains [2,38,39], and the Sachdev-Ye-Kitaev (SYK) model [40]. An alternative method widely used in the investigation of spectral statistics of chaotic systems is a field theoretical technique known as the nonlinear σ model, in supersymmetric [41–44], replica [45–47], and Keldysh [48] frameworks. This method allows the calculation of single-particle spectral correlations, but generalization to a microscopic analysis of many-body level statistics of interacting models [49–51] remains a challenge.

In this Letter, we show that the emergence of many-body quantum chaos from the noninteracting (single-particle chaotic) model can be generally understood in terms of dephasing of single-particle trajectories mediated by interactions. This dephasing mechanism has some parallels with the Altshuler-Aronov-Khmelnitskii dephasing [52] in the weak localization (WL) theory of disordered metals [53,54], but with the characteristic energy level separation [$\sim t^{-1}$ in the SFF, $K(t)$] playing the role of the temperature T in the context of WL. Specifically, we show that just like in the theory of WL, the behavior of the SFF is governed by a collective diffuson-like mode, which acquires an infrared cutoff in the presence of interactions as follows:

$$\mathcal{D}_0(\Delta\tau) \xrightarrow{\text{interactions}} e^{-F_\phi(\Delta\tau)}\mathcal{D}_0(\Delta\tau). \qquad (1)$$

Here $\mathcal{D}_0$ represents a gapless collective mode in the noninteracting case, which in the Fourier space has the form $\mathcal{D}_0(\omega, \mathbf{q}) \propto (-i\omega + Dq^2)^{-1}$ in the case of a disordered metal (D is the diffusion coefficient) and $\mathcal{D}_0(\omega) \propto 1/\omega$ in the RMT case of "zero-dimensional diffuson." $\Delta\tau$, ω, and q are the time, frequency, and momentum coordinates, respectively. The extra factor $e^{-F_\phi(\Delta\tau)}$ acquired by the interacting collective mode represents the exponential suppression in time stemming from the dephasing processes [the dephasing function $F_\phi(\Delta\tau \to \infty) = +\infty$]. The specific form of the dephasing function F_ϕ depends on the dimension and other details [55,56].

Below, we provide a brief qualitative review of the theory of WL and periodic-orbit theory of chaos and describe heuristic arguments underlying our main result. The second part of the paper is devoted to a microscopic σ-model analysis of an interacting many-body model, where we demonstrate the appearance of a dephasing cutoff of the relevant collective modes in the ergodic regime and the absence thereof in the integrable limit.

In a weakly disordered metal, the probability for a particle to diffuse from one point to another in the semiclassical limit can be expressed as [53,54]

$$\mathcal{P} = \left| \sum_p A_p e^{\frac{i}{\hbar} S_p} \right|^2 = \sum_{pq} A_p A_q^* e^{\frac{i}{\hbar} S_p - \frac{i}{\hbar} S_q}, \tag{2}$$

where the summation runs over all classical paths, labeled by p, connecting these two points. S_p and A_p represent, respectively, the action and amplitude of the path p. In Eq. (2), the diagonal terms ($p = q$) contribute to the classical probability and are associated with the Drude conductivity, while the off-diagonal terms ($p \neq q$) correspond to the quantum interference correction. The interference between a generic pair of paths vanishes after disorder averaging due to the strong sensitivity of the action S_p to the impurity potential. However, there exist pairs of coherent paths, such as the self-intersecting ones in Fig. 1(a), whose actions are identical in the absence of magnetic field, spin-orbit interactions, and particle-particle interactions. Their interference can no longer be neglected, and results in the WL correction to the conductivity. The WL correction to the conductivity is related to the probability of finding such self-intersecting paths [53,54]

$$\delta\sigma_{\rm WL} \propto -\int_{\tau_{\rm el}}^{\infty} d\tau C(\mathbf{r}, \mathbf{r}; \tau) \propto -\int_{\tau_{\rm el}}^{\infty} d\tau \frac{1}{(D\tau)^{d/2}}. \tag{3}$$

Here $C(\mathbf{r}, \mathbf{r}'; \tau)$ represents the Cooperon (or the diffuson in the particle-particle channel), which is the Green function of the diffusion equation. Specifically, $C(\mathbf{r}, \mathbf{r}; \tau)$ measures the return probability of a diffusing particle that starts at and returns to the same point $\mathbf{r}$ in time τ. $\tau_{\rm el}$ indicates the elastic scattering time and d is the dimension of the system. At this level, WL represents a single-particle effect and formally diverges in $d \leqslant 2$.

In the presence of interactions, the coherence between the time-reversed paths is reduced through the emissions of particle-hole pairs, and is completely destroyed when the traverse time exceeds the dephasing time τ_ϕ. It has been emphasized in Ref. [54–56] that the dephasing is dominated by real inelastic collisions with energy transfer $\tau_\phi^{-1} \ll \omega \ll T$. Inelastic processes with energy transfer $\omega \gg T$ are not allowed since quasiparticles with energy $\varepsilon \sim T$ measured from the Fermi surface cannot lose energy $\omega \gg T$ due to the Pauli blocking. When energy transfer $\omega \ll \tau_\phi^{-1}$, the action difference between the time-reversed pair can be ignored and therefore their coherence is preserved. Dephasing processes result in the appearance of a mass term in the Cooperon (τ_ϕ^{-1}), which cuts off the WL integral [Eq. (3)] in the infrared limit.

