Entanglement in Unruh, Hawking, and Cherenkov radiation from a quantum optical perspective

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Free quantum field theory in flat space-time is often believed to be well established, holding no surprises. We hope, by the end of this paper, to have demonstrated that surprises still exist. For example, we will show that a uniformly accelerated atom in Minkowski space-time emits entangled photon pairs in a squeezed state which mimics entanglement of the Minkowski vacuum in that the entanglement is between Minkowski modes which are dominantly in opposite causal wedges of the space-time. Similar emission of photon pairs occurs if an atom is held above the black hole event horizon. Namely, a ground-state atom becomes excited by emitting a "negative" -energy photon under the horizon and then spontaneously decays back to the ground state by emitting a positive-energy photon outside the horizon, which propagates away from the black hole.

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I. INTRODUCTION

The major results reported in this paper are given in Fig. 1. In Fig. 1(a) we depict a ground-state atom moving in the right Rindler wedge with an acceleration a which becomes excited by emitting a photon into a superposition of a right-propagating Unruh-Minkowski (UM) mode $F_{2\Omega}$, where $\Omega = \omega c/a$ and ω is the atomic transition frequency in the atom's frame, and a left-propagating mode $G_{2\Omega}$. The mode $F_{2\Omega}$ is mostly localized above the $t = z/c$ Rindler horizon, and $G_{2\Omega}$ is mostly localized below the $t = -z/c$ horizon. The excited atom then decays to the ground state by emitting a sum of a right-propagating photon into the UM mode $F_{1\Omega}$, which is mostly localized below the $t = z/c$ horizon, and a left-propagating mode $G_{1\Omega}$ mostly localized above the $t = -z/c$ horizon. The modes emitted via excitation of the atom are thus mostly located in regions of the space-time away from the atom, while the modes emitted during the deexcitation of the atom ($F_{1\Omega}$ and $G_{1\Omega}$) are largest in the region around the atom.

In the following we will concentrate on the calculation for the right-propagating modes (F_{Ω}), but the calculations for the left-propagating modes (G_{Ω}) are essentially identical. One could also make the atom sensitive to only the right-going modes by coupling the atom to $(\frac{1}{c}\frac{\partial}{\partial\tau} - \frac{\partial}{\partial\rho})\phi$, instead of to $\frac{\partial\phi}{\partial\tau}$, where τ is the proper time along the path of the atom and ρ is the orthogonal spatial coordinate at the atom's frame.

In Fig. 1(b) we depict a ground-state atom held fixed above the event horizon of a static black hole (BH) in Schwarzschild coordinates t, r. It is uniformly accelerated in the Kruskal-Szekeres coordinates (T, X) through the Hartle-Hawking vacuum (which could also be called the Kruskal vacuum, in analogy with the Minkowski vacuum). The atom becomes excited by emitting a photon into the Unruh-Schwarzschild (US) modes $F_{2\Omega}$ and $G_{2\Omega}$, which are mostly localized above the $T = X$ line or under the $T = -X$ line, which is behind the BH horizons. The excited atom then decays to the ground state by emitting a photon into the US modes $F_{1\Omega}$ and $G_{1\Omega}$, which are mostly localized outside the BH horizon. The right-propagating modes $F_{1\Omega}$ propagate away from the black hole to infinity.

In both the Unruh and Hawking radiation the combination of emission beyond the horizon associated with the $F_{2\Omega}$ mode and spontaneous emission into the $F_{1\Omega}$ mode yields entangled states of the two-photon pair. This two-photon squeezed-entangled state takes the form

$$|\Psi\rangle = (1 + \beta\hat{a}^{\dagger}_{2\Omega}\hat{a}^{\dagger}_{1\Omega})|0\rangle, \tag{1}$$

where β characterizes small but important correlation between the modes inside ($F_{2\Omega}$) and outside ($F_{1\Omega}$) the horizon associated with the photon creation operators $\hat{a}^{\dagger}_{2\Omega}$ and $\hat{a}^{\dagger}_{1\Omega}$, respectively.

From a quantum optical point of view, the Unruh effect can be thought of, and calculated, as a result of virtual photons made real due to accelerating an atom. The probability of finding the atom excited (and a plane-wave photon emitted) is found to be [1]

$$P = \frac{2\pi c g^2}{a\omega}\frac{1}{\exp\left(\frac{2\pi\omega c}{a}\right) - 1}, \tag{2}$$

where g is the atom-field coupling constant and a is the atom's acceleration.

However, there is more to the story. When a ground-state atom becomes excited by emitting an acceleration radiation

