Strange metallicity of moiré twisted bilayer graphene

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Recent measurements in several different laboratories report the observation of an approximately linear-in-temperature resistivity with a large twist-angle-dependent slope (or temperature coefficient) in moiré twisted bilayer graphene down to a few K and sometimes to much lower temperatures. In this paper, we theoretically discuss this "strange metal" linear-in-temperature transport behavior from the perspective of resistive scattering by acoustic phonons, emphasizing the aspects of the transport data, which are and which are not consistent with the phonon scattering mechanism. An extensive theoretical comparison with a recent experiment [A. Jaoui *et al.*, Nat. Phys. **18**, 633 (2022)] is the central new aspect of this work.

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I. INTRODUCTION AND BACKGROUND

An impressive recent experiment [1] reports extensive transport measurements for the electrical resistivity $\rho(T, n, \theta)$ of moiré twisted bilayer graphene (tBLG) as a function of temperature (T), twist angle (θ), and carrier/doping density (n). There already are existing works [2,3] that present similar tBLG transport results, but Ref. [1] provides more extensive resistivity data. This work [1] has attracted attention in the context of the much-discussed "strange metal" behavior in condensed matter physics [4]. The key message of Ref. [1] is that the linear-in-T behavior exists over a large temperature range (at least, for some specific doping-induced band fillings), making the behavior consistent with the putative strange metal behavior much discussed in the literature on strongly correlated materials [5–8]. Although "strange metallicity" is not a sharply defined concept with a universally accepted description, it is associated with a linear-in-T resistivity (which is often, but not always, also anomalously large) that exists over a substantial temperature range with the underlying scattering mechanism for the linearity that is not obviously apparent. For strange metals, therefore, the key question is what causes the large linear-in-T resistivity. In this note, we discuss the possibility that the linear-in-T resistivity observed in Ref. [1] arises (at least partially) from acoustic phonon scattering, using a minimal Dirac model for the tBLG band structure. This viewpoint was propounded earlier in our theoretical works [9,10] in the context of the tBLG experimental works in [2,3]. Reference [2] indeed concludes that the phonon scattering mechanism is the cause for the linear-in-T dependence of $\rho(T, n, \theta)$ in tBLG, whereas Ref. [3] suggests that the strangeness in tBLG arises from quantum criticality. The recent work in Ref. [1] claims that quantum criticality is the underlying mechanism for the tBLG strangeness, based mainly on the observation that the "metallic ground state features a T-linear resistivity extending over three decades in temperature, from 40 mK to 20 K, spanning a broad range of dopings including those where a correlation-driven Fermi surface reconstruction occurs" [1]. It is often stated in the literature that some unknown hidden quantum criticality may be giving rise to an observed linear-in-T resistivity. While such a possibility can never be ruled out, no known microscopic itinerant metallic quantum criticality has been shown to decisively lead to any linear-in-T resistivity. Neither quantitative nor qualitative arguments are provided for how and why quantum criticality leads to a linear-in-T resistivity at $T > 10$ K, or the nature of the putative quantum critical point leading to strangeness in tBLG. There is, in fact, no theoretically established and experimentally verified generic mechanism for producing an extended linear-in-T metallic resistivity, except for scattering by acoustic phonons.

The fact that acoustic phonon scattering leads to a linear-in-T metallic resistivity for $T > T^*$, where T^* is a characteristic temperature dependent on both phonon and electron parameters, has been known since the 1930s [11]. The basic idea is simple: At "high T," the phonons become classical with equipartition, and their thermal occupancy increases linearly with T, consequently producing a T-linear resistivity. In regular three-dimensional (3D) metals, this typically happens for $T > 40$ K or so, which, in the dimensionless electronic temperature units, implies an extremely low T/T_F of $O(10^{-3})$, where T_F is the metallic Fermi temperature. For $T \ll T^*$, the phonon-induced resistivity goes as T^4 (T^5) in 2D (3D) metals and is extremely small. The crossover temperature scales as $T^* \sim T_{BG}/6$ or $T_D/6$, depending on whether T_{BG} or T_D is