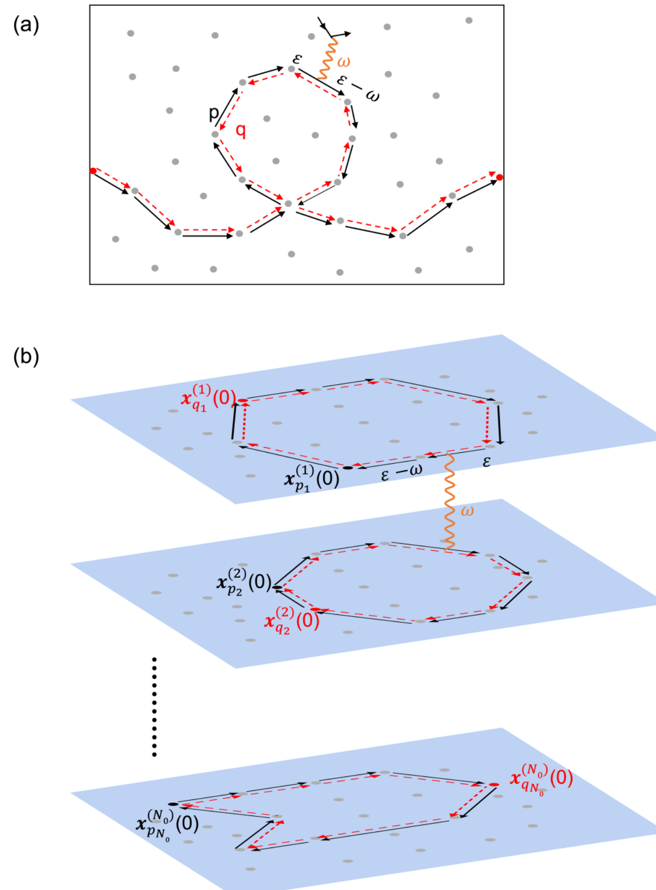

FIG. 1. Qualitative picture of the dephasing mechanism for (a) weak localization and (b) periodic-orbit theory of chaos. Panel (a) depicts two self-intersecting paths p (black lines) and q (red dashed line) of a particle moving in a disordered metal. The two paths coincide everywhere except for the loop, which is traversed in the opposite directions by them. The interference between such pair of paths leads to the WL correction to conductivity. In the presence of interactions, emissions of the electron-hole pairs (orange wavy line) result in the destruction of the phase coherence between the pair of paths and consequently a reduction in their quantum interference. Panel (b) shows two periodic paths $\{p\}$ and $\{q\}$ of a many-body system in a chaotic medium. The ith plane represents the phase space of the ith particle, and the black solid and red dashed lines in that plane describe the corresponding particle's trajectories for periodic paths p_i and q_i, respectively. Paths $\{p\}$ and $\{q\}$ are related by a "individual time translation" and differ only in the starting positions (denoted by black and red dots, respectively). In the noninteracting case, the interference between such pair of paths is essential to the exponential-in-t ramp in the SFF. In the presence of interactions, particle-particle collisions (orange wavy line) destroy the coherence unless the pair is related by a "global time translation", leading to the suppression of the exponential ramp.

To understand the effect of dephasing on spectral statistics, we now proceed with a sketch of the main idea of the periodic orbit derivation of the RMT spectral form factor (SFF) in the context of single-particle quantum chaos, see Refs. [18,57] for a more complete review. The SFF measures the correlation between two energy levels and is defined as

$$K(t) = \left\langle \mathrm{Tr}e^{-\frac{i}{\hbar}Ht}\mathrm{Tr}e^{\frac{i}{\hbar}Ht}\right\rangle, \tag{4}$$

where H is the Hamiltonian of the system and the angular bracket stands for the ensemble averaging. One can write the analytically continued partition function $\mathrm{Tr}e^{-\frac{i}{\hbar}Ht} = \int d^d\mathbf{r}\,\langle\mathbf{r}|\,e^{-\frac{i}{\hbar}Ht}\,|\mathbf{r}\rangle$ as an integral of quantum propagation amplitude over the real space position $\mathbf{r}$. The propagation amplitude, in the semiclassical limit, can be approximated as a summation over all classical paths that start and end at the same point $\mathbf{r}$ in time t. Integration over $\mathbf{r}$ further restricts the summation to be over those paths whose initial and final momenta are also identical. As a result, only periodic paths that return to its starting point in the phase space in fixed time t need to be retained. Grouping together all periodic paths that follow the same orbit and differ only in the starting points, $\mathrm{Tr}e^{-\frac{i}{\hbar}Ht}$ can be represented by a sum over periodic orbits of period t [18]:

$$\mathrm{Tr}e^{-\frac{i}{\hbar}Ht} = \sum_P A_P e^{\frac{i}{\hbar}S_P}, \tag{5}$$

with A_P and S_P being the amplitude and action of the orbit P, respectively. Here orbits that transverse the primitive orbit different numbers of times are considered as distinct, and the ones with multiple traversals will be ignored due to exponential proliferation of the primitive orbits [58]. We emphasize that each term in Eq. (5) represents the contribution from an infinite group of periodic paths that are related to each other by a time translation and share the same amplitude and action. As a result, the orbit's amplitude A_P contains a factor of t originating from the integration over all possible time shifts. Inserting Eq. (5) into the definition of the SFF [Eq. (4)], one obtains

$$K(t) = \left\langle \sum_{P,Q} A_P A_Q^* e^{\frac{i}{\hbar}S_P - \frac{i}{\hbar}S_Q}\right\rangle. \tag{6}$$

In the absence of any symmetry, the off-diagonal terms ($P \neq Q$) in Eq. (6) vanishes upon ensemble averaging because of the highly oscillatory nature of the phases [19]. One can therefore keep only the diagonal terms ($P = Q$), whose total contribution can be obtained using the Hannay-Ozorido de Almeida sum rule [58] $K(t) = \langle\sum_P |A_P|^2\rangle \simeq t/t_H$, valid for t larger than the ergodic time but smaller than the Heisenberg time t_H. This leads to the linear ramp of the SFF as predicted by RMT.

We emphasize that, unlike the diagonal term in Eq. (2), which is only responsible for the classical probability, the diagonal contribution to the SFF in Eq. (6) takes into account the quantum interference between periodic paths connected by a time translation. More specifically, for each periodic path p, one has to consider its interference with any path q that is related to the original one by $\mathbf{x}_q(t') = \mathbf{x}_p(t' + \Delta)$ for arbitrary time shift $\Delta \in (0, t)$. Here $\mathbf{x}_p(t')$ denotes the phase-space coordinate at time t' for the periodic path p, which obeys $\mathbf{x}_p(t) = \mathbf{x}_p(0)$. The factor of t in the SFF originates from the integration over all possible time shifts $\int_0^t d\Delta = t$. We can therefore see that the universal behavior of the spectral properties of quantum chaotic systems originates not from the specific details of the periodic orbits but from the quantum interference between periodic paths related by a time translation. For integrable systems, Eq. (6) can still be used to express the SFF except now the family of periodic paths contributing to each term in Eq. (5) stay on a high dimensional torus instead of a closed curve in the chaotic case. These periodic paths are not necessary related by a time translation, and the above argument, which relies on the integration over time shift is no longer applicable.