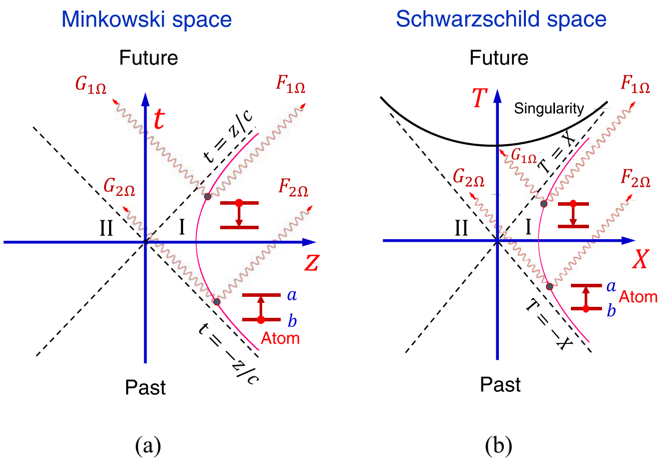

FIG. 1. (a) A ground-state atom accelerated in wedge I goes to the excited state $|a\rangle$ while emitting a photon into a right-propagating Unruh-Minkowski (UM) mode $F_{2\Omega}$, which is mostly located in the "future" wedge and wedge II by the ratio of the Boltzmann factor for temperature $a/(2\pi c)$, and a left-propagating mode $G_{2\Omega}$, which is mostly localized below the $t = -z/c$ horizon. Subsequently, the atom spontaneously decays to the ground state $|b\rangle$ emitting a photon into the UM modes $F_{1\Omega}$ and $G_{1\Omega}$, which are mostly located in the same wedge as the atom. (b) A ground-state atom held fixed above the horizon of a Schwarzschild black hole goes to the excited state while emitting a photon into the Unruh-Schwarzschild (US) mode $F_{2\Omega}$ which exists mostly in the "future" wedge below the event horizon and mode $G_{2\Omega}$, which is mostly localized under the $T = -X$ line. Subsequently, the atom spontaneously decays to the ground state emitting a photon into the US modes $F_{1\Omega}$ and $G_{1\Omega}$, which are located mostly above the horizon.

photon into, say, the left Rindler wedge, the atom can go back to the ground state by spontaneously emitting another photon into the right Rindler wedge. This entangled two-photon configuration can be (and is) well explained by nothing much more than operator algebra of the type discussed above. One may then well ask if there is a quantum optical explanation for the entangled photon pairs just as we have for the acceleration radiation. The answer is yes and is the subject of this paper.

To put this paper in perspective, we recall that the Unruh effect, that accelerated atoms see the vacuum as a thermal state, can be realized without introducing atoms at all. That is, we use the relation between Minkowski ($\hat{a}_\nu$) and Rindler ($\hat{b}_\Omega$) [2] photon operators as

$$\hat{b}_\Omega = \int_0^\infty d\nu[\alpha_{\Omega\nu}\hat{a}_\nu - \beta_{\Omega\nu}\hat{a}_\nu^\dagger], \tag{3}$$

where the Bogoliubov coefficients are [2,3]

$$\beta_{\Omega\nu} = -\frac{c}{2\pi a}\sqrt{\frac{\Omega}{\nu}}e^{-\frac{\pi c\Omega}{2a}}\left(\frac{c\nu}{a}\right)^{i\frac{c\Omega}{a}}\Gamma\left(-i\frac{c\Omega}{a}\right) \tag{4}$$

and $\Gamma(x)$ is the gamma function. The expectation value of the Rindler photon number operator for the Minkowski vacuum is then found to be

$$\langle 0_M|\hat{b}_\Omega^\dagger\hat{b}_\Omega|0_M\rangle = \int_0^\infty d\nu|\beta_{\Omega\nu}|^2 = \frac{1}{e^{2\pi c\Omega/a} - 1}. \tag{5}$$

Indeed, the entanglement of photons [4] between right and left Rindler wedges is a hallmark of acceleration radiation, having much in common with quantum optics. The Unruh effect can be understood as a process of superoscillations [5], in which a function, composed purely of modes with a limited range of frequencies, can have oscillations outside that range in certain regions. In this case, the limitation is that the frequency is limited to purely negative frequencies (for the modes associated with the creation operator of the fields). On excitation of the atom, the accelerated atom probes that region, and the positive frequencies associated with the creation operators cause an excitation of the field, a particle emission by the atom. The accelerated atom makes a transition from the ground state to an excited state while emitting a photon into a Minkowski vacuum [6]. From another point of view, Unruh radiation can be viewed as (Rindler) photons existing as a thermal bath in a uniformly accelerated reference frame [2,7,8]. The Rindler state vacuum can be obtained by operating on the Minkowski vacuum with UM creation operators [4]. From a quantum optics vantage, this amounts to applying a squeeze operator to the Rindler vacuum generating biphoton pairs of creation operators for photons corresponding to positively and negatively accelerated frames.

This is simply a feature of the Minkowski vacuum, when viewed from the modes naturally associated with the accelerated frame. There is, however, another effect, intimately involving the atom.

In quantum optics, correlated photon pairs can come from, for example, two-photon downconversion generating signal photons of frequency ν_1 and idler photons of frequency ν_2. That is, the photon pair operation is governed by the biphoton operator $\hat{a}_1^\dagger\hat{a}_2^\dagger$, where $\hat{a}_1^\dagger$ ($\hat{a}_2^\dagger$) is the creation operator of the idler (signal) photon.

In this paper we develop the theory of biphotons generated by atoms emitting acceleration radiation (described by operator $\hat{a}_2^\dagger$) and being excited in the process, and thereafter spontaneously emitting a photon (described by operator $\hat{a}_1^\dagger$). This creates an entangled pair of photons, ordinary Minkowski photons. We apply the same process to generate entangled pairs in Hawking [9] and Cherenkov [10] radiation.

The interplay between aspects of general relativity and quantum optics yields insights and flags open questions. See, e.g., recent work on acceleration radiation from an atom falling into a black hole [11], which is analogous to excitation of a fixed atom by a uniformly accelerated mirror in Minkowski space-time [12]. More recently [13], we have emphasized the following.

> "Emission of photons by atoms can occur into modes which extend into a region causally disconnected with the emitter. For example, a uniformly accelerated ground-state atom emits a photon into the Unruh-Minkowski mode which is exponentially larger in the causally disconnected region. This makes an impression that photon emission is acausal. Here we show that conventional quantum optical analysis yields that a detector atom will not detect the emitted photon in the region noncausally connected with the emitter. However, joint excitation probability of atoms in the causally disconnected regions can be correlated due to entanglement of Minkowski vacuum and be much larger than the product of independent excitation probabilities."

