Probing the spin structure of the fractional quantum Hall magnetoroton with polarized Raman scattering

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Starting from the Luttinger model for the band structure of GaAs, we derive an effective theory that describes the coupling of the fractional quantum Hall (FQH) system with photons in resonant Raman scattering experiments. Our theory is applicable in the regime when the energy of the photons ω_0 is close to the energy gap E_G, but $|\omega_0 - E_G|$ is much larger than the energy scales of the quantum Hall problem. In the literature, it is often assumed that Raman scattering measures the dynamic structure factor $S(\omega, \mathbf{k})$ of the FQH. However, in this paper, we find that the light scattering spectra measured in the experiments are proportional to the spectral densities of a pair of operators which we identified with the spin-2 components of the kinetic part of the stress tensor. In contrast with the dynamic structure factor, these spectral densities do not vanish in the long-wavelength limit $k \to 0$. We show that Raman scattering with circularly polarized light can measure the spin of the magnetoroton excitation in the FQH system. We give an explicit expression for the kinetic stress tensor that works on any Landau level and which can be used for numerical calculations of the spectral densities that enter the Raman scattering amplitudes. We propose that Raman scattering provides a way to probe the bulk of the $\nu = \frac{5}{2}$ quantum Hall state to determine its nature.

I. INTRODUCTION

The fractional quantum Hall effect (FQHE) was discovered in experiment [1]. Fractional quantum Hall (FQH) systems support a host of intriguing physical phenomena; they are also a playground for many exotic theoretical ideas ranging from anyons [2–4] to superconductivity [5], skyrmion [6], and bimetric gravity [7], to name a few. Anyonic excitations in nonabelian FQH states such as the Moore–Read state at filling fraction $\frac{5}{2}$ [8] may provide the building blocks for a topological quantum computer [9]. However, FQHE is still one of the most difficult and important unsolved problems of modern physics.

In a classic paper, Girvin *et al.* [10] proposed a single mode approximation for the FQHE in which the only excitation of the FQH system is a gapped charge-neutral mode called "magnetoroton." In the original treatment, the magnetoroton was interpreted as the charge density wave in the lowest Landau level (LLL). The dispersion relation of this neutral mode has a minimum at the wavelength of the order of the magnetic length ℓ_B, which imitates the behavior of the roton mode in superfluid ^{4}He. Experiments have confirmed the existence of the magnetoroton mode in light scattering experiments

[11–13].[1] In the widely accepted theoretical interpretation of these experiments, the light scattering intensity has been associated with the dynamical structure factor $S(\omega, \mathbf{k})$ in the LLL [15]. However, in the LLL limit, the dynamical structure factor goes to zero as k^4 [10] in the limit where the momentum of the magnetoroton k goes to zero. On the other hand, the Raman signal seems to persist down to $k = 0$, signaling that the identification of the intensity of Raman scattering with the dynamic structure factor may not be correct.

In this paper, we provide a theoretical treatment for Raman scattering experiments. We first note that the problem of Raman scattering involves many energy scales. The largest energy scale is that of the semiconductor gap E_G and the photon energy ω_0. The next scale is the distance between the Landau levels of the conduction-band electrons $\omega_c = eB/m^*c$, and the smallest energy scale is the Coulomb interacting energy between these electrons Δ. We assume a hierarchy

$$\Delta \ll \omega_c \ll E_G. \tag{1}$$

Only the physics at the scale Δ is "hard," i.e., nonperturbative or strongly correlated, while the physics at the scales E_G and ω_c are weakly coupled. To solve the Raman scattering problem effectively, one needs to separate out the nonperturbative, strongly correlated physics at the scale Δ from the perturbative, weak coupling physics at the other scales. (This is like the philosophy of "factorization" in quantum chromodynamics [16], where the perturbative physics at the hadronic scale

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[1]For an alternative interpretation of the mode, see Ref. [14].

is separated from the perturbative physics of higher energy scales.)

We perform this "factorization" procedure in two steps. The first is to integrate out the energy scale E_G. Starting with the Luttinger Hamiltonian for GaAs [17], we introduce a coupling between the lowest conduction band and highest valence band due to the interaction with light waves. We focus on the regime of resonant Raman scattering in which the frequency of the incoming light is close to the semiconductor energy gap $\omega_0 \approx E_G$. Under the assumption that the detuning between the frequency of light and the gap $|\omega_0 - E_G|$ is larger than both energy scales of the Hall effect ω_c and Δ, we integrate out valence bands to obtain the coupling of the conduction-band electron to the photon. The second step is to do projection to a single Landau level. The result is an effective coupling of the Raman photons to operators acting on a single Landau level.

Our result differs drastically from that of Ref. [15]. We find that, instead of measuring the spectral density of the density operator (the dynamic structure factor), the Raman scattering experiment measures the spectral densities of the operators that can be called the "kinetic stress tensor" operators $T_{ij}^{\text{kin}} = \frac{1}{m^*}\partial_i \psi^\dagger \partial_j \psi$. These operators are the components of the stress tensor that arise from the kinetic energy term in the many-body Hamiltonian. For simple model Hamiltonians leading to exact zero-energy trial wave functions, the kinetic stress tensor coincides with the full stress tensor, but that is not the case for the general case, including that of the Coulomb interaction. Moreover, in the LLL limit, in the long-wavelength limit, the only components of the kinetic stress tensor that have nonvanishing spectral densities are the two spin-2 components T_{zz}^{kin} and $T_{\bar{z}\bar{z}}^{\text{kin}}$ (the spin-0 component $T_{z\bar{z}}^{\text{kin}}$ has vanishing spectral density in the limit $k \to 0$).

Recent theoretical papers [18–21] have advanced a proposal on the interpretation of the magnetoroton. According to this proposal, the magnetoroton is the quantum dynamical metric, and at the long-wavelength regime, $k \sim 0$ has an angular momentum $= 2$ in the direction of the applied magnetic field [19]. To determine the spin of the magnetoroton, it has been suggested [19–21] that the spin of the magnetoroton can be detected through polarized Raman scattering.

We show in this paper how Raman scattering with circularly polarized light can indeed be used to confirm the spin of the magnetoroton, including the sign. The basic idea is very simple: if angular momentum is exactly conserved, an FQH state can absorb only one specific circular polarized photon to excite a spin-2 magnetoroton mode and emit a photon with opposite circular polarization. However, the real system does not have full rotational symmetry, but only C_4, so spins 2 and -2 are not distinct from the point of view of symmetry. In this paper, we carefully analyze the resonant Raman scattering using the Luttinger model of GaAs. We determine the magnitude of the effect of nonconservation of angular momentum through the Luttinger parameters and show that it is numerically small (of the order of $\frac{1}{40}$).

We organize the paper as follows. We start Sec. II by introducing our theoretical model for Raman scattering of an FQH state. In Sec. III, we present the calculation of the intensity of circularly polarized light scattering. We show that, in contrast with previous theoretical proposals, the peaks in scattering intensity, at the long-wavelength regime, was

obtained mainly due to the poles in the correlation functions of the kinetic part of stress tensor $\langle T^{\text{kin}} T^{\text{kin}} \rangle$. We relate the cross-section of Raman scattering by circularly polarized light with the spectral densities of the stress tensor. In Sec. IV, we derive the explicit formulas for the stress tensor projected to a Landau level, which can be used to evaluate numerically the spectral function. We then conclude the paper in Sec. V. The Appendices are devoted to the details of the calculation.

II. MODEL

In GaAs, the light hole and heavy hole bands make the main contribution to the Raman scattering process [15]. Thus, the spin-off bands can be ignored in this sense. We consider the effective Lagrangian, with only the conduction band ψ^α and $j = \frac{3}{2}$ valence bands (which is nothing but light hole and heavy hole bands) χ_i^α. In the notation, $\alpha = 1, 2$ represents the spinor index, and $i = 1, 2, 3$ represents three components of the p wave function. The ("Rarita–Schwinger") constraint is imposed

$$(\sigma^i)^\alpha{}_\beta \chi_i^\beta = 0, \tag{2}$$

which projects out the $j = \frac{1}{2}$ part from χ_i^α. Consider the Luttinger Hamiltonian for the heavy hole and light hole (within approximation $\mathbf{k} \approx 0$) [17]

$$H = \frac{1}{m}\left\{ \left(\gamma_1 + \frac{5\gamma_2}{2}\right)\frac{\mathbf{D}^2}{2} - \gamma_2 \sum_i J_i^2 D_i^2 - 2\gamma_3[\{J_x J_y\}\{D_x D_y\} \right.$$
$$\left. + \{J_y J_z\}\{D_y D_z\} + \{J_z J_x\}\{D_z D_x\}] + \frac{e}{c}\kappa \mathbf{J} \cdot \mathbf{B} \right\}, \tag{3}$$

where $\gamma_{1,2,3}$ are the Luttinger parameters, m is the mass of the electron, J_i are the $SO(3)$ generators: $(J_i)_{jk} = -i\epsilon_{ijk}$, $D_i \chi = (\partial_i - i\frac{e}{c}A_i)\chi$ is the covariant derivative, $\{\cdot\}$ denotes symmetrization, e.g., $\{D_x D_y\} \equiv \frac{1}{2}(D_x D_y + D_y D_x)$, and $e < 0$ is the electric charge of the electron. Using the equality

$$\{J_i J_j\}\{D_i D_j\} = (\mathbf{J} \cdot \mathbf{D})^2 - \frac{e}{2c}\mathbf{J} \cdot \mathbf{B}, \tag{4}$$

we can rewrite the Hamiltonian in Eq. (3) as

$$H = \frac{1}{m}\left[\left(\gamma_1 + \frac{5\gamma_2}{2}\right)\frac{\mathbf{D}^2}{2} + (\gamma_3 - \gamma_2)\sum_i J_i^2 D_i^2 - \gamma_3(\mathbf{J} \cdot \mathbf{D})^2 \right.$$
$$\left. + \frac{e}{c}\left(\kappa + \frac{\gamma_3}{2}\right)\mathbf{J} \cdot \mathbf{B} \right]. \tag{5}$$

The Lagrangian for the hole band is

$$\mathcal{L}_v = i\chi_{i\alpha}^\dagger \partial_t \chi_i^\alpha - \chi_{i\lambda}^\dagger H_{ij} \chi_j^\lambda + E_G \chi_{i\alpha}^\dagger \chi_i^\alpha, \tag{6}$$

where E_G is energy gap, and the covariant derivative is $D_i \psi \equiv (\partial_i - i\frac{e}{c}A_i)\psi$. We also have the Lagrangian of the conduction band

$$\mathcal{L}_c = i\psi_\alpha^\dagger \partial_t \psi^\alpha - \frac{D_i \psi_\alpha^\dagger D_i \psi^\alpha}{2m^*}, \tag{7}$$

and the coupling of the valence band and conduction band through interaction with light

$$\mathcal{L}_i = e\left(P^* \psi_\alpha^\dagger \chi_i^\alpha E_i + P \chi_{i\alpha}^\dagger \psi^\alpha E_i\right), \tag{8}$$

where P is the strength of the dipole transition between the conduction and valence bands (it will be related to the parameter usually denoted as E_p in the literature), and E_i is the electric field of electromagnetic wave.