The semiclassical representation of the SFF Eq. (6) can also be generalized to many-body systems with well-defined semiclassical limit, and the summation in this case runs over periodic orbits (or tori for integrable cases) in the many-body phase space. Before moving to many-body quantum chaos, let us now consider a system of noninteracting particles whose single-particle dynamics is chaotic. In the following, for an illustrative purpose, we ignore exchange statistics (indistinguishability of particles) and use a higher dimensional vector $\mathbf{X}_{\{p\}}(t') = [\mathbf{x}^{(1)}_{p_1}(t'), \mathbf{x}^{(2)}_{p_2}(t'), \ldots, \mathbf{x}^{(N_0)}_{p_{N_0}}(t')]$ to characterize the periodic path in the many-body phase space, where $\{p\}$ represents the group of path indices for all particles. Its element is the individual particle coordinate $\mathbf{x}^{(i)}_{p_i}(t')$ in its own phase space at time t', with i being the particle index and N_0 the total number of particles. The family of periodic paths belonging to the same periodic torus can be generated by individually time translate each single-particle path: $\mathbf{x}^{(i)}_{q_i}(t') = \mathbf{x}^{(i)}_{p_i}(t' + \Delta^{(i)})$, where each time shift $\Delta^{(i)}$ can take different value within the regime $[0, t)$. The special translation where all time shifts $\Delta^{(i)}$ are identical will be called "global time translation". All these periodic paths connected by "individual time translation" share the same amplitude and action, and as a result, it is essential to take into account the quantum interference between all of them, see Fig. 1(b). Counting the total number of periodic paths within the same torus, one would estimate that the SFF becomes a polynomial in t at order N_0 instead. This is consistent with the previous study, which evaluates the SFF of a system of noninteracting fermions populating the single-particle energy levels of a $N \times N$ random matrix from a Gaussian unitary ensemble (GUE), and finds a fast growing exponential-in-t ramp [59,60]. As explained in the Supplemental Material [60], this exponential ramp is an approximation to a $\lfloor N/2\rfloor$ order polynomial in the large N limit. Based on the periodic orbit discussion above, one can deduce that the contribution to the SFF is of the order of t^{N_0} (t^{N-N_0}) when the number of fermions is $N_0 \leqslant \lfloor N/2\rfloor$ ($N_0 > \lfloor N/2\rfloor$). Taking into account all possible configurations, we arrive at a polynomial of the form $\sum_{n=1}^{\lfloor N/2\rfloor} c_n t^n$, consistent with the analytical result [59,60]. See also Refs. [61,62] for a calculation of the SFF of an noninteracting Floquet model whose single-particle evolution over one period is governed by a Haar random unitary matrix.

Let us now introduce a generic type of interactions between the particles, which leads to a transition to many-body quantum chaos. In this case, interactions destroy the phase coherence between the periodic paths connected by "individual time translation" but not those related by "global time-translation," which simply implies conservation of total energy. The dephasing mechanism in this case is analogous to that in the theory of WL, and the coherence between the periodic paths connected by "individual time translation" is destroyed through the inelastic collisions between particles. However, the energy transfer ω of the dephasing processes are no longer required to be much smaller than T (as one considers now the statistical properties of entire spectrum and the notion of temperature does not appear in the theory). Since the number of coherent paths pairs is now significantly reduced due to the dephasing processes, the exponential ramp of the noninteracting model is suppressed to a linear one. As in the case of WL, the destruction of phase coherence is reflected by a mass acquired by the "inter-replica" diffuson—the correlator of a particle and a hole governed by the forward and backward time-evolution operator e^{-iHt} and e^{iHt}, respectively. The presence of the mass in the "inter-replica" diffuson suppresses its contribution to the SFF.

Focusing on the dephasing mechanism, we compare the level statistics of an ensemble of chaotic systems with that of integrable systems, both of which are described by the following complex $\mathrm{SYK}_2 + \mathrm{SYK}_2^2$ Hamiltonian:

$$H = \sum_{i,j=1}^{N} \psi_i^{\dagger} h_{ij} \psi_j + \frac{U}{2}\left(\sum_{i,j=1}^{N} \psi_i^{\dagger} V_{ij} \psi_j\right)^2. \tag{7}$$

For the chaotic ensemble, h and V are $\mathrm{N} \times \mathrm{N}$ random Hermitian matrices drawn independently from the Gaussian unitary ensemble (GUE) with the same distribution function

$$P(M) \propto \exp\left(-\frac{N}{2J^2}\mathrm{Tr}M^2\right), \quad M = h, V. \tag{8}$$

For the integrable case, $h = V$ for each system in the ensemble and is a $N \times N$ GUE matrix that also follows the distribution function Eq. (8). We consider weak enough interaction strength U and perform a perturbative calculation in U.

In a previous study [59], we investigated the SFF of the noninteracting model ($U = 0$) and found an exponential-in-t ramp. The eigenenergy E_0 of the noninteracting model is simply related to the eigenenergy E of the interacting integrable model [Eq. (7) with $h = V$] by

$$E^{(n)} = E_0^{(n)} + \frac{U}{2}\left(E_0^{(n)}\right)^2, \tag{9}$$

where the superscript n labels the energy level. In the case of $h = V$, if one transforms to the basis where the single-particle Hamiltonian h is diagonal, the many-body Hamiltonian then acquires the simple form

$$H = \sum_{i=1}^{N} \psi_i^{\dagger} \xi_i \psi_i + \frac{U}{2}\left(\sum_{i=1}^{N} \psi_i^{\dagger} \xi_i \psi_i\right)^2, \tag{10}$$

with ξ_i being the ith eigenvalue of the matrix h. One can see that the occupation number of each single-particle level is a conserved quantity, and the $h = V$ case is integrable. The two-body interactions of the current model is of the SYK_2^2 form and different from that of a previous work by the authors [63]. This explicit form is chosen so that the interactions can be Hubbard-Stratonovich decoupled via a scalar field (see details below), in an attempt to simplify the calculation. Moreover, it allows us to study in parallel the chaotic and integrable ensembles described by the same Hamiltonian Eq. (7).

