

Quantifying the presence of a neutron in the paths of an interferometer

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It is commonly assumed that no accurate experimental information can be obtained on the path taken by a particle when quantum interference between the paths is observed. However, recent progress in the measurement and control of quantum systems may provide the missing information by circumventing the conventional uncertainty limits. Here, we experimentally investigate the possibility that an individual neutron moving through a two-path interferometer may actually be physically distributed between the two paths. For this purpose, it is important to distinguish between the probability of finding the complete particle in one of the paths and the distribution of an individual particle over both paths. We accomplish this distinction by applying a magnetic field in only one of the paths and observing the exact value of its effect on the neutron spin in the two output ports of the interferometer. The results show that individual particles experience a specific fraction of the magnetic field applied in one of the paths, indicating that a fraction or even a multiple of the particle was present in the path before the interference of the two paths was registered. The obtained path presence equals the weak value of the path projector and is not a statistical average but applies to every individual neutron, verified by the recently introduced method of feedback compensation.

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I. INTRODUCTION

Young's single-particle double-slit experiment is at the heart of wave-particle duality [1,2], a particular form of complementarity [3], and one of the most fundamental concepts of quantum mechanics. The incompatibility between the wavelike behavior of quantum objects and the classical concept of a particle manifests in the observed aspect (wave or particle) and depends on the experimental context. When a position measurement to determine which slit a quantum particle traverses (particlelike property) is performed, in turn the interference pattern (wavelike property) vanishes; the more *which-way* information is extracted, the lower the visibility of the interference fringes and vice versa. This particular behavior is quantified in the Englert-Greenberger relation [4].

Einstein questioned this impossibility of determining the path taken by an individual particle in a double-slit interference experiment [5]. In his proposed scheme, *which-way* information is gained by measuring the recoil, seemingly without destroying the interference pattern. However, as pointed out by Bohr, Einstein's proposal was in conflict with the principles of quantum mechanics. Nevertheless,

this was an early version of a *welcher-weg* or *which-way* thought experiment. In the following years, different variants of such a *which-way* experiment were considered, demonstrating the mutual exclusivity of *which-way* information and interference.

A version of Einstein's *which-way* thought experiment that is compatible with the laws of quantum mechanics was developed by Wootters and Zurek in 1979 [6]. This work paved the way for so-called *delayed-choice experiments*, introduced by Wheeler [7,8], where both scenarios (*which-way* information or interference) are still possible after a photon has passed the first plate of a Mach-Zehnder interferometer by removing or inserting the last interferometer plate.

A recently widely discussed *which-way* experiment was proposed by Vaidman in Ref. [9], where *which-path* information is extracted from faint traces (with minimal perturbations) left along the beam path taken by the particle. Vaidman's so-called *past of a quantum particle* experiment was realized using photons [9] and later neutrons [10]. A recent *which-way* experiment, applying a slightly different approach to the *which-way* problem, is reported in Ref. [11].

Except for Vaidman's experiment, all these approaches evaluate the *which-path* information without any reference to the initial quantum state, neglecting any possible correlations between the *which-way* information and the interference effects that might be described by the initial state. This assumption is intuitively justified by the symmetry of conventional interference experiments, where both paths contribute equally to the interferences observed. However, this is not always the case. If there is an imbalance between the paths in the initial state, this imbalance includes

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correlations between which-way information and the outcome of an interference experiment, so the individual outcomes of the interference measurements provide some nontrivial which-way information in addition to the observation of interference. This possibility of using correlations of the input state to evaluate physical properties based on arbitrary measurement outcomes is expressed by the state-dependent uncertainties introduced by Ozawa in 2003 [12]. As explained by Hall, Ozawa's definition of measurement errors indicates that the *best estimate* of a physical property for any measurement is given by the real part of the weak value defined by the initial state and the measurement outcome [13]. When this theory is applied to the which-way problem, it is possible to evaluate the presence of a particle in the two paths based on the outcome of an interference measurement. Somewhat surprisingly, the theory attributes an uncertainty of zero to optimal estimates of the path presence which can be a fraction of one or even larger than one when the initial state is biased in favor of the path in which the presence is evaluated.

Originally it was thought that the Ozawa-Hall uncertainties have no experimentally observable consequences [14]. Previous experiments reconstructed the uncertainties based on statistical assumptions that were motivated by a theoretical analysis of the formalism, either using a tomographic reconstruction, i.e., three-state-method [15–20], or weak measurements [21,22]. However, it has recently been shown in Ref. [23] that Ozawa-Hall uncertainties can be directly observed as the uncertainty in the rotation of a probe qubit when the method of feedback compensation is used, cf. next section. We present an implementation of this method to the which-way problem of a neutron interferometer. The weak values of the path projection operator represent the measurement outcomes associated with the detection of the particles in one or the other output port of the interferometer. The results show that these weak values are quantitative estimates of the path presence with extremely low errors and therefore apply with precision to every particle that exits the interferometer through the respective output port. The error evaluation strongly indicates that a single particle can be physically distributed between the paths in the context of an interference measurement, demonstrating that the path of a particle strongly depends on the measurement context established in the final measurement performed in the output ports.

The remaining paper is organized as follows: In Sec. II, we introduce the theory of feedback compensation applied to a which-way measurement of a Mach-Zehnder interferometer. In Sec. III, we compare the path presences obtained in the interference experiment with the ones of a conventional which-way measurement. In Sec. IV, we present the results of our neutron interferometric experiment in the two different experimental contexts. In Sec. V, we discuss the obtained results and their connection to the theoretical framework of quantum errors introduced by Ozawa.

II. THEORY AND SETUP

The feedback compensation scheme [23], illustrated in Fig. 1(a), proposes to weakly couple a probe qubit to the observable of interest. Then the probe qubit carries information about the observable, and particularly precise information

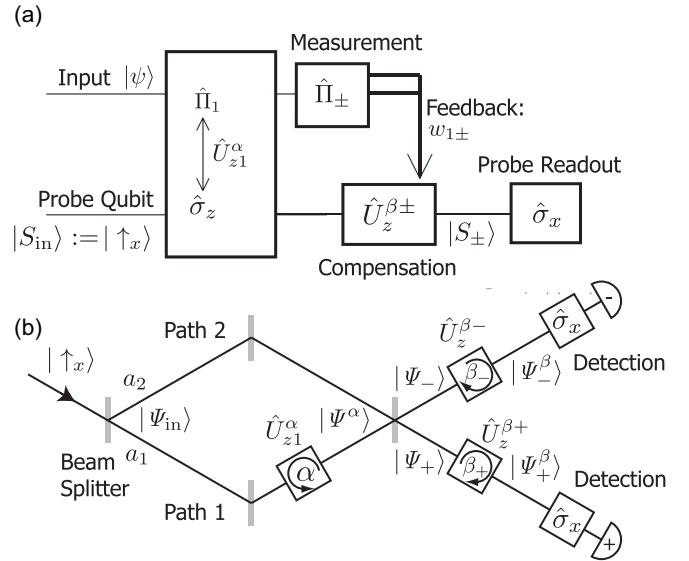


FIG. 1. (a) Scheme of *feedback compensation* from Ref. [23] as applied to a Mach-Zehnder interferometer (b). After a coupling $\hat{U}_{z1}^\alpha$ between object (interferometer paths) and probe system (spin), a compensation $\hat{U}_z^{\beta\pm}$ dependent on the output channel is applied, maintaining the original state $\langle\hat{\sigma}_x\rangle = 1$ of the probe qubit.

can be gained if the probe qubit is finally changed back to its original state by applying a compensation operation. The compensation depends on the measurement outcome of the observable and is therefore called feedback compensation. We explain the scheme in detail on the basis of our particular setup. The setup is schematically shown in Fig. 1(b) and the indicated quantum states are described in the following.

