Ginzburg effect in a dielectric medium with dispersion and dissipation

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As a quantum analog of Cherenkov radiation, an inertial photon detector moving through a medium with constant refractive index n may perceive the electromagnetic quantum fluctuations as real photons if its velocity v exceeds the medium speed of light c/n. For dispersive Hopfield-type media, we find this Ginzburg effect to extend to much lower v because the phase velocity of light is very small near the medium resonance. In this regime, however, dissipation effects become important. Via an extended Hopfield model, we present a consistent treatment of quantum fluctuations in dispersive and dissipative media and derive the Ginzburg effect in such systems. Finally, we propose an experimental test.

I. INTRODUCTION

As already realized by Fulling half a century ago, the particle concept in relativistic quantum field theories is not unique [1]. This nonuniqueness lies at the heart of many phenomena, such as Hawking radiation [2,3], cosmological particle creation [4–6], and acceleration radiation (a.k.a. the Unruh effect [7,8]). Adopting a quite pragmatic point of view, one could say that a particle is what makes a particle detector click. Even though this appears as a tautology, it summarizes the idea that particles are defined via their interplay with matter (e.g., in the form of detectors).

An inertial detector perceives the Minkowski vacuum as empty (of particles) because energy conservation implies that the detector must absorb a quantum from the field (which does not exist in the vacuum) in order to click. In more formal terms, the quantum vacuum fluctuations only contain positive frequencies with respect to the detector's proper time (consistent with the *spectrum condition* of the Wightman function [9–11]).

However, a nonzero response may occur, for instance, for a uniformly accelerated detector, which perceives the Minkowski vacuum as a thermal bath [7] because the quantum vacuum fluctuations, translated to the detector's proper time, also contain negative frequencies [12].

In theories with deformed or broken Lorentz invariance (e.g., due to a modified dispersion relation), even inertial detectors may click if they move at superluminal velocities—or, more precisely, at speeds faster than the phase velocity of the surrounding quantum vacuum fluctuations [13–20]. As a laboratory implementation, one could envision an inertial detector moving through a dielectric at a velocity greater than the medium speed of light. This phenomenon is referred to as the Ginzburg effect [21,22] and can be regarded as a quantum analog to Cherenkov radiation [23–32] which is further related to quantum friction [19,33–35] (see also Refs. [36–44]).

Actually observing the Ginzburg effect in a dielectric medium is hampered by the typically very large phase velocities. As a work-around idea, one could imagine using frequencies close to the medium resonance where the dielectric permittivity $\varepsilon(\omega)$ becomes very large and thus the phase velocity is sufficiently small (see also Refs. [45,46]). However, close to resonance, the imaginary part of $\varepsilon(\omega)$ also becomes very large such that dissipation starts to play an important role. Therefore studying the Ginzburg effect necessitates a consistent treatment of quantum fluctuations in the presence of dispersion and dissipation; see also Ref. [47]. In the following, we present such an approach based on a generalized version of the well-known Hopfield model.

II. MODEL OF ONE-DIMENSIONAL MEDIUM

We describe the dispersive and dissipative medium by the following Lagrangian ($\hbar = c = 1$),

$$L = \frac{1}{2} \int dx \, \{ [\partial_t A(t,x)]^2 - [\partial_x A(t,x)]^2$$
$$+ [\partial_t \Psi(t,x)]^2 - \Omega^2 \Psi^2(t,x) + 2gA(t,x)\partial_t \Psi(t,x) \}$$

$$+ \frac{1}{2} \int dx \, d\xi \, \{ [\partial_t \Phi(t,x,\xi)]^2 - [\partial_\xi \Phi(t,x,\xi)]^2 \}$$

$$+ G \int dx \, \Phi(t,x,\xi=0) \partial_t \Psi(t,x) . \tag{1}$$

The first two lines correspond to the well-known Hopfield model [48] in 1+1 dimensions—a generalization to 3+1 dimensions will be discussed in Sec. VII below. Here A represents the electromagnetic vector potential and Ψ denotes the polarization of the medium with the resonance frequency Ω while g is the coupling between them. The associated transition dipole moments are assumed to be fixed.

In order to include dissipation, the third line contains an environment field Φ propagating in an additional ξ direction (which could be an internal coordinate or an external dimension, e.g., y). This field Φ can carry away energy to $\xi \to \pm\infty$ and is coupled to the medium Ψ with the strength G in the fourth line [49]—providing a very simplified pathway for dissipation. While this is not the most general approach (see also Refs. [50–54]), it considers a simple and intuitive loss mechanism—see Appendix A for a comparison with existing approaches.

The equations of motion derived from Eq. (1) read

$$\left[\partial_t^2 - \partial_x^2 \right] A(t,x) = g \partial_t \Psi(t,x)$$

$$\left[\partial_t^2 + \Omega^2 \right] \Psi(t,x) = -g \partial_t A(t,x) - G \partial_t \Phi(t,x,0)$$

$$\left[\partial_t^2 - \partial_\xi^2 \right] \Phi(t,x,\xi) = G \partial_t \Psi(t,x) \delta(\xi) . \tag{2}$$

The last one can be solved in terms of the retarded Green's function of the d'Alembertian in 1+1 dimensions

$$\Phi(t,x,\xi) = \Phi_0(t,x,\xi) + \frac{G}{2} \Psi(t-|\xi|,x) , \tag{3}$$

where $\Phi_0(t,x,\xi)$ denotes the general (free-field) solution of the homogeneous problem $[\partial_t^2 - \partial_\xi^2]\Phi_0(t,x,\xi) = 0$. Inserting this solution (3) back into the second line of Eq. (2), we see that the (driven) harmonic oscillators Ψ representing the medium in the usual Hopfield model turn into damped (and driven) harmonic oscillators, i.e., $[\partial_t^2 + \Omega^2]\Psi \to [\partial_t^2 + 2\Gamma\partial_t + \Omega^2]\Psi$ with the effective damping rate $\Gamma = G^2/4$. In the following, we will focus on the underdamped regime of $\Gamma < \Omega$.

Finally, incorporating the first line of Eq. (2), we arrive at the following decoupled equation for A,

$$\left[\left(\partial_t^2 + \frac{G^2}{2} \partial_t + \Omega^2 \right) \left(\partial_t^2 - \partial_x^2 \right) + g^2 \partial_t^2 \right] A(t,x)$$

$$= -gG \partial_t^2 \Phi_0(t,x,\xi=0) . \tag{4}$$

III. QUANTIZATION

The homogeneous solution Φ_0 is just a free field in 2+1 dimensions $(x, \xi$ and $t)$, albeit propagating in ξ direction only, and can thus be quantized in the standard manner

$$\hat{\Phi}_0(t,x,\xi) = \int \frac{dk \, d\kappa}{2\pi} \frac{\hat{b}_{k\kappa} e^{i(kx+\kappa\xi-|\kappa|t)}}{\sqrt{2|\kappa|}} + \text{H.c.,} \tag{5}$$

where the frequency $|\kappa|$ is independent of the wave-number k in x direction and just reflects the wave-number κ in the

propagation direction ξ. As usual, $\hat{b}_{k\kappa}^\dagger$ and $\hat{b}_{k\kappa}$ denote bosonic creation and annihilation operators.

