Quantum refrigerator driven by nonclassical light

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We study a three-level quantum refrigerator which is driven by a generic light state, even a nonclassical one. With the help of P function expansion of the driving light, we obtain the heat current generated by different types of light states. It turns out all different input light states give the same coefficient of performance for this refrigerator, while the cooling power depends not only on the light intensity but also the specific photon statistics of the driving light. Comparing the coherent light with the same intensity, the driving light with super(sub)-Poissonian photon statistics could raise a smaller (stronger) cooling power. We find that this is because the bunching photons would first excite the system but then successively induce the stimulated emission, which draws the refrigerator back to the starting state of the cooling process and thus decreases the cooling current generation. This mechanism provides a more delicate control method via the high order coherence of the input light.

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I. INTRODUCTION

When a quantum heat engine runs between two reservoirs containing specific quantum coherences, the efficiency of the heat engine could exceed the Carnot limit between two canonical thermal baths [1–4]. But such exotic effects are restricted for practical applications since quantum coherences are usually quite fragile confronting the surrounding noises. In contrast, applying a quantum light to control or drive a quantum refrigerator is feasible under current techniques [5–8], promising intriguing properties. Some recent studies show that comparing the normal laser light with the same intensity using nonclassical squeezed light could help enhance the two-photon absorption rate [9] and exceed the cooling limit in laser cooling experiments [10,11].

Therefore, here we study a quantum refrigerator which is driven by different types of light states, especially the nonclassical lights. We focus on a typical quantum thermal machine composed of a three-level system in contact with two heat baths [12]. Applying a proper temperature difference to the two baths, a population inversion is generated between two levels, and the system could work as a heat engine, emitting laser light as its output work [13–16]. Reversely, when a driving light is input to the three-level system, it works as a refrigerator [Fig. 1(a)], moving the heat from the cold bath to the hot one [17]. It is also worth noting that the transition structure of this three-level refrigerator is analogous to many other physical systems [18–20], such as the laser cooling system [21–23] and photovoltaic systems [24,25].

When the driving light shining on the refrigerator is a generic quantum state, it is no longer enough to treat the driving light simply as a planar wave, which is a quasi-classical description in literature. To study the interaction with a nonclassical driving light, notice that, with the help of the P function representation, a generic light state can be regarded as the combination of many coherent states $|\alpha\rangle$ with $P(\alpha, \alpha^*)$ as the "quasiprobability," while the coherent states $|\alpha\rangle$ are the quantum correspondences for the classical planar waves [26–30]. Therefore, the full system dynamics can be obtained as the P function average of many evolution "branches," and each evolution branch can be obtained from the above quasiclassical approach, treating the driving light as a planar wave [31–34].

Based on this approach, we obtain the cooling power of this quantum refrigerator for different input light states. It turns out the coefficient of performance (COP) always remains as $e = \omega_c/(\omega_h - \omega_c) \leqslant T_c/(T_h - T_c)$, whose upper bound is just the Carnot limit for refrigerators. But the cooling powers generated by different driving lights depend on the specific photon statistics. Comparing the coherent light with the same intensity, the driving light with super(sub)-Poissonian photon statistics would raise a smaller (stronger) cooling power.

We find that this is because bunching photons would block the cooling current generation due to the stimulated emission they bring in. When a pair of bunching photons come together, the first photon would excite the refrigerator system up, but the second photon successively followed would induce the stimulated emission, drawing the system back to the previous state, and that blocks the generation of the cooling current flowing to the hot bath. Therefore, comparing the coherent

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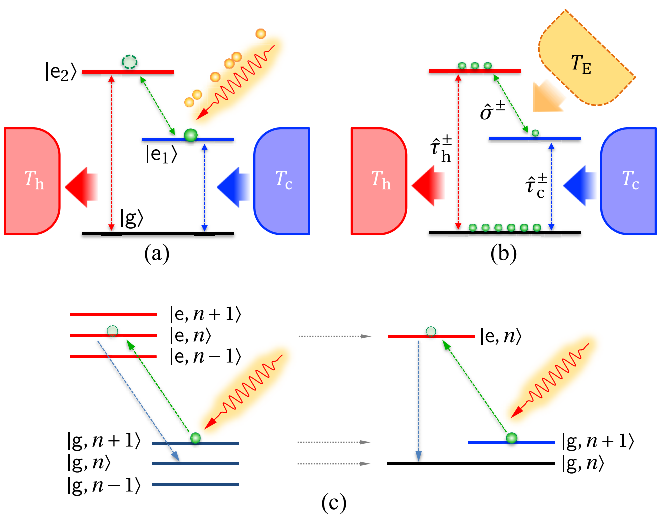

FIG. 1. Demonstrations for the the quantum refrigerator. (a) The transition $|\mathsf{e}_1\rangle \leftrightarrow |\mathsf{e}_2\rangle$ is driven by an idealistic single mode light, which may carry nonclassical photon statistics. (b) The the whole EM field is in the canonical thermal state with temperature T_{E}. (c) An analog between this three-level quantum refrigerator and the energy level structure in the sideband cooling systems, where the oscillation motion (of an ion or mechanical oscillator) is cooled down with the help of a two-level system ($|\mathsf{e}\rangle, |\mathsf{g}\rangle$) coupled with it, and $|n\rangle$ indicates the phonon states.

light where the photons come randomly both in bunches and individually, the bunching (antibunching) light could generate a smaller (stronger) cooling power under the same light intensity. Clearly the similar mechanism could also take effect in many other systems undergoing light driving.

As a comparison we also consider the situation where the whole multimode light field is in the thermal equilibrium state with a temperature T_{E} [35–38]. It turns out the thermal photon number must be larger than a certain threshold so as to make sure the system works as a refrigerator. Again that indicates the working status of the system is determined not only by the light intensity but also the specific quantum state of the light field.

The paper is arranged as follows. In Sec. II we show the basic properties of the three-level system quantum refrigerator under a coherent driving light. In Sec. III we discuss the situation that the driving light carries generic photon statistics. In Sec. IV we show that the driving light with antibunching statistics could enhance the cooling power. In Sec. V we consider the situation that the whole multimode EM field is in a thermal equilibrium state. The summary is drawn in Sec. VI, and some detailed derivations are placed in the Appendixes.

II. THE THREE-LEVEL QUANTUM REFRIGERATOR

The basic setup of the three-level quantum refrigerator is shown in Fig. 1(a), which is described by the Hamiltonian $\hat{H}_{\mathrm{s}} = \mathrm{E}_1|\mathsf{e}_1\rangle\langle\mathsf{e}_1| + \mathrm{E}_2|\mathsf{e}_2\rangle\langle\mathsf{e}_2|$ (the ground state energy is set as 0). The transition pathways $|\mathsf{e}_1\rangle \leftrightarrow |\mathsf{g}\rangle$ and $|\mathsf{e}_2\rangle \leftrightarrow |\mathsf{g}\rangle$ are coupled with two independent bosonic heat baths ($\hat{H}_{\mathrm{B},i} = \sum_k \omega_{i,k}\hat{b}^\dagger_{i,k}\hat{b}_{i,k}$, for $i = \mathrm{h,c}$), and their interaction Hamiltonians are $\hat{H}_{\mathrm{SB},i} = \hat{\tau}_i^+ \sum_k g_{i,k}\hat{b}_{i,k} + \mathrm{H.c.}$, with $\hat{\tau}^+_{\mathrm{h(c)}} := |\mathsf{e}_{2(1)}\rangle\langle\mathsf{g}| = [\hat{\tau}^-_{\mathrm{h(c)}}]^\dagger$ as the transition operator. We consider the two heat baths stay in the thermal equilibrium states with the temperatures $T_{\mathrm{h}} > T_{\mathrm{c}}$. It is worth noting that indeed the basic transition structure of the sideband cooling system is quite similar as this three-level model [Fig. 1(c)].

Here we show that, when using a light beam to drive the transition $|\mathsf{e}_1\rangle \leftrightarrow |\mathsf{e}_2\rangle$, such a three-level system could work as a quantum refrigerator, namely, the net energy flux would flow from the cold bath to the hot one [17,38].