We consider the regime of resonant Raman scattering where the photon energy is close to the gap $|\omega_0 - E_G| \ll E_G$. Here, we chose $\omega_0 = \frac{\omega_L + \omega_S}{2}$, with ω_L (ω_S) as the frequency of the incoming (scattered) photon. In this case, we can write

$$E_i = \tilde{E}_i e^{-i\omega_0 t} + \tilde{E}_i^* e^{i\omega_0 t}, \tag{9}$$

$$\chi_i^\alpha = \tilde{\chi}_i^\alpha e^{i\omega_0 t}, \tag{10}$$

where $\tilde{E}_i$ and $\tilde{\chi}$ are slowly varying fields (e.g., fields that vary with frequencies much smaller than ω_0). Substituting into the action and dropping the rapidly oscillating terms, the action can be rewritten as (for notational simplicity, we also drop the tildas in $\tilde{E}_i$ and $\tilde{\chi}$)

$$\begin{aligned}
\mathcal{L} = {} & i\psi_\alpha^\dagger \partial_t \psi^\alpha - \frac{D_i \psi_\alpha^\dagger D_i \psi^\alpha}{2m^*} + e\left(P^* \psi_\alpha^\dagger \chi_i^\alpha E_i + P \chi_{i\alpha}^\dagger \psi^\alpha E_i^*\right) \\
& + i\chi_{i\alpha}^\dagger \partial_t \chi_i^\alpha + (E_G - \omega_0)\chi_{i\alpha}^\dagger \chi_i^\alpha - \chi_{i\lambda}^\dagger H_{ij} \chi_j^\lambda \\
& + \lambda_\alpha^\dagger (\sigma^i)^\alpha_\beta \chi_i^\beta + \chi_{i\alpha}^\dagger (\sigma^i)^\alpha_\beta \lambda^\beta.
\end{aligned} \tag{11}$$

The last two terms are the Lagrangian multiplier for constraints in Eq. (2).

Integrating out χ and λ is equivalent to solving the field equations and the constraints

$$eP\psi^\alpha E_i^* + (E_G - \omega)\chi_i^\alpha + (\sigma^i)^\alpha_\beta \lambda^\beta + i\partial_t \chi_i^\alpha - H_{ij}\chi_j^\alpha = 0, \tag{12}$$

$$(\sigma^i)^\alpha_\beta \chi_i^\beta = 0. \tag{13}$$

We will focus on the regime where the photon energy is not too close to the gap. More precisely, we will assume that the detuning $|\omega_0 - E_G|$ is still much larger than the distance between the Landau levels in the bands

$$\omega_c \equiv \frac{|e|B}{mc} \ll |\omega_0 - E_G| \ll \omega_0. \tag{14}$$

In the FQH regime and holes, the typical momentum scale is $1/\ell_B$; one has

$$H \sim \frac{1}{m}D^2 \sim \frac{eB}{mc}. \tag{15}$$

This means that one can solve Eq. (12), ignoring the ∂_t and H terms,

$$\begin{aligned}
\chi_i^\alpha = {} & -\frac{eP}{3(E_G - \omega_0)}[2\psi^\alpha E_i^* + i\epsilon^{ijk}(\sigma^j)^\alpha_\beta \psi^\beta E_k^*] \\
& + O\left(\frac{\omega_c}{|E_G - \omega_0|}\right).
\end{aligned} \tag{16}$$

Substituting this solution into the action, we then find the effective action for ψ alone. In fact, since χ is the saddle point, an error of the order of $O(\omega_c/|E_G - \omega_0|)$ translated into an error $O(\omega_c^2/|E_G - \omega_0|^2)$ in the action; thus, it is justified to also keep the terms $\chi^\dagger \partial_t \chi$ and $\chi^\dagger H \chi$ when we do the substitution in Eq. (16).

To simplify the result, we assume that the electrons are fully polarized with spin $s_z = \frac{1}{2}$, so

$$\psi^\alpha = \begin{pmatrix} \psi \\ 0 \end{pmatrix}. \tag{17}$$

We assume the incoming and outgoing photons to have momenta along the z direction, so E_i are independent of x and y. In this case, the Lagrangian describing the interaction of the conduction-band electron with the Raman photons has the form

$$\mathcal{L}_{\text{int}} = V_{\alpha\beta}(E^*)^\alpha E^\beta, \tag{18}$$

where $V_{\alpha\beta}$ is some operators quadratic over ψ.

The photon spin then points along or opposite to the z axis, corresponding to the two circular polarizations. We will distinguish processes in which the direction of the spin of the photon flips from those in which the photon spin does not change direction. Introducing the complex coordinates (in the quantum Hall convention)

$$\begin{aligned}
z = x - iy, \quad \bar{z} = x + iy, \quad \partial_z = \tfrac{1}{2}(\partial_x + i\partial_y), \\
\partial_{\bar{z}} = \tfrac{1}{2}(\partial_x - i\partial_y),
\end{aligned} \tag{19}$$

so

$$E^z = E_x - iE_y, \quad E^{\bar{z}} = E_x + iE_y, \tag{20}$$

and

$$D_z = \tfrac{1}{2}(D_x + iD_y), \quad D_{\bar{z}} = \tfrac{1}{2}(D_x - iD_y), \tag{21}$$

the interaction Lagrangian can be written as

$$\mathcal{L}_{\text{int}} = V_{zz}(E^{\bar{z}})^* E^z + V_{\bar{z}\bar{z}}(E^z)^* E^{\bar{z}} + V_{z\bar{z}}(E^{\bar{z}})^* E^{\bar{z}} + V_{\bar{z}z}(E^z)^* E^z. \tag{22}$$

The terms responsible for scatterings with a switch in the photon helicity contain V_{zz} and $V_{\bar{z}\bar{z}}$. Direct calculation yields

$$\begin{aligned}
V_{\bar{z}\bar{z}} = {} & \frac{e^2|P|^2}{6(E_G - \omega_0)^2}[(\gamma_3 + \gamma_2)D_z\psi^\dagger D_z\psi \\
& - (\gamma_3 - \gamma_2)D_{\bar{z}}\psi^\dagger D_{\bar{z}}\psi],
\end{aligned} \tag{23}$$

$$\begin{aligned}
V_{zz} = {} & \frac{e^2|P|^2}{6(E_G - \omega_0)^2}[(\gamma_3 + \gamma_2)D_{\bar{z}}\psi^\dagger D_{\bar{z}}\psi \\
& - (\gamma_3 - \gamma_2)D_z\psi^\dagger D_z\psi].
\end{aligned} \tag{24}$$

Terms proportional to $\gamma_3 + \gamma_2$ in V_{zz} and $V_{\bar{z}\bar{z}}$ preserve rotational symmetry, while terms proportional to $\gamma_3 - \gamma_2$ break the angular momentum conservation by four.

It is convenient to introduce the "kinetic" stress tensor

$$T_{zz}^{\text{kin}} = \frac{1}{m^*}D_z\psi^\dagger D_z\psi, \quad T_{\bar{z}\bar{z}}^{\text{kin}} = \frac{1}{m^*}D_{\bar{z}}\psi^\dagger D_{\bar{z}}\psi,$$

$$T_{z\bar{z}}^{\text{kin}} = \frac{1}{2m^*}(D_z\psi^\dagger D_{\bar{z}}\psi + D_{\bar{z}}\psi^\dagger D_z\psi). \tag{25}$$

These components are the variation of the kinetic-energy part of the Hamiltonian over the external metric. This is different from the full stress tensor which contains also the variation of the potential energy over the metric. The effective operators

coupled to the Raman photon (and flip the direction of its spin) are

$$V_{\bar{z}z} = \frac{e^2 |P|^2 m^*}{6(E_G - \omega_0)^2} [(\gamma_3 + \gamma_2) T_{zz} - (\gamma_3 - \gamma_2) T_{\bar{z}\bar{z}}], \quad (26)$$

$$V_{zz} = \frac{e^2 |P|^2 m^*}{6(E_G - \omega_0)^2} [(\gamma_3 + \gamma_2) T_{\bar{z}\bar{z}} - (\gamma_3 - \gamma_2) T_{zz}]. \quad (27)$$