We start from the following fermionic path integral representation of the SFF [Eq. (4)] of the two ensembles described by Eq. (7):

$$K(t) = \left\langle \int \mathcal{D}(\bar{\psi}, \psi) \exp\left\{ i \sum_{a=\pm} \int_0^{t^a} dt' [\bar{\psi}_i^a(t') i\partial_{t'} \psi_i^a(t') - \zeta_a \bar{\psi}_i^a(t') h_{ij} \psi_j^a(t') - \zeta_a \frac{U}{2}\left(\bar{\psi}_i^a(t') V_{ij} \psi_j^a(t')\right)^2 \right]\right\}\right\rangle. \tag{11}$$

Here the fermionic field ψ_i^a carries a replica index $a = \pm$ (labels the forward/backward evolution) and a flavor index $i = 1, 2, \ldots, N$. It is subject to the antiperiodic boundary condition $\psi^a(t^a) = -\psi^a(0)$. $\zeta_a = \pm 1$ for $a = \pm$, and t^a is defined as $t^a = t \mp i\zeta_a 0^+$, where the infinitesimal imaginary increment $\mp i\zeta_a 0^+$ plays a key role selecting the appropriate saddle points via spontaneous breaking of unitarity [63]. To focus on the dephasing mechanism, the infinitesimal imaginary increment in t^a is ignored in the following. Through a standard derivation [60], we obtain a $\sigma-$model representation of the SFF:

$$K(t) = \frac{1}{Z} \int \mathcal{D}\phi e^{-S_0[\phi]} \int \mathcal{D}Q \exp(-S[Q, \phi]),$$
$$S_0[\phi] = -\sum_{a=\pm} \frac{i\zeta_a}{2U} \int_0^t dt' (\phi_{t'}^a)^2,$$
$$S[Q, \phi] = \frac{N}{2J^2}\mathrm{Tr}(QFQ) - N\mathrm{Tr}\ln(i\sigma^3\partial_t + iFQ). \tag{12}$$

Here, σ^3 denotes the third Pauli matrix in the replica space, Z is the normalization constant [60]. The real bosonic field ϕ is introduced to decouple the interactions and is subject to the periodic boundary condition $\phi_{t_1+t}^a = \phi_{t_1}^a$, while the Hermitian matrix field Q decouples the ensemble-averaging generated interactions and satisfies $Q_{t_1+t,t_2}^{ab} = -Q_{t_1,t_2}^{ab}$. F takes different forms for the chaotic and integrable cases:

$$F_{t_2 t_1, t_1' t_2'}^{ba, a'b'} = \begin{cases} \left(1 + \phi_{t_1}^a \phi_{t_2}^b\right) \mathbf{1}_{t_2 t_1, t_1' t_2'}^{ba, a'b'}, & h \neq V, \\ \left(1 + \phi_{t_1}^a\right)\left(1 + \phi_{t_2}^b\right) \mathbf{1}_{t_2 t_1, t_1' t_2'}^{ba, a'b'}, & h = V. \end{cases} \tag{13}$$

We emphasize that the seemingly small difference in F between the chaotic and integrable models is responsible for the strikingly contrasting behaviors of their SFFs.

In the integrable $h = V$ case, one can perform a time reparametrization:

$$Q_{t_1 t_2}^{ab} \to Q_{\tau^a(t_1), \tau^b(t_2)}^{ab}, \quad \tau^a(t_1) \equiv \int_0^{t_1} dt' (1 + \phi_{t'}^a), \tag{14}$$

after which the action becomes that of the noninteracting theory $S[Q, \phi = 0]$ with shifted time $t \to t + \bar{\phi}^{\pm}$. Here $\bar{\phi}^{\pm}$ is defined as $\bar{\phi}^{\pm} \equiv \int_0^t dt' \phi_{t'}^{\pm}$. The SFF of the integrable model is now related to that of the noninteracting model H_0 ($U = 0$) by

$$K(t) = \frac{\int \mathcal{D}\phi e^{-S_0[\phi]} \langle \mathrm{Tr} e^{-iH_0(t+\bar{\phi}^+)} \mathrm{Tr} e^{+iH_0(t+\bar{\phi}^-)} \rangle}{\int \mathcal{D}\phi e^{-S_0[\phi]}}, \tag{15}$$

and can be solved using the same approach employed in Ref. [59].

In Fig. 2, we show the numerical results of the SFFs in the integrable $h = V$ case for various interaction strengths U. For

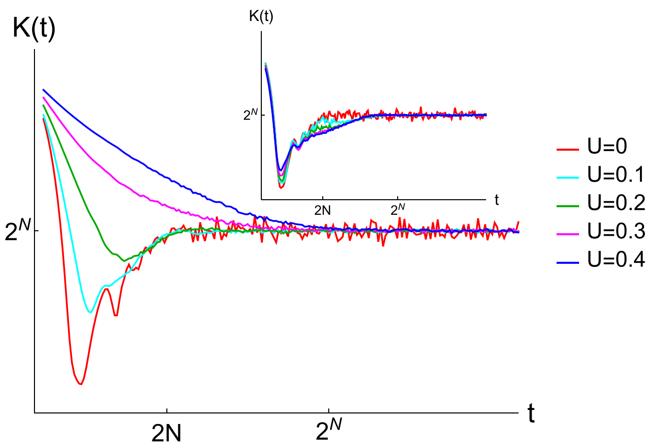

FIG. 2. Numerical results of the SFF $K(t)$ for the complex $SYK_2 + SYK_2^2$ model [Eq. (7)] in the integrable $h = V$ case and chaotic $h \neq V$ case (inset), plotted in the log-log scale. In both the main panel and the inset, the noninteracting SFF (red curve) is plotted for comparison. We set $N = 10$ and $J = 1$, and average over 1000 samples.

small interacting strength U, the SFF behaves similarly as its noninteracting counterpart (red curve), and exhibits a slope, an exponential ramp as well as a plateau that starts at time $t_p \sim N$. At larger U, the exponential ramp disappears and the SFF exhibits a nonuniversal slope approaching the plateau at a later time. This late plateau time is due to the fact that the nonuniversal slope contains a significant contribution from the slowly decaying disconnected SFF. The SFF for the $h \neq V$ chaotic case is plotted in the inset of Fig. 2. It exhibits a ramp, which grows much slower and reaches the plateau at a much later time. We note that, to see the linear in t dependence of the ramp, a computation of a larger system size is required, see for example Refs. [64,65] for numerical works studying the level statistics of similar models.