Generally, the driving light is modeled as a classical planar wave with a single frequency mode, and its interaction with the three-level system is described by [26–29]

$$\hat{V}(t) = -\hat{\boldsymbol{d}}\cdot\vec{E}_{\mathrm{d}}\,\sin(\omega_{\mathrm{d}}t - \phi_{\mathrm{d}}). \tag{1}$$

where ω_{d} is the driving frequency, $\hat{\boldsymbol{d}} := \vec{\wp}\hat{\sigma}^- + \mathrm{H.c.}$ is the dipole operator of the three-level system, with $\vec{\wp} := \langle\mathsf{e}_1|\hat{\boldsymbol{d}}|\mathsf{e}_2\rangle$ as the transition dipole moment, and $\hat{\sigma}^- := |\mathsf{e}_1\rangle\langle\mathsf{e}_2| := (\hat{\sigma}^+)^\dagger$.

Under the Born-Markovian-rotating-wave approximation [39], the dynamics of the system can be described by the following master equation[1]:

$$\begin{aligned}\partial_t\tilde{\rho} &= \frac{i}{\hbar}[\tilde{\rho},\tilde{V}(t)] + \mathcal{L}_{\mathrm{E}}[\tilde{\rho}] + \mathcal{L}_{\mathrm{c}}[\tilde{\rho}] + \mathcal{L}_{\mathrm{h}}[\tilde{\rho}],\\ \tilde{V} &= i\mathcal{E}\hat{\sigma}^+e^{i\Delta t} - i\mathcal{E}^*\hat{\sigma}^-e^{-i\Delta t},\end{aligned} \tag{2}$$

where $\Delta := \Omega - \omega_{\mathrm{d}}$ is the detuning between the driving light and the transition frequency $\hbar\Omega := \mathrm{E}_2 - \mathrm{E}_1$, $\mathcal{E} := e^{i\phi_{\mathrm{d}}}(\vec{\wp}\cdot\vec{E}_{\mathrm{d}})/2$ is the driving strength, and the dissipations terms are

$$\begin{aligned}\mathcal{L}_{\mathrm{E}}[\tilde{\rho}] &= \kappa\big(\hat{\sigma}^-\tilde{\rho}\hat{\sigma}^+ - \tfrac{1}{2}\{\tilde{\rho},\,\hat{\sigma}^+\hat{\sigma}^-\}_+\big),\\ \mathcal{L}_i[\tilde{\rho}] &= \gamma_i\bar{\mathsf{n}}_i\big(\hat{\tau}_i^+\tilde{\rho}\hat{\tau}_i^- - \tfrac{1}{2}\{\tilde{\rho},\,\hat{\tau}_i^-\hat{\tau}_i^+\}_+\big)\\ &\quad + \gamma_i(\bar{\mathsf{n}}_i+1)\big(\hat{\tau}_i^-\tilde{\rho}\hat{\tau}_i^+ - \tfrac{1}{2}\{\tilde{\rho},\,\hat{\tau}_i^+\hat{\tau}_i^-\}_+\big),\end{aligned} \tag{3}$$

for $i = \mathrm{h,c}$. Here $\mathcal{L}_{\mathrm{h(c)}}[\tilde{\rho}]$ gives the energy exchange with the hot (cold) bath, with $\bar{\mathsf{n}}_i := [\exp(\hbar\omega_i/k_BT_i) - 1]^{-1}$ from the Planck function (with $\hbar\omega_{\mathrm{h(c)}} \equiv \mathrm{E}_{1(2)}$), and $\mathcal{L}_{\mathrm{E}}[\tilde{\rho}_{\mathrm{s}}]$ gives the spontaneous emission to the EM field.

From the master equation (2), the changing rate of the system energy gives [40]

$$\begin{aligned}\partial_t\langle\hat{H}_{\mathrm{S}}\rangle &= \mathrm{tr}\Big\{\frac{i}{\hbar}[\tilde{\rho},\tilde{V}(t)]\hat{H}_{\mathrm{S}} + \mathcal{L}_{\mathrm{E}}[\tilde{\rho}]\hat{H}_{\mathrm{S}}\Big\} + \mathrm{tr}\{\mathcal{L}_{\mathrm{c}}[\tilde{\rho}]\hat{H}_{\mathrm{S}}\}\\ &\quad + \mathrm{tr}\{\mathcal{L}_{\mathrm{h}}[\tilde{\rho}]\hat{H}_{\mathrm{S}}\} := \mathcal{Q}_{\mathrm{E}} + \mathcal{Q}_{\mathrm{c}} + \mathcal{Q}_{\mathrm{h}},\end{aligned} \tag{4}$$

where $\mathcal{Q}_{\mathrm{E}}$, $\mathcal{Q}_{\mathrm{c}}$, $\mathcal{Q}_{\mathrm{h}}$ are the energy flux flowing to the system from the EM field, cold, and hot baths, respectively. In the steady state $\partial_t\langle\hat{H}_{\mathrm{S}}\rangle = 0$, the above energy flows can be obtained by solving master equation (2) (Appendix B). Here we consider the situation that the spontaneous emission rate κ is negligible comparing the coupling strengths with the two heat bath ($\gamma_{\mathrm{h,c}} \gg \kappa \to 0$), and that gives the heat flows as

[1]Hereafter, $\tilde{o}$ indicates the operator in the interaction picture of $\hat{H}_{\mathrm{s}}$.

(let $\gamma_{\rm h}=\gamma_{\rm c}\equiv\gamma$)

$$
\begin{aligned}
&\mathcal{Q}_{\rm h}=-\hbar\omega_{\rm h}J,\quad \mathcal{Q}_{\rm c}=\hbar\omega_{\rm c}J,\quad \mathcal{Q}_{\rm E}=\hbar\Omega\,J,\\
&J=\frac{|\vec{\wp}\cdot\vec{E}_{\rm d}|^2\,(\bar{\mathsf{n}}_{\rm c}-\bar{\mathsf{n}}_{\rm h})}{\Gamma_1\mathcal{N}+4\Delta^2/\Gamma_1+|\vec{\wp}\cdot\vec{E}_{\rm d}|^2\,\Gamma_2/\gamma^2}\\
&\quad\equiv\frac{\mathscr{A}|\vec{E}_{\rm d}|^2}{\mathscr{B}+\mathscr{C}|\vec{E}_{\rm d}|^2},
\end{aligned}\tag{5}
$$

where J is the population flux, $\mathscr{A}\equiv\wp_{\rm d}^2(\bar{\mathsf{n}}_{\rm c}-\bar{\mathsf{n}}_{\rm h})$, $\mathscr{B}\equiv\Gamma_1\mathcal{N}+4\Delta^2/\Gamma_1$, $\mathscr{C}\equiv\wp_{\rm d}^2\Gamma_2/\gamma^2$ are the abbreviated coefficients (denoting $\wp_{\rm d}:=|\vec{\wp}\cdot\hat{\mathrm{e}}_{\rm d}|$, with $\hat{\mathrm{e}}_{\rm d}$ as the unit direction of $\vec{E}_{\rm d}$), and

$$
\begin{aligned}
&\Gamma_1=\gamma(2+\bar{\mathsf{n}}_{\rm c}+\bar{\mathsf{n}}_{\rm h}),\quad \Gamma_2=\gamma(2+3\bar{\mathsf{n}}_{\rm c}+3\bar{\mathsf{n}}_{\rm h}),\\
&\mathcal{N}=1+2\bar{\mathsf{n}}_{\rm h}+2\bar{\mathsf{n}}_{\rm c}+3\bar{\mathsf{n}}_{\rm c}\bar{\mathsf{n}}_{\rm h}.
\end{aligned}\tag{6}
$$

Notice that, as long as $\bar{\mathsf{n}}_{\rm c}-\bar{\mathsf{n}}_{\rm h}\geqslant 0$ with $\mathcal{E}\neq 0$, we have $\mathcal{Q}_{\rm c}\geqslant 0$ and $\mathcal{Q}_{\rm h}\leqslant 0$, which means the heat is flowing across the system from the cold bath to the hot one. Namely, the incoming light is driving the system to work as a refrigerator, and the above inequality gives the cooling condition as $\omega_{\rm c}/T_{\rm c}\leqslant\omega_{\rm h}/T_{\rm h}$. When the driving strength is weak, the cooling power is proportional to the light intensity $J\simeq(\mathscr{A}/\mathscr{B})\,|\vec{E}_{\rm d}|^2$.

As a result, the coefficient of performance (COP) for this three-level refrigerator gives

$$
e\equiv\frac{|\mathcal{Q}_{\rm c}|}{|\mathcal{Q}_{\rm h}|-|\mathcal{Q}_{\rm c}|}=\frac{\omega_{\rm c}}{\omega_{\rm h}-\omega_{\rm c}}\leqslant\frac{T_{\rm c}}{T_{\rm h}-T_{\rm c}}.\tag{7}
$$

Therefore, this COP is just bounded by the Carnot limit for refrigerators. The equality holds when the energy flows [Eq. (5)] approach zero, which indicates the quasistatic and reversible process, leading to the zero power. When the spontaneous emission rate κ is a small but finite value, the upper bound of the COP would be smaller than the Carnot limit.