smaller. The Bloch-Grüneisen (BG) temperature is given by

$$T_{\rm BG} = 2\hbar k_{\rm F} v_{\rm ph}, \tag{1}$$

where $k_{\rm F}$ is the electronic Fermi momentum and $v_{\rm ph}$ is the acoustic phonon (i.e., longitudinal sound) velocity. The Debye temperature $T_{\rm D}$ of most materials is around a few-hundred kelvins and, in metals, by virtue of the very large $k_{\rm F}$, $T_{\rm BG} \gg T_{\rm D}$, so $T^* \sim T_{\rm D}/6 \sim$ 40-50 K. It has been argued [12] that many 2D strongly correlated materials tend to have very low carrier density, and hence $k_{\rm F} \sim n^{1/2}$ is small, leading to rather low values of $T^* \sim T_{\rm BG}/6 \sim$ 1-10 K (or even lower depending on the actual doping density), since for low-density systems $T_{\rm D} \gg T_{\rm BG}$. Thus, in principle, phonon scattering could lead to strange metallicity with a linear-in-T resistivity going down to very low temperatures in dilute systems, but whether it actually happens in any of the actively studied strange metals remains an open question. One thing is certain: if the linear-in-T behavior survives to arbitrarily low T, it is unlikely to arise entirely from phonon scattering since the resistive scattering by phonons must be strongly suppressed for $T \ll T^*$, where quantum degeneracy strongly suppresses the phonon thermal occupancy, changing the T-linear resistivity to an almost unobservable T^4 dependence (in 2D) with a complicated crossover regime (for $T < T^*$) in between the two different power-law behaviors.

Since scattering by acoustic phonons is the only known generic mechanism for producing strange metallicity, i.e., a linear-in-T resistivity over a large range of temperatures (but not extending to arbitrarily low temperatures), it behooves us to critically discuss the recent experimental findings of Ref. [1] using phonon scattering within a minimal flat band Dirac model (so that there is a minimum number of unknown free parameters) as the underlying mechanism, in order to discern what aspects of the data are consistent with the phonon mechanism and where new physics may be lurking. This is what we do in the current paper. We note that the current work is an expanded and enhanced application of our earlier transport theories on the electron-phonon interaction induced temperature dependence of graphene and tBLG resistivity [9,13]. The new aspects of the current work are applying the theory quantitatively to new experimental data over a much broader temperature and density range than before so as to critically examine where new theoretical thinking might be necessary.

II. THEORY AND RESULTS

Now, we focus on tBLG, where $T_{\rm BG} < T_{\rm D}$, for all doping densities, so $T^* \sim T_{\rm BG}/6 \sim n^{1/2}$, where n is the effective doping density. We note that in graphene, $T_{\rm BG}$ tends toward zero as the Dirac point or the charge neutrality point (CNP) is approached since the effective doping vanishes at the CNP. Thus, indeed, as a matter of principle, phonons could produce a T-linear resistivity in graphene for arbitrarily low T and for arbitrarily low carrier density. Of course, this linear-in-T phonon-induced resistivity may be overwhelmed, particularly at very low temperatures, by other resistive contributions (e.g., disorder and impurity scattering) and thus become unobservable. Also, such a phonon-induced linear-in-T resistive behavior, even if it exists, can only happen at very small doping near the CNP and cannot explain a very low-T linearity at large doping values with finite $k_{\rm F}$.

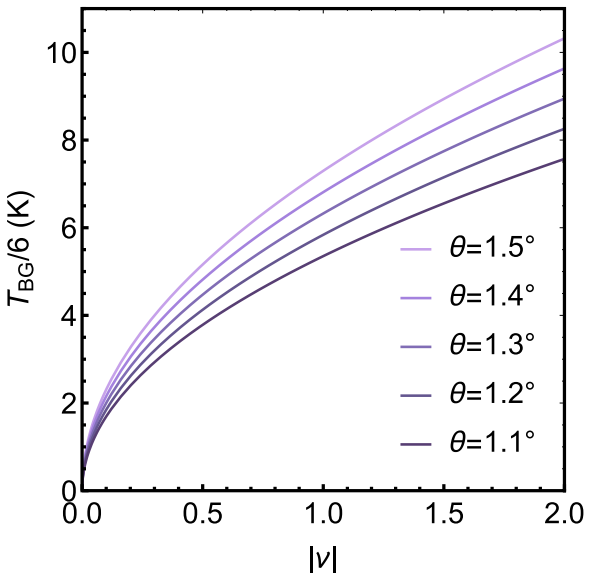

FIG. 1. The linear-in-T resistivity onset temperature $T^* \sim T_{\rm BG}/6$ as a function of the moiré band filling factor ν for different twist angles.

In Fig. 1, we show our calculated T^* for tBLG, for a few values of the twist angle, as a function of doping expressed in terms of the band filling ν, where $\nu = 0(4)$ indicates the unfilled empty moiré conduction band with the Fermi level at the CNP (the filled moiré conduction band with the Fermi level at the band edge). We show our results in terms of the band filling ν rather than the carrier density n in order to be consistent with the data presentation in Ref. [1]. Note that Fig. 1 stops at $\nu = 2$ simply because the Dirac approximation is manifestly invalid for large band filling beyond the van Hove singularities in the moiré band structure. In fact, our minimal model applies only for $\nu < 2$.