Inserting this expression (5) into the right-hand side of Eq. (4) and omitting the homogeneous solutions of Eq. (4) which are exponentially suppressed with damping time determined by G for late times (i.e., in a steady state), we find

$$\hat{A}(t,x) = \int \frac{d\kappa \, dk}{2\pi} \, gG \frac{\kappa^2 \hat{b}_{k\kappa} e^{ikx-i|\kappa|t}}{\zeta_{k\kappa} \sqrt{2|\kappa|}} + \text{H.c.,} \tag{6}$$

where $\zeta_{k\kappa}$ represents the spectral decomposition

$$\zeta_{k\kappa} = \left[\Omega^2 - \kappa^2 - \frac{iG^2|\kappa|}{2} \right] (k^2 - \kappa^2) - g^2\kappa^2 . \tag{7}$$

The integral in Eq. (6) is peaked around $\zeta_{k\kappa} = 0$, which corresponds to the complex dispersion relation between the wave-number k and the effective frequency $|\kappa|$. In the limit $G \to 0$, this peak becomes infinitely sharp and thus the integrand of Eq. (6) is supported along solutions of the real dispersion relation.

The steady-state solution (6) for $\hat{A}$ and similar expressions for the other fields diagonalize the bilinear system Hamiltonian $\hat{H}$ derived from the Lagrangian (1) via $: \hat{H} := \int dk \, d\kappa \, |\kappa| \hat{b}_{k\kappa}^\dagger \hat{b}_{k\kappa}$.

In Appendix B, we reconsider the above quantization procedure from the point of view of a moving inertial observer and discuss implications for the Ginzburg effect.

IV. FIELD CORRELATIONS

From the $\hat{A}$ field in Eq. (6), we arrive at the two-point function of the electric field operators $\hat{E} = -\partial_t \hat{A}$ in the vacuum state

$$\langle 0 | \hat{E}(t,x) \hat{E}(t',x') | 0 \rangle = \frac{g^2 G^2}{8\pi^2} \int \frac{d\kappa \, dk}{|\zeta_{k\kappa}|^2} |\kappa|^5 e^{ik\Delta x - i|\kappa|\Delta t} . \tag{8}$$

For small spacelike distances $\Delta x = x - x'$ and $\Delta t = t - t'$, we recover the free-field limit in the absence of a medium $\propto [(\Delta t)^2 + (\Delta x)^2]/[(\Delta t)^2 - (\Delta x)^2]^2$ as expected. For large distances, on the other hand, we have

$$\langle 0 | \hat{E}(t,x) \hat{E}(t',x') | 0 \rangle = -\frac{1}{2\pi n} \frac{n^2(\Delta x)^2 + (\Delta t)^2}{[n^2(\Delta x)^2 - (\Delta t)^2]^2}$$

$$- \frac{g^2 G^2}{\pi \Omega^4} \frac{|\Delta x|^3 [n^2(\Delta x)^2 + 5(\Delta t)^2]}{[n^2(\Delta x)^2 - (\Delta t)^2]^4}$$

$$+ \mathcal{O}(|\Delta x|^{-4}), \tag{9}$$

and see the impact of the medium. The leading-order term in the first line just reflects the (low-energy) refractive index of our medium $n = \sqrt{1 + g^2/\Omega^2}$ and is thus independent of dissipation. Damping just affects the subleading corrections in the second and third lines.

V. WIGHTMAN AXIOMS

It might be illuminating to compare the above two-point correlation (8) to the Wightman functions of relativistic quantum fields and their properties; see, e.g., Refs. [9–11]. Being the Fourier transform of an essentially rational function, the expression (8) obviously defines a *tempered distribution*.

However, unlike the correlators of relativistic quantum fields, it is not invariant under Lorentz boosts (in x direction) because the medium defines a distinguished reference frame. As a related point, the Fourier transform of (8) is supported in the entire half space of $k_0 > 0$ and not just in the forward light cone $k_0 \geqslant |k_1| = |k|$. Thus the correlation function (8) satisfies neither the *covariance* nor the *spectrum condition*, which will be important for the nonzero response of an inertial detector derived below.

Nevertheless, the correlator (8) is consistent with *locality* because the field commutator still vanishes at spacelike separations. The *positivity* condition just reflects the Hilbert space structure of quantum theory and is thus also satisfied here, as can be checked easily because Eq. (8) is the Fourier transform of a non-negative function. Another feature our steady-state correlation (8) for the electric fields E (not the vector potential A) has in common with relativistic quantum field theories is the *cluster property*, which here implies that the correlation vanishes at large spacelike separations, cf. Eq. (9).

VI. RESPONSE OF INERTIAL DETECTOR

Based on a two-point function analogous to (8), we may now calculate the response of a detector. We assume a point-like detector in the form of a two-level system (e.g., an atom) with the excitation energy ω moving along a prescribed trajectory $x[t] = vt$ with a constant velocity v. It is minimally coupled to the vector potential (as seen from its comoving rest frame) with dipole coupling strength λ. As usual, we start with a detector initially in the ground state and calculate its excitation probability by means of time-dependent perturbation theory (based on a power expansion in λ); see, e.g., Refs. [8,13,55,56] and Appendix C.

To this end, we parametrize the vector potential $\hat{A}(\gamma\tau, \gamma v\tau)$ seen by the detector along its trajectory in terms of its proper time $\tau = t/\gamma$, where γ is the Lorentz boost factor. Along with an additional factor $\lambda^2\omega^2/\gamma$, the Fourier transform of the resulting two-point correlator evaluated at the frequency $-\omega$ corresponding to the detector gap then yields the excitation probability (to lowest order in λ). For relativistic quantum fields, the Wightman axioms, especially *covariance* and *spectrum condition*, ensure that every inertial detector displays zero response in the vacuum (unless it is superluminal).

For our dielectric medium featuring dispersion and dissipation, however, these two conditions are violated resulting in a finite detector response for all nonzero velocities v. To lowest order in λ, we find the excitation probability per unit time

$$\frac{dP_\uparrow}{dt} = \frac{\lambda^2 g^2 G^2 \omega^2}{4\pi\gamma^2} \int \frac{d\kappa \, dk}{|\zeta_{k\kappa}|^2} |\kappa|^3 \delta\left(\frac{\omega}{\gamma} + |\kappa| - kv\right). \quad (10)$$

The Dirac delta just implements energy conservation in the detector frame. Due to $\omega > 0$, we only get contributions for $kv > |\kappa|$, i.e., when the detector velocity $|v|$ exceeds the phase velocity $|\kappa/k|$ of the modes inside the medium (in x direction). As an intuitive picture, the detector overtakes the propagating quantum fluctuations such that it effectively perceives them as going backward in time—which means that the roles of the creation and annihilation operators are effectively reversed. This inversion is often referred to as the anomalous Doppler

effect [21,22,57], which is also the underlying reason for classical (Cherenkov type) radiation phenomena.

