Quantum Jackiw-Teitelboim gravity, Selberg trace formula, and random matrix theory

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We show that the partition function of quantum Jackiw-Teitelboim (JT) gravity, including topological fluctuations, is equivalent to the partition function of a Maass-Laplace operator of large (imaginary) weight acting on noncompact, infinite-area, hyperbolic Riemann surfaces of arbitrary genus. The resulting spectrum of this open quantum system for a fixed genus is semiclassically exact and given by a regularized Selberg trace formula; namely, it is expressed as a sum over the lengths of primitive periodic orbits of these hyperbolic surfaces. By using semiclassical techniques, we compute analytically the spectral form factor and the variance of the Wigner time delay in the diagonal approximation. We find agreement with the random matrix theory prediction for open quantum chaotic systems. Our results show that full quantum ergodicity is a distinct feature of quantum JT gravity.

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I. INTRODUCTION

The study of quantum features of classically chaotic systems, usually termed quantum chaos, reveals a surprising degree of universality in the quantum dynamics. Universal features are observed specially at two time scales: the Ehrenfest time and the Heisenberg time. The former is a short time scale related to the buildup of quantum corrections to the classical motion. It is characterized by the exponential growth of certain correlation functions, related to the square of commutators that measure the uncertainty of an observable. Interestingly, this exponential growth is controlled by the classical Lyapunov exponent [1,2]. The Heisenberg time is a long time scale, the inverse of the mean level spacing, related to the time for a system to experience that the spectrum is discrete. In quantum chaotic systems, spectral correlations of neighboring eigenvalues are given by random matrix theory (RMT) [3–7]. Physically, this means that for sufficiently long times, the quantum dynamics is fully ergodic and only depends on the global symmetries of the system. Not surprisingly, quantum chaos ideas and techniques have been employed in many physical contexts. In nuclear physics, where they were originally introduced [3,8], it was shown that the spectral correlations of highly excited states are well described by RMT [3]. For a single particle in a random potential, and assuming that Anderson localization effects are not important, it was found analytically that the momentum uncertainty [1] grows exponentially for short times and that spectral correlations are given by RMT [9] for long times. For noninteracting chaotic systems with nonrandom deterministic motion, it was conjectured [10], and later demonstrated [11,12] in the semiclassical limit, that level correlations are also given by RMT. Similarly, in the limit of zero-dimensional QCD, the spectrum of the Dirac operator is also correlated according to the RMT prediction [13] for systems with chiral symmetry.

More recently, the proposal [14] of a universal bound in the Lyapunov exponent—one that controls the exponential growth mentioned above and its saturation in field theories with a gravity dual—has reinvigorated interest in quantum chaos. This saturation has been explicitly confirmed in the Sachdev-Ye-Kitaev (SYK) model [15–17] consisting of N Majorana fermions with infinite-range random interactions in zero spatial dimensions (see Refs. [8,18–22] for similar models with Dirac fermions). Interestingly, the infrared description of the SYK model is controlled by the Schwarzian action, which is also the effective description of the boundary dynamics of Jackiw-Teitelboim (JT) gravity [23–25]. It has also been found [26,27] that spectral correlations of the SYK model are given by RMT even for energies close to the ground state [28] where the model may have a gravity dual.

This observation of RMT spectral correlations in the SYK model together with the expected holographic duality with JT gravity, though encouraging, is not a demonstration that the latter has similar spectral correlations. It is far from clear whether the holographic duality survives up to time scales of the order of the Heisenberg time. More importantly, the very quantization of JT gravity, leading to a discrete spectrum, is still an open problem. Recently, a random matrix model for JT gravity has been proposed [29] (see also Refs. [30–32]) as a possible ultraviolet completion of the theory (see Ref. [33] for a generalization of this idea when fermions are included). More specifically, the approach of Ref. [29] shows that a possible boundary theory of JT gravity is a double scaling limit of a certain ensemble of random matrices. The connected part

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of n-point correlation functions of this random matrix model is written as a genus expansion of a JT gravity partition function with n boundaries. The latter can be evaluated explicitly by using Mirzakhani's recursion relation for Weil-Petersson volumes [34,35] and the fact that [36] this is equivalent to a topological recursion relation for correlation functions of the boundary ensemble of random matrices. This nice identity between random matrices and quantum JT gravity requires an ensemble average whose physical meaning on the gravity side is not straightforward.