For details of calculations, see Appendix A. For completeness, we also write down the operators that do not flip the photon spin

$$V_{\bar{z}z} = \frac{e^2 |P|^2}{(E_G - \omega_0)^2} \left\{ \left[\theta^* \omega_c - \frac{(E_G - \omega_0)}{2} + \frac{\beta^* \omega_c}{4} \right] \rho \right.$$
$$\left. + (2\alpha^* - \beta^* + \gamma'^*) T_{z\bar{z}} + \frac{i}{2} \psi^\dagger \partial_t \psi \right\}, \quad (28a)$$

$$V_{z\bar{z}} = \frac{e^2 |P|^2}{9(E_G - \omega_0)^2} \left\{ \left[-\theta^* \omega_c - \frac{9(E_G - \omega_0)}{2} - \frac{\beta^* \omega_c}{4} \right] \rho \right.$$
$$\left. + (6\alpha^* - 5\beta^* + 5\gamma'^*) T_{z\bar{z}} + \frac{9i}{2} \psi^\dagger \partial_t \psi \right\}, \quad (28b)$$

where

$$\alpha^* = \frac{m^*}{2m} \left(\gamma_1 + \frac{5\gamma_2}{2} \right), \quad \gamma'^* = \frac{m^*}{m} (\gamma_3 - \gamma_2), \quad (29)$$

$$\beta^* = \frac{m^*}{m} \gamma_3, \quad \theta^* = -\frac{m^*}{m} \left(\kappa + \frac{\gamma_3}{2} \right). \quad (30)$$

III. SCATTERING OF CIRCULARLY POLARIZED LIGHT IN THE FQH REGIME

In this section, we will calculate the Raman scattering on the FQH state. The magnitude of the effect can be characterized by the per-particle differential cross-section

$$\frac{d\sigma_{\lambda\lambda'}}{d\omega d\Omega}, \quad (31)$$

where ω is the difference between the energy of the incoming photon ω_L and the scattered photon ω_S: $\omega = \omega_L - \omega_S$, $d\Omega$ is the infinitesimal solid angle of the scattered photon, and λ and λ' are the indices denoting the polarization of the incoming and scattered photons. For simplicity, we consider the case when the incident and reflection light are directed perpendicularly to the sample. The light can pass through the sample, or as depicted in Fig. 1, be reflected from the sample. We will assume that both incident light and scattered lights have circular polarization, and λ and λ' can be either $+$ or $-$, depending on the projection of the proton spin on the z axis. For example, for σ_{++}, the incident light is left handed (in the "classical optics" convention, see, e.g., Ref. [22]), and so the incident photons have spin pointing along the direction of their momentum, and the scattered light is right handed, as in Fig. 1(a), where we have the formula for cross-section per electron [15,23]

$$\frac{d\sigma_{++}(\omega)}{d\omega d\Omega} = \frac{1}{N_e} \frac{\omega_S}{\omega_L} \omega_S^2 \omega_L^2 \sum_f |\langle f|V_{\bar{z}z}|i\rangle|^2 \delta(\epsilon_f - \epsilon_i - \hbar\omega)$$
$$\approx -\frac{1}{\bar{\rho}} \frac{\omega_0^4}{\pi} \text{Im} \langle V_{\bar{z}z}^\dagger V_{\bar{z}z} \rangle_{\omega,\mathbf{0}}, \quad (32)$$
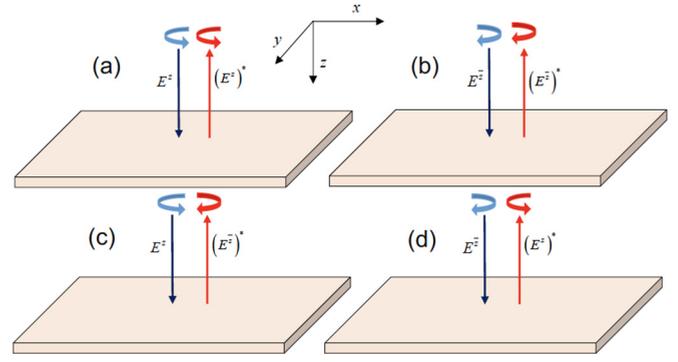
FIG. 1. Setup experiment for circular polarized light scattering.

with N_e being the total electron number in the conductance band, $\bar{\rho}$ is the electron density in the conductance band, ϵ_f, ϵ_i are energies of final and initial states, and

$$\langle A^\dagger A \rangle_{\omega,\mathbf{k}} \equiv \int dt \, d\mathbf{x} \, e^{i\omega t - i\mathbf{k}\cdot\mathbf{x}} \langle T A(t, \mathbf{x}) A^\dagger(0, \mathbf{0}) \rangle. \quad (33)$$

Thus, we need to calculate the spectral density of the operator $V_{\bar{z}z}(\omega, 0)$.

Similarly, in the case of setups in Figs. 1(b), 1(c) and 1(d), we have

$$\frac{d\sigma_{--}(\omega)}{d\omega d\Omega} = -\frac{1}{\bar{\rho}} \frac{\omega_0^4}{\pi} \text{Im} \langle V_{z\bar{z}}^\dagger V_{z\bar{z}} \rangle, \quad (34)$$

$$\frac{d\sigma_{+-}(\omega)}{d\omega d\Omega} = -\frac{1}{\bar{\rho}} \frac{\omega_0^4}{\pi} \text{Im} \langle V_{zz}^\dagger V_{zz} \rangle, \quad (35)$$

$$\frac{d\sigma_{-+}(\omega)}{d\omega d\Omega} = -\frac{1}{\bar{\rho}} \frac{\omega_0^4}{\pi} \text{Im} \langle V_{\bar{z}\bar{z}}^\dagger V_{\bar{z}\bar{z}} \rangle. \quad (36)$$

The intensity of the Raman scattering in these channels is proportional to the spectral densities of the operators $V_{z\bar{z}}$, V_{zz}, and $V_{\bar{z}\bar{z}}$.

Let us now show that, in the limit of negligible Landau level mixing, the spectral densities of the operators $V_{z\bar{z}}$ and $V_{\bar{z}z}$ are zero, implying that the processes depicted in Figs. 1(a) and 1(b) do not happen. For that, we note from Eq. (28) that the integrals of $V_{z\bar{z}}$ and $V_{\bar{z}z}$ over space are linear combinations of

$$\int d\mathbf{x} \, \rho, \quad \int d\mathbf{x} \, T_{z\bar{z}}^{\text{kin}}, \quad \int d\mathbf{x} \, i\psi^\dagger \partial_t \psi. \quad (37)$$

The first integral is the total number of particles N_e. As this quantity is conserved, it does not contribute to the spectral density. From Appendix D, we find the results for N^{th} Landau level

$$\int d\mathbf{x} \, T_{z\bar{z}}^{\text{kin}} = \left(N + \frac{1}{2} \right) \frac{\omega_c}{2} N_e, \quad (38)$$

$$\int d\mathbf{x} \, i\psi^\dagger \partial_t \psi = 2E - \left(N + \frac{1}{2} \right) \omega_c N_e, \quad (39)$$

where E is the total energy. Both integrals reduce to conserved quantities. Thus, the Raman processes that do not involve flipping the direction of the photon spin are suppressed.

In previous experiments [11,12], the momentum transfer to the electron gas is rather small $kl_B \lesssim 0.15$. This implies that these experiments mainly probe the transitions where the photon spin flips sign and effectively measure the spectral

densities of the traceless components of the kinetic stress tensor. The picture suggested here is different from the previous one suggested in Ref. [15], where the main coupling of the Raman photon to the electron liquids is through the $\psi^\dagger \psi A_i^2$ term in the Lagrangian. This coupling would lead to a vanishing Raman scattering at $k = 0$.

Let us introduce the short-hand notation for the spectral densities of the off-diagonal components of the stress tensor

$$-\operatorname{Im}\langle T_{zz}^{\mathrm{kin}} T_{\bar{z}\bar{z}}^{\mathrm{kin}}\rangle_{\omega,\mathbf{0}} = I_+(\omega), \tag{40}$$

$$-\operatorname{Im}\langle T_{\bar{z}\bar{z}}^{\mathrm{kin}} T_{zz}^{\mathrm{kin}}\rangle_{\omega,\mathbf{0}} = I_-(\omega). \tag{41}$$

These functions should be calculated numerically. The intensity of Raman scatterings can now be expressed as

$$\frac{d\sigma_{+-}}{d\omega d\Omega} = \frac{1}{\pi\bar\rho}\left[\frac{e^2|P|^2\omega_0^2 m^*}{6(E_0 - \omega_0)^2}\right]^2 [(\gamma_3 + \gamma_2)^2 I_+(\omega) \\ + (\gamma_3 - \gamma_2)^2 I_-(\omega)], \tag{42}$$

$$\frac{d\sigma_{-+}}{d\omega d\Omega} = \frac{1}{\pi\bar\rho}\left[\frac{e^2|P|^2\omega_0^2 m^*}{6(E_0 - \omega_0)^2}\right]^2 [(\gamma_3 + \gamma_2)^2 I_-(\omega) \\ + (\gamma_3 - \gamma_2)^2 I_+(\omega)]. \tag{43}$$

In Ref. [24], it was proven that $I_+(\omega) = 0$ for the trial ground states of model Hamiltonians with contact interactions. While there is no argument that I_+ should be zero for more general Hamiltonians, numerically, it was found that for Coulomb interaction I_+ is much smaller than I_- for the Laughlin $\nu = \frac{1}{3}$ state [20,25].

If one ignores I_+ compared with I_-, we find the ratio of scattered light intensity of experiment setups in Figs. 1(c) and 1(d)

$$\frac{I_{-+}(\omega = \Delta)}{I_{+-}(\omega = \Delta)} = \frac{(\gamma_3 + \gamma_2)^2}{(\gamma_3 - \gamma_2)^2}. \tag{44}$$

The ratio only depends on the Luttinger parameters. Moreover, the fact that I_{+-} will vanish if $\gamma_3 - \gamma_2 = 0$ suggests that the signal of I_{+-} is due to rotational symmetry breaking. These results confirm that at zero momentum ($\mathbf{k} = 0$), the magnetoroton excitation has spin 2 in the $\hat{z}$ direction. However, in the case of finite momentum, the magnetoroton excitation will be a mix of modes with spin $+2$ and spin -2 in the $\hat{z}$ direction, which was suggested in the previous paper [19].