In this paper we show that the vacuum entanglement referred to above can be demonstrated via the simple toy model depicted in Fig. 1(a). The message of Fig. 1(a) is that the atom (coming from infinity) is accelerated through wedge I. That is, the atom with high velocity travels along the z axis from infinity, slows down, stops, and and then keeps accelerating back out to infinity. The atom emits an acceleration radiation photon which is predominantly in wedge II and subsequently spontaneously emits a photon located dominantly in wedge I. As is shown below, these photons are entangled and are in a two-mode squeezed state.

In quantum optics [14] the two-mode squeezed state is generated by the action of the unitary squeeze operator

$$\hat{S} = e^{\xi^* \hat{a}_1 \hat{a}_2 + \xi \hat{a}_1^\dagger \hat{a}_2^\dagger}$$

on the two-photon vacuum state $|0_1 0_2\rangle$, where $\hat{a}_1(\hat{a}_2)$ and $\hat{a}_1^\dagger(\hat{a}_2^\dagger)$ are the usual annihilation and creation operators. In particular, for weak squeezing, such that $\xi \ll 1$, we have

$$|\Psi\rangle = [1 + \xi \hat{a}_1^\dagger \hat{a}_2^\dagger]|0_1 0_2\rangle.$$

The two-mode squeezed state can be generated in a nonlinear crystal by a parametric downconversion process in an optical cavity [15], in which a strong (classical) laser field produces a pair of photons 1 and 2 as described by the Hamiltonian

$$\hat{H} = \hbar g(\hat{a}_1^\dagger \hat{a}_2^\dagger + \hat{a}_1 \hat{a}_2),$$

where the coupling strength g is the effective Rabi frequency of the driving field. The Schrödinger equation yields that the state of the generated field is

$$|\Psi(t)\rangle = e^{-ig(\hat{a}_1 \hat{a}_2 + \hat{a}_1^\dagger \hat{a}_2^\dagger)t}|0_1 0_2\rangle,$$

which is a two-mode squeezed state. To lowest order, the downconversion process produces the "squeezed" state

$$|\Psi\rangle \cong (1 - ig\tau \hat{a}_1^\dagger \hat{a}_2^\dagger)|0_1 0_2\rangle,$$

where τ is the duration of the short pulse driving field in this simple model. In the squeezed state, the photon numbers in the modes are correlated. Namely, the photon numbers in each mode n_1 and n_2 fluctuate, obeying thermal distribution, but the difference $n_1 - n_2$ does not fluctuate. That is, if there are n photons in mode 1, then with unit probability there are n photons in mode 2.

It is interesting that the Minkowski vacuum $|0_M\rangle$ is a squeezed state in terms of Rindler photons; namely, in terms of Rindler states [4]

$$|0_M\rangle = \prod_{\nu>0}(1 - e^{-2\pi c\nu/a})e^{\exp(-\frac{\pi c\nu}{a})(\hat{b}_{R1\nu}^\dagger \hat{b}_{R2\nu}^\dagger + \hat{b}_{L1\nu}^\dagger \hat{b}_{L2\nu}^\dagger)}|0_R\rangle, \quad (6)$$

where $|0_R\rangle$ refers to Rindler vacuum, $\hat{b}_{R1\nu}^\dagger$ and $\hat{b}_{R2\nu}^\dagger$ ($\hat{b}_{L1\nu}^\dagger$ and $\hat{b}_{L2\nu}^\dagger$) are creation operators of Rindler photons in the right-(left-)propagating Rindler modes

$$\phi_{1\nu} = \sqrt{\frac{a}{\nu c}}(\mp z - ct)^{i\frac{\nu c}{a}}\theta(\mp z - ct) \quad (7)$$

and

$$\phi_{2\nu} = \sqrt{\frac{a}{\nu c}}(ct \pm z)^{-i\frac{\nu c}{a}}\theta(ct \pm z). \quad (8)$$

Here, $a > 0$ is a parameter which has dimension of acceleration. Rindler modes $\phi_{1\nu}$ and $\phi_{2\nu}$ are solutions of the wave equation and for $\nu > 0$ have positive norm (defined as the Klein-Gordon inner product). The mode functions (7) and (8) are nonzero in half of the t-z plane and form a complete basis set.

One should note that squeezed states also appear in cosmology. For example, relic gravitational waves are in squeezed states in which variances in the wave amplitude distribution are very large, while variances in the phase distribution are practically equal to zero [16].

In Sec. II we present a detailed analysis of the acceleration-radiation-induced entangling of UM modes $F_{1\Omega}$ and $F_{2\Omega}$, which is essentially a two-mode squeezed state. In Sec. III we present an analysis of the Hawking radiation which consists of entangled photon pairs localized above and below the BH event horizon. In Sec. IV we show how "squeezed-entangled" radiation is related to Cherenkov radiation. Section V is a discussion and conclusion.

II. GENERATION OF SQUEEZED PHOTON STATES BY ACCELERATED ATOMS

Next we consider an electrically neutral two-level atom with a transition angular frequency ω which moves along the trajectory $t(\tau), z(\tau)$ in a vacuum, where τ is the proper time of the atom. The atom is coupled to the electromagnetic field. For simplicity we approximate the field as a scalar field described by the operator $\hat{\Phi}(t, z)$ and consider dimension $1 + 1$. We will call the quanta of the scalar field photons and will assume the following form of the interaction Hamiltonian between the atom and the scalar field:

$$\hat{V}(\tau) = g(\hat{\sigma}e^{-i\omega\tau} + \hat{\sigma}^\dagger e^{i\omega\tau})\frac{\partial}{\partial\tau}\hat{\Phi}(t(\tau), z(\tau)), \quad (9)$$

where g is the atom-field coupling constant and $\hat{\sigma}$ and $\hat{\sigma}^\dagger$ are the atomic lowering and raising operators. Since the atom feels the local value of the field, the operator $\hat{\Phi}$ is evaluated at the atom's position $t(\tau), z(\tau)$. For simplicity we consider only the right-propagating waves. The results can be generalized to include the left-propagating waves in a straightforward way.