Another quantization of JT gravity has recently been introduced in Refs. [37,38] by noticing that bulk JT gravity maps to the dynamics of a quantum particle on a genus-zero hyperbolic space in the presence of an imaginary magnetic field. The resulting spectrum is still continuous [39], which prevents the study of quantum chaotic features by the analysis of spectral correlations of neighboring eigenvalues. Ultimately, this is due to the fact that the background geometry is still classical; namely, it is fixed by Einstein's equation of motion.

Motivated by these recent developments, we revisit the problem of quantization of JT gravity in a way reminiscent of Polyakov's approach to string theory [40]. In JT gravity, the two-dimensional space is rigid, of constant negative curvature, in the sense that it is just the solution of the classical equations of motion. The dynamics comes from the position of the boundary which acts as the physical boundary of the system. We relax the rigidness condition of the surface by allowing topological fluctuations of the geometry. This leads to a genus expansion, with genus $h \geqslant 1$, of noncompact Riemann surfaces of infinite area. We show that the JT gravity partition function can be written as a sum over genus of the partition function associated with the Maass-Laplace operator on the above surfaces in the limit of large (imaginary) weight of the holomorphic form. The area of the Riemann surfaces is infinite, therefore it is possible the existence of complex resonances in the spectrum, which arise as poles of the resolvent. We found that the spectrum or resonances can be computed exactly as a sum over classical primitive periodic orbits by using the Selberg zeta function and the generalized Selberg trace formula. The resulting spectrum or resonances only depend on classical information such as the primitive periodic orbits and the classical escape rate. We then compute, using semiclassical techniques, the spectral form factor and the variance of the Wigner time delay analytically without performing an ensemble average. We find that the result for both spectral correlators agrees with the RMT prediction. This indicates that for sufficiently long times, JT quantum gravity—in the above-mentioned limit—is fully ergodic. It also suggests that quantum chaos may be one of its features at all time scales.

II. QUANTIZATION OF JT GRAVITY

The classical Euclidean action for JT gravity is

$$I(g,\phi) = -2\pi\phi_0\,\chi - \left[\frac{1}{2}\int_{\mathcal{M}}\sqrt{g}\phi(R+2) + \int_{\partial\mathcal{M}}\phi_b(K-1)\right], \tag{1}$$

where ϕ stands for the deviation of the dilaton field from its value at zero temperature, ϕ_0, and χ is the Euler characteristic of the surface. The classical equation for the dilaton sets $R=-2$, which implies that $\mathcal{M}$ is a portion of rigid two-dimensional anti–de Sitter (AdS_2; Euclidean) space, which we will represent using the hyperbolic half-plane model $\mathbb{H} = \{z = x + iy \mid y > 0\}$ with line element $ds^2 = dx^2 + dy^2/y^2$. With the remaining term in the action, the classical solution for the dilaton can also be easily found [37]. The nontrivial dynamics of the model comes from the position of the physical boundary in the rigid $\mathbb{H}$ space. Boundary conditions for the metric and dilaton along the physical boundary are given by $ds|_{\mathrm{bdy}} = du\frac{\phi_r}{\epsilon}$ and $\phi|_{\mathrm{bdy}} = \frac{\phi_r}{\epsilon}$, respectively, where u is the boundary theory time and ϵ parametrizes the distance to the $\mathbb{H}$ boundary. In the $\epsilon \to 0$ limit, where the physical boundary approaches the rigid space boundary, the action in Eq. (1) becomes