The numerical values for the Luttinger parameters of GaAs are [26]

$$\gamma_1 = 6.9, \quad \gamma_2 = 2.1, \quad \gamma_3 = 2.9, \quad \kappa = 1.2. \tag{45}$$

Substituting these parameters into Eq. (44) yields the ratio of intensities

$$\frac{I_{-+}(\omega = \Delta)}{I_{+-}(\omega = \Delta)} \approx 40. \tag{46}$$

Note that this relies on the assumption that $I_+ = 0$, which is not expected to hold exactly for the Coulomb interaction. However, if I_+ is smaller than I_-, one still expects that $I_{-+} \ll I_{+-}$ for $\nu = \frac{1}{3}$ states. This is also expected for the Jain states $\nu = n/(2n + 1)$, in which the composite fermion theory implies that the magnetoroton has the same sign of spin as

in the $\nu = \frac{1}{3}$ state. In the particle-hole conjugate Jain states $\nu = (n + 1)/(2n + 1)$, in contrast, one expects that $I_{-+} \gg I_{+-}$ [27,28].

IV. STRESS TENSOR PROJECTED ON A LANDAU LEVEL

As we have seen from the previous section, to obtain the cross-section of polarized Raman scattering, we need to calculate the spectral function of the kinetic part of the stress tensor projected on a specific Landau level (the fractionally filled one). To enable future numerical calculations of these spectral functions, we need the expressions for the operators T_{ij}^{kin} after the projection to a Landau level. In this section, we will derive the explicit form of the projected kinetic stress tensor.

We summarize the result here. For a system of particles interacting through a two-body isotropic potential $V(|\mathbf{x} - \mathbf{y}|)$ on the Nth Landau level, the kinetic part of the stress tensor (at zero momentum) can be written as

$$\int d\mathbf{x}\, T_{ij}^{\mathrm{kin}} = \int_{\mathbf{q}} \frac{q_i q_j}{q} \frac{\partial}{\partial q}\{e^{-x_q}[L_N(x_q)]^2\}V(q)\bar\rho(\mathbf{q})\bar\rho(-\mathbf{q}), \tag{47}$$

where $\int_{\mathbf{q}} \equiv \int d\mathbf{q}/(2\pi)^2$,

$$x_q \equiv \frac{q^2\ell_B^2}{2}, \tag{48}$$

$L_N(x)$ is the Laguerre polynomial, ij can be either zz or $\bar{z}\bar{z}$, and $\bar\rho(\mathbf{q})$ is the projected density operator in momentum space [10].

The interpretation of the above equation is rather simple. Recall that the projected Hamiltonian of the system is

$$H = \int_{\mathbf{q}} e^{-x_q}[L_N(x_q)]^2 V(q)\bar\rho(\mathbf{q})\bar\rho(-\mathbf{q}), \tag{49}$$

where the form factor $e^{-x_q}[L_N(x_q)]^2$ arises from the projection to the Nth Landau level. Polarized Raman scattering, as explained above, has the effect of changing the effective metric in the kinetic term for the electron (making the effective mass m^* anisotropic). This makes the Landau orbit on the Nth Landau level anisotropic, and the effect of that is the operator $(q_i q_j/q)\partial_q$ acting on the form factor.

For $N = 0$, Eq. (47) reads

$$\int d\mathbf{x}\, T_{zz}^{\mathrm{kin}} = -\ell_B^2 \int_{\mathbf{q}} q_z^2 e^{-q^2\ell_B^2/2} V(q)\bar\rho(\mathbf{q})\bar\rho(-\mathbf{q}), \tag{50}$$

which is exactly the operator considered in Refs. [20,25]. Thus, the spectral densities computed in Refs. [20,25] are directly related to polarized Raman scattering on FQH states on the LLL.

For the next-to-LLL $N = 1$, we have

$$\int d\mathbf{x}\, T_{zz}^{\mathrm{kin}} = -\ell_B^2 \int_{\mathbf{q}} q_z^2 e^{-x_q}(1 - x_q)(3 - x_q) V(q)\bar\rho(\mathbf{q})\bar\rho(-\mathbf{q}). \tag{51}$$

The general expression for the kinetic stress tensor in Eq. (47) has also been found by Yang [29]. The similar spectral function for generalized pseudopotentials and quantum Hall

bilayers was introduced in Refs. [30,31] In the rest of this section, we provide a derivation of Eq. (47).

A. Preliminaries

We use the complex coordinates in Eq. (19) and the symmetric gauge $A_x = -\frac{1}{2}By$, $A_y = \frac{1}{2}Bx$. In the complex coordinates,

$$
A_z = \frac{1}{2}(A_x + iA_y) = i\frac{B}{4}\bar{z}, \quad A_{\bar{z}} = \frac{1}{2}(A_x - iA_y) = -i\frac{B}{4}z.
$$
(52)

Then in the symmetric gauge,

$$
D_z = \partial_z - \frac{ie}{c}A_z = \partial_z - \frac{\bar{z}}{4},
$$
(53)

$$
D_{\bar{z}} = \partial_{\bar{z}} - \frac{ie}{c}A_{\bar{z}} = \partial_{\bar{z}} + \frac{z}{4}.
$$
(54)

Note that

$$
[D_z, D_{\bar{z}}] = -\frac{eB}{2c} = \frac{1}{2\ell_B^2}.
$$
(55)

The (complex) guiding center coordinates are defined as

$$
Z = z - 2\ell_B^2 D_{\bar{z}} = \frac{z}{2} - 2\ell_B^2 \partial_{\bar{z}},
$$
(56)

$$
\bar{Z} = \bar{z} + 2\ell_B^2 D_z = \frac{\bar{z}}{2} + 2\ell_B^2 \partial_z,
$$
(57)

which satisfy

$$
[D_z, Z] = [D_z, \bar{Z}] = [D_{\bar{z}}, Z] = [D_{\bar{z}}, \bar{Z}] = 0,
$$
(58)

and

$$
[\bar{Z}, Z] = 2\ell_B^2.
$$
(59)

We define another set of coordinates: the relative coordinates which describe the motion around the guiding center

$$
\zeta = 2\ell_B^2 D_{\bar{z}} = \frac{z}{2} + 2\ell_B^2 \partial_{\bar{z}},
$$
(60)

$$
\bar{\zeta} = -2\ell_B^2 D_z = \frac{\bar{z}}{2} - 2\ell_B^2 \partial_z,
$$
(61)

which commute with Z and $\bar{Z}$ [Eq. (58)] and have the commutator

$$
[\bar{\zeta}, \zeta] = -2\ell_B^2.
$$
(62)

Then $z = Z + \zeta$, $\bar{z} = \bar{Z} + \bar{\zeta}$. We denote the two-dimensional (2D) vector whose complex coordinates are Z and $\bar{Z}$ as **R**, and the vector with complex coordinates ζ and $\bar{\zeta}$ as **r̃**. That means **x** = **R** + **r̃**.

One defines two sets of creation and annihilation operators. One set moves between different Landau levels

$$
a = \sqrt{2}\ell_B D_{\bar{z}} = \frac{\zeta}{\sqrt{2}\ell_B}, \quad a^\dagger = -\sqrt{2}\ell_B D_z = \frac{\bar{\zeta}}{\sqrt{2}\ell_B},
$$
(63)

and another set moves within a Landau level

$$
b = \frac{1}{\sqrt{2}\ell_B}\bar{Z}, \quad b^\dagger = \frac{1}{\sqrt{2}\ell_B}Z.
$$
(64)

The orbitals are obtained by the acting creation operator on the lowest state

$$
|M, m\rangle = \frac{1}{\sqrt{M!m!}}a^{\dagger M}b^{\dagger m}|0, 0\rangle,
$$
(65)

where

$$
\langle \mathbf{x}|0, 0\rangle \sim e^{-|z|^2/4\ell_B^2}.
$$
(66)

B. The kinetic stress tensor on a Landau level

Our task is to find the expression for the kinetic part of the stress tensor in the theory where the electrons live on one Landau level. This will be done through a field-theory formalism. The action describing electrons on the Nth Landau level is

$$
S = \int dt\, d\mathbf{x}\big[i\psi^\dagger \partial_t \psi + \chi^\dagger \big(2\ell_B^2 D_z D_{\bar{z}}\psi + N\psi\big)
$$
$$
+ \big(2\ell_B^2 D_{\bar{z}} D_z \psi^\dagger + N\psi^\dagger\big)\chi\big]
$$
$$
- \frac{1}{2}\int dt\, d\mathbf{x}\, d\mathbf{x}'\, V(\mathbf{x} - \mathbf{x}')\psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x}')\psi(\mathbf{x}')\psi(\mathbf{x}).
$$
(67)

The fields χ and $\chi^\dagger$ are simply the Lagrangian multipliers enforcing the constraint

$$
2\ell_B^2 D_z D_{\bar{z}}\psi + N\psi = 0,
$$
(68)

which is simply the condition that ψ lies on the Nth Landau level.