The wave-number k must also satisfy $kv > \omega/\gamma$ which corresponds to rather large $|k|$ in the nonrelativistic limit $|v| \ll 1$. In this limit, the above expression (10) can be power expanded in v and the lowest order reads

$$\frac{dP_\uparrow}{dt} = \int_0^\infty \frac{d\kappa}{2\pi(\kappa + \omega)^4} \frac{\lambda^2 g^2 G^2 \omega^2 \kappa^3 |v|^3}{(\kappa^2 - \Omega^2)^2 + \kappa^2 G^4/4}. \quad (11)$$

At first sight, the factor G^2 in the numerator might be a bit surprising since it seems to suggest that the detector response vanishes in the limit $G \to 0$ of zero dissipation. However, this naive expectation is wrong: In the weak-dissipation limit $G \to 0$, the integrand is peaked around the medium resonance $\kappa = \Omega$ and thus we find

$$\frac{dP_\uparrow}{dt} = \frac{\lambda^2 g^2 \omega^2}{2} \frac{\Omega}{(\Omega + \omega)^4} |v|^3. \quad (12)$$

As a function of ω (for fixed λ), this response is maximized for detector frequencies ω close to the medium resonance Ω. However, due to the scaling with $|v|^3$, the rate (12) rapidly decreases for small velocities v [58].

VII. MODEL OF THREE-DIMENSIONAL MEDIUM

As a step toward a realistic scenario, let us generalize our model (1) to three spatial dimensions

$$\begin{aligned}
L = \frac{1}{2} \int d^3r \, \{[\partial_t \boldsymbol{A} + \boldsymbol{\nabla}\varphi]^2 - [\boldsymbol{\nabla} \times \boldsymbol{A}]^2 \\
+ [\partial_t \boldsymbol{\Psi}]^2 - \Omega^2 \boldsymbol{\Psi}^2 + 2g\boldsymbol{A} \cdot \partial_t \boldsymbol{\Psi}\} \\
+ \frac{1}{2} \int d^3r \, d\xi \, \{[\partial_t \boldsymbol{\Phi}]^2 - [\partial_\xi \boldsymbol{\Phi}]^2\} \\
+ G \int d^3r \, \boldsymbol{\Phi}(t, \boldsymbol{r}, \xi = 0) \cdot \partial_t \boldsymbol{\Psi}(t, \boldsymbol{r}), \quad (13)
\end{aligned}$$

where we now include a three-dimensional (3D) vector potential $\boldsymbol{A}(t, \boldsymbol{r})$ and corresponding scalar potential $\varphi(t, \boldsymbol{r})$, as well as vector-valued polarization $\boldsymbol{\Psi}(t, \boldsymbol{r})$ and environment $\boldsymbol{\Phi}(t, \boldsymbol{r}, \xi)$ fields. Note that the damping field $\boldsymbol{\Phi}(t, \boldsymbol{r}, \xi)$ still propagates in 1D (the ξ direction) only. Here ξ represents an internal coordinate and constitutes an effective label for the pathway along which energy is carried away from the fields $\boldsymbol{A}$ and $\boldsymbol{\Psi}$ and eventually turned into heat (e.g., into phononic modes). The precise mechanism of energy transfer depends on the specific medium but could involve, for instance, many local electronic degrees of freedom which take energy from the $\boldsymbol{A}$ and $\boldsymbol{\Psi}$ fields and transfer it via Coulomb interactions. If this redistribution of energy occurs on length scales much shorter than the typical optical wavelengths of interest, picturing dissipation as energy transfer in an additional ξ direction should be a good approximation.

Analogous to free electrodynamics, the field momenta are not fully independent but subject to constraints which can be treated in the Dirac formalism [59]. The absence of $\dot{\varphi}$ in the above Lagrangian imposes a primary constraint $\Pi_\varphi = \partial\mathcal{L}/\partial\dot{\varphi} = 0$. The associated Euler-Lagrange equation yields the secondary constraint $\boldsymbol{\nabla} \cdot \boldsymbol{\Pi}_A = 0$, i.e., the Gauss law. Since these constraints are of first class, we may establish the temporal and Coulomb gauge conditions $\varphi = 0$ and $\boldsymbol{\nabla} \cdot \boldsymbol{A} =$

0. As a result, the vector potential becomes purely transversal and thus couples to the transversal components of $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$ only. Within this transversal subspace, we obtain basically the same equations of motion and solutions as in 1D.

Thus the quantization can be accomplished in analogy to the 1D case but with two polarization unit vectors $\boldsymbol{e}_{k,\sigma}$ obeying $\boldsymbol{k} \cdot \boldsymbol{e}_{k,\sigma} = 0$ and integrations over d^3k instead of dk. For example, Eq. (6) becomes

$$\hat{\boldsymbol{A}}(t,\boldsymbol{r}) = \sum_{\sigma} \int \frac{d\kappa \, d^3k}{(2\pi)^2} \, gG \, \frac{\kappa^2 \hat{b}_{k\kappa,\sigma} e^{i\boldsymbol{k}\cdot\boldsymbol{r}-i|\kappa|t}}{\zeta_{k\kappa}\sqrt{2|\kappa|}} \boldsymbol{e}_{k,\sigma} + \text{H.c.} \quad (14)$$

Note that the coupling $\boldsymbol{A} \cdot \boldsymbol{j}$ to the polarization charge-current-density $\boldsymbol{j} = g\partial_t \boldsymbol{\Psi}$ in Eq. (13) does not take into account the polarization charge-density $\varrho = -g\boldsymbol{\nabla} \cdot \boldsymbol{\Psi}$ of the medium, but the latter corresponds to the longitudinal sector and is not considered here.

In analogy to Eq. (12) we may derive the excitation rate $\dot{P}_{\uparrow}$ of an inertial detector with velocity v and obtain

$$\frac{dP_{\uparrow}}{dt} = \frac{\Lambda^2 g^2 \omega^2}{4\pi} \frac{\Omega}{(\Omega+\omega)^2} |v| \, ; \quad (15)$$

see Appendix D. In contrast to the 1D case (12), this rate scales linearly in $|v|$, which stems from the enlarged phase-space volume in 3D.

VIII. EXPERIMENTAL REALIZATION

Let us sketch a possible experimental realization; see also Refs. [21,22]. We choose crystalline silicon as a dielectric medium, which has a resonance at $\Omega \approx 3.3$ eV where the phase velocity is reduced to $0.15\,c$, corresponding to $\Re[n(\Omega)] \approx 6.8$ and a wave-number of $k \approx 22.4$ eV [60]. In order to perceive the quantum fluctuations of this mode as a real photon, the detector must be faster than the phase velocity of $0.15\,c$, so it can only be an atomic or subatomic particle.