$$I = -\int \mathrm{Sch}\left(\tan\frac{\varphi(u)}{2}, u\right) du, \tag{2}$$

where Sch stands for the Schwarzian and $\varphi(u)$ expresses the bulk time as a function of the boundary time [23]. Throughout the paper we will refer to the $\epsilon \to 0$ limit as the *Schwarzian limit*. This action results from the spontaneous and explicit breaking of conformal symmetry, down to $SL(2,\mathbb{R})$ symmetry, due to quantum and finite temperature effects. Different techniques and tools, from combinatorial analysis [28,41] based on the shared symmetry with the SYK model to an explicit evaluation of the path integral [42] and a mapping to a charged particle on $\mathbb{H}$ in the presence of an imaginary magnetic field [37,38], have been employed to compute spectral and thermodynamic properties of JT gravity. Here, we will focus on this latter approach, which can be summarized as follows (see Refs. [37,38] for details). After integrating over the dilaton field and using the Gauss-Bonnet theorem, the remaining term containing the extrinsic curvature in the action of Eq. (1) can be rewritten as $\int(K-1) = (2\pi + A[\mathbf{x}] - L)$, where $A[\mathbf{x}]$ is the area surrounded by the closed boundary curve of fixed length, $L = \beta\frac{\phi_r}{\epsilon}$. The area term can be interpreted as the flux of a uniform *electric* field on $\mathbb{H}$ of strength $q = \frac{\phi_r}{\epsilon}$, or equivalently a uniform imaginary *magnetic* field $b = iq$. It is worth noticing that the analogy of JT gravity to the charged particle is strictly valid at the classical level since quantum mechanically the path integrals seemingly have different properties, as explained in detail in Ref. [38]. This ambiguity is eliminated whenever we consider the *Schwarzian limit*, which in the present context involves considering large b. In the following, we will consider the case of finite b and take the limit at the end in the expressions whenever necessary.

At the quantum level, the path-integral quantization of JT gravity—using the analogy to the charged particle—is carried out including the constraint of trajectories of fixed proper length, which is related to the temperature. By considering an appropriate regularization prescription, it was found [37,38] that the partition function of JT gravity is equivalent to that of a charged particle on $\mathbb{H}$ in the presence of an imaginary magnetic field. The final form of the partition function is

$$Z_{\mathbb{H}}(b,\tau) = e^{2\pi(q+\phi_0)}e^{\tau/2(1/4-b^2)}\int \mathcal{D}\mathbf{x} \times \exp\left[-\int_0^{\tau} d\tau'\left(\frac{1}{2}\frac{\dot{x}^2+\dot{y}^2}{y^2} - b\frac{\dot{x}}{y}\right)\right], \tag{3}$$

where the gauge $A_x = -b/y$ was used. The quantum properties of this system are then governed by the Schrödinger equation $H\psi = E\psi$ where the Hamiltonian $2H = -D_m + b^2$ is the Maass-Laplace operator

$$D_m = y^2\left(\partial_x^2 + \partial_y^2\right) - imy\partial_x \tag{4}$$

on $\mathbb{H}$ for automorphic forms of weight $m = 2b$. The dynamics of the particle is influenced by the nonzero magnetic field such that for sufficiently large magnetic field, in addition to the monotonous continuous spectrum, we have a discrete spectrum associated with the Landau levels on $\mathbb{H}$ [39]. For an imaginary magnetic field, the spectrum of the Hamiltonian is always continuous [37,39] with energy $E = (1/4 + k^2 - q^2)/2$. In the Schwarzian limit, the spectral density for the Schwarzian action is recovered [37,38].

The above Hamiltonian is the starting point of our analysis. We note that the quantum gravity problem defined on a rigid half hyperbolic plane $\mathbb{H}$ with a dynamical physical boundary has been traded for the study of the spectral properties of the Maass-Laplace operator on $\mathbb{H}$, which resulted from the solution of the classical equations of motion. Typically, a distinctive feature of quantum systems is a discrete spectrum. However, the spectrum related to the system above is absolutely continuous. We also note that the quantization of gravity cannot be complete because the geometry is still given by the solution of the classical equations of motion. In the following, we consider quantization of JT gravity that will potentially lead to a discrete spectrum of the above model and to spectral correlations given by RMT which confirm the quantum chaotic nature of JT gravity even for long times of the order of the Heisenberg time.

A natural way to generalize the above arguments is by considering contributions of surfaces with nontrivial topology. This amounts to considering topological fluctuations in the model which are not solutions of the classical equations of motion. In the functional form of the JT gravity problem [Eq. (1)], the metric is still constrained to be of constant negative curvature at any point, although the latter enters as a delta function constraint [29]. It is then clear that the functional integral will have contributions from hyperbolic Riemann surfaces [29] with an asymptotic AdS_2 boundary [43] and arbitrary genus: Σ_h, $h \geqslant 1$. To be more precise, it will have contributions from bounded subregions of hyperbolic noncompact Riemann surfaces of infinite area and fixed boundary length where the boundary conditions of the JT gravity problem are met. Explicitly, we integrate the dilaton along an appropriate contour to obtain

$$Z_{\mathrm{JT}} = e^{2\pi\phi_0\chi}\int \mathcal{D}g_{\mu\nu}\,\delta(R+2)e^{-\phi_b\int(K-1)}. \tag{5}$$