We note that as expected, T^* increases (decreases) with increasing (decreasing) band filling, simply because a higher filling implies, within our minimal 2D Dirac model, a higher carrier density and consequently a higher $k_{\rm F}$ in Eq. (1). The variation with θ arises from the moiré band structure effect included in our theory within the minimal Dirac model. We note that our minimal model predicts that for $T > 10$ K and for $\theta = 1.1°$-$1.5°$ (the experimental range for Ref. [1]), the linear-in-T behavior arising from phonon scattering should prevail everywhere for Ref. [1] measurements, and we therefore focus on the resistivity behavior for $T > 10$ K, leaving the discussion of the intriguing low-T transport behavior toward the end of this paper.

In Fig. 2, we show the filling- and twist-angle-dependent linear temperature coefficient $A_{T,1} = d\rho(T)/dT$ for all the experimental data in Ref. [1], obtained for [14]. We emphasize that Fig. 2 is simply a succinct summary (for our purpose) of the resistivity experimental data from Ref. [1], shown as a temperature derivative of the resistivity plotted as a function of both density and twist angle [14]. Any theory trying to explain the strange metallicity reported in Ref. [1] must at least be able to make sense of this "higher-temperature" $T > 10$ K

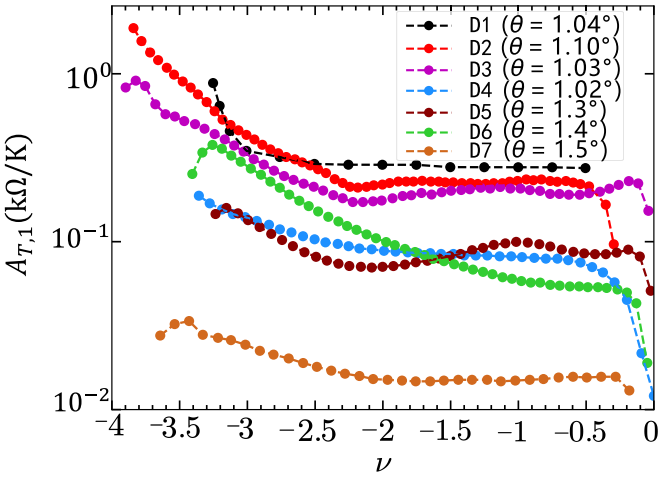

FIG. 2. The linear-in-T resistivity slope $A_{T,1} = d\rho/dT$ for $T >$ 10 K as a function of the filling factor ν in devices with different twist angles. The data are obtained from Ref. [1]. The vertical axis has a logarithmic scale.

data. The data for all seven samples of Ref. [1] are shown in Fig. 2, corresponding to seven different values of the twist angle between 1° and 1.5°. We first discuss several salient features of Fig. 2 before comparing our phonon theory with the data.

First, the density (angle) dependence of $A_{T,1}$ is weak (strong). This is absolutely obvious in Fig. 2, but not so apparent in Ref. [1], where $\rho(T)$ for many values of ν are shown displaced in the same figure and different θ values corresponding to different samples are plotted in different figures, making a comprehensive understanding of the density/angle dependence a challenge. Figure 2, which is a direct replot of the data in Ref. [1] (and kindly supplied to us by the experimentalists themselves [14]), makes it clear that the temperature derivative of resistivity, i.e., the slope of $\rho(T)$ with respect to T, is almost independent of density, but very strongly dependent on the twist angle, at least for all the high-T (>10 K) data. This qualitative behavior (i.e., weak/strong dependence on ν/θ) is easily explainable based on the phonon scattering mechanism, as discussed below and in our earlier work [9,10]. One should remember that each twist angle in Fig. 2 corresponds to a different sample, with likely unknown variations arising from disorder [15] and twist-angle fluctuations [16], making a direct quantitative comparison between the results at different angles difficult (and perhaps also explaining some of the nonmonotonicity and the crossing of various lines for $|\nu| > 1$). But the overall trend is clear: As the twist angle decreases from 1.5° to 1.1°, the resistivity coefficient $A_{T,1}$ is enhanced by more than a factor of 10 at, for example, $\nu = 0.5$ in Fig. 2. By contrast, $A_{T,1}$ remains essentially a constant as ν varies from 0.3 to 2 at the twist angle 1.5° and from 0.5 to 2.3 at the angle of 1.1°. Any theory must explain this dichotomy in the temperature coefficient of the measured tBLG resistivity with respect to its density and twist-angle dependence. Phonons can do this easily and naturally as discussed below. (We note that the drop-off at very low filling for all samples in Fig. 2 happens close to the CNP, where the carrier density is very small, and many other effects, including thermal carrier excitation and disorder, come into play. In addition, the CNP in the samples of Ref. [1] can have a small substrate-induced gap, which makes the temperature-dependent resistivity near CNP insulating in contrast to the metallic behavior of interest [14].)