In the usual quantization procedure, one splits the solutions to the field equations into two sets of Fourier modes. To make the system as simple as possible, we will examine a massless scalar field ϕ in $1 + 1$ dimensions, although the results, while more complex for massive fields, scalar or vector, apply in higher dimensions as well. The field equation of motion is taken as

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}\right)\phi(t, z) = 0. \quad (10)$$

The solutions of Eq. (10) can be written as sums of the Fourier transform modes

$$\phi_k(t, x) = e^{-i(\nu t - kz)} \quad (11)$$

with $\nu^2 = k^2$ or $\nu = \pm k$.

In the usual quantization procedure, one associates the modes with $\nu > 0$ with the annihilation operators for the field $\hat{a}_k$ and the modes with $\nu < 0$ with the creation operators $\hat{a}_k^\dagger$, which obey the commutation relations $[\hat{a}_k, \hat{a}_{k'}^\dagger] = \delta_{kk'}$. In order that the field strength ϕ and its conjugate momentum $\pi = c\partial L/\partial\dot{\phi} = (1/c)\partial\phi^*/\partial t$ obey the standard commutation relations, we must have

$$\frac{i}{2c}\int_{-\infty}^{\infty}\left(\phi_k^*\frac{\partial\phi_{k'}}{\partial t} - \frac{\partial\phi_k^*}{\partial t}\phi_{k'}\right)dz = \delta_{kk'}. \qquad (12)$$

For our purposes, however, we will choose a different set of modes, often called the Unruh-Minkowski (UM) modes. They were discovered by Unruh in 1976 [2]. The right-propagating UM modes are defined as [4]

$$F_{1\Omega}(t,z) = \frac{|t - z/c|^{i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{-\frac{\pi\Omega}{2}\mathrm{sgn}(t-z/c)}, \qquad (13)$$

$$F_{2\Omega}(t,z) = \frac{|t - z/c|^{-i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{\frac{\pi\Omega}{2}\mathrm{sgn}(t-z/c)}, \qquad (14)$$

where $\Omega > 0$ is a parameter which is proportional to the photon frequency in the Rindler space [17]. In wedge I, $z > ct$ and the potentially large factor $e^{\pi\Omega/2}$ appears in Eq. (13), which means that the mode $F_{1\Omega}$ is mostly in wedge I [see Fig. 1(a)] and has positive frequency in wedge II even though it is associated with a creation operator (negative norm). On the other hand, the mode $F_{2\Omega}$ is exponentially larger when $ct > z$, which is in wedge II. The left-moving modes are obtained from Eqs. (13) and (14) by replacing $z \to -z$.

The functions (13) and (14) are the limit for positive $\epsilon \to 0$ of the expression

$$F_{1\Omega}(t,z) = \frac{\left(t - \frac{z}{c} - i\epsilon\right)^{i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{-\frac{\pi\Omega}{2}},$$

$$F_{2\Omega}(t,z) = \frac{\left(t - \frac{z}{c} - i\epsilon\right)^{-i\Omega}}{\sqrt{2\Omega\sinh(\pi\Omega)}}e^{\frac{\pi\Omega}{2}}.$$

These functions are analytic and bounded in the upper half complex plane of the complex field argument $t - z/c$ for all values of $\Omega > 0$, just as the functions $e^{-i\nu(t-z/c)}$ are for $\omega > 0$. The factors $1/\sqrt{2\Omega\sinh(\pi\Omega)}$ are the normalization factors under the Klein-Gordon norm.

Expansion of the right-moving part of the field operator in terms of the UM modes (13) and (14) reads [4]

$$\hat{\Phi} = \sum_{\Omega>0}(F_{1\Omega}\hat{a}_{1\Omega} + F_{2\Omega}\hat{a}_{2\Omega} + F_{1\Omega}^*\hat{a}_{1\Omega}^\dagger + F_{2\Omega}^*\hat{a}_{2\Omega}^\dagger), \qquad (15)$$

where $\hat{a}_{1\Omega}$ and $\hat{a}_{2\Omega}$ are the UM photon annihilation operators. $\hat{a}_{1\Omega}$ and $\hat{a}_{2\Omega}$ can be written as sums of the plane-wave annihilation operators $\hat{a}_k$ introduced above. Thus the vacuum state for the UM photons is the usual Minkowski vacuum $|0_M\rangle$.

We consider a uniformly accelerated atom moving along the trajectory

$$t(\tau) = \frac{c}{a}\sinh\left(\frac{a\tau}{c}\right), \quad z(\tau) = \frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right) \qquad (16)$$

in Minkowski space-time. In Eq. (16), τ is the proper time of the atom. If $a > 0$ ($a < 0$), the atom moves in the right (left) Rindler wedge [see Fig. 1(a)].

The UM modes (13) and (14) are a convenient choice for the description of Unruh acceleration radiation. Namely, a ground-state atom with a transition frequency ω moving in the right Rindler wedge with an acceleration a [see Fig. 1(a)] in Minkowski vacuum $|0_M\rangle$ can become excited by emitting a right-propagating photon into the single UM mode $F_{2\Omega}$, where $\Omega = \omega c/a$ [4,18]. The excited atom can then decay to the ground state by emitting a right-propagating photon into the UM mode $F_{1\Omega}$. As a result of these processes, the final state of the field will have two photons in UM modes $F_{1\Omega}$ and $F_{2\Omega}$ which are entangled.