We send neutrons into the interferometer where we assume for universality and for illustrative reasons an asymmetric beam splitter. The neutron in the interferometer is then described by the state

$$|\psi\rangle = a_1|1\rangle + a_2|2\rangle, \quad (1)$$

where $|1\rangle$ and $|2\rangle$ denote the eigenstates of path 1 and 2, respectively. a_1 and a_2 are normalized real amplitudes, $a_1^2 + a_2^2 = 1$. The two exit ports of the interferometer are described by the states $|+\rangle$ and $|-\rangle$, respectively, which also take the phase χ between the two paths into account:

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|1\rangle \pm \exp(i\chi)|2\rangle). \quad (2)$$

We want to gain information about which path the neutron takes and use the neutron spin as the probe qubit. We prepare it initially in the $|\uparrow_x\rangle$ state and the total initial state reads

$$|\Psi_{\text{in}}\rangle = |\psi\rangle|\uparrow_x\rangle, \quad (3a)$$

$$|\uparrow_x\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle), \quad (3b)$$

where $|\uparrow_z\rangle$ and $|\downarrow_z\rangle$ denote the spin eigenstates in z direction. The z basis is the natural choice of spin representation since we use an external guide field in z direction.

We couple spin and path degree of freedom by weakly rotating the spin in path 1. We rotate it by a small angle α

about the z axis. The rotation is expressed by the operator $\hat{U}_z^\alpha$ applied only in path 1 or, equivalently, by the operator $\hat{U}_{z1}^\alpha$ applied to the total state of both paths,

$$\hat{U}_{z1}^\alpha = \exp\left(-\frac{i}{2}\alpha \hat{\sigma}_z \hat{\Pi}_1\right) = \hat{\Pi}_1 \hat{U}_z^\alpha + \hat{\Pi}_2, \quad (4a)$$

$$\hat{U}_z^\alpha = \exp\left(-\frac{i}{2}\alpha \hat{\sigma}_z\right) = \hat{\mathbb{1}} \cos \frac{\alpha}{2} - i\hat{\sigma}_z \sin \frac{\alpha}{2}, \quad (4b)$$

where $\hat{\Pi}_1$ and $\hat{\Pi}_2$ denote the path projection operators of paths 1 and 2, respectively. The identities between the exponentials and the right hand terms can be proven by series expansion of the exponential and the fact that $\hat{\sigma}_z^2 = \hat{\mathbb{1}}$ and $\hat{\Pi}_{1,2}^2 = \hat{\Pi}_{1,2}$. The state after the spin rotation reads

$$|\Psi^\alpha\rangle = \hat{U}_{z1}^\alpha |\Psi_{\text{in}}\rangle = \hat{U}_{z1}^\alpha |\psi\rangle |\uparrow_x\rangle. \quad (5)$$

The states of the two exit beams of the interferometer are given by the projection onto the exit states $|\pm\rangle$, respectively,

$$|\Psi_\pm\rangle = |\pm\rangle \langle \pm | \Psi^\alpha\rangle = \langle \pm | \hat{U}_{z1}^\alpha |\psi\rangle |\pm\rangle |\uparrow_x\rangle. \quad (6)$$

In the exit beams, we compensate the rotation of the spin by rotating it back by an angle β_+ in the $|+\rangle$ port and β_- in the $|-\rangle$ port. In general, different compensation angles in the two ports will be necessary for optimal compensation, indicating that the neutrons in the two exit ports have experienced different spin rotations in the interferometer. The compensated state reads

$$|\Psi_\pm^\beta\rangle = \hat{U}_z^{-\beta_\pm} |\Psi_\pm\rangle = \langle \pm | \hat{U}_z^{-\beta_\pm} \hat{U}_{z1}^\alpha |\psi\rangle |\pm\rangle |\uparrow_x\rangle. \quad (7)$$

By writing the operators as

$$\hat{U}_z^{-\beta_\pm} \hat{U}_{z1}^\alpha = \hat{\Pi}_1 \hat{U}_z^{-\beta_\pm} + \hat{\Pi}_2 \hat{U}_z^{-\beta_\pm}, \quad (8)$$

we clearly see that the spin of the path-1 component is rotated by $\alpha - \beta_\pm$ while the spin of the path-2 component is rotated by $-\beta_\pm$. The final components observed in the exit beams $|+\rangle$ and $|-\rangle$ can be written as

$$|\Psi_\pm^\beta\rangle = \langle \pm | \psi \rangle (\omega_{1\pm} \hat{U}_z^{-\beta_\pm} + \omega_{2\pm} \hat{U}_z^{-\beta_\pm}) |\pm\rangle |\uparrow_x\rangle, \quad (9)$$

where $\omega_{1\pm}$ and $\omega_{2\pm}$ denote the weak values [24,25] of the path projection operators, respectively,

$$\begin{aligned} \omega_{1\pm} &= \frac{\langle \pm | \hat{\Pi}_1 | \psi \rangle}{\langle \pm | \psi \rangle} = \frac{1}{1 \pm \frac{a_1}{a_2} \exp(-i\chi)}, \\ \omega_{2\pm} &= \frac{\langle \pm | \hat{\Pi}_2 | \psi \rangle}{\langle \pm | \psi \rangle} = \frac{1}{1 \pm \frac{a_1}{a_2} \exp(i\chi)} = 1 - \omega_{1\pm}. \end{aligned} \quad (10)$$

Note that in case of a symmetric beam splitter ($a_1 = a_2$) and $\chi = 0$, all neutrons reach the $|+\rangle$ state. The amplitude $\langle - | \psi \rangle$ of the $|-\rangle$ state vanishes and the weak values $\omega_{1,2\pm}$ diverge. The imbalance between a_1 and a_2 is therefore necessary to obtain clear results.

To read out the probe qubit, we analyze the spin in the two output ports. We factorize the state in each port into the path-dependent part $\langle \pm | \psi \rangle |\pm\rangle$ and the spin-dependent part $|S_\pm\rangle$:

$$|\Psi_\pm^\beta\rangle = \langle \pm | \psi \rangle |\pm\rangle |S_\pm\rangle, \quad (11a)$$

$$|S_\pm\rangle = (\omega_{1\pm} \hat{U}_z^{-\beta_\pm} + \omega_{2\pm} \hat{U}_z^{-\beta_\pm}) |\uparrow_x\rangle. \quad (11b)$$

We calculate the amplitudes of the $\hat{\sigma}_x$ eigenstates of the spin qubit:

$$\begin{aligned} s_{\uparrow x} &= \langle \uparrow_x | S_\pm \rangle = \omega_{1\pm} \cos \frac{\alpha - \beta_\pm}{2} + \omega_{2\pm} \cos \frac{\beta_\pm}{2}, \\ s_{\downarrow x} &= \langle \downarrow_x | S_\pm \rangle = i \left(-\omega_{1\pm} \sin \frac{\alpha - \beta_\pm}{2} + \omega_{2\pm} \sin \frac{\beta_\pm}{2} \right). \end{aligned} \quad (12)$$

With the substitutions

$$\begin{aligned} A_\pm \sin \frac{\beta_{0\pm}}{2} &:= \omega_{1\pm} \sin \frac{\alpha}{2}, \\ A_\pm \cos \frac{\beta_{0\pm}}{2} &:= \omega_{1\pm} \cos \frac{\alpha}{2} + \omega_{2\pm}, \end{aligned} \quad (13)$$

we see that the spin amplitudes $s_{\uparrow x}$ and $s_{\downarrow x}$ oscillate as function of the compensation angle $\beta_\pm$ with a certain phase $\beta_{0\pm}$ and with an oscillation amplitude $A_\pm$.

$$s_{\uparrow x} = A_\pm \cos \frac{\beta_\pm - \beta_{0\pm}}{2}, \quad s_{\downarrow x} = i A_\pm \sin \frac{\beta_\pm - \beta_{0\pm}}{2}, \quad (14)$$

$$\beta_{0\pm} = 2 \arctan \frac{\sin \frac{\alpha}{2}}{\frac{\omega_{2\pm}}{\omega_{1\pm}} + \cos \frac{\alpha}{2}}, \quad (15)$$

$$A_\pm^2 = 1 - 4\omega_{1\pm}\omega_{2\pm} \sin^2 \frac{\alpha}{4}. \quad (16)$$

Equations (14) show that perfect compensation is obtained if $\beta_\pm = \beta_{0\pm}$. Then $s_{\downarrow x}$ vanishes and the final spin state equals the initial state $|\uparrow_x\rangle$. The consequences are discussed further below.