We emphasize that, for both the chaotic and integrable ensembles, the action [Eq. (12)] remains invariant under the aforementioned "global time translation":

$$\phi^a_{t_1} \to \phi^a_{t_1+\Delta^a}, \quad Q^{ab}_{t_1,t_2} \to Q^{ab}_{t_1+\Delta^a,t_2+\Delta^b}, \tag{16}$$

where Δ^a can take different values for different replica index a. For the integrable case, the theory possesses a larger symmetry group. Under any $U(2)$ rotation applying to $\left[\begin{smallmatrix} \bar{Q}^{++}_{nn}/(t+\bar{\phi}^+) & \bar{Q}^{+-}_{nm}/\sqrt{(t+\bar{\phi}^+)(t+\bar{\phi}^-)} \\ \bar{Q}^{-+}_{mn}/\sqrt{(t+\bar{\phi}^+)(t+\bar{\phi}^-)} & \bar{Q}^{--}_{mm}/(t+\bar{\phi}^-) \end{smallmatrix}\right]$, the reparametrized action is invariant for any pair of energy indices n, m satisfying $\frac{n+1/2}{t+\bar{\phi}^+} = -\frac{m+1/2}{t+\bar{\phi}^-}$.

We next perform a perturbative calculation for both the chaotic and integrable models. We consider first the integration over the matrix field Q, in the presence of arbitrary ϕ. In this case, the problem becomes similar to that of a quantum dot in the presence of a time-dependent perturbation [66–69]. Because of the large overall factor N in the action $S[Q, \phi]$, we approximate the integral with the contribution from saddle points Q_{sp} and the Gaussian fluctuations δQ around them. Taking variation of the action with respect to Q, we find the saddle point equation

$$Q_{sp} = J^2(\sigma^3\partial_t + FQ_{sp})^{-1}. \tag{17}$$

The explicit expression for its solution up to order ϕ^2 is provided in the Supplemental Material [Eqs. (S38) and (S39)] [60]. At ϕ^0 order, the solution is the noninteracting saddle point, which in the Matsubara frequency space is given by

$$\left(Q^{(0)}_{sp}\right)^{aa'}_{nn'} = \delta_{nn'}\delta_{aa'}\frac{t}{2}\left(i\zeta_a\varepsilon_n + s^a_n\sqrt{4J^2 - (\varepsilon_n)^2}\right). \tag{18}$$

Here $\varepsilon_n = 2\pi(n + 1/2)/t$ is the fermionic Matsubara frequency. s^a_n can take values of $+1$ and -1 when $|\varepsilon_n| \leqslant 2J$, and for $|\varepsilon_n| > 2J$ is determined by the fact that $(Q^{(0)}_{sp})^{aa}_{nn} \to 0$ in the limit $J \ll |\varepsilon_n|$ [48,70]. There are various saddle points corresponding to different choices of $\{s^a_n\}$ [63]. Note that here we obtain an interacting saddle point Q_{sp} in the presence of arbitrary configuration of ϕ, in contrast to the noninteracting saddle (with $\phi = 0$) usually considered in the conventional Finkel'stein NLσM calculation (e.g., Refs. [63,71]).

The action for the Gaussian fluctuations around any saddle point Q_{sp} can be expressed as

$$\delta S[\delta\tilde{Q}, \phi] = \frac{N}{2}\mathrm{Tr}(\delta\tilde{Q}M\delta\tilde{Q}), \tag{19}$$

where $\delta\tilde{Q} = F\delta Q$ [72]. See Eqs. (S58) and (S59) in the Supplemental Material [60] for the explicit expressions for M for various saddle points. The inverse of the kernel M is related to the propagator of the Gaussian fluctuation:

$$\mathcal{D}^{ba,a'b'}_{t_2t_1,t_1't_2'} \equiv N\langle\delta\tilde{Q}^{ba}_{t_2t_1}\delta\tilde{Q}^{a'b'}_{t_1't_2'}\rangle = (M^{-1})^{ba,a'b'}_{t_2t_1,t_1't_2'}. \tag{20}$$

The corresponding contribution from the Gaussian fluctuations around Q_{sp} to the SFF is proportional to

$$\int \mathcal{D}\delta\tilde{Q}\exp\left(-\frac{N}{2}\int\delta\tilde{Q}M\delta\tilde{Q}\right) \propto \exp\left(\mathrm{Tr}\ln\mathcal{D}\right). \tag{21}$$

In the following, we will focus on the inter-replica fluctuations (δQ^{ab} with $a \neq b$) around a special saddle point $Q^{(\pm)}_{sp}$ with $s^a_n = \pm\zeta_a$ (for $|\varepsilon_n| \leqslant 2J$), whose kernel M assumes a simple form [60]. The fluctuations around other saddle points can be examined in an analogous way (although not as technically straightforward). In the noninteracting theory, the inter-replica fluctuations around $Q^{(\pm)}_{sp}$ are massless, while the intra-replica fluctuations, which contribute only to the nonuniversal disconnected SFF, are massive. In the presence of interactions, for the integrable case, the inter-replica fluctuation propagator (0-dim diffuson) follows an equation equivalent to that of the noninteracting theory:

$$\left[\frac{\mp 1}{2J^3}(\partial_{t_1} - \partial_{t_2})\right]\mathcal{D}^{ba,a'b'}_{t_2t_1,t_1't_2'} = \mathbf{1}^{ba,a'b'}_{t_2t_1,t_1't_2'}, \tag{22}$$

consistent with results from time-reparametrization [60]. By contrast, for the chaotic case, the diffuson equation becomes

$$\left[\frac{\mp 1}{2J^3}(\partial_{t_1} - \partial_{t_2}) + \frac{1}{2J^2}\left(\phi^a_{t_1} - \phi^b_{t_2}\right)^2\right]\mathcal{D}^{ba,a'b'}_{t_2t_1,t_1't_2'} = \mathbf{1}^{ba,a'b'}_{t_2t_1,t_1't_2'}. \tag{23}$$

We emphasize the close resemblance of Eq. (23) to the Cooperon equations for the interaction-induced dephasing of WL in disordered systems [52,54,73–75] and perturbation-induced dephasing of dynamical localization in driven quantum dots [66,67].