The minimal theory for acoustic phonon scattering induced carrier resistivity for moiré Dirac carriers gives the following [9] formula:

$$\rho = \frac{32F(\theta)D^2k_F}{g_s g_v g_l e^2 \rho_m v_F^{*2} v_{ph}} I\left(\frac{T}{T_{BG}}\right),$$

$$I(z) = \frac{1}{z}\int_0^1 dx x^4\sqrt{1-x^2}\frac{e^{x/z}}{(e^{x/z}-1)^2}, \quad (2)$$

where D is the deformation potential electron-phonon coupling, v_F^* is the Fermi velocity in the moiré band, $g_{s,v,l}$ are the degeneracy factors (all equal to 2), and ρ_m is the atomic mass density of graphene. The function $F(\theta)$ is a weakly varying function of θ, dependent on the detailed moiré band structure, which is discussed in detail in Ref. [9]. Typically, $0.5 < F(\theta) < 1$. Since our interest is in the equipartition regime of $T > T^*$, where the linear-in-T resistivity manifests, we can obtain from Eq. (2), for $T > T^*$, an approximate expression for $A_{T,1} = d\rho/dT$ as

$$A_{T,1} = d\rho/dT \propto \left[\frac{1}{v_F^{*2}}\left(\frac{D}{v_{ph}}\right)^2\right]T. \quad (3)$$

Thus the main features of the phonon-induced linear-in-T resistivity are the following: (1) it has no dependence on k_F, and hence on carrier density or band filling, and (2) it depends strongly on the moiré band velocity v_F^* through the $1/v_F^{*2}$ dependence—as the moiré band flattens approaching the magic angle, the flat-band-renormalized Fermi velocity v_F^* is strongly suppressed, enhancing the phonon-induced resistivity slope by a factor of v_F^{*-2}. Thus, the two most significant qualitative features of the data presented in Ref. [1] are completely explained by phonon scattering: *Weak* dependence on carrier density or band filling and *strong* dependence on the twist angle through the θ dependence of v_F^*. We know of no other theoretical model for tBLG transport, which can naturally explain these two key qualitative features of the tBLG strange metallicity. We also mention that Eq. (3) for the temperature coefficient of the linear-in-T tBLG resistivity is similar to the corresponding result for the untwisted monolayer graphene (MLG) [13], which has been experimentally verified [17], except for the appearance of the moiré flat-band Fermi velocity v_F^* in Eq. (3), whereas for regular untwisted graphene, the Fermi velocity is that for regular MLG, $v_F > v_F^*$.

To show the obvious large effect of the flat-band suppression of the Fermi velocity in the tBLG compared with untwisted MLG, we plot in Fig. 3 the calculated ratio v_F^*/v_F as a function of the twist angle, obtained using the standard Bistritzer-MacDonald (BM) band structure model for the moiré system [18]. Within the BM model, v_F^* vanishes at the magic angle ∼1°, and is as small as $v_F/4$ even for a large twist angle of 1.5°, which corresponds to a factor of 16 enhancement in the tBLG resistivity and its temperature coefficient compared with that in the untwisted MLG.

We use the full Eq. (2) to calculate the theoretical $d\rho/dT$ using the BM moiré band structure [18] to compare with

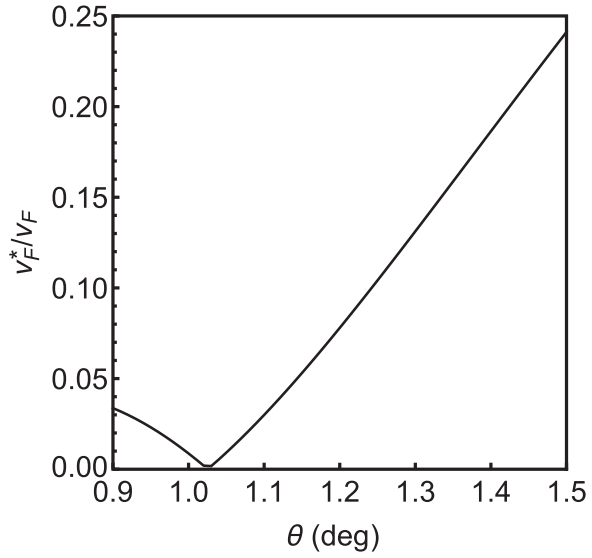

FIG. 3. The twist-angle dependence of v_F^*/v_F obtained from the BM model. v_F and v_F^* are the Fermi velocity at the Dirac points in MLG and tBLG, respectively.