III. THE DRIVING LIGHT WITH GENERIC PHOTON STATISTICS

Now we consider a more general situation where the input light driving the refrigerator could carry different kinds of photon statistics, but still has a quite small linewidth and can be regarded as a monochromatic light.

In this situation it is no longer enough to treat the driving light only as a classical planar wave which cannot reflect the photon statistics of the input light. Here we consider the EM field is fully quantized, which is described by the multimode Hamiltonian $\hat{H}_{\rm E}=\sum\hbar\omega_{\mathbf{k}}\hat{a}^\dagger_{\mathbf{k}\varsigma}\hat{a}_{\mathbf{k}\varsigma}$. The driving mode stays in a specific quantum state, while all the other field modes are in the vacuum state. Generally the quantum state of the driving mode always can be represented as the following P function:

$$
\hat{\varrho}_{\rm d}=\int d^2\alpha\,P(\alpha,\alpha^*)\,|\alpha\rangle\langle\alpha|.\tag{8}
$$

Formally this density state $\hat{\varrho}_{\rm d}$ of the driving mode can be regarded as the combination of many coherent states $|\alpha\rangle$, with $P(\alpha,\alpha^*)$ as the quasiprobability. For the quantized EM field, a single mode coherent state $|\alpha\rangle$ corresponds to a classical planar wave, since the electric field operator $\hat{\mathbf{E}}(\mathbf{x},t)\equiv\hat{\mathbf{E}}_-+\hat{\mathbf{E}}_+$ gives

$$
\begin{aligned}
&\hat{\mathbf{E}}_+(\mathbf{x},t)=(\hat{\mathbf{E}}_-)^\dagger\equiv i\sum_{\mathbf{k}\varsigma}\hat{\mathrm{e}}_{\mathbf{k}\varsigma}\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2\epsilon_0V}}\,\hat{a}_{\mathbf{k}\varsigma}e^{i\mathbf{k}\cdot\mathbf{x}-i\omega_{\mathbf{k}}t}\\
&\langle\alpha|\hat{\mathbf{E}}(\mathbf{x},t)|\alpha\rangle=\vec{E}_\alpha\sin(\omega_{\rm d}t-\mathbf{k}_{\rm d}\cdot\mathbf{x}-\phi_\alpha),
\end{aligned}\tag{9}
$$

where $\hat{\mathbf{E}}_\pm$ are the field operators with positive and negative frequencies, and $\vec{E}_\alpha:=\hat{\mathrm{e}}_{\rm d}\,|\alpha|\sqrt{2\hbar\omega_{\rm d}/\epsilon_0V}$.

In this sense, the generic state $\hat{\varrho}_{\rm d}$ of the driving light could be regarded as a "probabilistic" combination of many classical planar waves. Therefore, the system dynamics also could be obtained as the combination of many evolution branches, and in each branch the system is driven by the planar wave given by Eq. (9) that is (a rigorous proof is shown in Appendix A)

$$
\begin{aligned}
&\tilde{\rho}(t)=\int d^2\alpha\,P(\alpha,\alpha^*)\,\tilde{\rho}^{(\alpha)}(t),\\
&\partial_t\tilde{\rho}^{(\alpha)}=\frac{i}{\hbar}[\tilde{\rho}^{(\alpha)},\tilde{V}_\alpha(t)]+\mathcal{L}[\tilde{\rho}^{(\alpha)}],
\end{aligned}\tag{10}
$$

where the master equation for $\tilde{\rho}^{(\alpha)}(t)$ has the same form as the above Eq. (2) for the quasiclassical driving [31–34]. Here the driving light in $\tilde{V}_\alpha$ should be replaced by the planar wave in Eq. (9) determined by the coherent state $|\alpha\rangle$, and the driving strength in Eq. (2) now should be modified as $\mathcal{E}_\alpha:=e^{i\phi_\alpha}(\vec{\wp}\cdot\vec{E}_\alpha)/2$, respectively. Correspondingly, the expectations of the system observables are obtained as

$$
\langle\hat{o}_{\rm S}(t)\rangle\equiv\mathrm{tr}[\hat{o}_{\rm S}\cdot\hat{\rho}(t)]=\int d^2\alpha\,P(\alpha,\alpha^*)\,\langle\hat{o}_{\rm S}^{(\alpha)}(t)\rangle,\tag{11}
$$

where $\langle\hat{o}_{\rm S}^{(\alpha)}(t)\rangle:=\mathrm{tr}[\hat{o}_{\rm S}\cdot\hat{\rho}^{(\alpha)}(t)]$.

Namely, the full system evolution $\langle\hat{o}_{\rm S}(t)\rangle$ could be regarded as the probabilistic summation of many evolution branches $\langle\hat{o}_{\rm S}^{(\alpha)}(t)\rangle$, with $P(\alpha,\alpha^*)$ as their probabilities, and each branch $\langle\hat{o}_{\rm S}^{(\alpha)}(t)\rangle$ can be obtained from the above master equation with the quasiclassical driving.

Based on this method, now we study the heat flows of the above quantum refrigerator when the driving light carries generic photon statistics. Similar to the energy-flow conservation relation (4), the three heat flows are given as $\overline{\mathcal{Q}}_{\rm h}=-\hbar\omega_{\rm h}\overline{J}$, $\overline{\mathcal{Q}}_{\rm c}=\hbar\omega_{\rm c}\overline{J}$, and $\overline{\mathcal{Q}}_{\rm E}=\hbar\Omega\overline{J}$, where $\overline{J}$ is the population flux obtained from the P function average (11), namely,

$$
\overline{J}=\int d^2\alpha\,P(\alpha,\alpha^*)\,J_\alpha,\quad J_\alpha\equiv\frac{\mathscr{A}|\vec{E}_\alpha|^2}{\mathscr{B}+\mathscr{C}|\vec{E}_\alpha|^2}.\tag{12}
$$

Here J_α is the same as the above Eq. (5), which indicates the steady state flux when the driving light is a classical planar wave corresponding to the coherent state as Eq. (9).

As a result, in spite of the driving light state, the COP for this refrigerator is still $e=\omega_{\rm c}/(\omega_{\rm h}-\omega_{\rm c})$ as long as $\overline{J}\geqslant 0$, which would lead to the same cooling condition as Eq. (7).

IV. ENHANCING THE COOLING POWER BY ANTIBUNCHING PHOTONS

As long as the driving light state $P(\alpha,\alpha^*)$ is known, the heat flows of the refrigerator can be obtained from the above Eq. (12). Here we consider some typical examples of different driving light.

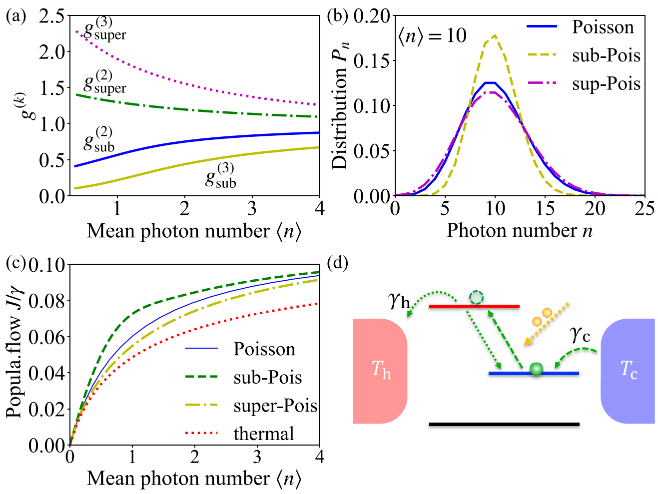

FIG. 2. (a) The optical coherence $g^{(k)}$ for the photon distributions (15). (b) Demonstration for the Poissonian, sub- and super-Poissonian statistics (15) with the same mean photon number $\langle n\rangle = 10$. (c) The cooling power under a different driving light obtained by Eq. (12). Here we set $\xi_0/\gamma = 1.1$ and $\bar{\mathsf{n}}_c = 0.5, \bar{\mathsf{n}}_h = 1$. (d) Demonstration for the heat current blockage by a pair of bunching photons. When two photons come together, the first photon would excite the system up, but the second photon successively followed would induce the stimulated emission, which draws down the energy back before it flows to the hot bath, and that decreases the cooling power.