As a detector, we choose metastable hydrogen atoms in the 2s state. Thus the detector gap ω is an energy of the Balmer series starting at $\omega \approx 1.9$ eV. To excite the atom, the Doppler shift $\boldsymbol{v} \cdot \boldsymbol{k}$ must lift a quantum fluctuation with the negative frequency $\kappa = -\Omega$ up to the positive detector frequency ω (with the Lorentz factor γ)

$$\boldsymbol{v} \cdot \boldsymbol{k} = \frac{\omega}{\gamma} + \Omega \, , \quad (16)$$

which is analogous to the argument of the Dirac delta function in Eq. (10). With the above values, we find that this requires velocities of $v \approx c/4$.

Such velocities can be achieved for protons from an accelerator with an energy of about 30 MeV. By sending them through a foil, gas cell, or jet of liquid helium, they may capture an electron and form hydrogen atoms [61,62]. At such high energies, the capture cross-section is quite small (about 10^{-29} cm^2 [63,64]) such that only a small fraction $\mathcal{O}(10^{-8})$ of the protons actually forms atoms. As a typical ion accelerator might deliver a current of about 1 mA, this corresponds to $\mathcal{O}(10^8)$ atoms per second. The protons may be deflected out of the beam by a magnetic field such that only the hydrogen atoms remain. After a propagation length of several meters, most of the hydrogen atoms will have decayed to the 1s ground state but some fraction (a small percentage [63]) will

reach the 2s metastable state (with a lifetime of about 0.1 s). Atoms formed in long-lived higher excited (Rydberg) states may be filtered out by field ionization (and the magnetic field).

For a hydrogen atom actually moving *inside* the silicon crystal, we find the probability of spontaneous excitation $\dot{P}_{\uparrow}$ to be of order 10^{-3} per centimeter path length. However, as sending an atom beam directly through the crystal is probably not feasible, we envision sending them through a small hole or along the surface instead. Obviously the relevant modes with $k \approx 22.4$ eV cannot propagate outside the crystal (i.e., in vacuum) but will leak out of the crystal in the form of evanescent fields with an e-folding length of about 9 nm, which is much larger than the size of an atom; see Appendix E. As a result, only atoms passing the surface at distances of order 9 nm may get excited. For example, if a homogeneous beam of $\mathcal{O}(10^6)$ atoms per second (in the 2s state) passes through a 1-mm-diameter hole in a silicon crystal, about 5×10^{-3} atoms per second and centimeter length of the medium should get excited to the 3p state—as detailed in Appendix F.

These atoms will then decay under the emission of a Lyman β photon at 103 nm after a radiative lifetime of 18 ns which corresponds to a propagation length of 1.3 m for hydrogen atoms with a velocity of $c/4$. The detector to be placed behind the silicon crystal could thus consist of a cylindrical array of microchannel plates of about 1 m length that detect the Lyman β photons.

A similar experiment is possible with other atoms with resonance frequencies in the visible or near ultraviolet regime; however, much higher ion energies would be required to generate neutral atoms of velocities $v = c/4$. For example, lithium would require about 200 MeV Li$^+$ ions. While the relevant transition energy $\omega \approx 1.8$ eV is similar to the hydrogen case, the final photon count might be larger (for the same initial current) because excitations would start from the atomic ground state rather than from a weakly populated excited state.

IX. CONCLUSIONS

Via an extension of the well-known Hopfield model which can easily be generalized to three spatial dimensions, we provide a consistent microscopic treatment of quantum electrodynamics in a dielectric medium featuring dispersion and dissipation. The simple structure and well-defined asymptotic behavior (e.g., at $\xi \to \pm\infty$) of our model allow us to diagonalize the full system Hamiltonian in terms of steady-state solutions and to derive two-point functions of the quantum field operators, both without invoking further assumptions (such as the Markov approximation).

In contrast with most existing models for dissipative dielectrics [50–54], our approach neither involves an empirically given dielectric permittivity $\varepsilon(\omega)$ nor uses infinitely many baths of harmonic oscillators as a pathway for dissipation. Instead it provides a canonical quantization procedure for the famous Lorentz oscillator model known from classical electrodynamics [65,66] and hence constitutes a simple and clean approach to the quantum dynamics in dissipative media—especially in the case of Lorentzian resonances.

Comparison with the Wightman functions of relativistic quantum fields reveals that the two-point function in such a

medium does neither satisfy *covariance* nor the *spectrum condition*. The latter property is closely related to the Ginzburg effect, i.e., the nonzero response of an inertial detector moving faster than the phase velocity of some mode. As an intuitive picture, the detector overtakes this mode and thus experiences its oscillation as going backward in time—thereby reversing the roles of positive and negative frequencies and thus creation and annihilation operators. In other words, the Doppler shift vk or $\boldsymbol{v} \cdot \boldsymbol{k}$ lifts quantum fluctuations with negative frequencies up to positive frequencies which are then able to excite the detector.

In a complete analogy to acceleration radiation (a.k.a. the Unruh effect), Ginzburg-type clicking of superluminal detectors can be understood as the absorption of a quantum ground-state fluctuation and is always accompanied by the emission of a real photon into the medium. In the frame of the detector, this emitted photon effectively carries negative energy (see Appendix B) which compensates the positive energy required to excite the detector. In the laboratory frame, both have positive energy and thus this process depletes the kinetic energy of the detector, leading to a small frictional force [19,33–35], which constitutes a special type of quantum friction (see also Refs. [36–44]).

Near the medium resonance, the phase velocity becomes very small, i.e., k becomes very large, such that even low detector velocities v may trigger excitations. For these modes, dissipation effects become important, which we take into account consistently with our model. We find that dissipation does not impede the Ginzburg effect.

Actually, our idealized model incorporates modes with unlimited k near the resonance, such that the Ginzburg effect would formally extend to arbitrarily small detector velocities v. However, for decreasing velocities v, the relevant phase space of modes with $\boldsymbol{k} \cdot \boldsymbol{v} > \omega/\gamma + \Omega$ shrinks, which explains the scaling of the excitation probability with $|v|$ or $|v|^3$ in 3D and 1D, respectively.

Of course, there are several limitations to describing real media by our simplified model. Apart from the points already discussed above, our Lagrangian density is local in space and time and thus does not incorporate any wave-number or frequency cutoff. However, the underlying approximations (e.g., the dipole approximation) limit the validity of this effective continuum description to finite band widths in both wave-number and frequency. Typically real media feature not just a single resonance frequency Ω but also may have several resonances (e.g., described by the Sellmeier coefficients [67]). Depending on the local environment of the atoms/molecules, their individual resonance frequencies Ω may also vary around a mean value causing inhomogeneous broadening. In media where the resonance frequencies Ω correspond to gaps in the electronic band structure (e.g., between the valence and the conduction bands in semiconductors), dispersion within those bands can also lead to resonance broadening and peak deformation.