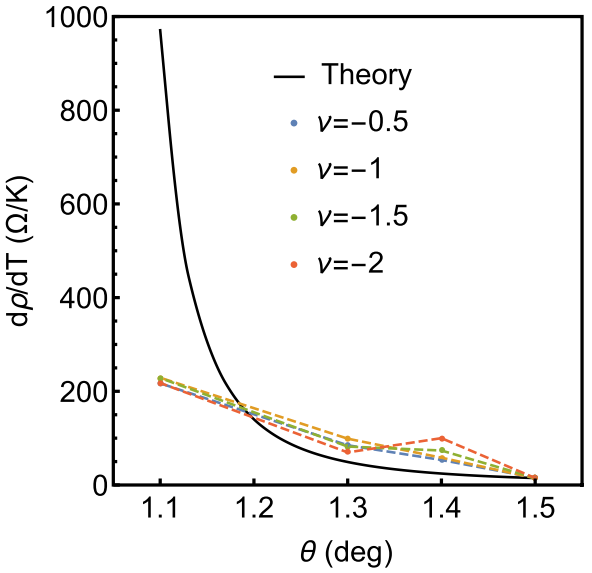

FIG. 4. Comparison between theoretical (black curve) and experimental (colored dots) slope $d\rho/dT$ for $T > T^*$. The experimental data are replotted from Fig. 2. The theoretical curve is calculated based on Eq. (2) with a fixed $D = 100$ eV.

the experimental data shown in Fig. 2. The only unknown, which we use as an adjustable parameter, is the deformation potential coupling D in tBLG, whose value we fix by demanding agreement between theory and experiment at the largest twist angle $\theta = 1.5^\circ$ (and then keep it fixed throughout). Note that D just determines the overall scale of the resistivity, not its functional dependence on carrier density and twist angle. Thus, the qualitative finding of a weak (strong) density (angle) dependence is generic in the phonon theory and independent of the value of D. We use the accepted graphene values for the sound velocity and mass density for the transport calculation, $\rho_m = 7.6 \times 10^{-8}$ g/cm^2 and $v_{\rm ph} = 2 \times 10^6$ cm/s. In Eqs. (2) and (3) for the resistivity of tBLG, the Fermi velocity v_F^* takes the renormalized value due to moiré superlattices, but the mass density and sound velocity take the same values as those in monolayer graphene. The sound velocity of longitudinal acoustic phonons in tBLG is found to be not significantly altered compared to that in monolayer graphene, as shown in Ref. [19]. Therefore, we take $v_{\rm ph}$ to be the sound velocity of monolayer graphene. ρ_m is taken to be the mass density of monolayer graphene in Eq. (2), while the additional layer degree of freedom in tBLG is taken into account by the factor g_l. A detailed description of electron-phonon coupling in tBLG can be found in Ref. [9].

In Fig. 4, we provide a direct comparison of the theoretical results for the calculated temperature coefficient $A_{T,1} = d\rho/dT$ as a function of the twist angle with the corresponding experimental results from Ref. [1], as shown in Fig. 2. The experimental data for different band fillings (i.e., different densities) are replotted from Fig. 2 for clarity, and the solid line is the theoretical calculation within the minimal Dirac model using the BM band structure.

The deformation potential fit at $\theta = 1.5^\circ$ gives $D = 100$ eV, which is what we use throughout in Fig. 4. It is reassuring that the same value of D (∼80-100 eV) is necessary here for agreement with the tBLG data of Ref. [1] as what was needed [9] to obtain quantitative agreement with the earlier tBLG transport results in Refs. [2] and [3]. For regular MLG, the quoted value for D ranges between 20 and 40 eV [9,13], and for tBLG, we need roughly a D which is 2–3 times larger. It is possible that the twisted graphene layers indeed have larger values of deformation potential coupling than regular untwisted graphene, but, for us, D is simply one free parameter of the theory, which we fix at the largest twist angle, for which Ref. [1] reports its experimental results. It may be useful to point out in this context that, in general, the deformation potential coupling is often unknown in materials, and different ways of estimating the coupling constant often differ by factors of 2–3. For example, even in extensively studied GaAs, the deformation potential coupling differs by a factor of 2 between optical and transport measurements, and typically its precise value for calculating the resistivity of doped GaAs has to be fixed by comparing with the transport data [20]. It is thus quite possible for the moiré structure to modify the deformation potential coupling by a factor of 2–4 in the tBLG system. More work should be carried out to definitively settle this question. In fact, it has been argued [21] that an effective "Purcell effect" associated with the compression of the Wannier orbitals in tBLG strongly enhances the electron-phonon coupling, which is certainly consistent with our finding of an effective enhanced deformation potential necessary for quantitatively explaining the tBLG resistivity data.