We first consider the driving light is carrying the thermal statistics, whose P function is given by $P_{\text{th}}(\alpha) = (\pi\bar{n}_{\text{th}})^{-1}\exp[-|\alpha|^2/\bar{n}_{\text{th}}]$, with $\bar{n}_{\text{th}}$ as the mean photon number. Such a distribution is also consistent with the classical picture for the chaotic light, which is regarded as the probabilistic combination of planar waves, whose intensities satisfy the negative exponential distribution [41,42]. In this case, from Eq. (12), the population flow in the steady state gives

$$\bar{J}_{\text{th}} = \int d^2\alpha\, \frac{e^{-|\alpha|^2/\bar{n}_{\text{th}}}}{\pi\bar{n}_{\text{th}}} \frac{\xi_0^2|\alpha|^2(\bar{\mathsf{n}}_c - \bar{\mathsf{n}}_h)}{\mathscr{B} + \xi_0^2|\alpha|^2\Gamma_2/\gamma^2}. \tag{13}$$

Here $\xi_0 := \wp_d\sqrt{2\hbar\omega_d/\epsilon_0 V}$ is the single photon coupling strength, where V takes the coherence volume of the driving light [41]. And the intensity of the driving light (the Poynting vector) is $I_E \equiv 2c\epsilon_0\langle\hat{E}_-\hat{E}_+\rangle = (c/V)\,\bar{n}_{\text{th}}\hbar\omega_d$.

It turns out the monochromatic "thermal" light is also driving the system to work as a refrigerator rather than warming it up. But comparing the coherent driving [Eq. (5)] under the same light intensity, the driving light with thermal statistics generates a smaller cooling power [see Fig. 2(c)], while the COP keeps the same as Eq. (7).

A more attracting situation is when the driving light has nonclassical photon statistics, e.g., the antibunching light. For nonclassical light states, their P functions may contain negative parts, or could be highly singular functions, and thus cannot be regarded as legal probability distributions [26–30]. In these cases, besides adopting the specific P function directly, the above average (12) also can be calculated in the following way: since the P function average is equal to the normal order expectation by making the replacement $\alpha, \alpha^* \to \hat{a}, \hat{a}^\dagger$, $|\vec{E}_\alpha|^2 \to \hat{E}_-\hat{E}_+$ [43,44], the above population flow becomes [29,34,45]

$$\begin{aligned}\bar{J} &= \Big\langle :\frac{\mathscr{A}\hat{E}_-\hat{E}_+}{\mathscr{B} + \mathscr{C}\hat{E}_-\hat{E}_+}:\Big\rangle \\ &= \Big\langle :\frac{\mathscr{A}}{\mathscr{C}} - \frac{\mathscr{A}}{\mathscr{C}}\int_0^\infty ds\, e^{-s(1+\frac{\mathscr{C}}{\mathscr{B}}\hat{E}_-\hat{E}_+)}:\Big\rangle.\end{aligned} \tag{14}$$

Here $\langle : \hat{J}(\hat{a}^\dagger, \hat{a}) : \rangle$ denotes the normal order expectation. Thus, once the generation function $F(\tilde{s}) \equiv \langle : \exp(-\tilde{s}\hat{a}^\dagger\hat{a}) : \rangle$ is obtained for the driving light statistics, the heat flows of the refrigerator can be calculated from the above integral.

Here we consider two examples of the photon statistics for the driving light, i.e.,

$$P_n^{(-)} = \frac{1}{Z_-}\frac{\lambda^n}{(2n)!}, \quad P_n^{(+)} = \frac{1}{Z_+}\frac{\lambda^n}{(n+2)!}, \tag{15}$$

with $Z_\pm(\lambda)$ as the normalization factors (see Appendix C). By checking the mean photon number $\langle n\rangle$ and variance $\langle\delta n^2\rangle$, we can verify $P_n^{(+)}$ and $P_n^{(-)}$ are super- and sub-Poissonian distributions, respectively [see Figs. 2(a) and 2(b)].

The cooling currents generated by these two types of photon statistics are shown in Fig. 2(c). Comparing the coherent light with the same intensity, the driving light with super(sub)-Poissonian photon statistics produces a smaller (stronger) cooling power; with the increase of the driving light intensity, the cooling powers generated by the super- and sub-Poissonian lights both converge to the one generated by the coherent light. In contrast, the cooling power generated by the thermal light always keeps a finite difference lower than the coherent light situation.

This result can be explained with the help of the following expansion for the above normal order expectation (14),

$$\begin{aligned}\bar{J} &= \mathscr{A}\sum_{k=1}^{\infty}\frac{(-\mathscr{C})^{k-1}}{\mathscr{B}^k}\langle\hat{E}_-^k\hat{E}_+^k\rangle \\ &= \frac{\mathscr{A}}{2c\epsilon_0\mathscr{B}}I_E - \frac{\mathscr{A}\mathscr{C}}{(2c\epsilon_0\mathscr{B})^2}I_E^2 g^{(2)} + \cdots.\end{aligned} \tag{16}$$

Here $g^{(k)} \equiv \langle\hat{E}_-^k\hat{E}_+^k\rangle/\langle\hat{E}_-\hat{E}_+\rangle^k$ is the k-order coherence function of the driving light (zero time), and $I_E \equiv 2c\epsilon_0\langle\hat{E}_-\hat{E}_+\rangle$ is the light intensity.

It turns out, when the light intensity is not too strong, the heat flow induced by the driving light is proportional to the light intensity I_E in spite of the photon statistics [see the first term of Eq. (16)]. With the increase of the light intensity I_E, the high order coherence $g^{(k)}$ $(k \geqslant 2)$ of the driving light also effects the heat flow. From the minus sign of the second term in Eq. (16), comparing the situation of coherent driving $g^{(2)} = 1$, the driving lights with the sub-Poisson $(g^{(2)} < 1)$ and super-Poisson $(g^{(2)} > 1)$ statistics would generate larger and smaller flows, respectively.

Moreover, it is worth noting that for the two types of photon statistics (15), they both give $g^{(2)}_{\text{sub/super}} \to 1$ when the light intensities grow large. Therefore, the cooling powers they generate converge to the coherent light situation. In contrast, the thermal light always gives $g^{(2)}_{\text{th}} = 2$ in spite of the light intensity, thus the cooling power generated by the thermal light always keeps a finite difference lower than the coherent light situation.

This effect can be understood by the demonstration in Fig. 2(d). Generally one incoming photon would excite the system up $|\mathsf{e}_1\rangle \to |\mathsf{e}_2\rangle$, and then generate an energy flow to the hot bath. But if a pair of bunching photons come together, after the system is excited to $|\mathsf{e}_2\rangle$ by the first photon, the second photon successively followed could immediately induce the simulated emission and draw back the system to $|\mathsf{e}_1\rangle$, which prevents the energy flowing to the hot bath. Such a blockage effect depends on the competition between the releasing rate to the hot bath ($\sim\gamma_\mathrm{h}$) and the stimulated emission rate due to the incoming photons ($\sim\xi_0$).

For a driving light with the Poissonian statistics, the photons come to the system randomly, either in bunches or individually, while the bunching photon pairs would decrease the cooling current flowing from the cold bath to the hot one. Therefore, comparing the coherent light with the Poissonian statistics, the bunching (antibunching) light, which has the super(sub)-Poissonian statistics, would contribute more (less) blocking effect due to the stimulated emission, and thus produces a smaller (stronger) cooling power. It should be emphasized that here we focus on the situation that the linewidth of the driving light is negligible compared with the decay rates $\gamma_{\mathrm{h,c}}$, thus the corrections from the finite bandwidth of the driving light are omitted. If the finite linewidth is taken into consideration, a more rigorous treatment is needed, e.g., by adopting proper stochastic equations [31–33,46].

V. THE WHOLE LIGHT FIELD AS A THERMAL BATH

As a comparison, here we consider another situation where the multimode EM field as a whole is a heat bath [35–38], staying in the thermal equilibrium state $\boldsymbol{\rho}_\mathrm{E} \propto \exp[-\hat{H}_\mathrm{E}/k_\mathrm{B}T_\mathrm{E}]$ (T_E is the temperature), and there is no other driving light beam [see Fig. 1(b)].