While the inter-replica fluctuation remains massless for the integrable case, it acquires a mass arising from the ϕ-dependent term in Eq. (23) for the chaotic case. For simplicity, let us now consider the large t limit. The solution to Eq. (23) can now be expressed as

$$\mathcal{D}^{ba,a'b'}_{t_2t_1,t'_1t'_2} = \exp\left(-S_D[\phi]\right)(\mathcal{D}_0)^{ba,a'b'}_{t_2t_1,t'_1t'_2},$$
$$S_D[\phi] = \mp\frac{J}{2}\int_{\tau'}^{\tau} dv(\phi^a(u+v/2)-\phi^b(u-v/2))^2,$$
$$(\mathcal{D}_0)^{ba,a'b'}_{t_2t_1,t'_1t'_2} = \mp s' J^3\Theta(s'(\tau-\tau'))\delta(u-u')\delta_{aa'}\delta_{bb'}, \quad (24)$$

where $t_{1,2} = u \pm \tau/2$ and $t'_{1,2} = u' \pm \tau'/2$. $\mathcal{D}_0$ represents the solution to the noninteracting diffuson equation [Eq. (22)], and $s' = \mp 1$ is determined by the boundary condition that $\mathcal{D}$ is nondivergent when $|\tau - \tau'| \to \infty$. We can therefore see that, in contrast to the integrable case, the Gaussian fluctuation propagator $\mathcal{D}$ in the chaotic case acquires an exponential factor $\exp(-S_D[\phi])$, which decays with increasing $|\tau - \tau'|$ for almost all possible configurations of ϕ (except when $\phi^a_{t'}$ is time and replica index independent).

Substituting the Fourier transform of ϕ in $S_D[\phi]$ [Eq. (24)], we find

$$S_D[\phi] = \mp\frac{J}{2t^2}\sum_m \left(|\phi^a_m|^2 + |\phi^b_m|^2 - 2\phi^a_m\phi^b_m e^{i2\omega_m u}\right)(\tau-\tau')$$
$$\mp\frac{J}{t^2}\sum_{m\neq m'}\frac{1}{i\omega_{m-m'}}\left(e^{i\omega_{m-m'}u}\phi^a_m\phi^a_{-m'} + e^{-i\omega_{m-m'}u}\phi^b_{-m}\phi^b_{m'}\right.$$
$$\left.-e^{i\omega_{m+m'}u}2\phi^a_m\phi^b_{m'}\right)\left(e^{i\omega_{m-m'}\tau/2} - e^{i\omega_{m-m'}\tau'/2}\right). \quad (25)$$

At large $\Delta\tau = |\tau - \tau'|$ and $|u|$, the dominant contribution to S_D comes from the nonoscillating term $S_D[\phi] \approx |\tau - \tau'|/\tau_\phi$, which therefore determines the dephasing function $F_\phi[\Delta\tau] \approx \Delta\tau/\tau_\phi$ from Eq. (1). The dephasing rate is as follows

$$\tau_\phi^{-1} = \frac{J}{2t^2}\left[\sum_{m\neq 0}(|\phi^+_m|^2 + |\phi^-_m|^2) + (\phi^+_0 - \phi^-_0)^2\right]. \quad (26)$$

From Eq. (21), we can see that the diffuson's contribution to the SFF is suppressed due to the extra exponential decaying factor $e^{-S_D[\phi]}$ acquired by the interacting diffuson in the chaotic case. This decaying factor is a manifestation of the dephasing effect due to interactions. The suppression of the exponential ramp is a necessary prerequisite for the expected transition from Poisson to RMT statistics, indicating a significant role played by the dephasing in the emergence of many-body quantum chaos. The detailed calculation of the SFF for the chaotic model would require averaging det $\mathcal{D}$ over the fluctuation of decoupling field ϕ, where det $\mathcal{D}$ arises from integration over the Gaussian fluctuations [Eq. (21)]. This is out of the scope of current study and is left to future work.

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[1] O. Bohigas, M.-J. Giannoni, and C. Schmit, Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws, Phys. Rev. Lett. **52**, 1 (1984).

[2] P. Kos, M. Ljubotina, and T. Prosen, Many-Body Quantum Chaos: Analytic Connection to Random Matrix Theory, Phys. Rev. X **8**, 021062 (2018).

[3] E. P. Wigner, Characteristic vectors of bordered matrices with infinite dimensions, Ann. Math. **62**, 548 (1955).

[4] F. J. Dyson, Statistical theory of the energy levels of complex systems. I, J. Math. Phys. **3**, 140 (1962).

[5] M. L. Mehta, *Random Matrices* (Elsevier, Amsterdam, 2004)

[6] T. Guhr, A. MüllerGroeling, and H. A. Weidenmüller, Random-matrix theories in quantum physics: Common concepts, Phys. Rep. **299**, 189 (1998).

[7] V. K. B. Kota, *Embedded Random Matrix Ensembles in Quantum Physics* Vol. 3 (Springer, New York, 2014).

[8] M. Aßmann, J. Thewes, D. Fröhlich, and M. Bayer, Quantum chaos and breaking of all anti-unitary symmetries in Rydberg excitons, Nat. Mater. **15**, 741 (2016).

[9] A. Frisch, M. Mark, K. Aikawa, F. Ferlaino, J. L. Bohn, C. Makrides, A. Petrov, and S. Kotochigova, Quantum chaos in ultracold collisions of gas-phase erbium atoms, Nature (London) **507**, 475 (2014).

[10] W. Zhou, Z. Chen, B. Zhang, C. H. Yu, W. Lu, and S. C. Shen, Magnetic Field Control of the Quantum Chaotic Dynamics of Hydrogen Analogs in an Anisotropic Crystal Field, Phys. Rev. Lett. **105**, 024101 (2010).