Another possibility is that the sound velocity $v_{\rm ph}$ is somehow suppressed by the moiré superlattices, decreasing it from its nominal graphene value of 2×10^6 cm/s. Since $A_{T,1} \propto (D/v_{\rm ph})^2$, a decrease of $v_{\rm ph}$ is equivalent to an increase in the deformation potential constant compared with its nominal MLG value. It is also possible that both D and $v_{\rm ph}$ are affected by the moiré superlattices, leading to an enhancement of the effective coupling. These are all possibilities that future work should explore, but, for our purpose, it suffices to take the

overall scale as a phenomenological tuning parameter and fix it by comparing with the experiment at the highest available twist angle of 1.5° in Ref. [1]. We note that a suppressed v_{ph} in the twisted moiré system would also decrease the crossover temperature scale T^* since $T_{BG} \propto v_{ph}$, leading to the linear-in-T resistivity extending to lower temperature scales of the order of 1 K or less. Our calculation assumes that the phonons are unaffected by the moiré superlattice and retain their pristine values for MLG. It is, however, possible that the twisting strongly affects the acoustic phonons through strain and other effects. If so, D/v_{ph} could increase considerably from their MLG value, providing a unified explanation for why the resistivity is linear down to low-T and why it is so strongly enhanced. We do not, however, pursue such a line of argument further in this paper since our goal is not seeking a precise quantitative agreement with experiments, but to understand which aspects of the data in Ref. [1] are generically and qualitatively in agreement with the phonon scattering mechanism so that we can, in the future, focus on the unknown mechanisms perhaps in play here beyond phonon scattering.

The results presented in Fig. 4 show that the phonon mechanism explains the experimental data of Ref. [1] for all samples for twist angle down to ∼1.17°, but then the theory predicts much larger resistivity than the experiment. Fine tuning the value of D to obtain the best recursive fit to the experimental data may extend the regime of agreement down to ∼1.15°, but we do not believe that this is a useful exercise as the experiment and theory most definitely disagree at low enough twist angle somewhere just below 1.2°. A trivial possibility, which we do not consider, is that D somehow decreases for $\theta < 1.2°$, for example, experiment and theory would agree well at $\theta = 1.1°$ if we arbitrarily reduce D (at 1.1° twist angle) to 30 eV. However, this is just data fitting with no justification.

We believe that the systematic quantitative failure of our transport theory at lower twist angle, apparent in Fig. 4, arises from two reasons: (1) as the moiré band flattens with decreasing angle, our Dirac approximation becomes increasingly inaccurate quantitatively, and, perhaps more importantly, (2) for small twist angles, the moiré band Fermi velocity v_F^* becomes comparable to the phonon velocity v_{ph}, leading to the inapplicability of the Migdal theorem [22], which is the basis of our minimal Boltzmann transport theory. To demonstrate this point more concretely, we show in Fig. 5 the calculated ratio v_F^*/v_{ph} as a function of the twist angle. We emphasize that for regular MLG, i.e., for untwisted graphene (or, equivalently, for very large twist angles), this ratio goes to the very high value of 50 since $v_{ph} \sim 2 \times 10^6$ cm/s and $v_F \sim 10^8$ cm/s for regular graphene, and the Migdal approximation is essentially exact [23]. But with decreasing v_F^*/v_{ph} in tBLG, as shown in Fig. 5, the Migdal approximation, and hence the leading-order transport theory, becomes increasingly inaccurate [24]. In Fig. 5, we denote, on the abscissa, the points where $v_F^* = v_{ph}$ as well as the "magic angle" where the moiré band is perfectly flat within the BM approximation producing $v_F^* = 0$. Since $v_F^* \sim v_{ph}$ already at $\theta \sim 1.1°$, we expect our transport approximation to work only for $\theta > 1.1°$. If we arbitrarily set the lower limit of the Migdal approximation to be $v_F^* > 3v_{ph}$, then the theory should give reasonable quantitative