With the above formulas, we can evaluate feedback compensation effects for any interaction strength α . However, larger values of α would reduce the visibility of interference fringes and make it more difficult to identify the universal features that characterize weak interactions [26]. Here, we focus on the limit of a small interaction strength α . Series expansions by α show that the optimal compensations $\beta_{0\pm}$ are to first order determined by the weak values $\omega_{1\pm}$ and that the output probabilities $A_\pm$ are to first order unchanged.

$$\beta_{0\pm} = \omega_{1\pm} \alpha + \frac{1}{24} \omega_{1\pm} \omega_{2\pm} (\omega_{1\pm} - \omega_{2\pm}) \alpha^3 + O(\alpha^5), \quad (17)$$

$$A_\pm = 1 - \frac{1}{8} \omega_{1\pm} \omega_{2\pm} \alpha^2 + O(\alpha^4). \quad (18)$$

In other words, in the limit of weak coupling (small α) the weak value of the path projection operator is given by the ratio of the spin rotation angle α and the optimal compensation $\beta_{0\pm}$:

$$\omega_{1\pm} = \beta_{0\pm} / \alpha. \quad (19)$$

This can also be concluded from a first order series expansion of the operators:

$$\begin{aligned} \hat{U}_z^{-\beta_\pm} \hat{U}_{z1}^\alpha &= \exp\left(-\frac{i\alpha}{2} \left(\hat{\Pi}_1 - \frac{\beta_\pm}{\alpha}\right) \hat{\sigma}_z\right) \\ &\approx \hat{\mathbb{1}} - \frac{i\alpha}{2} \hat{\sigma}_z \left(\hat{\Pi}_1 - \frac{\beta_\pm}{\alpha}\right). \end{aligned} \quad (20)$$

Optimal compensation requires a vanishing total rotation angle, so the value of $\beta_\pm / \alpha$ must match the value associated with the operator $\hat{\Pi}_1$. Application of the path state input and the state representing the output beam shows that this is again the weak value of the operator $\hat{\Pi}_1$.

The weak values $\omega_{1\pm}$ can be determined experimentally by analyzing the spin expectation values $\langle\hat{\sigma}_x\rangle$, $\langle\hat{\sigma}_y\rangle$, and $\langle\hat{\sigma}_z\rangle$ (see Appendix 1 for a detailed derivation):

$$\langle\hat{\sigma}_x\rangle = \frac{\cos(\text{Re}\beta_{0\pm} - \beta_{\pm})}{\cosh \text{Im}\beta_{0\pm}}, \quad (21a)$$

$$\langle\hat{\sigma}_y\rangle = \frac{\sin(\text{Re}\beta_{0\pm} - \beta_{\pm})}{\cosh \text{Im}\beta_{0\pm}}, \quad (21b)$$

$$\langle\hat{\sigma}_z\rangle = \tanh \text{Im}\beta_{0\pm}. \quad (21c)$$

A second-order series expansion by α gives

$$\langle\hat{\sigma}_x\rangle = \cos(\beta_{\pm} - \alpha \text{Re}\omega_{1\pm}) \left(1 - \frac{1}{2}(\alpha \text{Im}\omega_{1\pm})^2\right), \quad (22a)$$

$$\langle\hat{\sigma}_y\rangle = \sin(\beta_{\pm} - \alpha \text{Re}\omega_{1\pm}) \left(1 - \frac{1}{2}(\alpha \text{Im}\omega_{1\pm})^2\right), \quad (22b)$$

$$\langle\hat{\sigma}_z\rangle = \alpha \text{Im}\omega_{1\pm}. \quad (22c)$$

In a conventional experiment, no compensation is applied ($\beta_{\pm}=0$) and the weak value is determined for example as $\omega_1 \approx (-\langle\sigma_y\rangle + i\langle\sigma_z\rangle)/\alpha$ [25,27,28]. In order not to disturb the interference, the angle α is kept small, and the measurement is called a weak measurement. Due to the complementarity of which-way information and visibility of interference [4], only little information can be gained per event and many events have to be measured. Since the spin state is close to $|\uparrow_x\rangle$, the variances of $\langle\sigma_y\rangle$ and $\langle\sigma_z\rangle$ are close to their maximum (cf. Appendix 2), and the obtained weak values are clearly a statistical average over the whole ensemble. They don't reveal anything about an individual neutron.

In the feedback compensation scheme, we don't measure individual spin components but determine the spin rotation relative to its initial state. This is done by applying the compensation, i.e., an estimated back rotation by $\beta_{\pm}$, and measuring the spin in the original direction x . The estimate is varied until $\langle\sigma_x\rangle$ reaches a maximum. In an ideal case $\max(\langle\sigma_x\rangle) = 1$ and the variance of $\langle\sigma_x\rangle$ between events vanishes completely. Complementarity is still valid in the sense that many events are needed to determine the optimal compensation. But once the compensation is adjusted, every neutron verifies its correctness and the determined weak value can be attributed to every individual neutron.

The same conclusion can be drawn using the Ozawa-Hall theory. Ozawa [12] introduced a general concept of measurement errors and it was shown by Hall that these errors correspond to the uncertainty of an estimate of a physical property based on the outcome of an arbitrary measurement [13]. This uncertainty is given by the statistical deviation between the operator of interest and the estimated value of that operator. In our case, it reads

$$\varepsilon^2(\hat{\Pi}_1) = \langle\psi| \left[\hat{\Pi}_1 - \sum_{\pm} \frac{\beta_{\pm}}{\alpha} |\pm\rangle\langle\pm| \right]^2 |\psi\rangle, \quad (23)$$

where the estimated value is given by $\beta_{\pm}/\alpha$. As shown in Ref. [23], this estimate corresponds to a feedback of $\beta_{\pm}$ that can compensate a weak interaction of α . The Ozawa-Hall error given by Eq. (23) then describes the error of the feedback compensation and can be observed experimentally in the reduction of the spin component $\langle\sigma_x\rangle$ from its original

value of 1 to

$$\langle\sigma_x\rangle = 1 - \frac{1}{2}\alpha^2\varepsilon^2(\hat{\Pi}_1). \quad (24)$$

In our experiment, the measurement basis $\{|+\rangle, |-\rangle\}$ is complete and orthogonal and the operator $\hat{\Pi}_1$ is self-adjoint. Then the uncertainty Eq. (23) is determined only by the differences between the estimates and the weak values,

$$\begin{aligned} \varepsilon^2(\hat{\Pi}_1) &= \sum_{\pm} \langle\psi| \left(\hat{\Pi}_1 - \frac{\beta_{\pm}}{\alpha} \right) |\pm\rangle\langle\pm| \left(\hat{\Pi}_1 - \frac{\beta_{\pm}}{\alpha} \right) |\psi\rangle \\ &= \sum_{\pm} p_{\pm} \left| w_{1\pm} - \frac{\beta_{\pm}}{\alpha} \right|^2, \end{aligned} \quad (25)$$

where $p_{\pm}$ denotes the statistical probability in the undisturbed system of finding the neutron in the final state $|\pm\rangle$:

$$p_{\pm} = |\langle\pm|\psi\rangle|^2 = \frac{1}{2} \pm a_1 a_2 \cos \chi. \quad (26)$$

This means the uncertainty vanishes completely if the compensations $\beta_{\pm}/\alpha$ applied in the output ports $|+\rangle$ and $|-\rangle$, respectively, equal the corresponding weak values $\omega_{1\pm}$. Then the compensations are no longer just estimates of the probe qubit rotations but precisely determine the effects of the weak interactions on each individual qubit. A successful feedback compensation thus realizes a precise evaluation of the weak values.