Altogether, our simplified model should only be applied in a finite range of frequencies and up to a certain cutoff wave-number $k_{\max}$. Via Eq. (16), this would then imply a minimum detector velocity $v_{\min}$ necessary to observe the Ginzburg effect. For crystalline silicon, the wave-number $k \approx 22.4\,\mathrm{eV}$ corresponding to the measured refractive index

$\Re[n(\Omega)] \approx 6.8$ at resonance $\Omega \approx 3.3\,\mathrm{eV}$ may serve as a conservative estimate for the cutoff $k_{\max}$. Thus an experimental test of the Ginzburg effect requires velocities $v \approx c/4$. Using, for instance, fast hydrogen atoms as detectors we find that such an experiment is challenging but not out of reach.

APPENDIX A: COMPARISON TO OTHER APPROACHES

In using a scalar field Φ as an environment, our model substantially differs from existing works building upon the established Huttner-Barnett approach for dissipative media [50,51]. As opposed to our Lagrangian (1), the Huttner-Barnett model incorporates dissipation by attaching infinitely many baths of harmonic oscillators with continuous eigen-frequencies ω and frequency-dependent coupling strengths $G(\omega)$ to all points of a given medium. By diagonalizing the resulting Hamiltonian, the Huttner-Barnett model yields an effective dielectric permittivity $\varepsilon(\omega)$, which is connected to the coupling strengths $G(\omega)$ via some integral relation [50]. Simplified versions of the Huttner-Barnett approach even yield exactly solvable Heisenberg equations of motion [68] but they typically assume special profiles for the coupling strengths $G(\omega)$. The successful microscopic derivation by Huttner and Barnett has further inspired the framework of *macroscopic quantum electrodynamics* [53], which provides a phenomenological quantization scheme based on an *empirically given* complex dielectric permittivity $\varepsilon(\omega)$. In contrast to these approaches, our model just involves the three parameters Ω, g and G and—without further assumptions—allows for deriving an effective permittivity $\varepsilon(\omega)$ with a Lorentzian resonance (see also our previous work [49]). Further note that our Lagrangian (1) generates consistent quantum dynamics even in media with explicitly space-time-dependent parameters Ω, g, and G.

On a microscopic level, the Huttner-Barnett formalism and our approach can be understood as opposite idealizations of the same (realistic) loss mechanism. As an exemplary pathway for dissipation, we consider a scenario where the medium polarization $\Psi(t, x)$—being a smoothed average over a small neighborhood $\mathcal{M}_x$ of the point x—locally couples to many microscopic degrees of freedom (e.g., to the nuclei and electrons in the region $\mathcal{M}_x$). Via this interaction, energy can be transferred from the A and Ψ fields to a microscopic environment. In the Huttner-Barnett model this *environment* is described by the aforementioned baths of harmonic oscillators (which all couple to the medium polarization but not to each other). This implicitly assumes the environmental degrees of freedom to be decoupled and allows them to store but not to permanently absorb energy. In fact, energy *dissipated* to the harmonic baths can couple back into the A and Ψ fields (see also Ref. [68]). In

realistic media, however, interactions on the microscopic level (e.g., Coulomb forces among the discrete electrons and nuclei) generally allow energy to be directly redistributed between the environmental degrees of freedom. Weakly interacting environments may give rise to rather complicated dynamics—for instance, energy transferred to the environment at one point x might couple back into the A and Ψ fields elsewhere. We explore the opposite limit where energy is redistributed on short time scales such that all energy transferred to the environment is almost immediately lost for the fields A and Ψ. Under this assumption, it seems well justified to model dissipation at each point x with a scalar field $\Phi(t, x, \xi)$ that propagates in a perpendicular (and potentially artificial) ξ direction.

APPENDIX B: MOVING REFERENCE FRAME

To gain further insights into the Ginzburg mechanism, we briefly reconsider the quantization procedure from Sec. III in the Lorentz boosted rest frame of an inertial observer moving at a constant velocity v along the medium. In the corresponding Lorentz transformation, we interpret the A field as the single component of a vector potential oriented perpendicularly to the medium. The polarization field Ψ (up to a scalar prefactor) corresponds to a length measured parallel to the A field and Φ is treated either as the single component of a second vector potential parallel to A (if ξ refers to a real spatial dimension) or as a scalar field (if ξ constitutes an internal coordinate).

As before, our quantization procedure yields straightforward steady-state solutions which now produce a diagonalized Hamiltonian,

$$:\hat{H}:= \int dk\, d\kappa\, \gamma(|\kappa| - kv)\, \hat{b}^{\dagger}_{k\kappa}\hat{b}_{k\kappa}, \qquad (B1)$$

with the same creation and annihilation operators $\hat{b}^{\dagger}_{k\kappa}$ and $\hat{b}_{k\kappa}$ as before but with the Lorentz boosted energy $\gamma(|\kappa| - kv)$, where γ denotes the usual Lorentz factor. For pairs of κ and k with $|\kappa| < kv$, this boosted energy grows negative such that creating the excited state $\hat{b}^{\dagger}_{k\kappa}|0\rangle$ from the quantum vacuum $|0\rangle$ could actually set free an energy $-\gamma(|\kappa| - kv) > 0$ in the rest frame of the moving observer. Absorbing this energy could make an inertial particle detector click—which explains how the Ginzburg effect complies with energy conservation.

The condition $|\kappa| < kv$ is satisfied whenever our observer moves superluminally with respect to the phase velocity $|\kappa/k|$ of the corresponding excitation $\hat{b}^{\dagger}_{k\kappa}|0\rangle$. Since relativistic quantum fields do not allow for superluminal motion, the Ginzburg mechanism has no counterpart in relativistic field theories.

APPENDIX C: DETECTOR MODEL

The following paragraph provides further details on the model used to describe the two-level photon detectors considered in Secs. VI and VII. It applies the established Unruh-DeWitt model [8,13,55,56] for pointlike particle detectors in background fields to the concrete example of a hydrogen atom (which is the detector studied in Sec. VIII).

We assume the nucleus to follow a prescribed inertial trajectory $x[\tau] = \gamma v\tau$ or $\mathbf{r}[\tau] = \gamma v\tau\mathbf{e}_z$ for the 1D setup in Sec. VI or the 3D scenario of Sec. VII, respectively, where

$\tau = t/\gamma$ denotes the atom's proper time. In a comoving reference frame, the electron undergoes dynamic motion with the position and momentum operators $\hat{\mathbf{q}}(\tau)$ and $\hat{\mathbf{p}}(\tau)$.