To find the stress tensor, we first rewrite the action by integration by part

$$
S = \int dt\, d\mathbf{x}\big[i\psi^\dagger \partial_t \psi - 2\ell_B^2 D_z \chi^\dagger D_{\bar{z}}\psi - 2\ell_B^2 D_z \psi^\dagger D_{\bar{z}}\chi
$$
$$
+ N(\chi^\dagger \psi + \psi^\dagger \chi)\big] - \frac{1}{2}\int dt\, d\mathbf{x}\, d\mathbf{x}'\, V(\mathbf{x} - \mathbf{x}')\psi^\dagger(\mathbf{x})\psi^\dagger
$$
$$
\times (\mathbf{x}')\psi(\mathbf{x}')\psi(\mathbf{x}),
$$
(69)

then the kinetic part of the stress tensor can be calculated from Noether's theorem:

$$
T^i{}_j = -\frac{\partial L}{\partial(\partial_i \phi_a)}\partial_j \phi_a,
$$
(70)

where one sums over all fields ϕ_a, which in our case encompass $\psi, \phi^\dagger, \chi$, and $\chi^\dagger$. For the polarized Raman experiment with perpendicularly incoming and outgoing photons, with a flipping of the photon spin, one only needs the traceless part of the stress tensor, integrated over space:

$$
\int d\mathbf{x}\, T_{zz}^{\text{kin}} = -\ell_B^2 \int d\mathbf{x}\, \big(\chi^\dagger D_z^2 \psi + D_z^2 \psi^\dagger \chi\big),
$$
(71)

$$
\int d\mathbf{x}\, T_{\bar{z}\bar{z}}^{\text{kin}} = -\ell_B^2 \int d\mathbf{x}\, \big(\chi^\dagger D_{\bar{z}}^2 \psi + D_{\bar{z}}^2 \psi^\dagger \chi\big).
$$
(72)

We can expand χ as a sum over Landau levels: $\chi = \chi_0 + \chi_1 + \chi_2 + \cdots$. We recall that, when acting on ψ and χ, D_z raises and $D_{\bar{z}}$ lowers the Landau level index, while when acting on $\psi^\dagger$ and $\chi^\dagger$, they switch roles. Due to the orthogonality of wave functions on different Landau levels, only the parts of χ that are on the $(N + 2)$th and (if $N \geqslant 2$) $(N - 2)$th Landau

levels contribute to the integrals in Eq. (71). We then have, for $N \geqslant 2$,

$$\int d\mathbf{x}\, T_{zz}^{\mathrm{kin}} = -\ell_B^2 \int d\mathbf{x}\, \big(\chi_{N+2}^\dagger D_z^2 \psi + D_z^2 \psi^\dagger \chi_{N-2}\big), \quad (73)$$

$$\int d\mathbf{x}\, T_{\bar{z}\bar{z}}^{\mathrm{kin}} = -\ell_B^2 \int d\mathbf{x}\, \big(\chi_{N-2}^\dagger D_{\bar{z}}^2 \psi + D_{\bar{z}}^2 \psi^\dagger \chi_{N+2}\big), \quad (74)$$

and for $N = 0$ or 1,

$$\int d\mathbf{x}\, T_{zz}^{\mathrm{kin}} = -\ell_B^2 \int d\mathbf{x}\, \chi_{N+2}^\dagger D_z^2 \psi, \quad (75)$$

$$\int d\mathbf{x}\, T_{\bar{z}\bar{z}}^{\mathrm{kin}} = -\ell_B^2 \int d\mathbf{x}\, D_{\bar{z}}^2 \psi^\dagger \chi_{N+2}. \quad (76)$$

The equation determining χ is

$$0 = \frac{\delta S}{\delta \psi^\dagger} = i\partial_t \psi + \big(2\ell_B^2 D_z D_{\bar{z}} + N\big)\chi + W(\mathbf{x}), \quad (77)$$

where we denoted

$$W(\mathbf{x}) = -\int d\mathbf{x}'\, V(\mathbf{x} - \mathbf{x}')\psi^\dagger(\mathbf{x}')\psi(\mathbf{x}')\psi(\mathbf{x}). \quad (78)$$

For $n \neq N$, Eq. (77) implies

$$\chi_n = \frac{W_n}{n - N}. \quad (79)$$

Specifically,

$$\chi_{N+2} = \tfrac{1}{2}W_{N+2}, \quad (80)$$

$$\chi_{N-2} = -\tfrac{1}{2}W_{N-2} \qquad (N \geqslant 2), \quad (81)$$

and therefore,

$$\int d\mathbf{x}\, T_{zz}^{\mathrm{kin}} = -\frac{1}{2}\ell_B^2 \int d\mathbf{x}\, \big(W^\dagger D_z^2 \psi - D_z^2 \psi^\dagger W\big), \quad (82)$$

$$\int d\mathbf{x}\, T_{\bar{z}\bar{z}}^{\mathrm{kin}} = \frac{1}{2}\ell_B^2 \int d\mathbf{x}\, \big(W^\dagger D_{\bar{z}}^2 \psi - D_{\bar{z}}^2 \psi^\dagger W\big), \quad (83)$$

where we have used the orthogonality of the functions on different Landau levels to replace W_{N+2} and W_{N-2} by simply W.

Using formulas from Appendix C, we then find

$$\int d\mathbf{x}\, T_{zz}^{\mathrm{kin}} = -\frac{1}{2} \int_{\mathbf{q}} \ell_B^2 q_z^2 e^{-x_q} L_N(x_q)\big[L_N^2(x_q) - L_{N-2}^2(x_q)\big]$$
$$\times V(q)\bar{\rho}(\mathbf{q})\bar{\rho}(-\mathbf{q}),$$
$$x_q \equiv \frac{q^2 \ell_B^2}{2}, \quad (84)$$

and a similar equation where T_{zz} is replaced by $T_{\bar{z}\bar{z}}$ and q_z by $q_{\bar{z}}$. Here, L_N is the Laguerre polynomial, and L_N^2 is not the square of L_N but the associated Laguerre polynomial L_N^k with $k = 2$, and for the uniformity of the equation, we have defined $L_{-1}^2 = L_{-2}^2 = 0$.

These equations can be brought to an alternative form by using the following identities involving the associated Laguerre polynomials:

$$L_N^k(x) = L_N^{k+1}(x) - L_{N-1}^{k+1}(x), \quad \frac{d}{dx}L_N^k(x) = -L_{N-1}^{k+1}(x). \quad (85)$$

One can show that, for $N \geqslant 2$,

$$L_N^2(x) - L_{N-2}^2(x) = L_N(x) - 2\frac{d}{dx}L_N(x), \quad (86)$$

while one can also check directly that

$$L_0(x)[L_0(x) - 2L_0'(x)] = L_0(x)L_0^2(x),$$
$$L_1(x)[L_1(x) - 2L_1'(x)] = L_1(x)L_1^2(x). \quad (87)$$

We then can rewrite the kinetic part of the stress tensor for a general Landau level N as

$$\int d\mathbf{x}\, T_{zz}^{\mathrm{kin}} = -\frac{\ell_B^2}{2} \sum_{\mathbf{q}} q_z^2 e^{-x_q} L_N(x_q)[L_N(x_q) - 2L_N'(x_q)]$$
$$\times V(q)\bar{\rho}(\mathbf{q})\bar{\rho}(-\mathbf{q}), \quad (88)$$

and another equation with the replacement $T_{zz}^{\mathrm{kin}} \to T_{\bar{z}\bar{z}}^{\mathrm{kin}}$ and $q_z \to q_{\bar{z}}$. This can be further transformed to Eq. (47).

Some remarks are in order. The kinetic part of stress tensor operators in Eq. (88) for the LLL share the same form as the spin-2 operators in Ref. [20], in which the authors calculated the normalized spectral functions. One can employ the same approach to obtain the spectral density of the stress tensor for higher Landau levels. The result will provide the estimation for Raman scattering intensity of a FQH system at higher Landau levels in our theoretical model. In Appendix E, we give the expression for the full stress tensor operators, including the contribution from the interaction. This can be used to calculate the spectral function of the LLL stress tensor and check the sum rules derived in Ref. [19].

V. CONCLUSIONS

In this paper, we have derived the coupling of the electrons in a single Landau level with applied electromagnetic waves, which effectively captures the essential physics of Raman scattering on FQH systems. We show that the electron operator responsible for Raman scattering is not the density operator, but the "kinetic stress tensor," and we derive the expression of the latter after projection to a single Landau level. We then show that, in the long-wavelength regime, the light scattering intensity in Raman experiments measures the spectral function of the kinetic part of stress tensors. Our calculation explains the scattering intensity peaks at zero momentum without relying on any momentum-nonconserving processes.

In addition, we proposed experimental setups to verify the spin-2 hypothesis of magnetoroton mode in FQH systems using Raman scattering with circularly polarized light. We show that, for a magnetoroton with a well-defined sign of spin, the ratio between light scattering intensities of different configurations of circularly polarized Raman experiments only depends on Luttinger parameters, which are well known. Measuring those ratios can confirm our theoretical model and unveil the spin of the magnetoroton excitations in a FQH state.

Using the explicit form of the stress tensor operator derived in this paper, one can perform the numerical calculation to obtain the spectral function of the stress tensors. One can then use the numerical results to verify the LLL sum rules proposed in Ref. [19] to predict the result of Raman scattering on states on higher Landau levels.

Raman scattering may help resolve the question about the nature of the $\nu = \frac{5}{2}$ state. In recent experiments [32,33], the thermal Hall conductance at the edge of the $\nu = \frac{5}{2}$ state was determined to be consistent with the PH-Pfaffian state [34], but not the Pfaffian [8] or the anti-Pfaffian state [35,36], seemingly contradicting the results of numerical simulations [37]. Theoretical proposals aiming to explain this discrepancy include a disorder-stabilized thermal metal phase which is adiabatically connected to the PH-Pfaffian phase [38–41] and an incomplete thermalization on the edge [42–45]. Raman scattering provides a way to probe directly the bulk of the $\nu = \frac{5}{2}$ state. The magnetoroton in the Pfaffian (Moore-Read) state [8] must have a spin of the same sign as in the $\nu = \frac{1}{3}$ Lauglin state, while in the anti-Pfaffian state [35,36], it must have the opposite sign. The PH-Pfaffian state [34], in the absence of Landau-level mixing, is particle-hole symmetric; hence, the Raman scattering probabilities I_{+-} and I_{-+} must be the same. However, it is not clear how significant the effect of Landau level mixing would be in this case. In the preprint by Haldane *et al.* [46], the spectral functions I_+, I_- for $\nu = \frac{5}{2}$ states was calculated numerically. The results support the conclusions in our paper.