[11] L. Vina, M. Potemski, and W. Wang, Signatures of quantum chaos in the magneto-excitonic spectrum of quantum wells, Phys.-Usp. **41**, 153 (1998).

[12] G. E. Mitchell, A. Richter, and H. A. Weidenmüller, Random matrices and chaos in nuclear physics: Nuclear reactions, Rev. Mod. Phys. **82**, 2845 (2010).

[13] H.-J. Stöckmann and J. Stein, "Quantum" Chaos in Billiards Studied by Microwave Absorption, Phys. Rev. Lett. **64**, 2215 (1990).

[14] T. Zimmermann, H. Köppel, L. S. Cederbaum, G. Persch, and W. Demtröder, Confirmation of Random-Matrix Fluctuations in Molecular Spectra, Phys. Rev. Lett. **61**, 3 (1988).

[15] S. W. McDonald and A. N. Kaufman, Spectrum and Eigenfunctions for a Hamiltonian with Stochastic Trajectories, Phys. Rev. Lett. **42**, 1189 (1979).

[16] G. Casati, F. Valz-Gris, and I. Guarnieri, On the connection between quantization of nonintegrable systems and statistical theory of spectra, Lett. Nuovo Cimento **28**, 279 (1980).

[17] M. V. Berry, Quantizing a classically ergodic system: Sinai's billiard and the KKR method, Ann. Phys. **131**, 163 (1981).

[18] M. C. Gutzwiller, *Chaos in Classical and Quantum Mechanics*, Vol. 1 (Springer Science & Business Media, New York, 2013).

[19] M. V. Berry, Semiclassical theory of spectral rigidity, Proc. R. Soc. London, Ser. A **400**, 229 (1985).

[20] N. Argaman, F.-M. Dittes, E. Doron, J. P. Keating, A. Y. Kitaev, M. Sieber, and U. Smilansky, Correlations in the Actions of

Periodic Orbits Derived from Quantum Chaos, Phys. Rev. Lett. **71**, 4326 (1993).
[21] M. Sieber and K. Richter, Correlations between periodic orbits and their role in spectral statistics, Phys. Scr. **T90**, 128 (2001).
[22] M. Sieber, Leading off-diagonal approximation for the spectral form factor for uniformly hyperbolic systems, J. Phys. A **35**, L613 (2002).
[23] S. Müller, S. Heusler, P. Braun, F. Haake, and A. Altland, Semiclassical Foundation of Universality in Quantum Chaos, Phys. Rev. Lett. **93**, 014103 (2004).
[24] S. Müller, S. Heusler, P. Braun, F. Haake, and A. Altland, Periodic-orbit theory of universality in quantum chaos, Phys. Rev. E **72**, 046207 (2005).
[25] N. Argaman, Y. Imry, and U. Smilansky, Semiclassical analysis of spectral correlations in mesoscopic systems, Phys. Rev. B **47**, 4440 (1993).
[26] R. A. Smith, I. V. Lerner, and B. L. Altshuler, Spectral statistics in disordered metals: A trajectories approach, Phys. Rev. B **58**, 10343 (1998).
[27] R. Dubertrand and S. Müller, Spectral statistics of chaotic many-body systems, New J. Phys. **18**, 033009 (2016).
[28] M. Akila, D. Waltner, B. Gutkin, P. Braun, and T. Guhr, Semiclassical Identification of Periodic Orbits in a Quantum Many-Body System, Phys. Rev. Lett. **118**, 164101 (2017).
[29] T. Engl, J. D. Urbina, and K. Richter, Periodic mean-field solutions and the spectra of discrete bosonic fields: Trace formula for Bose-Hubbard models, Phys. Rev. E **92**, 062907 (2015).
[30] T. Engl, J. Dujardin, A. Argüelles, P. Schlagheck, K. Richter, and J. D. Urbina, Coherent Backscattering in Fock Space: A Signature of Quantum Many-Body Interference in Interacting Bosonic Systems, Phys. Rev. Lett. **112**, 140403 (2014).
[31] A. Chan, A. De Luca, and J. T. Chalker, Solution of a Minimal Model for Many-Body Quantum Chaos, Phys. Rev. X **8**, 041019 (2018).
[32] A. Chan, A. De Luca, and J. T. Chalker, Spectral Statistics in Spatially Extended Chaotic Quantum Many-Body Systems, Phys. Rev. Lett. **121**, 060601 (2018).
[33] A. J. Friedman, A. Chan, A. De Luca, and J. T. Chalker, Spectral Statistics and Many-Body Quantum Chaos with Conserved Charge, Phys. Rev. Lett. **123**, 210603 (2019).
[34] S. J. Garratt and J. T. Chalker, Local Pairing of Feynman Histories in Many-Body Floquet Models, Phys. Rev. X **11**, 021051 (2021).
[35] S. J. Garratt and J. T. Chalker, Many-Body Delocalization as Symmetry Breaking, Phys. Rev. Lett. **127**, 026802 (2021).
[36] A. Chan, A. De Luca, and J. T. Chalker, Spectral Lyapunov exponents in chaotic and localized many-body quantum systems, Phys. Rev. Research **3**, 023118 (2021).
[37] B. Bertini, P. Kos, and T. Prosen, Random matrix spectral form factor of dual-unitary quantum circuits, Commun. Math. Phys. **387**, 597 (2021).
[38] B. Bertini, P. Kos, and T. Prosen, Exact Spectral Form Factor in a Minimal Model of Many-Body Quantum Chaos, Phys. Rev. Lett. **121**, 264101 (2018).
[39] D. Roy and T. Prosen, Random matrix spectral form factor in kicked interacting fermionic chains, Phys. Rev. E **102**, 060202(R) (2020).
[40] P. Saad, S. H. Shenker, and D. Stanford, A semiclassical ramp in SYK and in gravity, arXiv:1806.06840.
[41] K. Efetov, Supersymmetry and theory of disordered metals, Adv. Phys. **32**, 53 (1983).
[42] A. V. Andreev, O. Agam, B. D. Simons, and B. L. Altshuler, Quantum Chaos, Irreversible Classical Dynamics, and Random Matrix Theory, Phys. Rev. Lett. **76**, 3947 (1996).
[43] A. V. Andreev and B. L. Altshuler, Spectral Statistics beyond Random Matrix Theory, Phys. Rev. Lett. **75**, 902 (1995).
[44] A. Andreev, B. Simons, and B. Altshuler, Energy level correlations in disordered metals: Beyond universality, J. Math. Phys. **37**, 4968 (1996).
[45] A. Kamenev and M. Mézard, Wigner-Dyson statistics from the replica method, J. Phys. A **32**, 4373 (1999).
[46] I. V. Yurkevich and I. V. Lerner, Nonperturbative results for level correlations from the replica nonlinear σ model, Phys. Rev. B **60**, 3955 (1999).
[47] A. Kamenev and M. Mézard, Level correlations in disordered metals: The replica σ model, Phys. Rev. B **60**, 3944 (1999).
[48] A. Altland and A. Kamenev, Wigner-Dyson Statistics from the Keldysh σ-Model, Phys. Rev. Lett. **85**, 5615 (2000).
[49] A. Altland and D. Bagrets, Quantum ergodicity in the SYK model, Nucl. Phys. B **930**, 45 (2018).
[50] T. Micklitz, F. Monteiro, and A. Altland, Nonergodic Extended States in the Sachdev-Ye-Kitaev Model, Phys. Rev. Lett. **123**, 125701 (2019).
[51] F. Monteiro, T. Micklitz, M. Tezuka, and A. Altland, Minimal model of many-body localization, Phys. Rev. Research **3**, 013023 (2021).
[52] B. L. Altshuler, A. G. Aronov, and D. E. Khmelnitsky, Effects of electron-electron collisions with small energy transfers on quantum localisation, J. Phys. C: Solid State Phys. **15**, 7367 (1982).
[53] B.L. Altshuler and A.G. Aronov, in *Electron-Electron Interaction in Disordered Systems*, edited by A.L. Efros and M. Pollak (North-Holland, Amsterdam, 1985).
[54] I. L. Aleiner, B. L. Altshuler, and M. E. Gershenson, Interaction effects and phase relaxation in disordered systems, Waves Random Media **9**, 201 (1999).
[55] F. Marquardt, J. von Delft, R. A. Smith, and V. Ambegaokar, Decoherence in weak localization. I. Pauli principle in influence functional, Phys. Rev. B **76**, 195331 (2007).
[56] J. von Delft, F. Marquardt, R. A. Smith, and V. Ambegaokar, Decoherence in weak localization. II. Bethe-Salpeter calculation of the cooperon, Phys. Rev. B **76**, 195332 (2007).
[57] F. Haake, *Quantum Signatures of Chaos* (Springer-Verlag, Berlin, 2010).
[58] J. H. Hannay and A. M. O. D. Almeida, Periodic orbits and a correlation function for the semiclassical density of states, J. Phys. A **17**, 3429 (1984).
[59] Y. Liao, A. Vikram, and V. Galitski, Many-Body Level Statistics of Single-Particle Quantum Chaos, Phys. Rev. Lett. **125**, 250601 (2020).
[60] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevResearch.4.L012037 for the detailed derivation of the spectral form factor of the chaotic and integrable ensembles as well as a brief discussion about the ramp in the noninteracting theory.
[61] P. Diaconis and S. N. Evans, Linear functionals of eigenvalues of random matrices, Trans. Am. Math. Soc. **353**, 2615 (2001).
[62] G. Köstenberger, Weingarten calculus, arXiv:2101.00921.