In this case, the incoherent thermal lights are injected into the system from all different directions with different frequencies. The system dynamics is now described by the following master equation [39]:

$$\begin{aligned}\partial_t\tilde{\rho} &= \mathcal{L}'_\mathrm{E}[\tilde{\rho}] + \mathcal{L}_\mathrm{c}[\tilde{\rho}] + \mathcal{L}_\mathrm{h}[\tilde{\rho}],\\ \mathcal{L}'_\mathrm{E}[\tilde{\rho}] &= \kappa\bar{\mathsf{n}}_\mathrm{E}\big(\hat{\sigma}^+\tilde{\rho}\hat{\sigma}^- - \tfrac{1}{2}\{\tilde{\rho},\,\hat{\sigma}^-\hat{\sigma}^+\}_+\big)\\ &\quad + \kappa(\bar{\mathsf{n}}_\mathrm{E}+1)\big(\hat{\sigma}^-\tilde{\rho}\hat{\sigma}^+ - \tfrac{1}{2}\{\tilde{\rho},\,\hat{\sigma}^+\hat{\sigma}^-\}_+\big),\end{aligned} \tag{17}$$

where $\bar{\mathsf{n}}_\mathrm{E} := [\exp(\hbar\Omega/k_\mathrm{B}T_\mathrm{E}) - 1]^{-1}$ is the mean thermal photon number, and $\mathcal{L}_{\mathrm{h,c}}[\tilde{\rho}]$ are the same as Eq. (3) indicating the dissipation due to the coupling with the hot and cold baths.

Similarly as Eq. (5), the energy flowing from the system to the hot, cold, EM baths are given by the above master equation, i.e., $\partial_t\langle\hat{H}_\mathrm{S}\rangle = \mathcal{Q}'_\mathrm{E} + \mathcal{Q}'_\mathrm{c} + \mathcal{Q}'_\mathrm{h}$. In the steady state they give

$$\begin{aligned}&\mathcal{Q}'_\mathrm{h} = -\hbar\omega_\mathrm{h}J', \quad \mathcal{Q}'_\mathrm{c} = \hbar\omega_\mathrm{c}J', \quad \mathcal{Q}'_\mathrm{E} = \hbar\Omega J',\\ &J' = \frac{\kappa[\bar{\mathsf{n}}_\mathrm{E}(\bar{\mathsf{n}}_\mathrm{c} - \bar{\mathsf{n}}_\mathrm{h}) - \bar{\mathsf{n}}_\mathrm{h}(\bar{\mathsf{n}}_\mathrm{c}+1)]}{\mathcal{N} + \frac{\kappa}{\gamma}\mathcal{M}},\\ &\mathcal{M} = \bar{\mathsf{n}}_\mathrm{E}(3\bar{\mathsf{n}}_\mathrm{h} + 3\bar{\mathsf{n}}_\mathrm{c} + 2) + \bar{\mathsf{n}}_\mathrm{h} + 2\bar{\mathsf{n}}_\mathrm{c} + 1,\end{aligned} \tag{18}$$

and $\mathcal{N}$ is the same as Eq. (6).

To make the system work as a refrigerator, namely, the heat flows from the cold bath to the hot one, the above heat flows require $J' > 0$, and that gives

$$\bar{\mathsf{n}}_\mathrm{c} - \bar{\mathsf{n}}_\mathrm{h} \geqslant \frac{\bar{\mathsf{n}}_\mathrm{h}(\bar{\mathsf{n}}_\mathrm{c}+1)}{\bar{\mathsf{n}}_\mathrm{E}} \quad \Leftrightarrow \quad \frac{\omega_\mathrm{c}}{T_\mathrm{c}} + \frac{\Omega}{T_\mathrm{E}} \leqslant \frac{\omega_\mathrm{h}}{T_\mathrm{h}} \tag{19}$$

by substituting the Planck functions. When this condition is not satisfied, the heat flows from the hot bath to the cold one, and the system is not working as a refrigerator. Since $\omega_\mathrm{h} - \omega_\mathrm{c} \equiv \Omega$, this cooling condition requires that the three temperatures must satisfy $T_\mathrm{c} < T_\mathrm{h} \leqslant T_\mathrm{E}$.

Clearly the cooling condition here is different from the situation where the refrigerator is driven by a monochromatic light beam, which only requires $\bar{\mathsf{n}}_\mathrm{c} - \bar{\mathsf{n}}_\mathrm{h} \geqslant 0$ and nonzero intensity for the driving light [Eq. (5)]. Unlike the above monochromatic driving case, here the incoherent thermal lights are coming to the system from all different directions with different frequencies. It turns out the cooling condition here requires that the mean thermal photon number $\bar{\mathsf{n}}_\mathrm{E}(\Omega, T_\mathrm{E})$ in the EM bath must be larger than a certain threshold $\bar{\mathsf{n}}_\mathrm{E} \geqslant \bar{\mathsf{n}}_\mathrm{h}(\bar{\mathsf{n}}_\mathrm{c}+1)/(\bar{\mathsf{n}}_\mathrm{c} - \bar{\mathsf{n}}_\mathrm{h})$. And the COP of this refrigerator is

$$e = \frac{|\mathcal{Q}'_\mathrm{c}|}{|\mathcal{Q}'_\mathrm{h}| - |\mathcal{Q}'_\mathrm{c}|} = \frac{\omega_\mathrm{c}}{\omega_\mathrm{h} - \omega_\mathrm{c}} \leqslant \frac{T_\mathrm{c} - T_\mathrm{h}T_\mathrm{c}/T_\mathrm{E}}{T_\mathrm{h} - T_\mathrm{c}}, \tag{20}$$

whose upper bound is smaller than the above monochromatic driving case [Eq. (7)]. Such an upper bound also has been obtained in some previous studies about the quantum absorption refrigerators working between three thermal baths [35–37]. Clearly the working status of the refrigerator is significantly dependent on the the specific quantum state of the EM field, not only on the incoming light intensity.

VI. DISCUSSION

In the paper we study a three-level quantum refrigerator which is driven by a generic light state, even a nonclassical one. Since the driving light input to the refrigerator could be a generic quantum state, it is no longer enough to treat the driving light simply as a planar wave, which is a quasiclassical description in literature. With the help of the P function representation, a generic driving state can be regarded as the combination of many coherent states $|\alpha\rangle$ with $P(\alpha, \alpha^*)$ as the quasiprobability, while the coherent input states could well return the quasiclassical driving description. Therefore, the full system dynamics can be obtained as the P function average of many evolution branches, and each evolution branch is obtained from the quasiclassical approach by treating the driving light as a planar wave.

Based on this approach, it turns out all different input light states give the same COP for this refrigerator, while the cooling power depends not only on the light intensity but also the specific photon statistics of the driving light. Comparing the coherent light with the same intensity, the driving light with super(sub)-Poissonian photon statistics could raise a smaller (stronger) cooling power. We find that this is because the bunching photons could block the cooling current generation due to the the spontaneous emission they enhanced. This mechanism could provide a more delicate control method via the high order coherence of the input light.

As a comparison we also consider the situation where the multimode EM field as a whole is in the thermal equilibrium state, and the incoherent thermal lights with different frequencies are injected into the system from all different directions. It turns out the thermal photon number in the EM bath must be larger than a certain threshold so as to make the system work as a refrigerator. Therefore, the working status of the refrigerator is significantly dependent on the photon statistics and frequency distribution of the EM field, not only on the incoming light intensity.

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APPENDIX A: THE SYSTEM DYNAMICS UNDER GENERIC DRIVING LIGHT

When the photon statistics of the driving light is taken into consideration, it is no longer enough to treat the driving light only as a classical planar wave, and the light state should be described by the fully quantized EM field. In this situation the classical planar wave corresponds to a coherent state in the driving mode. Considering the whole EM field is in a multimode coherent state $\hat{\boldsymbol{\rho}}_{\mathrm{B}}^{\{\boldsymbol{\alpha}\}} := |\{\boldsymbol{\alpha}\}\rangle\langle\{\boldsymbol{\alpha}\}| = \otimes_{\mathbf{k}\varsigma}|\alpha_{\mathbf{k}\varsigma}\rangle\langle\alpha_{\mathbf{k}\varsigma}|$, the expectation of the electric field operator [Eq. (9)] gives

$$\begin{aligned}\langle\hat{\mathbf{E}}(\mathbf{x},t)\rangle &= \sum_{\mathbf{k}\varsigma}\hat{\mathrm{e}}_{\mathbf{k}\varsigma}\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2\epsilon_0 V}}\left(i\,\alpha_{\mathbf{k}\varsigma}e^{i\mathbf{k}\cdot\mathbf{x}-i\omega_{\mathbf{k}}t}+\text{H.c.}\right)\\ &:= \vec{E}_{\boldsymbol{\alpha}}(\mathbf{x},t),\end{aligned} \tag{A1}$$

which just corresponds to a classical wave package composed of many field modes, with the amplitude and phase of each mode $(\mathbf{k}\varsigma)$ determined by $\alpha_{\mathbf{k}\varsigma}$. For an idealistic monochromatic driving light, only the driving mode is in the coherent state $|\alpha\rangle$ while all the other field modes are in the vacuum states, then the electric field gives $\vec{E}_{\boldsymbol{\alpha}}(\mathbf{x},t)\to\vec{E}_{\mathrm{d}}\sin(\mathbf{k}_{\mathrm{d}}\cdot\mathbf{x}-\omega_{\mathrm{d}}t-\phi_\alpha)$, with the amplitude $\vec{E}_{\mathrm{d}}\equiv\hat{\mathrm{e}}_{\mathrm{d}}\,|\alpha|\sqrt{2\hbar\omega_{\mathrm{d}}/\epsilon_0 V}$ and $\phi_\alpha=\arg\alpha$.