To derive the coupling of the Raman photons to FQH electron liquid, we have assume that the detuning $|\omega_0 - E_G|$ is much larger than the cyclotron energy ω_c. This allows us to perform the first step of our "factorization" procedure—integrating out the holes—without having to think about the effect of the magnetic field on the conduction-band electrons. We suspect that our final result is valid under a weaker assumption—that the detuning is larger than the energy scale of the FQHE, i.e., of the Coulomb interaction between the conduction-band electrons. A derivation of this result would need to be a one-step procedure—integrating out the valence bands and the projecting to one Landau level at the same time. We defer this to a future paper.

APPENDIX A: DETAILED DERIVATION OF RAMAN SCATTERING COUPLING

Here, we present the detailed derivation of the coupling of the FQH system with a photon. We define the parameters

$$\alpha = \frac{1}{2m}\left(\gamma_1 + \frac{5\gamma_2}{2}\right), \qquad \gamma' = \frac{1}{m}(\gamma_3 - \gamma_2), \qquad \text{(A1)}$$

$$\beta = \frac{1}{m}\gamma_3, \qquad \theta = -\frac{1}{m}\left(\kappa + \frac{\gamma_3}{2}\right). \qquad \text{(A2)}$$

After integrating out fields in the valence band, we derive the effective Lagrangian for the conduction band

$$\mathcal{L} = i\psi_\alpha^\dagger \partial_t \psi^\alpha - \frac{D_i\psi_\alpha^\dagger D_i\psi^\alpha}{2m^*} + i\chi_{i\alpha}^\dagger \partial_t \chi_i^\alpha - (E_G - \omega_0)\chi_{i\alpha}^\dagger \chi_i^\alpha$$
$$- \chi_{k'\lambda}^\dagger \left[\alpha\mathbf{D}^2 + \gamma' \sum_i J_i^2 D_i^2 - \beta(\mathbf{J}\cdot\mathbf{D})^2 - \frac{e}{c}\theta\mathbf{J}\cdot\mathbf{B} \right]_{k'k}$$
$$\times \chi_k^\lambda. \qquad \text{(A3)}$$

All terms which contain valence band field χ_i^α can be considered as coupling of conduction band field ψ^α with the electric field through substitution of Eq. (16). We define

$$\mathcal{I}_0 = i\chi_{i\lambda}^\dagger \partial_t \chi_i^\lambda, \qquad \text{(A4)}$$

$$\mathcal{I}_1 = -(E_G - \omega_0)\chi_{i\alpha}^\dagger \chi_i^\alpha, \qquad \text{(A5)}$$

$$\mathcal{I}_\alpha = -\alpha\chi_{k\lambda}^\dagger \mathbf{D}^2 \chi_k^\lambda, \qquad \text{(A6)}$$

$$\mathcal{I}_\beta = \beta\chi_{k'\lambda}^\dagger [(\mathbf{J}\cdot\mathbf{D})^2]_{k'k}\chi_k^\lambda, \qquad \text{(A7)}$$

$$\mathcal{I}_{\gamma'} = -\gamma'\chi_{k'\lambda}^\dagger \sum_i (J_i^2)_{k'k} D_i^2 \chi_k^\lambda, \qquad \text{(A8)}$$

$$\mathcal{I}_\theta = \theta\frac{e}{c} \sum_{k,k',\lambda} \chi_{k'\lambda}^\dagger (\mathbf{J}_{k'k}\cdot\mathbf{B})\chi_k^\lambda. \qquad \text{(A9)}$$

Consequently, the effective Lagrangian can be rewritten as

$$\mathcal{L}_{\text{eff}} = i\psi^\dagger \partial_t \psi - \frac{(D_i\psi)^\dagger D_i\psi}{2m^*} + \mathcal{I}_0 + \mathcal{I}_1 + \mathcal{I}_\alpha + \mathcal{I}_\beta$$
$$+ \mathcal{I}_{\gamma'} + \mathcal{I}_\theta. \qquad \text{(A10)}$$

Substitution of Eq. (16) for χ_i^α in Eq. (A5) yields

$$\mathcal{I}_1 = -\frac{e^2|P|^2}{3(E_G - \omega_0)}(2\psi^\dagger \psi |E|^2 + i\epsilon^{ijk}\psi^\dagger \sigma^j \psi E_k^* E_i). \qquad \text{(A11)}$$

The first term in $\mathcal{I}_1$ is the interaction of light with charge density; the second term is the interaction of light with spin density. Considering that the electrons in the conduction band, under strong magnetic field in the $\hat{z}$ direction, only have the spin component $s_z = \frac{1}{2}$, we can rewrite

$$\mathcal{I}_1 = -\frac{e^2|P|^2}{3(E_G - \omega_0)}[2E_i E_i^* + i(E_2^* E_1 - E_1^* E_2)]\rho, \qquad \text{(A12)}$$

where $\rho = \psi^\dagger \psi$. Since E_i^* is a slow varying field under the redefinition in Eq. (9), the term with $\partial_t E_i^*$ in $\mathcal{I}_0$ is smaller than $\mathcal{I}_1$. We then have

$$\mathcal{I}_0 = i\frac{e^2|P|^2}{3(E_G - \omega_0)^2}[2E_i E_i^* + i(E_2^* E_1 - E_1^* E_2)]\psi^\dagger \partial_t \psi. \qquad \text{(A13)}$$

To understand the next interaction terms (the Luttinger terms), we recall the formula for the kinetic part of the stress energy tensor

$$T_{ij}^{\text{kin}} = \frac{(D_i\psi)^\dagger D_j\psi}{2m^*} + \frac{(D_j\psi)^\dagger D_i\psi}{2m^*}. \qquad \text{(A14)}$$

Under above assumption of spin state of electrons in the conduction band and $k_3 = 0$ (we consider a 2D system in the

xy plane, and the applied magnetic field is in the $\hat{z}$ direction), we can rewrite the Luttinger terms as

$$\mathcal{I}_\alpha = \frac{\alpha^* e^2 |P|^2}{3(E_G - \omega_0)^2}[2E_i E_i^* + i(E_2^* E_1 - E_1^* E_2)](T_{11}^{\text{kin}} + T_{22}^{\text{kin}}), \tag{A15}$$

$$\mathcal{I}_\beta = -\frac{\beta^* e^2 |P|^2}{9(E_G - \omega_0)^2}\Big\{T_{11}^{\text{kin}}[5E_2 E_2^* + 5E_3 E_3^* + 2E_1 E_1^* + i(E_2 E_1^* - E_1 E_2^*)]$$

$$+ T_{22}^{\text{kin}}[5E_1 E_1^* + 5E_3 E_3^* + 2E_2 E_2^* + i(E_2 E_1^* - E_1 E_2^*)]$$

$$- 3T_{12}^{\text{kin}}(E_1 E_2^* + E_2 E_1^*) + i\frac{\omega_c}{2c}[5(E_2 E_1^* - E_1 E_2^*) - 4iE_a E_a^* - 2iE_3 E_3^*]\rho\Big\}, \tag{A16}$$

$$\mathcal{I}_{\gamma'} = \frac{\gamma'^* e^2 |P|^2}{9(E_G - \omega_0)^2}\Big\{T_{11}^{\text{kin}}[5E_2 E_2^* + 5E_3 E_3^* + 2E_1 E_1^* + i(E_2 E_1^* - E_1 E_2^*)] + T_{22}^{\text{kin}}[5E_1 E_1^* + 5E_3 E_3^* + 2E_2 E_2^* + i(E_2 E_1^* - E_1 E_2^*)]\Big\}, \tag{A17}$$

$$\mathcal{I}_\theta = i\theta^* \omega_c \frac{e^2 |P|^2}{9(E_G - \omega_0)^2}[5(E_1 E_2^* - E_2 E_1^*) + 4iE_a E_a^* + 2iE_3 E_3^*]\rho, \tag{A18}$$

where we have defined the parameters

$$\beta^* = \beta m^*, \quad \alpha^* = \alpha m^*, \tag{A19}$$

$$\gamma'^* = \gamma' m^*, \quad \theta^* = \theta m^*, \tag{A20}$$

and the cyclotron frequency

$$\omega_c = -\frac{eB}{cm^*} = \frac{|eB|}{cm^*}. \tag{A21}$$

The effective Lagrangian for the conduction band includes the coupling of the electric field E_i with charge density ρ and the kinetic part of stress energy tensor T_{ij}^{kin}. We have the effective interaction of the conduction band with light through $\mathcal{I}_0, \mathcal{I}_1, \mathcal{I}_\alpha, \mathcal{I}_\beta, \mathcal{I}_{\gamma'}$, and $\mathcal{I}_\theta$. We can rewrite the interaction term in the convenient form for the circular polarized light scattering experiment setup in Fig. 1. In this case, we can consider $E_3 = 0$ and $E_3^* = 0$. Going to the complex coordinates in Eqs. (19) and (20), in which

$$T_{zz}^{\text{kin}} = \tfrac{1}{4}\big(T_{xx}^{\text{kin}} - T_{yy}^{\text{kin}} + 2iT_{xy}^{\text{kin}}\big), \quad T_{\bar{z}\bar{z}}^{\text{kin}} = \tfrac{1}{4}\big(T_{xx}^{\text{kin}} - T_{yy}^{\text{kin}} - 2iT_{xy}^{\text{kin}}\big), \quad T_{z\bar{z}}^{\text{kin}} = \tfrac{1}{4}\big(T_{xx}^{\text{kin}} + T_{yy}^{\text{kin}}\big), \tag{A22}$$

we can rewrite the interaction terms as

$$\mathcal{I}_0 = i\frac{e^2 |P|^2}{6(E_G - \omega_0)^2}[3E^{\bar{z}}(E^{\bar{z}})^* + E^z(E^z)^*]\psi^\dagger \partial_t \psi, \tag{A23}$$

$$\mathcal{I}_1 = -\frac{e^2 |P|^2}{6(E_G - \omega_0)}[3E^{\bar{z}}(E^{\bar{z}})^* + E^z(E^z)^*]\rho, \tag{A24}$$