The functional integral is reduced to the boundary term, which according to the previous discussion—the genus-zero case—is equivalently described by the path integral of a charged particle in an imaginary magnetic field on $\mathbb{H}$. For nonzero genus, we use Riemann uniformization to argue that the full path integral involves considering contributions of noncompact Riemann surfaces of infinite area and arbitrary genus, where a charged particle propagates. In addition, each of these contributions is weighted by a term proportional to the Euler characteristic $\chi = 1 - 2h$. The final form of the functional integral is

$$\begin{aligned} Z_{\mathrm{JT}} &= Z_{\mathbb{H}}(b,\tau) + e^{\tau/2(1/4+b^2)} \\ &\quad\times \sum_{h\geqslant 1} e^{2\pi(q+\phi_0)\chi}\int_{\mathrm{moduli}}\int \mathcal{D}\mathbf{x}e^{-I_h(\mathbf{x},b)} \\ &= Z_{\mathbb{H}}(b,\tau) + e^{\tau/2(1/4+b^2)}\sum_{h\geqslant 1} e^{2\pi(q+\phi_0)\chi}\int_{\mathrm{moduli}} Z^h_{\Gamma\setminus\mathbb{H}}(b,\tau), \end{aligned} \tag{6}$$

where $Z_{\mathbb{H}}$ is the partition function associated with the Poincaré half-plane [Eq. (3)] and $Z^h_{\Gamma\setminus\mathbb{H}}(b,\tau)$ is the analog for the higher-genus hyperbolic Riemann surfaces, the remaining integral accounting for their moduli with the metric given by the Weil-Petersson metric [29]. At genus zero, the quantization of this system has been studied in the literature [37,38]. However, the spectrum is continuous and monotonous, which prevents the calculation of spectral correlations. By contrast, for higher-genus surfaces, the resulting spectrum is amenable to a level statistics analysis. We will use spectral theory techniques concerning trace formulas in order to compute these higher-genus partition functions, associated with the spectrum of the Maass Laplacian (4) on the corresponding Riemann surfaces.

We will see below, using the "equivalence" in Eq. (6), that the spectral density associated with Z_{JT} (basically its Laplace transform) has a highly oscillating term which is expressed as a Selberg trace formula, namely, as a sum over the classical primitive periodic orbits of the surface. An explicit analytical evaluation of density-density spectral correlations, such as the spectral form factor, will reveal that the spectrum of JT gravity is quantum chaotic and described by RMT.

Let us briefly clarify the notation we have used. Formally, noncompact hyperbolic Riemann surfaces of infinite area are obtained using Riemann uniformization. Namely, the Riemann surface Σ_h is represented "as" the right coset of the Poincaré upper half-plane model $\mathbb{H}$ by a Fuchsian group Γ of the second kind [44]. The latter is a discrete subgroup of $\mathrm{PSL}(2,\mathbb{R})$ with proper discontinuous action. To be more specific, we consider a fundamental domain $\mathcal{F} \subset \mathbb{H}$, the geodesic boundaries of which are paired by elements γ of Γ generating a surface $X = \Gamma \setminus \mathbb{H}$ homeomorphic to Σ_h. In addition to the infinite-area condition, the surfaces we will be considering are connected, orientable, and geometrically finite (finite Euler characteristic). Moreover, the above Riemann surfaces are characterized by their hyperbolic ends. In the absence of cusps, geometric finiteness constrains the end space to be a funnel (infinite-area elementary surface). It is then clear that every noncompact hyperbolic Riemann surface of infinite area can be decomposed into separate pieces involving a bordered Riemann surface (compact core) and a funnel (see Fig. 1). Explicitly, the decomposition is written as $X = F \cup K_h$, where F and K_h denote the funnel and compact core subregions, respectively.