Since the compensation angles can only be real numbers, perfect compensation is only possible if the imaginary parts of the weak values vanish. This can be achieved by adjusting the phase χ between the two paths to zero, cf. Eq. (10). In the following considerations and in the experiment, we will focus on this case only.

III. PATH PRESENCE

Let us assume that the real part of the weak values ω_1 and ω_2 of the path projectors describe physical reality, namely, the presence of a neutron in the respective path. The effective spin rotation is then given by the angle α times the path presence in path 1, and it is no surprise that this rotation can be compensated by $\beta_{0\pm} = \alpha \omega_{1\pm}$.

As shown before, the path presence is not a statistical average but can be attributed to every single detected neutron. We can exclude the possibility that some neutrons have taken only one path and other neutrons have taken only the other path and that they are distributed over the paths only in a statistical way. The probe qubit would carry on the noise of the path presence, which is not the case. The path presence is precisely measured and it quantifies how each individual neutron was distributed between the paths.

The path presence should not be confused with the amplitudes a_1 and a_2 of the initial state. The latter determine the detection probabilities $p_{1,2} = |a_{1,2}|^2$ in case a neutron detector is placed directly in one or the other path. This would be a naive which-way measurement which destroys the interference. In contrast, the path presence proposed here can be precisely measured while full interference is maintained. Being a weak value, the path presence always links an initial state to a final state. This means one still cannot predict the path a neutron will take, but once the neutron has been detected in

one or the other exit beams, one can in retrospect infer its presence in paths 1 and 2, respectively.

While we cannot predict the path, we can calculate the statistical probability of detecting the neutron in path 1, which is given by the expectation value of the path projector $\hat{\Pi}_1$. In the context of the which-way measurement, we measure in the eigenbasis of the operator and obtain the expectation value by summing over the eigenvalues:

$$\langle \psi | \hat{\Pi}_1 | \psi \rangle = |\langle \psi | 1 \rangle|^2 \times 1 + |\langle \psi | 2 \rangle|^2 \times 0 = p_1. \quad (27)$$

Since an expectation value can be calculated by any complete basis, we chose the $|\pm\rangle$ basis for the interference context. Now the weak values play the role of the eigenvalues, and the expectation value is given by the averaged weak value [29,30]:

$$\begin{aligned} p_1 &= \langle \psi | \hat{\Pi}_1 | \psi \rangle = |\langle \psi | + \rangle|^2 \times \omega_{1+} + |\langle \psi | - \rangle|^2 \times \omega_{1-} \\ &= p_+ \omega_{1+} + p_- \omega_{1-} = \bar{\omega}_1. \end{aligned} \quad (28)$$

We see that the detection probability p_1 can also be interpreted as an averaged weak value $\bar{\omega}_1$.

Also, the variance can be calculated in either way. In the which-way context, we get

$$\Delta^2(p_1) = p_1(1 - p_1)^2 + p_2(0 - p_1)^2 = p_1 p_2 \quad (29)$$

and in the interference context

$$\Delta^2(\bar{\omega}_1) = p_+(\omega_{1+} - \bar{\omega}_1)^2 + p_-(\omega_{1-} - \bar{\omega}_1)^2 = p_1 p_2. \quad (30)$$

In short, the statistics of the input state is described equally by eigenvalues in the which-way context and by weak values in the interference context [31]. We can also define weak values in the which-way context $\omega_{pf} = \langle f | \hat{\Pi}_p | \psi \rangle / \langle f | \psi \rangle$, where p denotes the path (1 or 2) of the weak value and f the path (1 or 2) where the neutron has been detected. However, these weak values coincide with the eigenvalues $\omega_{pf} = \delta_{pf}$. Weak values are the more general concept, respecting the actual outcomes of a measurement even if they are not represented by eigenstates. Since they reproduce the correct statistics of the particle presence $\hat{\Pi}_p$ in the initial state, it seems reasonable to interpret them as a valid quantitative measure of the neutron presence in the respective path.

Table I shows a comparison of (a) the initial state, (b) the description in the interference context, and (c) in the which-way context. As an example, we have calculated the numbers for a 4 : 1 beam splitter. In the interference context, the neutrons reach the $|+\rangle$ and $|-\rangle$ ports with the probabilities $p_+ = 0.9$ and $p_- = 0.1$ respectively. A neutron ending in the $|+\rangle$ port has been to two-thirds in path 1 and to one-third in path 2. However, a neutron ending in the $|-\rangle$ port had a path-1 presence of 2 and a path-2 presence of -1 . These are so-called anomalous weak values which lie outside the eigenvalue spectrum of the path projection operator. The present experiment confirms that such anomalous weak values are necessary [29,32] to describe the path presence created by the input state Eq. (28) and its fluctuations Eq. (30) by the outcome probabilities of the interference experiment.

The path presences can be used to correctly calculate, e.g., the average spin rotation angle in either context:

$$\begin{aligned} \bar{\alpha} &= p_+ \omega_{1+} \alpha + p_- \omega_{1-} \alpha \\ &= p_1 \omega_{11} \alpha + p_2 \omega_{12} \alpha = \bar{\omega}_1 \alpha. \end{aligned} \quad (31)$$

TABLE I. Path presences calculated for a 4 : 1 beam splitter. (a) Initial preparation, (b) path presences in the interference context, (c) path presences in a which-way measurement.

(a)		Path 1	Path 2
Initial amplitudes		$a_1 = \frac{2}{\sqrt{5}}$	$a_2 = \frac{1}{\sqrt{5}}$
Initial probabilities		$p_1 = \frac{4}{5}$	$p_2 = \frac{1}{5}$
(b)		Presence in path 1	Presence in path 2
Final $ +\rangle$	$p_+ = \frac{9}{10}$	$\omega_{1+} = \frac{2}{3}$	$\omega_{2+} = \frac{1}{3}$
Final $ -\rangle$	$p_- = \frac{1}{10}$	$\omega_{1-} = 2$	$\omega_{2-} = -1$
Average		$\bar{\omega}_1 = \frac{4}{5}$	$\bar{\omega}_2 = \frac{1}{5}$
Variance		$\Delta(\bar{\omega}_1) = \frac{2}{5}$	$\Delta(\bar{\omega}_2) = \frac{2}{5}$
(c)		Presence in path 1	Presence in path 2
Final $ 1\rangle$	$p_1 = \frac{4}{5}$	$\omega_{11} = 1$	$\omega_{21} = 0$
Final $ 2\rangle$	$p_2 = \frac{1}{5}$	$\omega_{12} = 0$	$\omega_{22} = 1$
Average		$\bar{\omega}_1 = \frac{4}{5}$	$\bar{\omega}_2 = \frac{1}{5}$
Variance		$\Delta(\bar{\omega}_1) = \frac{2}{5}$	$\Delta(\bar{\omega}_2) = \frac{2}{5}$