$$\mathcal{I}_\alpha = \alpha^* \frac{e^2 |P|^2}{3(E_G - \omega_0)^2}[6E^{\bar{z}}(E^{\bar{z}})^* + 2E^z(E^z)^*]T_{z\bar{z}}^{\text{kin}}, \tag{A25}$$

$$\mathcal{I}_\beta = -\frac{\beta^* e^2 |P|^2}{9(E_G - \omega_0)^2}\Big\{-3E^z(E^{\bar{z}})^* T_{zz}^{\text{kin}} - 3E^{\bar{z}}(E^z)^* T_{\bar{z}\bar{z}}^{\text{kin}} + [5E^z(E^z)^* + 9E^{\bar{z}}(E^{\bar{z}})^*]T_{z\bar{z}}^{\text{kin}} + \frac{\omega_c}{4}[E^z(E^z)^* - 9E^{\bar{z}}(E^{\bar{z}})^*]\rho\Big\}, \tag{A26}$$

$$\mathcal{I}_{\gamma'} = \frac{\gamma'^* e^2 |P|^2}{9(E_G - \omega_0)^2}\Big\{-\frac{3}{2}[E^z(E^{\bar{z}})^* + E^{\bar{z}}(E^z)^*]\big(T_{zz}^{\text{kin}} + T_{\bar{z}\bar{z}}^{\text{kin}}\big) + [5E^z(E^z)^* + 9E^{\bar{z}}(E^{\bar{z}})^*]T_{z\bar{z}}^{\text{kin}}\Big\}, \tag{A27}$$

$$\mathcal{I}_\theta = \theta^* \frac{\omega_c e^2 |P|^2}{18(E_G - \omega_0)^2}[E^z(E^z)^* - 9E^{\bar{z}}(E^{\bar{z}})^*]\rho. \tag{A28}$$

We can easily check that only $\mathcal{I}_{\gamma'}$ violates rotational invariance.

APPENDIX B: THE DIPOLE-TRANSITION COEFFICIENT P

Here, we follow Ref. [47] to derive an expression for the dipole-transition coefficient P through the electron Bloch wave functions. The first term of Eq. (8) absorbs a photon and creates a hole in the valence band and adds an electron to the conductance band. We can rewrite this term in the Hamiltonian as

$$-\int dx\, eP_{ij}^* \psi^{\alpha\dagger} \chi_\alpha^i E^j, \tag{B1}$$

with $P_{ij}^* = P^* \delta_{ij}$. Comparing with eq. (11.23) in Ref. [47], we see that[2]

$$-eP_{ij}^* i\omega_{ph}\langle n_{ph}-1|A^j|n_{ph}\rangle = -\frac{e}{m}\langle \psi_{\mathbf{k}}^\alpha, n_{ph}-1|A^j p_j|\chi_{i\alpha,\mathbf{k}}, n_{ph}\rangle, \tag{B2}$$

with m being the free electron mass, and p_j is the free electron momentum operator. Then we have

$$P_{ij}^* = -\frac{i}{m\omega_{ph}}\langle \psi_{\mathbf{k}}^\alpha|p_j|\chi_{i\alpha,\mathbf{k}}\rangle, \tag{B3}$$

where $\psi_{\mathbf{k}}^\alpha$ is the Bloch wave function of the electron in the conductance band (s band)

$$\psi_{\mathbf{k}}^\alpha = \frac{1}{\sqrt{N_{\text{site}}}} \sum_a e^{i\mathbf{k}\cdot\mathbf{x}} u_c^\alpha(\mathbf{x}-\mathbf{R}_a), \tag{B4}$$

and $\chi_{i\alpha,\mathbf{k}}$ is the Bloch wave function of the electron in the valence bands (p band)

$$\chi_{i\alpha,\mathbf{k}} = \frac{1}{\sqrt{N_{\text{site}}}} \sum_a e^{i\mathbf{k}\cdot\mathbf{x}} u_v^{i\alpha}(\mathbf{x}-\mathbf{R}_a). \tag{B5}$$

We have

$$\langle \psi_{\mathbf{k}}^\alpha|p_j|\chi_{i\alpha,\mathbf{k}}\rangle = \frac{1}{N_{\text{site}}} \sum_{a,b} \int d^3\mathbf{x} \left[k_j (u^\alpha)_c^*(\mathbf{x}-\mathbf{R}_a) u_v^{i\alpha}(\mathbf{x}-\mathbf{R}_b) \right.$$
$$\left. - i(u^\alpha)_c^*(\mathbf{x}-\mathbf{R}_a)\partial_j u_v^{i\alpha}(\mathbf{x}-\mathbf{R}_b) \right], \tag{B6}$$

The first term vanishes due to the orthogonality of the linear combination of atomic orbitals. The second term can be written as

$$\langle \psi_{\mathbf{k}}^\alpha|p_j|\chi_{i\alpha,\mathbf{k}}\rangle$$
$$= -i \sum_{a,b} \int_{\text{unit cell}} d^3\mathbf{x}\, (u^\alpha)_c^*(\mathbf{x}-\mathbf{R}_a)\partial_j u_v^{i\alpha}(\mathbf{x}-\mathbf{R}_b). \tag{B7}$$

Due to the angular momentum conservation, we have

$$\langle \psi_{\mathbf{k}}^\alpha|p_j|\chi_{i\alpha,\mathbf{k}}\rangle$$
$$= -i\delta_{ij} \sum_{a,b} \int_{\text{unit cell}} d^3\mathbf{x}\, (u^\alpha)_c^*(\mathbf{x}-\mathbf{R}_a)\partial_k u_v^{k\alpha}(\mathbf{x}-\mathbf{R}_b). \tag{B8}$$

Following Ref. [47], we obtain

$$\langle \psi_{\mathbf{k}}^\alpha|p_i|\chi_{i\alpha,\mathbf{k}}\rangle = |\hat{e}\cdot\vec{p}_{cv}|, \tag{B9}$$

where $\hat{e}$ is any unit vector, and $p_{cv}^x = p_{cv}^y = p_{cv}^z$ with the definition

$$p_{cv}^i = -i \sum_{a,b} \int_{\text{unit cell}} d^3\mathbf{x}\, (u^\alpha)_c^*(\mathbf{x}-\mathbf{R}_a)\partial_k u_v^{k\alpha}(\mathbf{x}-\mathbf{R}_b). \tag{B10}$$

We also have the relation between $|\hat{e}\cdot\vec{p}_{cv}|$ and the parameter E_p often used in the photonics literature [47]

$$|\hat{e}\cdot\vec{p}_{cv}|^2 = \frac{E_p m}{2}. \tag{B11}$$

[2]We use the Coulomb gauge $A_0 = 0$ here.

We then obtain

$$P^* = -\frac{i}{m\omega_{ph}}\sqrt{\frac{E_p m}{2}}. \tag{B12}$$

Note that the coupling depends on the frequency of photon. However, if we rewrite the coupling with electric field to coupling with gauge potential, there will be no dependence on the photon frequency. We can rewrite the coupling with photon in Eq. (8) as

$$\mathcal{L}_i = e\sqrt{\frac{E_p}{2m}}\left(\psi_\alpha^\dagger \chi_i^\alpha A_i + \chi_{i\alpha}^\dagger \psi^\alpha A_i^*\right). \tag{B13}$$

APPENDIX C: HOW TO PROJECT TO A LANDAU LEVEL

Here, we provide the detailed calculation for the kinetic part of the stress tensor in a specific Landau level. The electron field can be expanded in orbitals

$$\psi(\mathbf{x}) = \sum_{Mm} \langle \mathbf{x}|Mm\rangle c_{Mm}, \tag{C1}$$

where M labels the Landau level, and m labels the states within the Landau level. To obtain the kinetic part of the stress tensor, we need to compute

$$\int d\mathbf{x}\, W^\dagger(\mathbf{x}) D_z^2 \psi(\mathbf{x})$$
$$= -\int d\mathbf{x}\, d\mathbf{x}'\, \psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x}')V(\mathbf{x}-\mathbf{x}')\psi(\mathbf{x}')D_z^2\psi(\mathbf{x}). \tag{C2}$$

Inserting the expansion over modes, and limiting to the Nth Landau level, this becomes

$$-\int d\mathbf{x}\, d\mathbf{x}' \sum_{mnm'n'} \langle Nm|\mathbf{x}\rangle \langle Nm'|\mathbf{x}'\rangle V(\mathbf{x}-\mathbf{x}')\langle \mathbf{x}'|Nn'\rangle D_z^2$$
$$\times \langle \mathbf{x}|Nn\rangle c_{Nm}^\dagger c_{Nm'}^\dagger c_{Nn'} c_{Nn}. \tag{C3}$$

Introducing the Fourier transform of the potential

$$V(\mathbf{x}-\mathbf{x}') = \int \frac{d\mathbf{q}}{(2\pi)^2} e^{i\mathbf{q}\cdot(\mathbf{x}-\mathbf{x}')} V(\mathbf{q}), \tag{C4}$$

the expression becomes

$$-\sum_{mnm'n'} \int_{\mathbf{q}} V(\mathbf{q}) \int d\mathbf{x}\, \langle Nm|\mathbf{x}\rangle e^{i\mathbf{q}\cdot\mathbf{x}} D_z^2 \langle \mathbf{x}|Nn\rangle \int d\mathbf{x}'\, \langle Nm'|\mathbf{x}'\rangle$$
$$\times e^{-i\mathbf{q}\cdot\mathbf{x}'} \langle \mathbf{x}'|Nn'\rangle c_m^\dagger c_{m'}^\dagger c_{n'} c_n. \tag{C5}$$