To study the interaction with such an EM field in a multimode coherent state, we start from the general interaction between a two-level system and the fully quantized EM field, which is (interaction picture)

$$\begin{aligned}\tilde{H}_{\mathrm{SB}} &= -\tilde{\boldsymbol{d}}(t)\cdot\hat{\mathbf{E}}(\mathbf{x}_0,t)\\ &= -\tilde{\boldsymbol{d}}(t)\sum_{\mathbf{k}\varsigma}\hat{\mathrm{e}}_{\mathbf{k}\varsigma}\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2\epsilon_0 V}}\\ &\quad\times\left(i\,\hat{a}_{\mathbf{k}\varsigma}\,e^{i\mathbf{k}\cdot\mathbf{x}_0-i\omega_{\mathbf{k}}t}+\text{H.c.}\right),\end{aligned} \tag{A2}$$

with $\mathbf{x}_0$ as the position of the two-level system (hereafter we set $\mathbf{x}_0=0$). The field operators can be divided as the summation of their mean values and quantum fluctuations, i.e., $\hat{a}_{\mathbf{k}\varsigma}\equiv\langle\hat{a}_{\mathbf{k}\varsigma}\rangle+\delta\hat{a}_{\mathbf{k}\varsigma}=\alpha_{\mathbf{k}\varsigma}+\delta\hat{a}_{\mathbf{k}}$ and $\hat{\mathbf{E}}\equiv\vec{E}_{\boldsymbol{\alpha}}+\delta\hat{\mathbf{E}}$, then the above interaction also can be divided into two parts $\tilde{H}_{\mathrm{SB}}=\tilde{V}_{\boldsymbol{\alpha}}(t)+\tilde{H}_{\mathrm{SB}}^{(0)}$, where

$$\begin{aligned}\tilde{V}_{\boldsymbol{\alpha}}(t) &= -\tilde{\boldsymbol{d}}(t)\cdot\vec{E}_{\boldsymbol{\alpha}}(\mathbf{x}_0,t),\\ \tilde{H}_{\mathrm{SB}}^{(0)} &= -\tilde{\boldsymbol{d}}(t)\sum_{\mathbf{k}\varsigma}\hat{\mathrm{e}}_{\mathbf{k}\varsigma}\sqrt{\frac{\hbar\omega_{\mathbf{k}}}{2\epsilon_0 V}}\left(i\,\delta\hat{a}_{\mathbf{k}\varsigma}\,e^{-i\omega_{\mathbf{k}}t}+\text{H.c.}\right).\end{aligned} \tag{A3}$$

Notice that $\tilde{V}_{\boldsymbol{\alpha}}(t)$ just indicates the interaction between the electric dipole and a planar wave $\vec{E}_{\boldsymbol{\alpha}}(\mathbf{x}_0,t)$.

Furthermore, the dynamics of the system can be obtained by taking the integral iteration of the von Neumann equation (interaction picture), that is,

$$\begin{aligned}\partial_t\tilde{\boldsymbol{\rho}}_{\mathrm{SB}}(t) &= \frac{i}{\hbar}[\tilde{\boldsymbol{\rho}}_{\mathrm{SB}}(t),\,\tilde{V}_{\boldsymbol{\alpha}}(t)]+\frac{i}{\hbar}\big[\tilde{\boldsymbol{\rho}}_{\mathrm{SB}}(t),\,\tilde{H}_{\mathrm{SB}}^{(0)}(t)\big]\\ &= \frac{i}{\hbar}[\tilde{\boldsymbol{\rho}}_{\mathrm{SB}}(t),\,\tilde{V}_{\boldsymbol{\alpha}}(t)]+\frac{i}{\hbar}\big[\tilde{\boldsymbol{\rho}}_{\mathrm{SB}}(0),\,\tilde{H}_{\mathrm{SB}}^{(0)}(t)\big]\\ &\quad-\frac{1}{\hbar^2}\int_0^t ds\,\big[\big[\tilde{\boldsymbol{\rho}}_{\mathrm{SB}}(s),\,\tilde{V}_{\boldsymbol{\alpha}}(s)+\tilde{H}_{\mathrm{SB}}^{(0)}(s)\big],\,\tilde{H}_{\mathrm{SB}}^{(0)}(t)\big].\end{aligned} \tag{A4}$$

Here $\tilde{H}_{\mathrm{SB}}^{(0)}$ only contains the contribution of pure quantum fluctuation $\delta\hat{a}_{\mathbf{k}\varsigma}$, which gives the same result as the situation dealing with the spontaneous emission in the vacuum field. Thus, applying the Born-Markov RWA, the master equation for the system dynamics is obtained as [34,39]

$$\begin{aligned}\partial_t\tilde{\rho}_{\mathrm{S}}^{\{\boldsymbol{\alpha}\}} &= \frac{i}{\hbar}\big[\tilde{\rho}_{\mathrm{S}}^{\{\boldsymbol{\alpha}\}},\tilde{V}_{\boldsymbol{\alpha}}(t)\big]+\kappa\big(\hat{\sigma}^-\tilde{\rho}_{\mathrm{S}}^{\{\boldsymbol{\alpha}\}}\hat{\sigma}^+\\ &\quad-\tfrac{1}{2}\hat{\sigma}^+\hat{\sigma}^-\tilde{\rho}_{\mathrm{S}}^{\{\boldsymbol{\alpha}\}}-\tfrac{1}{2}\tilde{\rho}_{\mathrm{S}}^{\{\boldsymbol{\alpha}\}}\hat{\sigma}^+\hat{\sigma}^-\big).\end{aligned} \tag{A5}$$

The superscript $\{\boldsymbol{\alpha}\}$ indicating the initial state of the EM field is the specific multimode coherent state $\hat{\boldsymbol{\rho}}_{\mathrm{B}}^{\{\boldsymbol{\alpha}\}}$. This is just the master equation dealing with quasiclassical driving widely adopted in literature.

The above derivations also indicate that if the initial state of the EM field is not a coherent state but a general one, the above master equation (A5) for quasiclassical driving is not sufficient enough to enclose the photon statistics of the driving light. In this situation, generally, the field state always can be represented as the following multimode P function:

$$\begin{aligned}\hat{\boldsymbol{\rho}}_{\mathrm{B}}(0) &= \int d\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\}\,\boldsymbol{P}(\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\})\,|\{\boldsymbol{\alpha}\}\rangle\langle\{\boldsymbol{\alpha}\}|\\ &= \int d\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\}\,\boldsymbol{P}(\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\})\,\hat{\boldsymbol{\rho}}_{\mathrm{B}}^{\{\boldsymbol{\alpha}\}}.\end{aligned} \tag{A6}$$