Also, the expectation value $\langle \hat{\sigma}_x \rangle$ measured in our experiment depends on the final state. In case we could not distinguish between the two exit beams, we could apply only a common compensation angle β and measure only the averaged expectation value, denoted by $\langle \hat{\sigma}_x \rangle_{\pm}$ in the interference context and $\langle \hat{\sigma}_x \rangle_{12}$ in the which-way context:

$$\langle \hat{\sigma}_x \rangle_{\pm} = p_+ \langle \hat{\sigma}_x \rangle_+ + p_- \langle \hat{\sigma}_x \rangle_-, \quad (32a)$$

$$\langle \hat{\sigma}_x \rangle_{12} = p_1 \langle \hat{\sigma}_x \rangle_1 + p_2 \langle \hat{\sigma}_x \rangle_2. \quad (32b)$$

The detailed expressions are given in Appendix 3. In the limit of small α both expectation values converge to

$$\langle \hat{\sigma}_x \rangle_{\pm} = \langle \hat{\sigma}_x \rangle_{12} = \left(1 - \frac{1}{2} p_1 p_2 \alpha^2\right) \cos(\beta - \bar{\alpha}) \quad (33)$$

and have exactly the form predicted by Eq. (24). Comparing both equations, we identify a measurement error of $\varepsilon^2(\hat{\Pi}_1) = p_1 p_2 = \Delta^2(p_1)$. This means, by not distinguishing between the measurement outcomes, we end up with a measurement error equal to the original path uncertainty Eq. (29).

IV. EXPERIMENT

In the experiment, we verified the path-1 presences listed in Table I and the reduction of $\max(\langle \sigma_x \rangle)$ expressed in Eq. (33) in case the exit beams are not distinguished.

The experimental realization of the operations described in the theory (Sec. II) are mostly straightforward and are depicted in Fig. 2. The neutrons are polarized by a magnetic prism which deflects the spin-down neutrons out of the Bragg acceptance angle of the interferometer crystal. The spin rotator DC1 rotates the remaining spin-up neutrons by $\pi/2$ into the initial $|\uparrow_x\rangle$ state. The spin rotator consists of a DC coil which creates a magnetic field pointing in the $-y$ direction. In this region, the spin precesses about the y axis due to Larmor precession within the DC coil. The asymmetry of the beam splitter is realized by a partial absorber in path 2. The spin

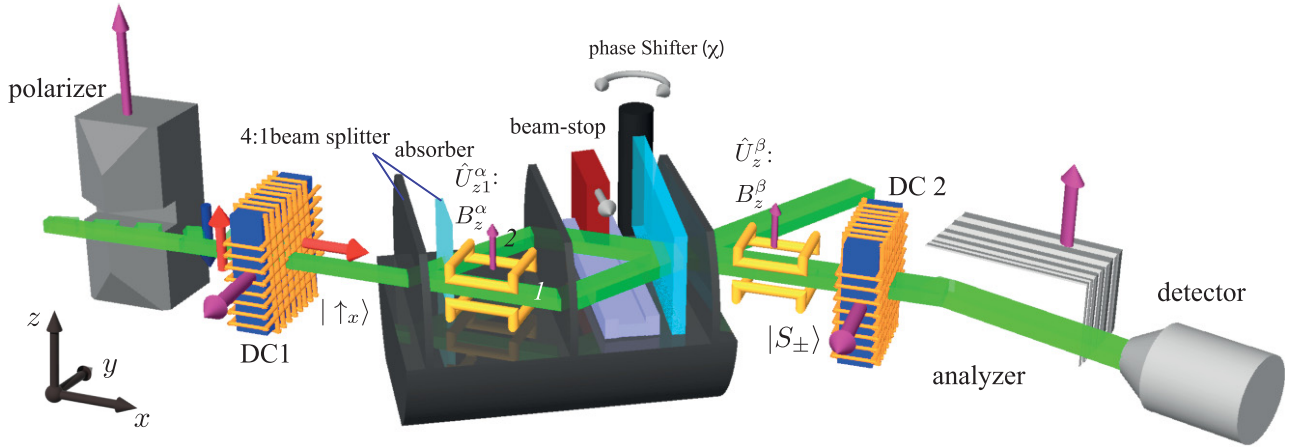


FIG. 2. Polarized monochromatic neutrons enter the interferometer and are split into paths $|1\rangle$ and $|2\rangle$ at the first interferometer plate at a ratio of 1:4. Before the interferometer, the probe qubit is prepared in the initial spin state $|S_{in}\rangle = |s_{\uparrow x}\rangle$ by a $\pi/2$ direct-current spin rotator (DC 1). In path 1, the spin is rotated by an angle α . Beam stop and phase shifter are required for *which-path* or *interference* measurements, respectively. Behind the interferometer, the compensation is applied, that is, a spin rotation by angle $\beta_{\pm}$, dependent of the respective measurement context. The spin is analyzed in $\pm x$ direction by the combination of a $\pi/2$ direct-current spin rotator (DC 2) and the magnetic supermirror. The neutrons are counted in a ^3He detector.

rotations α and β are realized by small Helmholtz coils which modify the external overall B_z guide field (not depicted in Fig. 2) such that the spin precession in the x - y -plane changes. The precession angle is given by $-2\mu B_z^j \tau / \hbar$, where τ is the neutron's transit time in the magnetic field region, with $j = \alpha, \beta$. The β compensation and the spin analysis is realized only in the forward exit beam, normally corresponding to the $|+\rangle_{\chi=0}$ state. By changing the phase χ between the beam paths from 0 to π we can however flip the meaning of the exit beams and thereby analyze also the $|-\rangle_{\chi=0}$ state. The spin analysis is realized by a spin-dependent reflection from a Co-Ti supermirror array. The magnetic supermirror array consists of a stack of slightly bent glass plates coated with alternating layers of (magnetic) Cobalt and (non-magnetic) Titanium embedded in the vertical field of permanent magnets. The combination of materials is chosen such that the sum of the nuclear scattering length and the magnetic scattering length of Cobalt for one spin component equals the scattering length of Titanium. Then the layer structure is invisible for this spin component and it will not be reflected. Consequently, the supermirror only reflects the $|\uparrow_z\rangle$ state and the $|\downarrow_z\rangle$ state is absorbed. In combination with a $\pi/2$ spin rotator (DC2) it analyzes the $|\uparrow_x\rangle$ state required here. The position of the DC2 coil is adjusted in beam direction to catch the precessing spin at the correct angle required for the $\pi/2$ rotation.

The experiment was carried out at the neutron interferometer instrument S18 at the high-flux reactor of the Institute Laue-Langevin (ILL) in Grenoble, France. A monochromatic beam with mean wavelength $\lambda = 1.91 \text{ \AA}$ ($\delta\lambda/\lambda \sim 0.02$) and $5 \times 7 \text{ mm}^2$ beam cross section was used. The experimental data can be found on the ILL data server in Ref. [33].

A. Which-way context

Formally, the *which-way* measurement is described by the observable $\hat{\Pi}_1$, i.e., the projector onto path 1. Experimentally the projector $\hat{\Pi}_{1,2}$ is realized by blocking path 2, 1, which

is schematically illustrated in Fig. 2. Depending on which projection is measured the respective compensation operation $\hat{U}_z^{\beta_{1,2}} = \exp(-i(\beta_{0,1,2})/2 \hat{\sigma}_z)$ is applied, with $\beta_{01} = -\alpha$ for path 1 and $\beta_{02} = 0$ for path 2.