Now we have

$$\int d\mathbf{x}'\, \langle Nm'|\mathbf{x}'\rangle e^{-i\mathbf{q}\cdot\mathbf{x}'}\langle \mathbf{x}'|Nn'\rangle = \langle Nm'|e^{-i\mathbf{q}\cdot\hat{\mathbf{x}}}|Nn'\rangle$$
$$= \langle N|e^{-i\mathbf{q}\cdot\tilde{\mathbf{r}}}|N\rangle \langle m'|e^{-i\mathbf{q}\cdot\mathbf{R}}|n'\rangle, \tag{C6}$$

but

$$\langle N|e^{-i\mathbf{q}\cdot\tilde{\mathbf{r}}}|N\rangle$$
$$= \left\langle N \left| \exp\left[-\frac{i\ell_B}{\sqrt{2}}(q^z a^\dagger + q^{\bar{z}} a) \right] \right| N \right\rangle$$

$$= e^{-x_q/2} \left\langle N \right| \exp\left(-\frac{i\ell_B}{\sqrt{2}} q^z a^\dagger\right) \exp\left(-\frac{i\ell_B}{\sqrt{2}} q^{\bar{z}} a\right) |N\rangle$$

$$= e^{-x_q/2} L_N(x_q), \quad x_q = \frac{q^2\ell_B^2}{2}; \tag{C7}$$

therefore,

$$\sum_{m'n'} \int d\mathbf{x}' \langle Nm'|\mathbf{x}'\rangle e^{-i\mathbf{q}\cdot\mathbf{x}'} \langle \mathbf{x}'|Nn'\rangle c_{m'}^\dagger c_{n'} = e^{-x_q/2} L_N(x_q) \bar\rho(\mathbf{q}). \tag{C8}$$

Analogously,

$$\int d\mathbf{x} \langle Nm|\mathbf{x}\rangle e^{i\mathbf{q}\cdot\mathbf{x}} D_z^2 \langle \mathbf{x}|Nn'\rangle = \frac{1}{2} \langle Nm|e^{i\mathbf{q}\cdot\hat{\mathbf{x}}} \frac{a^{\dagger 2}}{\ell_B^2}|Nn\rangle$$

$$= \frac{1}{2\ell_B^2} \sqrt{(N+1)(N+2)} \langle N|e^{i\mathbf{q}\cdot\bar{\mathbf{r}}}|N+2\rangle \langle m|e^{i\mathbf{q}\cdot\mathbf{R}}|n\rangle = -q_z^2 e^{-x_q/2} L_N^2(x_q) \langle m|e^{i\mathbf{q}\cdot\mathbf{R}}|n\rangle; \tag{C9}$$

therefore,

$$\sum_{mn} \int d\mathbf{x} \langle Nm|\mathbf{x}\rangle e^{-i\mathbf{q}\cdot\mathbf{x}} D_z^2 \langle \mathbf{x}|Nn\rangle c_m^\dagger c_n = -q_z^2 e^{-x_q/2} L_N^2(x_q) \bar\rho(-\mathbf{q}). \tag{C10}$$

Finally, we obtain

$$\int d\mathbf{x}\, W^\dagger(\mathbf{x}) D_z^2 \psi(\mathbf{x}) = \int \frac{d\mathbf{q}}{(2\pi)^2} q_z^2 e^{-x_q} L_N(x_q) L_N^2(x_q) V(q) \bar\rho(\mathbf{q}) \bar\rho(-\mathbf{q}). \tag{C11}$$

Similarly, for $N \geqslant 2$,

$$\int d\mathbf{x}\, W^\dagger(\mathbf{x}) D_{\bar z}^2 \psi(\mathbf{x}) = \int \frac{d\mathbf{q}}{(2\pi)^2} q_{\bar z}^2 e^{-x_q} L_N(x_q) L_{N-2}^2(x_q) V(q) \bar\rho(\mathbf{q}) \bar\rho(-\mathbf{q}), \tag{C12}$$

while for $N = 0$ or 1, the expression is obviously zero due to the presence of two lowering operators $D_{\bar z}$ acting on ψ. Equations (C11) and (C12) are used in Sec. IV to obtain the explicit form of the kinetic part of the stress tensor on a specific Landau level.

APPENDIX D: $i\psi^\dagger \partial_t \psi$ AND $T_{z\bar z}^{\mathrm{kin}}$

Here, we derive the explicit form of $i\psi^\dagger \partial_t \psi$ in the LLL. The field equation reads

$$i\partial_t \psi(\mathbf{x}) = -\frac{1}{m^*}(D_z D_{\bar z} + D_{\bar z} D_z)\psi(\mathbf{x})$$
$$+ \int d\mathbf{x}'\, V(\mathbf{x} - \mathbf{x}')\psi^\dagger(\mathbf{x}')\psi(\mathbf{x}')\psi(\mathbf{x}). \tag{D1}$$

We then use the constraint equation $2\ell_B^2 D_z D_{\bar z}\psi = -N\psi$ and the commutator in Eq. (55). From that, we get

$$i\int d\mathbf{x}\, \psi^\dagger \partial_t \psi = I_0 + \left(N + \frac{1}{2}\right)\omega_c N_e, \tag{D2}$$

with N_e being the total electron number in the conductance band and

$$I_0 = \int d\mathbf{x}\, d\mathbf{x}'\, V(\mathbf{x} - \mathbf{x}')\psi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x}')\psi(\mathbf{x}')\psi(\mathbf{x})$$
$$= \int_{\mathbf{q}} V(\mathbf{q}) e^{-q^2\ell_B^2/2} \bar\rho(\mathbf{q})\bar\rho(-\mathbf{q}). \tag{D3}$$

The first term on the right-hand side of Eq. (D2) is twice the interacting energy, and the second term is the kinetic energy of electrons.[3] We also have

$$\int d\mathbf{x}\, T_{z\bar z}^{\mathrm{kin}} = \int d\mathbf{x}\, \frac{1}{2m^*}(D_z\psi^\dagger D_{\bar z}\psi + D_{\bar z}\psi^\dagger D_z\psi)$$

$$= \int d\mathbf{x}\, \frac{1}{2m^*}\left(-2\psi^\dagger D_z D_{\bar z}\psi - \frac{eB}{2c}\psi^\dagger\psi\right)$$

$$= \frac{\omega_c}{2}\left(N + \frac{1}{2}\right)\int d\mathbf{x}\, \rho$$

$$= \left(N + \frac{1}{2}\right)\frac{N_e \omega_c}{2}, \tag{D4}$$

where we used the constraint $2\ell_B^2 D_z D_{\bar z}\psi = -N\psi$ to obtain the last equality.

APPENDIX E: TWO SUM RULES

This Appendix is not directly related to Raman scattering but contains some exact sum rules. First, we write down formulas for the full stress tensor. For the model Hamiltonian, at the long-wavelength regime $\mathbf{k} \sim 0$, the full stress tensor is the same as T^{kin} [24]. For the general case, one needs to consider the potential-energy term in the Lagrangian. When the metric varies with time (but remains uniform in space), the potential changes according to

$$V(\mathbf{x}) \rightarrow V(\sqrt{g_{ij}x^i x^j}), \tag{E1}$$

---

[3] In Ref. [24], we eliminate the second term in the LLL case by introducing the Landé factor $\mathfrak{g} = 2$.

and so the Fourier transform changes as

$$V(\mathbf{q}) \to V(\sqrt{g^{ij}q_i q_j}). \tag{E2}$$

Since the stress tensor is given by $\delta S = \frac{1}{2} \int dx \, T_{ij} \delta g_{ij}$, the potential part of the stress-energy tensor is then

$$\int d\mathbf{x} \, T_{ij}^{\text{pot}} = \frac{1}{2} \int_{\mathbf{q}} \frac{q_i q_j}{q} V'(q) \rho(\mathbf{q}) \rho(-\mathbf{q}). \tag{E3}$$

This can be projected to the Nth Landau level to become

$$\int d\mathbf{x} \, T_{ij}^{\text{pot}} = \frac{1}{2} \int_{\mathbf{q}} e^{-x_q} [L_N(x_q)]^2 \frac{q_i q_j}{q} V'(q) \bar{\rho}(\mathbf{q}) \bar{\rho}(-\mathbf{q}). \tag{E4}$$

It is interesting to compare the formula with that of the kinetic stress tensor, Eq. (47): the "stretching operator" $(q_i q_j/q)\partial_q$ now acts on the potential $V(q)$ instead of acting on the form factor. The full stress tensor is the sum of the kinetic stress tensor, Eq. (88), and the potential stress tensor, Eq. (E4). It can be written as

$$\int d\mathbf{x} \, T_{ij}^{\text{full}} = \frac{1}{2} \sum_{\mathbf{q}} \frac{q_i q_j}{q} \frac{\partial}{\partial q} \{ e^{-x_q} [L_N(x_q)]^2 V(q) \} \bar{\rho}(\mathbf{q}) \bar{\rho}(-\mathbf{q}). \tag{E5}$$

We define the spectral densities [19]

$$\rho_T(\omega) = \frac{1}{N} \sum_n |\langle n| \int d\mathbf{x} \, T_{zz} |0\rangle|^2 \delta(\omega - E_n), \tag{E6}$$

$$\bar{\rho}_T(\omega) = \frac{1}{N} \sum_n |\langle n| \int d\mathbf{x} \, T_{\bar{z}\bar{z}} |0\rangle|^2 \delta(\omega - E_n), \tag{E7}$$

where N is the total number of electrons, $|0\rangle$ is the ground state, the sum is taken over all excited states $|n\rangle$ in the lowest Landau level, and E_n is the energy of the state $|n\rangle$. The two spectral densities satisfy the sum rules [19]

$$\int_0^\infty \frac{d\omega}{\omega^2} [\rho_T(\omega) - \bar{\rho}_T(\omega)] = \frac{\bar{s}}{4}, \tag{E8}$$

$$\int_0^\infty \frac{d\omega}{\omega^2} [\rho_T(\omega) + \bar{\rho}_T(\omega)] = S_4, \tag{E9}$$

where $\bar{s}$ is the "guiding center spin" [18], which is equal to $(\mathcal{S} - 1)/2$ on the LLL, where $\mathcal{S}$ is the shift, and S_4 is the coefficient in front of $(k\ell_B)^4$ in the static structure factor.

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