For the 1D setup in Sec. VI, we use the following interaction Hamiltonian,

$$\hat{H}_{\text{int}}(\tau) = \frac{e_0}{m_e}\,\hat{p}(\tau)\hat{A}(\gamma\tau, \gamma v\tau), \qquad (C1)$$

to model minimal coupling between the atom (idealized as a point particle) and the electromagnetic field along its trajectory, where e_0 denotes the elementary charge and m_e corresponds to the electron mass. The corresponding operator for the 3D case involves the scalar product between the 3D momentum $\hat{\mathbf{p}}(\tau)$ and the vector potential $\hat{\mathcal{A}}(\tau)$ in the detector frame, which has the Cartesian components $\hat{A}_x(\gamma\tau, \gamma v\tau\mathbf{e}_z)$, $\hat{A}_y(\gamma\tau, \gamma v\tau\mathbf{e}_z)$, and $\gamma\hat{A}_z(\gamma\tau, \gamma v\tau\mathbf{e}_z)$ along the detector worldline. Converting the interaction Hamiltonian $\hat{H}_{\text{int}}(\tau)$ to the laboratory time $t = \gamma\tau$ produces an additional factor γ^{-1} [69]. The detector itself, in principle, evolves according to the Hamiltonian $\hat{H}_{\text{det}}$ of a hydrogen atom but we adopt a two-level approximation where $\hat{H}_{\text{det}}$ just has two eigenstates $|0\rangle_{\text{det}}$ and $|1\rangle_{\text{det}}$ that are separated by an energy-gap ω.

Following the standard treatment of Unruh-DeWitt detectors, we assume our two-level system and the surrounding field to initially occupy their respective ground states. To leading-order time-dependent perturbation theory, we then calculate the probability per (laboratory) time $dP_{\uparrow}/dt$ (or $\dot{P}_{\uparrow}$) at which the detector gets spontaneously excited.

For the 1D scenario in Sec. VI, the resulting expression

$$\frac{dP_{\uparrow}}{dt} = \lambda^2\omega^2 \int \frac{d\tau}{\gamma}\, \langle 0|\hat{A}(\gamma\tau, \gamma v\tau)\hat{A}(0, 0)|0\rangle\, e^{-i\omega\tau} \qquad (C2)$$

involves a two-point function analogous to the field correlation (8) and the coupling strength λ corresponds to the magnitude of the transition matrix element $d^{(01)}$ of the dipole operator $e_0\hat{q}$ evaluated with respect to the detector eigen-states $|0\rangle_{\text{det}}$ and $|1\rangle_{\text{det}}$. The matrix element $q^{(01)}$ of the position operator $\hat{q}$ is related to the corresponding quantity for the momentum $\hat{p}$ via $p^{(01)} = -im_e\omega q^{(01)}$.

In 3D, the dipole operator $e_0\hat{\mathbf{q}}$ has matrix elements $d_i^{(01)}$ for all three spatial dimensions and $\dot{P}_{\uparrow}$ adopts the form

$$\frac{dP_{\uparrow}}{dt} = \omega^2 \sum_{i,j} d_i^{(01)}d_j^{(10)} \int \frac{d\tau}{\gamma}\, \langle 0|\hat{\mathcal{A}}_i(\tau)\hat{\mathcal{A}}_j(0)|0\rangle\, e^{-i\omega\tau}, \qquad (C3)$$

where $\hat{\mathcal{A}}_i(\tau)$ labels the components $\hat{A}_x(\gamma\tau, \gamma v\tau\mathbf{e}_z)$, $\hat{A}_y(\gamma\tau, \gamma v\tau\mathbf{e}_z)$, and $\gamma\hat{A}_z(\gamma\tau, \gamma v\tau\mathbf{e}_z)$ of the Lorentz boosted vector potential $\hat{\mathcal{A}}(\tau)$ occurring before.

After evaluating all quantum mechanical expectation values in Eqs. (C2) and (C3), the τ integrals produce Dirac deltas. For the 1D case, we arrive at the expression (10) in the main text. The more complicated expression for 3D is further analyzed in Appendix D.

APPENDIX D: DETECTOR IN 3D

In order to obtain an explicit result for the excitation probability (C3), we parametrize momentum space with spherical coordinates $k_x = k\sin\theta\cos\phi$, $k_y = k\sin\theta\sin\phi$, and

$k_z = k \cos\theta$ and evaluate all quantum mechanical expectation values as well as the τ and ϕ integrals. After substituting $\eta = \cos\theta$, we obtain the expression

$$\frac{dP_\uparrow}{dt} = \frac{g^2 G^2 \omega^2}{8\pi^2 \gamma^2} \int_{-\infty}^{\infty} d\kappa \int_0^\infty dk \int_{-1}^1 d\eta \, \frac{|\kappa|^3 k^2}{|\zeta_{k\kappa}|^2}$$
$$\times \left[\frac{1+\eta^2}{2} \left(|d_x^{(01)}|^2 + |d_y^{(01)}|^2 \right) + (1-\eta^2) |d_z^{(01)}|^2 \right]$$
$$\times \delta \left(\frac{\omega}{\gamma} + |\kappa| - \eta k v \right), \tag{D1}$$

which has a structure quite similar to Eq. (10) for a 1D medium. The Dirac delta, once again, imposes the conservation of energy and the term $\eta k v = k_z v$ in its argument accounts for the projection $\mathbf{k} \cdot \mathbf{v}$ of the wave-vector $\mathbf{k}$ onto the detector speed $\mathbf{v}$.

Analogous to the 1D case, we next evaluate the k integral and then adopt the nonrelativistic limit, in which the remaining κ and η integrals decouple. Using straightforward symmetry arguments, we restrict those integrals to the domains of positive κ and $\eta \in [0, 1]$. As before, the κ integral is peaked at $\kappa = \Omega$ for weakly dissipative media (formally for $G \to 0$). The η integrals read $\int_0^1 d\eta \, \eta(1+\eta^2)/2 = 3/8$ for the $d_x^{(01)}$ and $d_y^{(01)}$ contributions to $\dot{P}_\uparrow$, and $\int_0^1 d\eta \, \eta(1-\eta^2) = 1/4$ for the $d_z^{(01)}$ term. Eventually, we arrive at the following excitation probability for small detector speeds v and sufficiently weak dissipation G

$$\frac{dP_\uparrow}{dt} = \frac{g^2 \omega^2}{4\pi} \frac{\Omega}{(\Omega+\omega)^2} |v| \left[\frac{3}{8} \left(|d_x^{(01)}|^2 + |d_y^{(01)}|^2 \right) + \frac{1}{4} |d_z^{(01)}|^2 \right]. \tag{D2}$$

In SI units (where Ω, g, and G^2 would be measured in s^{-1} and v in m/s), this expression involves another factor $\mu_0/(\hbar c^2)$ with μ_0 denoting the magnetic vacuum permeability. Under the assumption of identical matrix elements, $|d_i^{(01)}| = \Lambda$, for all spatial directions, the above result reduces to Eq. (15) in the main text.