To the lowest order, we find that the final state of the field is

$$|\Psi\rangle = |0_M\rangle - \left(\frac{g}{\hbar}\right)^2\int_{-\infty}^{\infty}d\tau' e^{-i\omega\tau'}\frac{e^{i\alpha\Omega\tau'}e^{\frac{\pi\Omega}{2}}}{\sqrt{2\Omega\sinh(\pi\Omega)}}$$
$$\times\int_{-\infty}^{\tau'}d\tau'' e^{i\omega\tau''}\frac{e^{-i\alpha\Omega\tau''}e^{-\frac{\pi\Omega}{2}}}{\sqrt{2\Omega\sinh(\pi\Omega)}}|1_{2\Omega}1_{1\Omega}\rangle, \qquad (17)$$

where $\alpha = a/c$. One can write Eq. (17) as

$$|\Psi\rangle = \left(1 + \frac{\Lambda}{2\Omega\sinh(\pi\Omega)}\hat{a}_{2\Omega}^\dagger\hat{a}_{1\Omega}^\dagger\right)|0_M\rangle, \qquad (18)$$

where Λ is a factor containing the time integral

$$\Lambda = -\left(\frac{g}{\hbar}\right)^2\int_{-\infty}^{\infty}d\tau' e^{-i\omega\tau'}e^{i\alpha\Omega\tau'}\int_{-\infty}^{\tau'}d\tau'' e^{i\omega\tau''}e^{-i\alpha\Omega\tau''}. \qquad (19)$$

The factor Λ is large if the resonant condition $\Omega = \omega c/a$ is satisfied.

III. GENERATION OF SQUEEZED PHOTON STATES BY ATOMS HELD ABOVE THE BLACK HOLE HORIZON

The Schwarzschild metric of a nonrotating black hole of mass M in $1+1$ dimensions is given by

$$ds^2 = \left(1 - \frac{r_g}{r}\right)c^2dt^2 - \frac{1}{1 - \frac{r_g}{r}}dr^2, \qquad (20)$$

where $r_g = 2GM/c^2$ is the gravitational radius, G is the gravitational constant, and t, r are Schwarzschild coordinates.

Kruskal-Szekeres coordinates on a black hole geometry are defined in terms of the Schwarzschild coordinates t, r as

$$T = \sqrt{\frac{r}{r_g} - 1}\,e^{\frac{r}{2r_g}}\sinh\left(\frac{ct}{2r_g}\right), \qquad (21)$$

$$X = \sqrt{\frac{r}{r_g} - 1}\,e^{\frac{r}{2r_g}}\cosh\left(\frac{ct}{2r_g}\right), \qquad (22)$$

for $r > r_g$, and

$$T = \sqrt{1 - \frac{r}{r_g}}\,e^{\frac{r}{2r_g}}\cosh\left(\frac{ct}{2r_g}\right), \qquad (23)$$

$$X = \sqrt{1 - \frac{r}{r_g}}\,e^{\frac{r}{2r_g}}\sinh\left(\frac{ct}{2r_g}\right), \qquad (24)$$

for $0 < r < r_g$.

In $1+1$ dimensions, in the Kruskal-Szekeres coordinates, the Schwarzschild metric is conformally invariant to the Minkowski metric

$$ds^2 = \frac{4r_g^3}{r}e^{-\frac{r}{r_g}}(dT^2 - dX^2), \qquad (25)$$

where r is a function of $T^2 - X^2$ which has a value of r_g when $T^2 - X^2 = 0$. Thus one can choose mode functions in the

Kruskal-Szekeres coordinates as functions of $T \pm X$. Again, although the atom couples to both the right- and left-moving modes ($F_{\{1,2\}\Omega}$ and $G_{\{1,2\}\Omega}$), we again concentrate on the right movers.

Here, we consider a two-level [a is the excited level (not to be confused with the acceleration), and b is the ground state] atom with transition angular frequency ω near an eternal nonrotating BH of mass M. If the atom does not move in the Schwarzschild coordinates, that is, the atom's trajectory is $r(t) = r_0 > r_g$, then in the Kruskal-Szekeres coordinates the atom is uniformly accelerated along the trajectory

$$T(t) = \sqrt{2\alpha r_0/c - 1}e^{\alpha r_0/c} \sinh(\alpha t), \quad (26)$$

$$X(t) = \sqrt{2\alpha r_0/c - 1}e^{\alpha r_0/c} \cosh(\alpha t), \quad (27)$$

where $\alpha = c/2r_g$. One can write these equations as

$$T(\tau) = \sqrt{2\alpha r_0/c - 1}e^{\alpha r_0/c} \sinh\left(\frac{\alpha\tau}{\sqrt{1 - r_g/r_0}}\right), \quad (28)$$

$$X(\tau) = \sqrt{2\alpha r_0/c - 1}e^{\alpha r_0/c} \cosh\left(\frac{\alpha\tau}{\sqrt{1 - r_g/r_0}}\right), \quad (29)$$

where $\tau = t\sqrt{1 - r_g/r_0}$ is the proper time of the atom.