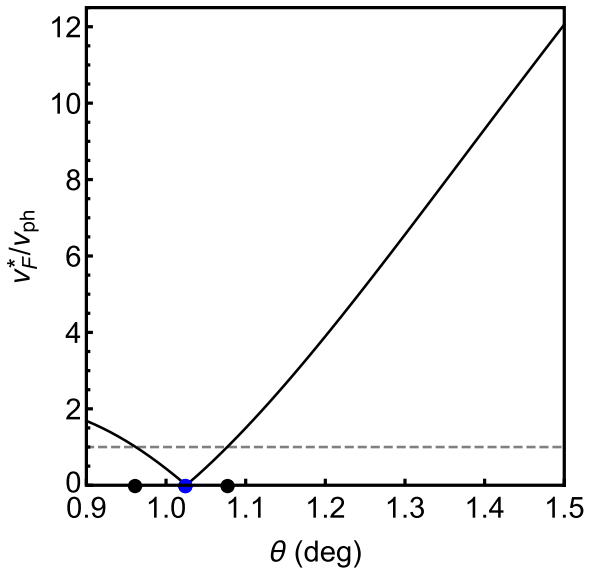

FIG. 5. The twist-angle dependence of v_F^*/v_{ph} obtained from the BM model. v_F^* is calculated using the BM model, and v_{ph} is assumed to be independent of the twist angle in the calculation. The blue dot marks the magic angle at which v_F^* vanishes. The two black dots indicate the angles where $v_F^* = v_{ph}$.

results for $\theta > 1.15°$, which is approximately consistent with Fig. 4. We emphasize that there is no controlled way to go beyond the minimal theory when the Migdal approximation no longer applies since all diagrams in the electron-phonon coupling to all orders contribute to the resistivity in a hopelessly strong-coupling manner since the electron-phonon coupling itself is also strong by virtue of the small Fermi velocity v_F^* at small angles, exactly where the Migdal approximation also breaks down.

III. DISCUSSION AND CONCLUSION

We have established a reasonable case that the tBLG linear-in-T resistivity data of Ref. [1] are well explained by acoustic phonon scattering at higher temperatures (>10 K) and at higher twist angles (>1.15°), suggesting the failure of the Migdal theorem as the reason for the quantitative failure of our theory at lower twist angles. In addition to providing a reasonable quantitative description of the data [1], the theory has the great intrinsic advantage of providing a natural *qualitative* explanation for the weak density (i.e., band filling) and the strong twist-angle dependence of the temperature coefficient of the observed linear-in-T resistivity. We know of no alternative theory which is capable of providing a qualitative understanding of the experimentally observed (see Fig. 2 and also the results in Ref. [1]) weak (strong) density (twist-angle) dependence of the resistivity as the phonon theory does naturally.

The open question we have not discussed yet is the low-temperature behavior of the resistivity, below 10 K, and actually even below 1 K. We emphasize a key aspect of the experimental data which all theories must address. The slope of $d\rho/dT$ in the observed linear-in-T resistivity can remain constant throughout the low-T and high-T regimes