By substituting $\omega \mapsto -\omega$ and $v = 0$ into Eq. (D1), we can further calculate the leading (zero velocity) contribution to the corresponding decay rate $\dot{P}_\downarrow$. In the limit of weak dissipation $G \to 0$ and for $\omega < \Omega$, we obtain

$$\frac{dP_\downarrow}{dt} = \frac{\omega^3}{3\pi} \sqrt{1 + \frac{g^2}{\Omega^2 - \omega^2}} \sum_i |d_i^{(01)}|^2. \tag{D3}$$

Comparing the rates $\dot{P}_\uparrow$ and $\dot{P}_\downarrow$ for the specific scenario, where a hydrogen atom (with quantization axis parallel to its motion) is excited from the metastable 2s state with quantum numbers $n = 2$, $l = m = 0$ to a 3p state with $n = 3$, $l = 1$, $m = \pm 1$ yields the ratio

$$\frac{\dot{P}_\uparrow}{\dot{P}_\downarrow} = \frac{9}{32} \sqrt{1 - \frac{g^2}{\Omega^2 - \omega^2 + g^2}} \frac{g^2}{(\Omega+\omega)^2} \frac{\Omega}{\omega} |v|. \tag{D4}$$

Although the $d_x^{(01)}$, $d_y^{(01)}$, and $d_z^{(01)}$ contributions to $\dot{P}_\uparrow$ all seem to be of the same order of magnitude, additional care is required when considering realistic experiments because

our idealized model does not yet have an ultraviolet cutoff. More specifically, the Dirac delta in Eq. (D1) requires very large wave-numbers $k = |\mathbf{k}|$ in the nonrelativistic limit of $|v| \ll 1$, which do not exist in real media. As a simple work-around, one could truncate the k integral in Eq. (D1) at a suitable cutoff $k_{\max}$—corresponding, e.g., to the maximum wave-number compatible with measured dispersion relations for the medium of interest [70].

After introducing a wave-number cutoff $k_{\max}$, the k integral in Eq. (D1) yields an additional Heaviside function $\Theta(\eta k_{\max} v - \omega/\gamma - |\kappa|)$ coupling the remaining κ and η integrals. Contributions to the result (D2) should hence stem from points (κ, η) satisfying the condition $\eta k_{\max} v > \omega/\gamma + |\kappa|$ (with $\gamma \approx 1$ for $|v| \ll 1$) in order to be physically reliable. Since the κ integrand peaks at rather large frequencies close to Ω, the above condition will usually be satisfied only if $\eta = \cos\theta = k_z/k$ is sufficiently close to sgn(v) or, more specifically, if $|\eta| \gtrsim \eta_{\min} \approx (\omega + \Omega)/(|v| k_{\max})$. This restriction decreases both prefactors 3/8 and 1/4 in Eq. (D2) but the factor 1/4 is affected much more significantly because it stems from an η integral whose integrand $\eta(1 - \eta^2)$ vanishes at $\eta = 1$. {The integrand $\eta(1 + \eta^2)/2$ corresponding to the factor 3/8, on the other hand, monotonically increases for $\eta \in [0, 1]$ and adopts its maximum at $\eta = 1$.} In a refined version of Eq. (D2), the $d_z^{(01)}$ contribution would thus be negligible compared with the $d_x^{(01)}$ and $d_y^{(01)}$ terms and the prefactor 3/8 would be replaced with $3/8 - \eta_{\min}^2/4 - \eta_{\min}^4/8$ (or 0 if $\eta_{\min} > 1$).

Although the result (D2) has been derived in the nonrelativistic limit of $|v| \ll 1$ and for vanishing dissipation $G \to 0$, it remains a reasonable approximation (at least for order-of-magnitude estimates) in scenarios with moderately relativistic detector speeds v and weakly dissipative media. In the case of moderately relativistic velocities (such as $v = c/4$), one could refine the above result by adding higher-order corrections to our small velocity expansion. However, compared with the lowest-order result, the first nonvanishing correction already involves an additional factor v^2 which is still small, such that we expect corrections on the order of percent. For media with finite dissipation G, the peak of the κ integral at $|\kappa| \approx \Omega$ is not infinitely sharp but adopts a finite width proportional to G^2. As long as the remaining integrand does not change too much in this κ range, the above result remains approximately valid.

APPENDIX E: EVANESCENT MODES

Since sending particle detectors right through bulk dielectrics seems unfeasible in real experiments, one would rather send them through a small hole in a crystal or along the surface of a dielectric plate. To understand how such a change in geometry affects the order of magnitude of our results, we consider the situation of a dielectric half space next to a vacuum environment. Using the same detector trajectory $\mathbf{r}[t] = v t \mathbf{e}_z$ as in Appendix D, we will assume the dielectric to fill the region with $x < -d$—resulting in a constant medium-detector separation d.

Inside the dielectric, the electromagnetic field is described by our steady-state solution (14) for the vector potential $\hat{\mathbf{A}}$ plus additional contributions $\hat{\mathbf{A}}_0$ satisfying the correspond-

ing homogeneous wave equation (i.e., the 3D counterpart of Eq. (4) without the Φ_0 term). Analogous to the 1D field (6), the 3D steady-state solution $\hat{\mathbf{A}}$ for weakly dissipative media (with sufficiently small G) approximately satisfies the dispersion relation $|\kappa| = \omega_{\text{med}}(|\mathbf{k}|)$ encoded in the identity $\zeta_{\mathbf{k}\kappa} = 0$. The homogeneous $\hat{\mathbf{A}}_0$ solution exactly obeys the same identity. In the y and z directions, all contributions to the $\hat{\mathbf{A}}$ and $\hat{\mathbf{A}}_0$ fields oscillate with real wave-numbers k_y and k_z. Outside the medium, the vector potential $\hat{\mathbf{A}}$ can be expanded in terms of exponentials $\exp\{i\tilde{\mathbf{k}} \cdot \mathbf{r} - i\omega_{\text{vac}}(\tilde{\mathbf{k}})t\}$ obeying the relativistic dispersion relation $\omega_{\text{vac}}(\tilde{\mathbf{k}}) = \pm\sqrt{\tilde{\mathbf{k}}^2}$. At the medium-vacuum interface, translation invariance in the coordinates t, y, and z causes modes to conserve their frequencies ω and wave-vector components k_y and k_z.