As in the case of the Rindler horizon in the flat Minkowski space-time, the black hole event horizon $T = X$ divides the Schwarzschild space-time for right-running waves into two regions. The outgoing positive-frequency modes, which are analogous to the right-running UM modes of the previous section on acceleration radiation, are given by [2]

$$F_{1\Omega}(T, X) = \frac{|T - X|^{i\Omega}}{\sqrt{2\Omega \sinh(\pi\Omega)}} e^{-\frac{\pi\Omega}{2}\mathrm{sgn}(T-X)}, \quad (30)$$

$$F_{2\Omega}(T, X) = \frac{|T - X|^{-i\Omega}}{\sqrt{2\Omega \sinh(\pi\Omega)}} e^{\frac{\pi\Omega}{2}\mathrm{sgn}(T-X)}, \quad (31)$$

where $\Omega > 0$. The corresponding photon annihilation operators we denote as $\hat{A}_{1\Omega}, \hat{A}_{2\Omega}$. We assume that the state of the field is a vacuum state with respect to the Unruh-Schwarzschild (US) modes (30) and (31). We denote it as $|0_K\rangle$. For simplicity we consider only the outgoing modes.

Along the atom's trajectory (28) and (29), the US mode functions (30) and (31) are

$$F_{1\Omega}(T(\tau), X(\tau)) = (2\alpha r_0/c - 1)^{\frac{i\Omega}{2}} \frac{e^{\frac{\pi\Omega}{2}} e^{i\alpha r_0\Omega/c}}{\sqrt{2\Omega \sinh(\pi\Omega)}} e^{-\frac{i\alpha\Omega\tau}{\sqrt{1-r_g/r_0}}}, \quad (32)$$

$$F_{2\Omega}(T(\tau), X(\tau)) = (2\alpha r_0/c - 1)^{-\frac{i\Omega}{2}} \frac{e^{-\frac{\pi\Omega}{2}} e^{-i\alpha r_0\Omega/c}}{\sqrt{2\Omega \sinh(\pi\Omega)}} e^{\frac{i\alpha\Omega\tau}{\sqrt{1-r_g/r_0}}}; \quad (33)$$

that is, from the atom's perspective, the photon described by the mode function $F_{1\Omega}$ has positive frequency, while the photon $F_{2\Omega}$ has negative frequency. The photon is resonant with the atom for $\Omega = \sqrt{1 - r_g/r_0}\omega/\alpha$, where ω is the atomic transition frequency. As a result, the ground-state atom can become excited by emitting a photon into the mode $F_{2\Omega}$. Subsequently, the atom can go to the ground state by spontaneously emitting a photon into the mode $F_{1\Omega}$ [see Fig. 1(b)]. As in the case of an atom accelerated in Minkowski space-time, these two processes generate a two-photon entangled state

$$|\Psi\rangle = \left(1 + \frac{\Lambda}{2\Omega \sinh(\pi\Omega)} \hat{A}^\dagger_{2\Omega}\hat{A}^\dagger_{1\Omega}\right)|0_K\rangle, \quad (34)$$

where Λ is a factor containing the time integral

$$\Lambda = -\left(\frac{g}{\hbar}\right)^2 \int_{-\infty}^{\infty} d\tau' e^{-i\omega\tau'} e^{\frac{i\alpha\Omega\tau'}{\sqrt{1-r_g/r_0}}} \int_{-\infty}^{\tau'} d\tau'' e^{i\omega\tau''} e^{-\frac{i\alpha\Omega\tau''}{\sqrt{1-r_g/r_0}}}. \quad (35)$$

According to Eq. (31), the mode function $F_{2\Omega}$ is exponentially ($e^{\pi a\Omega}$) larger in the region $T > X$, and hence the probability is high that this photon falls into the black hole singularity rather than propagating to infinity. In contrast, the mode function $F_{1\Omega}$ of the spontaneously emitted photon is exponentially larger by the same factor in the region $X > T$, and therefore the spontaneously emitted photon most likely escapes from the black hole region to infinity. In three dimensions, if the frequency Ω is less than $\alpha(l + 1)/2\pi$, where l is the angular momentum of the photon, the outward directed photon will, with very high probability, reflect back into the black hole, bouncing off the momentum or curvature barrier for photons propagating out from the black hole horizon at $r = r_g$. It is only those photons with a high enough energy to get over those barriers which get out to infinity. It is also important to note that far away from the black hole, the "natural" modes are the equivalent of the Rindler modes, with time dependence $e^{-i\nu t}$, not τ, which represent what most would call the natural particle modes. The natural vacuum for the Hartle-Hawking (Kruskal) modes is, and for these Schwarzschild modes will be, a thermal state, both incoming and outgoing, with temperature $(1/8\pi M)(\hbar c^3/Gk_B)$, the Hawking temperature.

IV. GENERATION OF SQUEEZED PHOTON STATES THROUGH THE CHERENKOV EFFECT

Entangled photon pairs can be generated by a similar mechanism if the ground-state atom is moving through a medium with a constant velocity V greater than the speed of light in the medium. For the description of Cherenkov radiation it is convenient to choose mode functions as plane waves which in the medium with refractive index n read (in the laboratory frame)

$$\varphi_k(t, z) = e^{-i\frac{c|k|}{n}t + ikz}, \quad (36)$$

where k is the photon wave vector. Here, we consider dimensions $1 + 1$, and k can be both positive and negative. We denote operators of the plane-wave photons (36) as $\hat{c}_k$. At the location of the atom moving with a constant velocity $V > 0$ along the trajectory

$$t(\tau) = \frac{\tau}{\sqrt{1 - \frac{V^2}{c^2}}}, \quad z(\tau) = \frac{V\tau}{\sqrt{1 - \frac{V^2}{c^2}}},$$

the mode function φ_k takes the form $\varphi_k(t(\tau), z(\tau)) = e^{-i\nu\tau}$; that is, the atom senses harmonic oscillations of the field with frequency

$$\nu = \frac{\frac{c|k|}{n} - Vk}{\sqrt{1 - \frac{V^2}{c^2}}}.$$

From the perspective of the moving atom if $V > c/n$, photons propagating in the same direction as the atom's velocity ($k > 0$) have negative frequency. This leads to Cherenkov radiation. That is, the atom can become excited by emitting a photon in the forward direction with a wave vector k_2 such that

$$k_2 = \frac{\omega\sqrt{1-\frac{V^2}{c^2}}}{V-\frac{c}{n}}, \tag{37}$$

where ω is the atomic transition frequency in the atom's frame. A negative frequency of the Cherenkov photon in the atom's frame is analogous to the negative frequency of the UM photon in the frame of an accelerated atom.