[63] Y. Liao and V. Galitski, Emergence of many-body quantum chaos via spontaneous breaking of unitarity, arXiv:2104.05721.
[64] P. H. C. Lau, C.-T. Ma, J. Murugan, and M. Tezuka, Correlated disorder in the SYK2 model, J. Phys. A **54**, 095401 (2021).
[65] F. Sun, Y. Yi-Xiang, J. Ye, and W. M. Liu, Universal ratio in random matrix theory and chaotic-to-integrable transition in type-I and type-II hybrid Sachdev-Ye-Kitaev models, Phys. Rev. B **104**, 235133 (2021).
[66] D. M. Basko, M. A. Skvortsov, and V. E. Kravtsov, Dynamic Localization in Quantum Dots: Analytical Theory, Phys. Rev. Lett. **90**, 096801 (2003).
[67] M. A. Skvortsov, Quantum correction to the Kubo formula in closed mesoscopic systems, Phys. Rev. B **68**, 041306(R) (2003).
[68] M. G. Vavilov and I. L. Aleiner, Conductance fluctuations of open quantum dots under microwave radiation, Phys. Rev. B **64**, 085115 (2001).
[69] M. G. Vavilov, V. Ambegaokar, and I. L. Aleiner, Charge pumping and photovoltaic effect in open quantum dots, Phys. Rev. B **63**, 195313 (2001).
[70] M. Winer, S.-K. Jian, and B. Swingle, Exponential Ramp in the Quadratic Sachdev-Ye-Kitaev Model, Phys. Rev. Lett. **125**, 250602 (2020).
[71] A. Finkel'stein, Influence of Coulomb interaction on the properties of disordered metals, Zh. Eksp. Teor. Fiz. **84**, 168 (1983) [Sov. Phys. JETP **57**, 97 (1983)].
[72] After the transformation $\tilde{Q} = FQ$, the saddle point $\tilde{Q}_{sp}$ is related to the self-energy of the ensemble-averaged fermionic Green's function, while the propagator of Gaussian fluctuations $\mathcal{D} = M^{-1}$ is associated with the 0-dim diffuson defined by an infinite series of "impurity" ladder diagrams (see Supplemental Material [60] for details).
[73] Y. Liao and M. S. Foster, Dephasing Catastrophe in $4 - \epsilon$ Dimensions: A Possible Instability of the Ergodic (Many-Body-Delocalized) Phase, Phys. Rev. Lett. **120**, 236601 (2018).
[74] S. M. Davis and M. S. Foster, Non-Markovian dephasing of disordered quasi-one-dimensional fermion systems, Phys. Rev. B **102**, 155101 (2020).
[75] S. Chakravarty and A. Schmid, Weak localization: The quasiclassical theory of electrons in a random potential, Phys. Rep. **140**, 193 (1986).