Analogous to the energy constraint in Eq. (D1), a detector with energy-gap ω moving in vacuum close to a medium has to satisfy the condition

$$\frac{\omega}{\gamma} + \omega_{\text{vac}}(\tilde{\mathbf{k}}) - \tilde{k}_z v = 0 \qquad \text{(E1)}$$

in order to get excited. Since $\omega_{\text{vac}}(\tilde{\mathbf{k}}) = \omega_{\text{med}}(k)$, $k_y = \tilde{k}_y$, and $k_z = \tilde{k}_z$, this is essentially the same requirement as for a detector moving through a bulk medium. For the latter scenario, we already know Ginzburg excitations to involve wave vectors $\mathbf{k}$ that are almost parallel to the detector velocity $\mathbf{v}$ (such that $k_z \approx \text{sgn}(v)|\mathbf{k}|$ and $k_x, k_y \approx 0$). Apart from that, $|\mathbf{k}|$ has to be large, which requires frequencies $\omega_{\text{vac}}(\tilde{\mathbf{k}})$ close to the medium resonance Ω. Therefore the modes $\exp\{i\tilde{\mathbf{k}} \cdot \mathbf{r} - i\omega_{\text{vac}}(\tilde{\mathbf{k}})t\}$ potentially exciting our detector outside the medium need to satisfy the condition $\omega_{\text{vac}}^2(\tilde{\mathbf{k}}) = \tilde{k}_x^2 + \tilde{k}_y^2 + \tilde{k}_z^2 \approx \Omega^2$. Approximating $k_y \approx 0$, we thus obtain $\tilde{k}_x \approx i\sqrt{k_z^2 - \Omega^2}$ which is purely imaginary for media with subluminal dispersion.

As a result, the relevant modes seen by our detector are evanescent fields and undergo exponentially decay with the e-folding length

$$\ell(k_z) = \frac{1}{\sqrt{k_z^2 - \Omega^2}}. \qquad \text{(E2)}$$

Previous studies of atoms moving next to medium-vacuum interfaces [33,34] have found similar scaling behavior with respect to the distance d and further predict the Ginzburg effect to be exponentially suppressed for small detector speeds v. By just comparing our Eqs. (E1) and (E2), one would draw the same conclusion here: If v grows very small, $\tilde{k}_z = k_z$ needs to be large to facilitate Ginzburg excitations. Thus the e-folding length $\ell(k_z)$ shrinks for decreasing velocities v. However, in real media, there usually exist medium-specific maximum values k_{max} for $|\mathbf{k}|$ and k_z, such that inertial detectors need to exceed certain boundary velocities v_{min} in order to get excited. Other works on quantum friction have predicted rational scaling $\sim v^n/d^m$ with velocity v and distance d, where n and m denote system-specific integers. However, these works either study other friction mechanisms, such as pairwise photon production [33,34], or involve *detectors* with richer internal dynamics, such as metallic nanoparticles [42].

APPENDIX F: ORDER-OF-MAGNITUDE ESTIMATES

In Sec. VIII of the main text, we propose an experimental test of the Ginzburg effect which uses inertial hydrogen atoms in the metastable 2s state as photon detectors and crystalline silicon as a dielectric medium. The detector transition $2s \mapsto 3p$ of minimum energy then corresponds to a detector gap $\omega = 1.9\,\text{eV}$ and the resonance frequency of crystalline silicon reads $\Omega = 3.3\,\text{eV}$ [60]. As a cutoff wave-number $k_{\text{max}} = 22.4\,\text{eV}$, we use the maximum value of $|\mathbf{k}|$ compatible with the real part of the refractive index at resonance $\Re[n(\Omega)] \approx 6.8$ reported in Ref. [60]. The coupling strength g between the electromagnetic field and the medium polarization can be extracted from the low-energy refractive index $n = 3.4$ of silicon via $n = \sqrt{1 + g^2/\Omega^2}$ and the parameters $d_i^{(01)}$ are just the transition dipole matrix elements for the $2s \mapsto 3p$ excitation in an unperturbed hydrogen atom (within the scope of our perturbative treatment).

For a single hydrogen atom moving through a bulk of silicon, Eq. (D2)–after replacing $3/8 \mapsto \max\{3/8 - \eta_{\text{min}}^2/4 - \eta_{\text{min}}^4/8, 0\}$ and $1/4 \mapsto 0$ as explained in Appendix D—yields an excitation probability $dP_\uparrow/d(|v|t)$ of order $10^{-4}\,\text{cm}^{-1}$ per path $|v|t$ of the moving atom. Although Eq. (D2) produces a ratio $\dot{P}_\uparrow/|v|$ independent of the detector speed v, the velocity-dependent factor $\eta_{\text{min}} = (\omega + \Omega)/(|v|k_{\text{max}})$ enters the result when imposing a wave-number cutoff k_{max}. As explained in Appendix D, η_{min} becomes the lower boundary of the integrals $\int_0^1 d\eta\, \eta(1 + \eta^2)/2$ and $\int_0^1 d\eta\, \eta(1 - \eta^2)$ and has to be less than 1 to allow the Doppler shift $\mathbf{k} \cdot \mathbf{v}$ with $|\mathbf{k}| < k_{\text{max}}$ to lift quantum vacuum fluctuations above the detector gap. Here this imposes a minimum detector velocity $v_{\text{min}} \approx c/4$ that has to be met in order to facilitate spontaneous Ginzburg excitations.

As a proposal for a real experiment, we suggest sending a beam of atoms either parallel to a dielectric plate or through a hole in a silicon crystal. According to Appendix E, the relevant field modes that may excite inertial atoms in such scenarios form evanescent fields. For detector speeds v just slightly above the velocity threshold $v_{\text{min}} \approx c/4$, Ginzburg excitations require the wave-vector component k_z in v direction to be of order k_{max}. Via Eq. (E2), we hence obtain an e-folding length of $\ell(k_{\text{max}}) \approx 9\,\text{nm}$ (in SI units), which causes inertial atoms with the atom-medium separation d to perceive a field amplitude that has been reduced by a factor of order $\exp\{-d/\ell(k_{\text{max}})\}$. The corresponding excitation rate is reduced by a factor of order $\exp\{-2d/\ell(k_{\text{max}})\}$ since $\dot{P}_\uparrow$ involves a two-point correlation of the field.

In a scenario where a homogeneous beam of 10^6 metastable H(2s) atoms per second passes through a hole of diameter $2R = 1\,\text{mm}$ in a silicon crystal, we incorporate this exponential decay by averaging the factor $\exp\{-2d/\ell(k_{\text{max}})\}$ (where d now denotes the radial separation from the hole's boundary) over the beam surface πR^2—producing a factor of 2×10^{-5}.

Combining this factor with the flux of 10^6 atoms per second and with our previous estimate for the excitation rate $dP_\uparrow/d(|v|t)$ of a single atom in a bulk of silicon, we eventually estimate approximately 5×10^{-3} atoms to get spontaneously excited per second and centimeter path inside the hole.

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[58] Recall that as the damping goes to zero, the resonance gets sharper, and thus there always exists a regime in which the phase velocity is less than the detector velocity.

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[69] This factor automatically arises when reparameterizing the Schrödinger equation $i\,\partial_\tau|\varphi(\tau)\rangle = \hat{H}(\tau)|\varphi(\tau)\rangle$ (here given with respect to the detector's proper time τ) in terms of the laboratory time $t = \gamma\tau$ (for a more profound discussion in Heisenberg picture representation, see Sec. II.B. of Ref. [71]).

[70] This choice could still be an underestimate because empirical data are typically discrete and do not necessarily capture the point of maximum $|k|$.

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