Photons emitted in the backward direction ($k < 0$) have positive frequency. Thus an excited atom can spontaneously decay to the ground state by emitting a photon in the backward direction with a wave vector k_1 such that

$$k_1 = -\frac{\omega\sqrt{1-\frac{V^2}{c^2}}}{V+\frac{c}{n}}. \tag{38}$$

As in the case of an atom accelerated in Minkowski spacetime, these processes generate a two-photon entangled state

$$|\Psi\rangle = \left(1+\Lambda c^\dagger_{k_2} c^\dagger_{k_1}\right)|0_M\rangle, \tag{39}$$

where Λ is a factor containing the time integral

$$\Lambda = -\left(\frac{g}{\hbar}\right)^2 \int_{-\infty}^{\infty} d\tau' e^{-i\omega\tau'} e^{\frac{\frac{c|k_1|}{n}-Vk_1}{\sqrt{1-\frac{V^2}{c^2}}}\tau'} \times \int_{-\infty}^{\tau'} d\tau'' e^{i\omega\tau''} e^{\frac{\frac{c|k_2|}{n}-Vk_2}{\sqrt{1-\frac{V^2}{c^2}}}\tau''}. \tag{40}$$

Λ is large if the resonance conditions (37) and (38) are satisfied. The Cherenkov effect yields entanglement generation between photons propagating in the forward and backward directions.

V. DISCUSSION AND SUMMARY

To put Sec. II in perspective, we recall that the Bogoliubov relations allow us to obtain an expression for the Minkowski vacuum $|0_M\rangle$ in terms of excitation states of the Rindler vacuum $|0_R\rangle$. That is, we use the relations between operators for the Rindler modes $\hat{b}_\nu$ and the UM modes $\hat{a}_\nu$ [13]

$$\hat{b}_{1\nu} = \frac{\hat{a}_{1\nu} + e^{-\pi c\nu/a}\hat{a}^\dagger_{2\nu}}{\sqrt{1-e^{-2\pi c\nu/a}}}, \quad \hat{b}_{2\nu} = \frac{\hat{a}_{2\nu} + e^{-\pi c\nu/a}\hat{a}^\dagger_{1\nu}}{\sqrt{1-e^{-2\pi c\nu/a}}} \tag{41}$$

and the the identities [13]

$$\hat{a}_1 e^{\gamma\hat{a}^\dagger_1\hat{a}^\dagger_2} = e^{\gamma\hat{a}^\dagger_1\hat{a}^\dagger_2}(\hat{a}_1 + \gamma\hat{a}^\dagger_2), \tag{42}$$

$$\hat{a}_2 e^{\gamma\hat{a}^\dagger_1\hat{a}^\dagger_2} = e^{\gamma\hat{a}^\dagger_1\hat{a}^\dagger_2}(\hat{a}_2 + \gamma\hat{a}^\dagger_1), \tag{43}$$

where $\gamma = e^{-\pi c\nu/a}$. Multiplying both sides of Eq. (42) by $|0_M\rangle$, we obtain

$$\hat{a}_1 e^{\gamma\hat{a}^\dagger_1\hat{a}^\dagger_2}|0_R\rangle = e^{\gamma\hat{a}^\dagger_1\hat{a}^\dagger_2}(\hat{a}_1 + \gamma\hat{a}^\dagger_2)|0_R\rangle \propto e^{\gamma\hat{a}^\dagger_1\hat{a}^\dagger_2}\hat{b}_1|0_R\rangle = 0. \tag{44}$$

That is, if $|0_R\rangle$ is the vacuum state for the Rindler photon operator $\hat{b}_1$, then

$$|0_M\rangle = \frac{1}{N} e^{\gamma\hat{a}^\dagger_1\hat{a}^\dagger_2}|0_R\rangle \tag{45}$$

is the vacuum state for the UM operator $\hat{a}_1$, where N is a normalization constant. Applying the identity (43), one can show that Eq. (45) is also the vacuum state for the operator $\hat{a}_2$. Multiplying both sides of Eq. (45) by $Ne^{-\gamma\hat{a}^\dagger_1\hat{a}^\dagger_2}$, we find

$$|0_R\rangle = Ne^{-\gamma\hat{a}^\dagger_1\hat{a}^\dagger_2}|0_M\rangle. \tag{46}$$

The value of N is determined by the normalization $\langle 0_R|0_R\rangle = 1$, which yields

$$N = \frac{1}{\sqrt{\sum_{n=0}^{\infty}\gamma^{2n}}} = \sqrt{1-\gamma^2}.$$

For $\gamma \ll 1$, Eq. (46) approximately can be written as

$$|0_R\rangle \approx (1 - e^{-\pi c\nu/a}\hat{a}^\dagger_1\hat{a}^\dagger_2)|0_M\rangle, \tag{47}$$

which has the same exponential factor as Eq. (18) in the limit $\Omega \gg 1$,

$$|\Psi\rangle \approx \left(1 + \frac{\Lambda}{\Omega}e^{-\pi\Omega}\hat{a}^\dagger_{R2\Omega}\hat{a}^\dagger_{R1\Omega}\right)|0_M\rangle. \tag{48}$$