for certain density ranges at magic-angle devices, hinting at one underlying universal transport mechanism in play since it is unlikely that two independent mechanisms would produce the same slope. In our phonon theory, this constant slope is naturally explained as arising from the phonon equipartition physics coming from the high-T regime. Any quantum critical or other competing theories must address why the resistivity remains linear to very high T ($\sim$50 K) without changing the slope since quantum criticality is a $T = 0$ phenomenon. We emphasize that our focus has been on the experimental resistivity of Ref. [1] for $T > 10$ K, where the linear-in-T resistivity is fairly generic in all samples essentially for most values of band filling. Our Fig. 2 is a precise replotting of the experimental data for $T > 10$ K in Ref. [1], shown as $d\rho/dT$ for different ν and θ. Unfortunately, the experimental data of Ref. [1] at low temperatures do not reflect a universal behavior, with the temperature dependence of $\rho(T)$ manifesting different power laws at different band fillings, as can be clearly seen in Fig. 1(d) as well as Fig. S2(c) of Ref. [1]. This is in fact to be expected since, at low temperatures for $T < T^*$, the phonon scattering gets quenched and phonon-induced resistivity changes from the linear-in-T behavior to a T^4 behavior, decreasing by four orders of magnitude and becoming essentially unobservable. In this situation, many different scattering mechanisms come into play, including phonon drag, impurity scattering, disorder scattering by twist-angle fluctuations, electron-electron Baber and/or umklapp scattering, and possible scattering by the quantum fluctuations associated perhaps with any quantum critical point dominating the $T = 0$ quantum phase diagram. We comment that scattering by spin or isospin (e.g., valley) fluctuations leads to a rather weak resistivity, which manifests a power law higher than linear, thus making these mechanisms unlikely to be the underlying resistive scattering mechanisms. A possible way to distinguish experimentally between quantum criticality and phonon scattering mechanisms for tBLG transport is to directly demonstrate the existence of critical fluctuations by measuring a diverging susceptibility, which, to the best of our knowledge, has not yet been established in tBLG systems. We do not anticipate any universal temperature dependence in this regime because of the presence of multiple different scattering processes competing with each other. Reference [1] reports resistivity power laws on T varying from 1 to 2, depending on the band filling, in the $T < 1$ K regime. It is unlikely that any linear-in-T resistivity in the $T < 1$ K regime could arise from the electron-phonon scattering mechanism of interest in the current work since, according to Fig. 1, the crossover temperature scale for phonon scattering to manifest a linear-in-T resistivity is $\sim$1 K even for a band filling as low as $\nu = 0.1$. We do not therefore have any explanation based on our minimal phonon scattering theory for any linear-in-T behavior arising in the $T < 1$ K regime in Ref. [1]. We do, however, mention that low-temperature (< 10 K) resistivity of even regular normal metals is not explicable based on any single scattering mechanism, and typically one must combine several different scattering sources in a rather arbitrary manner to make sense of the low-T resistivity of metals, in sharp contrast to the high-T ($>$40 K for metals, $>$10 K for tBLG) linear-in-T resistivity which is generic and universal, and is caused by phonon scattering [25].

There are known situations in semiconductor-based 2D systems [26–30] where an approximate low-temperature linear-T resistivity may manifest in dilute carrier systems, arising from the nonphonon mechanism of the interplay between disorder and screening effects [31], but this mechanism is very unlikely to play any role in graphene or tBLG. Also, a Hubbard-type strongly correlated model may produce a linear-in-T resistivity from umklapp electron-electron scattering at very "high" temperatures, but this is essentially a classical effect of energy equipartition with absolutely nothing "strange" in this linearity. It is unlikely that the linear-in-T resistivity behavior observed in tBLG [1–3] has anything to do with the high-T Hubbard model properties. In fact, we believe that any umklapp electron-electron interaction is likely to produce a T^2 low-temperature resistivity in tBLG itself, as has recently been argued in the literature [32,33]. Thus, the appearance of a linear-in-T resistivity in tBLG down to arbitrarily low temperatures for some specific band fillings, as reported in Ref. [1] (and earlier in Ref. [3]), remains a mystery at this stage, but the observed higher-T linear-in-T resistivity most likely arises from the phonon scattering that is strongly enhanced by flat-band moiré effects.

Before concluding, we mention that a pronounced linear-in-T resistivity has also recently been reported [34,35] in moiré twisted double bilayer graphene (tDBLG), as predicted earlier in [36] based on phonon scattering considerations, with a strong flat-band-induced enhancement of the temperature coefficient of the resistivity with respect to the untwisted regular DBLG [37]. Thus, the phenomenon of phonon-induced pronounced strange metallic behavior may be a generic property of all moiré systems where the carrier Fermi velocity is strongly suppressed by the moiré band structure. We should also comment on the so-called Planckian behavior of the phonon physics [3–8]. Converting the electron-phonon deformation potential D into an effective dimensionless coupling constant λ, we get $\lambda \sim 0.5$-1 for tBLG [9], which should be contrasted with a $\lambda \sim 0.0001$ for regular untwisted MLG [13]. Thus, the scattering rate $1/\tau$ in tBLG for small twist angles, $\hbar/\tau = 2\pi\lambda k_B T$, is roughly 3–6 times the temperature (similar to the situation in strong electron-phonon coupling metallic systems such as Pb), making tBLG a super-Planckian metal, strongly violating the Planckian bound of $\hbar/\tau < k_B T$, as has been discussed in the literature [1,3,8]. The super-Planckian behavior in our theory is of course neither strange nor mysterious (for $T > 10$ K at least) since it arises from enhanced phonon scattering under the moiré flat-band conditions.

To conclude, acoustic phonon scattering provides a reasonable generic explanation for the observed temperature dependence of the tBLG resistivity reported in Ref. [1], but cannot explain the transport properties for $T < 1$ K. More work would be necessary to figure out the scattering mechanisms in tBLG at low temperatures below 1 K, where phonon scattering is likely strongly suppressed because of low thermal phonon occupancy in the Bloch-Grüneisen regime.

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