Formally, the density state $\hat{\boldsymbol{\rho}}_{\mathrm{B}}(0)$ could be regarded as the combination of different coherent states $\hat{\boldsymbol{\rho}}_{\mathrm{B}}^{\{\boldsymbol{\alpha}\}}$ with $\boldsymbol{P}(\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\})$ as the quasiprobability, and each $\hat{\boldsymbol{\rho}}_{\mathrm{B}}^{\{\boldsymbol{\alpha}\}}$ corresponds to a classical wave package $\vec{E}_{\boldsymbol{\alpha}}(\mathbf{x},t)$. Then the system dynamics $\hat{\rho}_{\mathrm{S}}(t)=\mathrm{tr}[\hat{\boldsymbol{\rho}}_{\mathrm{SB}}(t)]$ can be written as

$$\begin{aligned}\hat{\rho}_{\mathrm{S}}(t) &= \mathrm{tr}_{\mathrm{B}}\big[\mathcal{U}\,\hat{\rho}_{\mathrm{S}}(0)\otimes\hat{\boldsymbol{\rho}}_{\mathrm{B}}(0)\,\mathcal{U}^\dagger\big]\\ &= \int d\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\}\,\boldsymbol{P}\big(\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\}\big)\,\mathrm{tr}_{\mathrm{B}}\big[\mathcal{U}\,\hat{\rho}_{\mathrm{S}}(0)\otimes\hat{\boldsymbol{\rho}}_{\mathrm{B}}^{\{\boldsymbol{\alpha}\}}\,\mathcal{U}^\dagger\big]\\ &= \int d\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\}\,\boldsymbol{P}\big(\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\}\big)\,\hat{\rho}_{\mathrm{S}}^{\{\boldsymbol{\alpha}\}}(t).\end{aligned} \tag{A7}$$

Here $\hat{\rho}_{\mathrm{S}}^{\{\boldsymbol{\alpha}\}}(t)\equiv\mathrm{tr}_{\mathrm{B}}[\mathcal{U}\,\hat{\rho}_{\mathrm{S}}(0)\otimes\hat{\boldsymbol{\rho}}_{\mathrm{B}}^{\{\boldsymbol{\alpha}\}}\,\mathcal{U}^\dagger]$ indicates the system evolution when the initial state of the EM field is $\hat{\boldsymbol{\rho}}_{\mathrm{B}}^{\{\boldsymbol{\alpha}\}}=$

$|\{\boldsymbol{\alpha}\}\rangle\langle\{\boldsymbol{\alpha}\}|$, which just can be given by the above master equation (A5). Correspondingly, the observable expectations of the system also can be obtained as a P function average, that is,

$$\langle\hat{o}_{\rm S}(t)\rangle = \int d\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\}\,\boldsymbol{P}\big(\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\}\big)\,\langle\hat{o}_{\rm S}^{(\boldsymbol{\alpha})}(t)\rangle, \tag{A8}$$

where $\langle\hat{o}_{\rm S}^{(\boldsymbol{\alpha})}(t)\rangle := {\rm tr}_{\rm S}[\hat{o}_{\rm S}\hat{\rho}_{\rm S}^{\{\boldsymbol{\alpha}\}}(t)]$ can be obtained as the master equation (A5). Namely, the fully system dynamics $\langle\hat{o}_{\rm S}(t)\rangle$ could be regarded as the probabilistic combination of many evolution branches $\langle\hat{o}_{\rm S}^{(\boldsymbol{\alpha})}(t)\rangle$ with $\boldsymbol{P}(\{\boldsymbol{\alpha},\boldsymbol{\alpha}^*\})$ as the quasiprobability, and each branch can be obtained by the quasiclassical driving approach.

APPENDIX B: THE ENERGY FLOW UNDER COHERENT DRIVING

Here we show the results for the steady state solution of the master equation (2), which describes the system driven by the coherent light (modeled by a planar wave). Denoting $\Delta \equiv \Omega - \omega_{\rm d}$ as the detuning between the driving light and the transition frequency $\hbar\Omega \equiv {\rm E}_2 - {\rm E}_1$, under the interaction picture defined by $\hat{H}_\Delta := \hat{H}_S - \Delta|{\rm e}_2\rangle\langle{\rm e}_2|$, the expectations of $\hat{\rm N}_{1(2)} \equiv |{\rm e}_{1(2)}\rangle\langle{\rm e}_{1(2)}|$, $\hat{\rm N}_{\rm g} \equiv |{\rm g}\rangle\langle{\rm g}|$, and $\hat{\tau}_{\rm E}^+ \equiv (\hat{\tau}_{\rm E}^-)^\dagger \equiv |{\rm e}_2\rangle\langle{\rm e}_1|$) form a closed set of time-independent equations, which read (setting $\gamma_{\rm h} = \gamma_{\rm c} \equiv \gamma$)

$$\begin{aligned}
\partial_t\langle\hat{\rm N}_1\rangle &= -[\mathcal{E}\langle\hat{\tau}_{\rm E}^+\rangle + \mathcal{E}^*\langle\hat{\tau}_{\rm E}^-\rangle] - \gamma(\bar{\rm n}_{\rm c}+1)\langle\hat{\rm N}_1\rangle \\
&\quad + \kappa\langle\hat{\rm N}_2\rangle + \gamma\bar{\rm n}_{\rm c}\langle\hat{\rm N}_{\rm g}\rangle,\\
\partial_t\langle\hat{\rm N}_2\rangle &= [\mathcal{E}\langle\hat{\tau}_{\rm E}^+\rangle + \mathcal{E}^*\langle\hat{\tau}_{\rm E}^-\rangle] - \gamma(\bar{\rm n}_{\rm h}+1)\langle\hat{\rm N}_2\rangle \\
&\quad - \kappa\langle\hat{\rm N}_2\rangle + \gamma\bar{\rm n}_{\rm h}\langle\hat{\rm N}_{\rm g}\rangle,\\
\partial_t\langle\hat{\tau}_{\rm E}^+\rangle &= i\Delta\langle\hat{\tau}_{\rm E}^+\rangle + \mathcal{E}^*[\langle\hat{\rm N}_1\rangle - \langle\hat{\rm N}_2\rangle] \\
&\quad - \tfrac{1}{2}[\gamma(\bar{\rm n}_{\rm c}+1) + \gamma(\bar{\rm n}_{\rm h}+1) + \kappa]\langle\hat{\tau}_{\rm E}^+\rangle,
\end{aligned} \tag{B1}$$

and $\langle\hat{\rm N}_1\rangle + \langle\hat{\rm N}_2\rangle + \langle\hat{\rm N}_{\rm g}\rangle \equiv 1$. In the steady state $t\to\infty$, the system becomes stationary and the above time derivatives give zero. Here we consider the situation where the spontaneous emission rate κ is negligible comparing the coupling strengths with the two heat baths, i.e., $\gamma \gg \kappa \simeq 0$, and then the steady state solution gives

$$\begin{aligned}
\langle\hat{\rm N}_1\rangle &= \frac{4\Delta^2\gamma^2\bar{\rm n}_{\rm c}(1+\bar{\rm n}_{\rm h}) + 4|\mathcal{E}|^2\gamma(\bar{\rm n}_{\rm c}+\bar{\rm n}_{\rm h})\Gamma_1 + \gamma^2\bar{\rm n}_{\rm c}(1+\bar{\rm n}_{\rm h})\Gamma_1^2}{\Phi},\\
\langle\hat{\rm N}_2\rangle &= \frac{4\Delta^2\gamma^2\bar{\rm n}_{\rm h}(1+\bar{\rm n}_{\rm c}) + 4|\mathcal{E}|^2\gamma(\bar{\rm n}_{\rm c}+\bar{\rm n}_{\rm h})\Gamma_1 + \gamma^2\bar{\rm n}_{\rm h}(1+\bar{\rm n}_{\rm c})\Gamma_1^2}{\Phi},\\
\langle\hat{\tau}_{\rm E}^+\rangle &= \big[\langle\hat{\tau}_{\rm E}^-\rangle\big]^* = \frac{4i\Delta\,\mathcal{E}^*\gamma^2(\bar{\rm n}_{\rm c}-\bar{\rm n}_{\rm h}) + 2\mathcal{E}^*\gamma^2(\bar{\rm n}_{\rm c}-\bar{\rm n}_{\rm h})\Gamma_1}{\Phi},
\end{aligned} \tag{B2}$$

where $\Phi = 4\Delta^2\gamma^2\mathcal{N} + \Gamma_1(\Gamma_1\gamma^2\mathcal{N} + 4|\mathcal{E}|^2\Gamma_2)$ and

$$\Gamma_1 = \gamma(2+\bar{\rm n}_{\rm c}+\bar{\rm n}_{\rm h}),\quad \Gamma_2 = \gamma(2+3\bar{\rm n}_{\rm c}+3\bar{\rm n}_{\rm h}),\quad \mathcal{N} = 1+2\bar{\rm n}_{\rm h}+2\bar{\rm n}_{\rm c}+3\bar{\rm n}_{\rm c}\bar{\rm n}_{\rm h}. \tag{B3}$$

Based on the master equation, the heat flows defined in Eq. (4) give

$$\mathcal{Q}_{\rm h(c)} = \hbar\omega_{\rm h(c)}\cdot\gamma_{\rm h(c)}[\bar{\rm n}_{\rm h(c)}\langle\hat{\rm N}_{\rm g}\rangle - (\bar{\rm n}_{\rm h(c)}+1)\langle\hat{\rm N}_{2(1)}\rangle],\quad \mathcal{Q}_{\rm E} = \hbar\Omega[\mathcal{E}\langle\hat{\tau}_{\rm E}^+\rangle + \mathcal{E}^*\langle\hat{\tau}_{\rm E}^-\rangle] - \hbar\Omega\kappa\langle\hat{\rm N}_2\rangle. \tag{B4}$$

By substituting the steady state solutions (B2) into the above flows, they give the steady heat flows as Eq. (5) in the main text.