The obtained results of which-way measurements are plotted in Fig. 3(a) for $\alpha = \pi/4$ (strong interaction) and Fig. 3(b) for $\alpha = \pi/16$ (weak interaction). The experimentally observed phase shifts $\beta_{0,1,2}^{\text{meas}}$ (see caption of Fig. 3) reproduce the theoretically predicted values from Eq. (22a), given by

$$\langle \hat{\sigma}_x \rangle_1 = \cos(\beta_1 - \omega_{11} \alpha), \quad \langle \hat{\sigma}_x \rangle_2 = \cos(\beta_2 - \omega_{12} \alpha) \quad (34)$$

(within the error-bars), accurately yielding $\omega_{11} = 1$ and $\omega_{12} = 0$. The experimentally obtained values of the path presence ω_1 are summarized in Fig. 5 for the interaction strengths $\alpha = \pi/4, \pi/8$, and $\alpha = \pi/16$.

If the same compensation is applied in both paths, we observe fringes, shown in green in the figure, which have a reduced visibility as predicted by Eq. (33). The visibility reads

$$v = 1 - \frac{1}{2} \alpha^2 \Delta^2(p_1) \quad (35)$$

and is determined by the initial path uncertainty $\Delta^2(p_1) = p_1 p_2 = 0.16$. For the two interaction strengths $\alpha = \pi/4$ and $\alpha = \pi/8$, respectively, the theoretical values are $v_{\alpha/4} = 0.951$ and $v_{\alpha/8} = 0.997$, indicated by the red lines in the figure. The experimentally determined values are $v_{\alpha/4}^{\text{meas}} = 0.964(7)$ and $v_{\alpha/8}^{\text{meas}} = 0.993(10)$. The fringes have been calculated by averaging the data points of both paths according to Eq. (32b) using $p_1 = 0.8$ and $p_2 = 0.2$. To obtain data points at equal β values, we calculated for each data point of one curve a corresponding value and error from the fit of the other curve.

B. Interference context

In the *interference* context, the optimal correction is given by $\beta_{0\pm} = -\omega_{1\pm} \alpha$ as explained in the previous sections. Path presences of $w_{1+} = 2/3$ and $w_{1-} = 2$, with probabilities

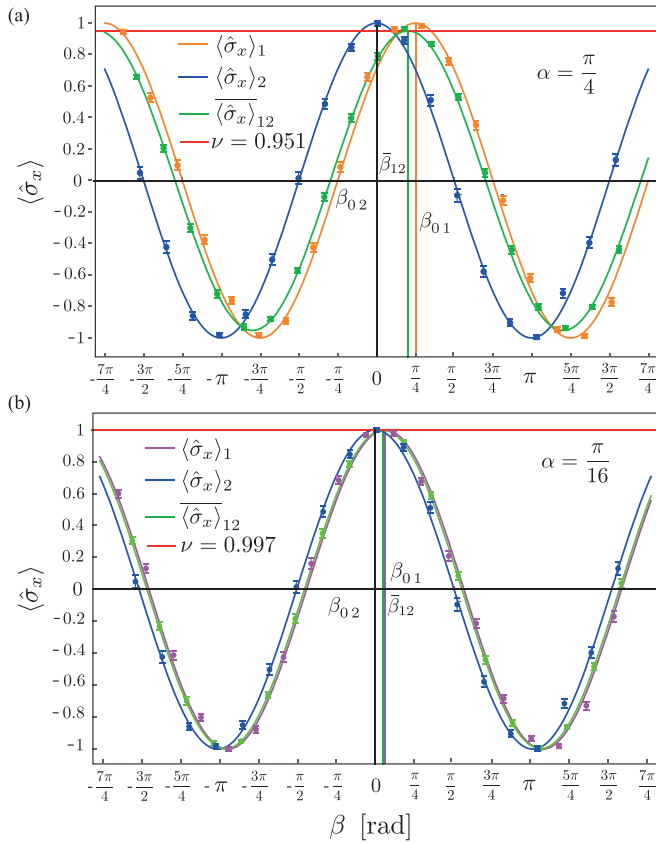


FIG. 3. Experimental results of feedback compensation in which-way context. The expectation values $\langle \sigma_x \rangle_{1,2}$ are measured in paths 1 and 2 respectively, and the average is calculated according to Eq. (32b). Solid curves indicate theoretical predictions. The optimal compensation angles are given by the positions of the maxima, obtained by fitting the data points and indicated by vertical lines. The obtained values are $\beta_{02}^{\text{meas}} = -0.0012(38)\pi$ and (a) for $\alpha = \pi/4$: $\beta_{01}^{\text{meas}} = 0.2533(61)\pi$ and (b) for $\alpha = \pi/16$: $\beta_{01}^{\text{meas}} = 0.0646(66)\pi$. The error bars represent ± 1 standard deviation.

$p_+ = 0.9$ and $p_- = 0.1$, respectively, are obtained in the limit of weak coupling (small α).

Applying the respective setup from Fig. 2, the total combined state, consisting of object system (path) and probe system (spin), behind the phase shifter flag reads

$$|\Psi(\chi)\rangle_{\text{tot}} = \frac{2}{\sqrt{5}}|1\rangle \exp\left(-i\frac{\alpha}{2}\hat{\sigma}_z\right)|s_{\uparrow x}\rangle + e^{i\chi}\frac{1}{\sqrt{5}}|2\rangle|s_{\uparrow x}\rangle, \quad (36)$$

where χ is the relative phase between the (path) eigenstates, adjusted by a phase shifter. The phases $\chi = 0$ and $\chi = \pi$ account for the postselection of states $|+\rangle_{\chi=0}$ and $|-\rangle_{\chi=0}$, respectively. Depending on the selected state, the respective compensation operation $\hat{U}_z^{\beta_{\pm}} = \exp(-i(\beta_{0\pm})/2 \hat{\sigma}_z)$ is applied behind the interferometer.

The experimentally observed phase shifts $\beta_{0\pm}^{\text{meas}}$ (see caption of Fig. 4) correctly reproduce the feedback fringes

$$\langle \hat{\sigma}_x \rangle_+ = \cos(\beta_+ - \omega_{1+} \alpha), \quad \langle \hat{\sigma}_x \rangle_- = \cos(\beta_- - \omega_{1-} \alpha), \quad (37)$$

predicted by Eq. (22a). The obtained measurement results for measurement strength $\alpha = \pi/4$ are plotted in Fig. 4 (a). The corresponding output port probabilities are $p_+ = 0.8696$