Having demonstrated in Sec. III that the *Unruh-Schwarzschild* mode expansion applied to the BH problem is very analogous to the acceleration radiation with *Unruh-Minkowski* modes, we conclude by making a key but somewhat subtle point. Namely, the negative-energy photon, which is associated with the mode $F_{2\Omega}$, in both cases is localized below the "horizon." This was already emphasized in Ref. [4]. That is, the $F_{2\Omega}$ mode into which the atom emits the first photon is exponentially large in the left wedge ($t > z/c$) in the case of Unruh acceleration radiation, and is likewise exponentially large below the BH horizon ($T > X$). In the case of Hawking radiation the photon does not so much "fall" into the BH, but rather is created under the event horizon because the mode function $F_{2\Omega}$ is localized in this region.

It would be interesting to look for similar effects in various analogs of the Unruh acceleration radiation [19–21] and Cherenkov radiation. For example, a ground-state atom moving above a metal surface [see Fig. 2(a)] can become excited by emitting a surface plasmon with wave vector in the direction of the atom's motion [22]. Surface plasmons are collective excitations of the electromagnetic field and metal electrons which propagate along the surface. The electromagnetic field in the surface plasmon exponentially decays away from the surface as shown in Fig. 2(a).

From the perspective of the moving atom, the surface plasmon frequency is Doppler shifted, and for large wave numbers k the frequency becomes negative [see Fig. 2(b)]. In the moving frame associated with the atom, the ground-state atom becomes excited by emitting a surface plasmon with negative frequency, which insures energy conservation. The excited atom can then decay to the ground state by emitting a surface plasmon with positive frequency, e.g., with a wave vector opposite to the atom's velocity. Thus the generated pair of surface plasmons is entangled.

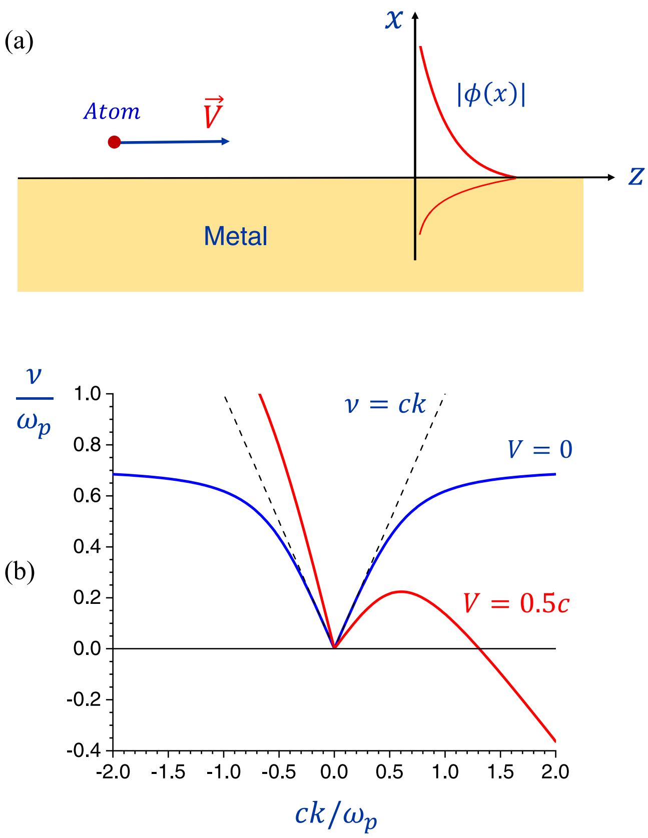

FIG. 2. (a) An atom is moving near a metal surface with constant velocity. The atom can become excited by emitting a surface plasmon with negative frequency and then spontaneously decay to the ground state by emitting a surface plasmon with positive frequency. (b) Frequency of surface plasmons in the atom's frame moving with velocity V as a function of the surface plasmon wave vector $\mathbf{k}$ for $V = 0$ and $V = 0.5c$. The frequency of modes with $k > \frac{\omega_p}{V}\sqrt{\frac{c^2-V^2}{2c^2-V^2}}$ is negative, where ω_p is the electron plasma frequency.

We note that this also forms the basis for the analog gravity program, in which the black hole is modeled by a trans-sonic fluid flow and the quantum field is modeled by sound (or other waves) in the fluid. Effects such as the Cherenkov effect discussed above are also important here, where the fluid flow being faster than the wave velocity plays a crucial role (see, for example, the paper by Schuetzhold and Unruh [23] and diagrams and references therein). This also sheds light on the Landau critical velocity, where the frequency of the sound waves in liquid He becomes negative, as for the Cherenkov situation discussed above. An impurity coupled to the sound waves will create particle pairs (squeezing) yielding a quantum friction force on the impurity.

One should also mention that an atom can generate entangled photon pairs in a de Sitter universe for which horizon is provided by the universe exponential expansion. If the atom does not follow a cosmic fluid geodesic, but remains at constant proper distance from the horizon, then it is "accelerated" with respect to locally inertial observers. Consequently, the atom perceives the de Sitter vacuum as a thermal bath with a temperature associated with the surface gravity of the Hubble sphere, much as a uniformly accelerated detector in Minkowski space-time measures a temperature associated with its acceleration [24]. Thus the ground-state atom can become excited by emitting a photon followed by spontaneous decay with emission of a second photon. This yields generation of a two-mode squeezed photon state in the de Sitter space-time.

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