APPENDIX C: THE NORMAL ORDER EXPECTATION FOR THE HEAT FLOW GENERATED BY GENERIC PHOTON STATISTICS

Here we show how to calculate the normal order expectation for the population flow in Eq. (14). Notice that the population flow has been turned into the integral of the normal order expectation $\langle: e^{-\tilde{s}\hat{a}^\dagger\hat{a}} :\rangle \equiv F(\tilde{s})$, which is the critical part to be calculated. Here we consider the density state of the monochromatic driving light is diagonal in the Fock basis, i.e., $\rho = \sum P_n|n\rangle\langle n|$ with P_n as the photon number distribution, thus the characteristic function $F(\tilde{s})$ gives

$$F(\tilde{s}) = \sum_{k=0}^{\infty}\frac{(-\tilde{s})^k}{k!}\langle(\hat{a}^\dagger)^k\hat{a}^k\rangle = \sum_{k=0}^{\infty}\sum_{m,n=0}^{\infty}\rho_{mn}\frac{(-\tilde{s})^k}{k!}\langle n|(\hat{a}^\dagger)^k\hat{a}^k|m\rangle = \sum_{n=0}^{\infty}\sum_{k=0}^{n}\rho_{nn}\frac{(-\tilde{s})^k n!}{k!(n-k)!} = \sum_n P_n(1-\tilde{s})^n. \tag{C1}$$

For the examples of the Poisson, sub-Poisson, and super-Poisson statistics discussed in the main text, they are

$$\begin{gathered}
\text{Poisson: } P_n = \frac{1}{e^\lambda}\frac{\lambda^n}{n!},\quad \Rightarrow\ F(\tilde{s}) = e^{-\tilde{s}\lambda},\quad \text{sub-Poisson: } P_n^{(-)} = \frac{1}{Z_-(\lambda)}\frac{\lambda^n}{(2n)!},\\
\Rightarrow\ F(\tilde{s}) = \frac{Z_-\big(\lambda(1-\tilde{s})\big)}{Z_-(\lambda)},\quad Z_-(\lambda) \equiv \cosh\sqrt{\lambda},\,.\\
\text{super-Poisson: } P_n^{(+)} = \frac{1}{Z_+(\lambda)}\frac{\lambda^n}{(n+2)!}\quad \Rightarrow\ F(\tilde{s}) = \frac{Z_+\big(\lambda(1-\tilde{s})\big)}{Z_+(\lambda)},\quad Z_+(\lambda) \equiv \frac{e^\lambda - \lambda - 1}{\lambda^2}
\end{gathered} \tag{C2}$$

Then the population flow (14) can be further calculated from the integral numerically, i.e.,

$$\bar{J} = \Big\langle :\frac{\mathscr{A}\hat{E}_-\hat{E}_+}{\mathscr{B}+\mathscr{C}\hat{E}_-\hat{E}_+}: \Big\rangle = \Big\langle :\frac{\mathscr{A}}{\mathscr{C}} - \frac{\mathscr{A}}{\mathscr{C}}\int_0^\infty ds\, e^{-s(1+\frac{\mathscr{C}}{\mathscr{B}}\hat{E}_-\hat{E}_+)} : \Big\rangle$$
$$= \frac{\mathscr{A}}{\mathscr{C}} - \frac{\mathscr{A}}{\mathscr{C}}\int_0^\infty ds\, e^{-s} F\Big(\frac{\mathscr{C}}{\mathscr{B}}\frac{\hbar\omega_d}{2\epsilon_0 V}s\Big). \tag{C3}$$

APPENDIX D: THE ENERGY FLOW GENERATED BY THE MULTIMODE THERMAL FIELD

Here we show the results for the energy flow when the whole multimode EM field is in the thermal equilibrium state $\boldsymbol{\rho}_{\text{E}} \propto \exp[-\hat{H}_{\text{E}}/k_B T_{\text{E}}]$ with T_{E} as the temperature. In this case the incoherent thermal lights with different frequencies are injected into the system from all different directions, and there is no other driving light beam. The system dynamics is described by the master equation (17), and that gives

$$\partial_t\langle\hat{\text{N}}_1\rangle = \kappa[(\bar{\text{n}}_{\text{E}}+1)\langle\hat{\text{N}}_2\rangle - \bar{\text{n}}_{\text{E}}\langle\hat{\text{N}}_1\rangle] - \gamma[(\bar{\text{n}}_{\text{c}}+1)\langle\hat{\text{N}}_1\rangle - \bar{\text{n}}_{\text{c}}\langle\hat{\text{N}}_{\text{g}}\rangle],$$
$$\partial_t\langle\hat{\text{N}}_2\rangle = -\kappa[(\bar{\text{n}}_{\text{E}}+1)\langle\hat{\text{N}}_2\rangle - \bar{\text{n}}_{\text{E}}\langle\hat{\text{N}}_1\rangle] - \gamma[(\bar{\text{n}}_{\text{h}}+1)\langle\hat{\text{N}}_2\rangle - \bar{\text{n}}_{\text{h}}\langle\hat{\text{N}}_{\text{g}}\rangle]. \tag{D1}$$

In the steady state $t\to\infty$, $\partial_t\langle\hat{\text{N}}_{1,2}\rangle = 0$ and $\langle\hat{\text{N}}_1\rangle + \langle\hat{\text{N}}_2\rangle + \langle\hat{\text{N}}_{\text{g}}\rangle \equiv 1$ give the solution as

$$\langle\hat{\text{N}}_1\rangle = \frac{\bar{\text{n}}_{\text{c}}(\bar{\text{n}}_{\text{h}}+1) + \frac{\kappa}{\gamma}(\bar{\text{n}}_{\text{E}}+1)(\bar{\text{n}}_{\text{h}}+\bar{\text{n}}_{\text{c}})}{\mathcal{N}+\frac{\kappa}{\gamma}\mathcal{M}},$$
$$\langle\hat{\text{N}}_2\rangle = \frac{\bar{\text{n}}_{\text{h}}(\bar{\text{n}}_{\text{c}}+1) + \frac{\kappa}{\gamma}(\bar{\text{n}}_{\text{E}}+1)(\bar{\text{n}}_{\text{h}}+\bar{\text{n}}_{\text{c}})}{\mathcal{N}+\frac{\kappa}{\gamma}\mathcal{M}},$$
$$\mathcal{M} \equiv \bar{\text{n}}_{\text{E}}(3\bar{\text{n}}_{\text{h}}+3\bar{\text{n}}_{\text{c}}+2) + \bar{\text{n}}_{\text{h}} + 2\bar{\text{n}}_{\text{c}} + 1, \tag{D2}$$

and $\mathcal{N}$ is the same as Eq. (6) in the coherent driving case. With the help of the master equation (17), the energy flowing from the three reservoirs to the system is defined from the conservation relation $\partial_t\langle\hat{H}_S\rangle = \mathcal{Q}'_{\text{E}} + \mathcal{Q}'_{\text{c}} + \mathcal{Q}'_{\text{h}}$. In the steady state they give

$$\mathcal{Q}'_{\text{h}} = -\hbar\omega_{\text{h}}J', \quad \mathcal{Q}'_{\text{c}} = \hbar\omega_{\text{c}}J', \quad \mathcal{Q}'_{\text{E}} = \hbar\Omega J',$$
$$J' = \frac{\kappa[\bar{\text{n}}_{\text{E}}(\bar{\text{n}}_{\text{c}}-\bar{\text{n}}_{\text{h}}) - \bar{\text{n}}_{\text{h}}(\bar{\text{n}}_{\text{c}}+1)]}{\mathcal{N}+\frac{\kappa}{\gamma}\mathcal{M}}, \tag{D3}$$

When $J'>0$, the heat current flows from the cold bath to the hot one, and thus the system works as a refrigerator, and that requires the cooling condition (19) in the main text.

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