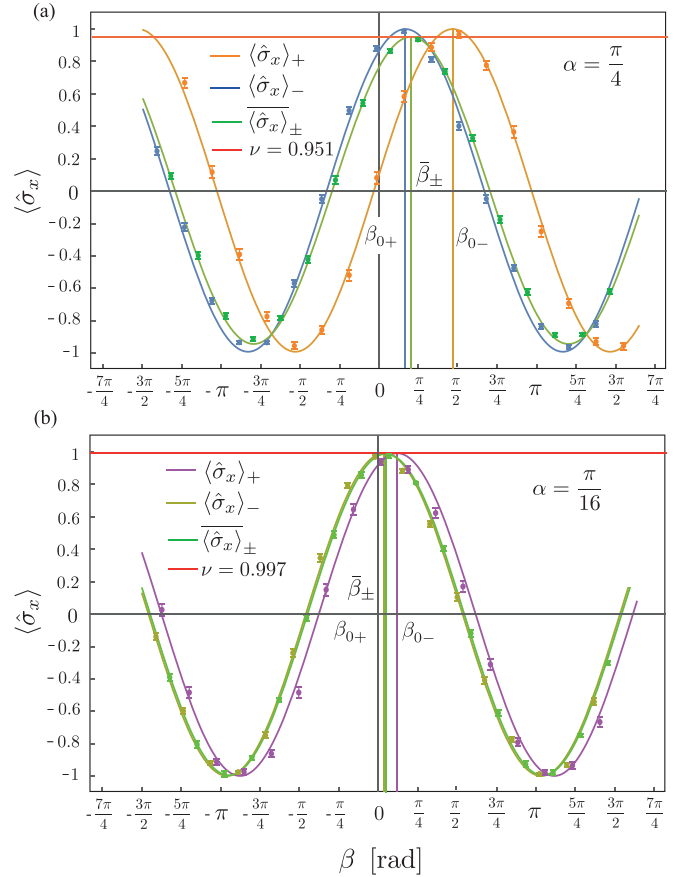


FIG. 4. Experimental results of feedback compensation in interference context. The expectation values $\langle \sigma_x \rangle_{\pm}$ are measured and the average is calculated according to Eq. (32a). Solid curves indicate theoretical predictions. The optimal compensation angles are given by the positions of the maxima, obtained by fitting the data points and indicated by vertical lines. The obtained values are (a) for $\alpha = \pi/4$: $\beta_{0+}^{\text{meas}} = 0.1671(61)\pi$, $\beta_{0-}^{\text{meas}} = 0.4727(35)\pi$, and (b) for $\alpha = \pi/16$: $\beta_{0+}^{\text{meas}} = 0.0449(54)\pi$, $\beta_{0-}^{\text{meas}} = 0.1229(54)\pi$. The error bars represent ± 1 standard deviation.

and $p_- = 0.1304$. The fringe shifts are given by $\omega_{1\pm} \alpha$. For $\alpha = \pi/4$, the predicted values of the path presence are $\omega_{1+} = 0.6686$ at $p_+ = 0.8696$ and $\omega_{1-} = 1.8701$ at $p_- = 0.1304$ and are evidently reproduced in the experiment, which can be seen from the values in the figure captions.

The green fringes in the figure show the statistical average over both output ports according to Eq. (32a) using $p_+ = 0.9$ and $p_- = 0.1$. As predicted by Eq. (33), the fringe visibility then shrinks to the same value as in the which-way context indicated by the red lines in the figure. The experimentally determined values are $\nu_{\alpha/4}^{\text{meas}} = 0.943(11)$ and $\nu_{\alpha/16}^{\text{meas}} = 0.984(10)$.

A plot of the optimal compensation β_0/α versus interaction strength α is given in Fig. 5. The measured path presences converge for small α toward the weak values. For large α , the optimal compensation angle is no longer given by the first order term of Eq. (17) but by the complete expression Eq. (15), shown by the dotted line in Fig. 5. As mentioned in Sec. II, this change is, in principle, accompanied by a change in the probabilities of finding the particles in the exits beams,

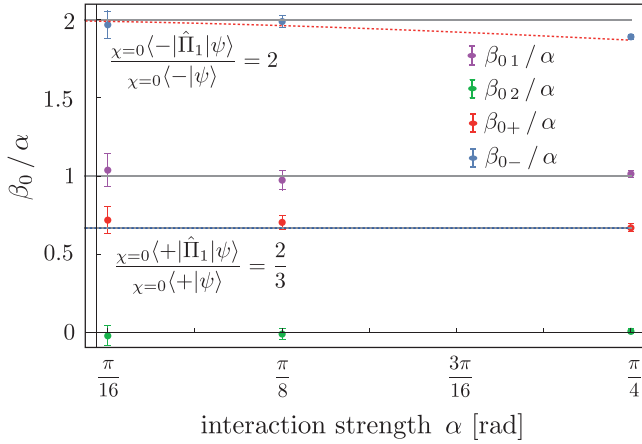


FIG. 5. Optimal compensation β_0/α versus interaction strength α for $\alpha = \pi/4$, $\pi/8$ and $\pi/16$. Solid lines represent the predicted weak values $\omega_{12} = 0$, $\omega_{11} = 1$, $\omega_{1+} = 2/3$, $\omega_{1-} = 2$. Dotted lines represent theoretical predictions Eq. (15) of the optimal compensation for larger values of α .

indicating that the probe qubit perturbs the measurement and modifies the output beams in a non-trivial manner. However, the results shown in Fig. 5 indicate that this perturbation of the interference effect is sufficiently small in the full range of interaction strengths covered in the present experiment.

V. DISCUSSION

Our experimental results show that spin rotations applied in one of the paths in a two-path neutron interferometer result in proportional rotations of the spin in the output ports of the interferometer. Each of the two different rotation angles observed in the output ports are precisely defined and show no fluctuations. We therefore conclude that the spin rotations define a precise value of the presence of a particle in the path in which the rotation was applied. This result indicates that the particles have a precisely defined fractional presence in the path when interference effects are observed in the output ports.

We would like to point out that this result is consistent with Ozawa's theory of measurement errors, which predicts an error of zero when the measurement results are given by the weak values defined by a pure input state and a precise projective measurement. The experiment confirms that the error measure introduced by Ozawa corresponds to experimentally observable fluctuations in the effects of weak interactions, strongly suggesting that the weak values are an accurate description of the dependence of physical properties on the measurement context [34]. It seems to be highly significant that the feedback compensation approach establishes the consistency of Ozawa uncertainties, weak values and the quantum mechanics of weak unitary interactions. The experimental results show that weak values and Ozawa uncertainties provide an accurate description of the experimentally observable statistics of weak interactions.

In neutron interferometry, we always have clear experimental evidence that the interference is based on self-interference of individual particles. First, neutrons are fermions and would never occupy the same state, and second

the beam intensity is so low that there are virtually never two particles simultaneously in the setup. Still, the visibility of interference itself requires a whole ensemble of neutrons, allowing alternative interpretations of quantum mechanics which assume locality of single particles. The present experiment rules out such statistical interpretations due to the vanishing error of the path presence determined by the weak values.

VI. CONCLUSION

The precise analysis of the effects of sufficiently small spin rotations applied in only one path of a two-path neutron interferometer shows that the presence of the neutrons in the paths during an interference experiment is accurately described by fractional values corresponding to the weak values associated with the output ports of the interferometer, where the precision of the results is given by the Ozawa uncertainties. We have experimentally verified that these fluctuations are close to zero in the present case, providing the first experimental evidence that the partial path presences described by weak values apply to each individual neutron detected in the corresponding output beam of the interferometer. Since partial path presences are not observed when a which-path measurement is performed, our results also demonstrate that the way in which particles propagate through an interferometer depends on the measurements performed in the output. The same initial uncertainty of the particle presence that appears as a statistical distribution of detection events in a which-path measurement will appear as the fluctuation of the individual partial path presences when interference effects are detected instead.

It should be emphasized that all results are completely consistent with standard quantum theory. The conclusion that particles can be physically delocalized between paths in which no strong interactions occur and that the localization or delocalization is decided by a measurement that takes place after the particles have propagated along the paths is a possibility inherent in the paradoxical aspects of quantum superpositions. In the present paper, we demonstrated that standard quantum theory predicts precise and specific effects of the presence of a particle in a path, even when the particle only undergoes a very weak interaction on its way through the interferometer.

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APPENDIX

1. Spin expectation values

The spin expectation values in the directions $d \in \{x, y, z\}$ are given by

$$\langle \hat{\sigma}_d \rangle = \frac{(+1) \times |s_{\uparrow d}|^2 + (-1) \times |s_{\downarrow d}|^2}{|s_{\uparrow d}|^2 + |s_{\downarrow d}|^2}, \quad (\text{A1a})$$

$$s_{\uparrow d} = \langle \uparrow_d | (\omega_1 \hat{U}_z^{\alpha-\beta_{\pm}} + \omega_2 \hat{U}_z^{-\beta_{\pm}}) | \uparrow_x \rangle, \quad (\text{A1b})$$

$$s_{\downarrow d} = \langle \downarrow_d | (\omega_1 \hat{U}_z^{\alpha-\beta_{\pm}} + \omega_2 \hat{U}_z^{-\beta_{\pm}}) | \uparrow_x \rangle, \quad (\text{A1c})$$

and with $\beta_{0\pm}$ and $A_{\pm}$ defined in Eqs. (15) and (16), respectively, and

$$|s_{\uparrow x}|^2 = \frac{|A_{\pm}|^2}{2} [\cos(\beta_{\pm} - \text{Re}\beta_{0\pm}) + \cosh \text{Im}\beta_{0\pm}], \quad (\text{A2a})$$

$$|s_{\downarrow x}|^2 = \frac{|A_{\pm}|^2}{2} [-\cos(\beta_{\pm} - \text{Re}\beta_{0\pm}) + \cosh \text{Im}\beta_{0\pm}], \quad (\text{A2b})$$

$$|s_{\uparrow y}|^2 = \frac{|A_{\pm}|^2}{2} [-\sin(\beta_{\pm} - \text{Re}\beta_{0\pm}) + \cosh \text{Im}\beta_{0\pm}], \quad (\text{A2c})$$

$$|s_{\downarrow y}|^2 = \frac{|A_{\pm}|^2}{2} [\sin(\beta_{\pm} - \text{Re}\beta_{0\pm}) + \cosh \text{Im}\beta_{0\pm}], \quad (\text{A2d})$$

$$|s_{\uparrow z}|^2 = \frac{|A_{\pm}|^2}{2} \exp(\text{Im}\beta_{0\pm}), \quad (\text{A2e})$$

$$|s_{\downarrow z}|^2 = \frac{|A_{\pm}|^2}{2} \exp(-\text{Im}\beta_{0\pm}), \quad (\text{A2f})$$

we obtain the results

$$\langle \hat{\sigma}_x \rangle = \frac{\cos(\text{Re}\beta_{0\pm} - \beta_{\pm})}{\cosh \text{Im}\beta_{0\pm}}, \quad (\text{A3a})$$

$$\langle \hat{\sigma}_y \rangle = \frac{\sin(\text{Re}\beta_{0\pm} - \beta_{\pm})}{\cosh \text{Im}\beta_{0\pm}}, \quad (\text{A3b})$$

$$\langle \hat{\sigma}_z \rangle = \tanh \text{Im}\beta_{0\pm}. \quad (\text{A3c})$$

2. Variances of spin expectation values

The variances $\Delta(\langle \hat{\sigma}_d \rangle)$ of the expectation values are given by

$$\Delta(\langle \hat{\sigma}_d \rangle) = p_{\uparrow d}(1 - \langle \hat{\sigma}_d \rangle)^2 + p_{\downarrow d}(-1 - \langle \hat{\sigma}_d \rangle)^2, \quad (\text{A4a})$$

$$p_{\uparrow d} = \frac{|s_{\uparrow d}|^2}{|s_{\uparrow d}|^2 + |s_{\downarrow d}|^2}, \quad (\text{A4b})$$

$$p_{\downarrow d} = \frac{|s_{\downarrow d}|^2}{|s_{\uparrow d}|^2 + |s_{\downarrow d}|^2}, \quad (\text{A4c})$$

and we obtain

$$\Delta^2(\langle \hat{\sigma}_x \rangle) = 1 - \frac{\cos^2(\text{Re}\beta_{0\pm} - \beta_{\pm})}{\cosh^2 \text{Im}\beta_{0\pm}}, \quad (\text{A5a})$$

$$\Delta^2(\langle \hat{\sigma}_y \rangle) = 1 - \frac{\sin^2(\text{Re}\beta_{0\pm} - \beta_{\pm})}{\cosh^2 \text{Im}\beta_{0\pm}}, \quad (\text{A5b})$$

$$\Delta^2(\langle \hat{\sigma}_z \rangle) = \frac{1}{\cosh^2 \text{Im}\beta_{0\pm}}. \quad (\text{A5c})$$

For optimal compensation ($\chi = 0$ and $\beta_{\pm} = \text{Re}\beta_{0\pm}$), we find that $\Delta(\langle \hat{\sigma}_x \rangle)$ vanishes completely while $\Delta(\langle \hat{\sigma}_y \rangle)$ and $\Delta(\langle \hat{\sigma}_z \rangle)$ are maximal and equal unity. For small α and $\beta_{\pm} = 0$ this turns into

$$\Delta(\langle \hat{\sigma}_x \rangle) = \alpha |\omega_{1\pm}|, \quad (\text{A6a})$$

$$\Delta(\langle \hat{\sigma}_y \rangle) = 1 - \alpha^2 \text{Re}\omega_{1\pm}^2/2, \quad (\text{A6b})$$

$$\Delta(\langle \hat{\sigma}_z \rangle) = 1 - \alpha^2 \text{Im}\omega_{1\pm}^2/2. \quad (\text{A6c})$$

3. Expectation values averaged over exit beams

In the interference context, the expectation values $\langle \sigma_x \rangle$ in the two exit beams $|+\rangle$ and $|-\rangle$ are given by Eq. (21a). Using a common compensation angle β for both exit beams, the average can be written as

$$\overline{\langle \hat{\sigma}_x \rangle}_{\pm} = p_+ \langle \hat{\sigma}_x \rangle_+ + p_- \langle \hat{\sigma}_x \rangle_- = v_{\pm} \cos(\beta - \bar{\beta}_{\pm}), \quad (\text{A7a})$$

$$\bar{\beta}_{\pm} = \arctan \frac{p_+ \sin \beta_{0+} + p_- \sin \beta_{0-}}{p_+ \cos \beta_{0+} + p_- \cos \beta_{0-}}, \quad (\text{A7b})$$

$$v_{\pm}^2 = 1 - \frac{(p_1 - p_2)^2}{2} [1 - \cos(\beta_{0+} - \beta_{0-})], \quad (\text{A7c})$$

calculated for real weak values and $\chi = 0$. A series expansion by α gives

$$\bar{\beta}_{\pm} = p_1 \alpha + O(\alpha^3), \quad (\text{A8a})$$

$$v_{\pm} = 1 - \frac{1}{2} p_1 p_2 \alpha^2 + O(\alpha^4). \quad (\text{A8b})$$

In the which-way context, the expectation values in paths 1 and 2 read $\langle \sigma_x \rangle_1 = \cos(\alpha - \beta)$ and $\langle \sigma_x \rangle_2 = \cos(\beta)$, respectively, and the average is given by

$$\overline{\langle \hat{\sigma}_x \rangle}_{12} = p_1 \langle \hat{\sigma}_x \rangle_1 + p_2 \langle \hat{\sigma}_x \rangle_2 = v_{12} \cos(\beta - \bar{\beta}_{12}), \quad (\text{A9a})$$

$$\bar{\beta}_{12} = \arctan \frac{p_1 \sin \alpha}{p_2 + p_1 \cos \alpha}, \quad (\text{A9b})$$

$$v_{12}^2 = 1 - 2p_1 p_2 (1 - \cos \alpha). \quad (\text{A9c})$$

A series expansion by α gives

$$\bar{\beta}_{12} = p_1 \alpha + O(\alpha^3) \quad (\text{A10a})$$

$$v_{12} = 1 - \frac{1}{2} p_1 p_2 \alpha^2 + O(\alpha^4). \quad (\text{A10b})$$

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