Moreover, a quantity that will come very useful in the forthcoming analysis is the exponent of convergence of a Fuchsian group Γ, which is defined by

$$\delta(\Gamma) := \inf\left\{ s \geqslant 0 : \sum_{\gamma\in\Gamma} e^{-s\,d(z,\gamma z')} < \infty \right\}, \tag{7}$$

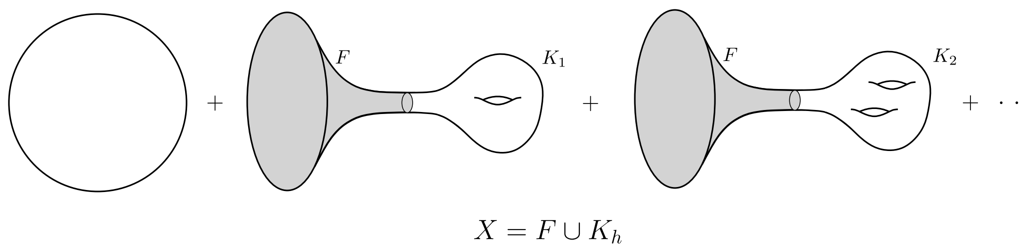

FIG. 1. Sketch of the leading topological corrections to the classical (leftmost) geometry of JT gravity.

where $d(z, z')$ is the distance on $\mathbb{H}$. For a Fuchsian group Γ of the second kind and geometrically finite we have that the above exponent satisfies $\delta(\Gamma) \in (0, 1)$ and equals the Hausdorff dimension of the limit set of Γ. For infinite-area surfaces, the Hausdorff dimension of the limit set is associated with the dynamics of geodesics, which either flow around in the compact core or escape to infinity.

Role of moduli space

Notice that the equivalence between the partition function of JT gravity and $Z^h_{\Gamma\backslash\mathbb{H}}$ (in the *Schwarzian limit*) is valid up to the integral over the moduli, as seen from Eq. (6). For a genus-h Riemann surface, there is a continuous (but finite) number of parameters characterizing (inequivalent) Riemann surfaces with the same underlying topology. The integral in Eq. (6) accounts for all these contributions to the JT partition function. As we will see below, the partition function $Z^h_{\Gamma\backslash\mathbb{H}}$ receives contributions from continuous as well as discrete parts of the spectrum. Throughout this paper we will focus on the discrete part of the spectrum. The explicit computation of the moduli integral even in this case is not straightforward since it involves performing the integration on functions depending on primitive periodic orbits of the surface [45]. We will not attempt to address this problem here, since the aim of this work is not the detailed knowledge of the spectrum but rather the study of the dynamics of the primitive periodic orbit flow which is, in a sense, independent of Riemann surface deformations, which will enable us to study the quantum chaotic features of the spectrum of JT gravity. More specifically, the (discrete) spectrum associated with the Riemann surfaces of genus h is characterized by $\delta > 0$ defined above and the length spectrum, namely, the set of primitive periodic orbits. In all cases, it has been demonstrated that there is an exponential proliferation of long primitive geodesics [46,47], which is a signature of classical chaos and also a key feature [48] to establish the quantum chaotic nature of the spectrum. It is worth emphasizing that a proper understanding of other quantum features of JT gravity will necessarily involve the integration over the moduli space. Another feature that requires additional comment is the role of the moduli space in the computation of two-level and higher-order spectral correlation functions. Here, in principle, one should have to consider not only the correlation of eigenvalues for a given genus-h surface but also correlations of eigenvalues among the entire set of surfaces in the moduli space. However, at least to leading order, this interference effect is likely to be negligible. The reason being that, as we shall see below, the length spectrum of each surface is unique. According to the Selberg trace formula, the spectral correlations are expressed as products of highly oscillating sums representing different surfaces. The leading term will be given by autocorrelations, called the diagonal approximation, namely, correlations between primitive periodic orbits of a given genus-h surface. Even at next-to-leading order we expect that it will be dominated by correlations between different periodic orbits of the same surface. It is unclear whether correlations among geodesics of different genus-h surfaces could play a role in these subleading contributions, but in any case it would not affect our results.

We can summarize the discussion above by saying that, even though for other observables one must necessarily account for the contributions of the entire set of Riemann surfaces of given genus in the moduli space, this is not very relevant with respect to level statistics since all of them have similar features. In this vein, we can then argue that the calculation of the path integral over geometries accepts a saddle-point solution that effectively picks a Riemann surface for each topological class, defined by a certain δ and its length spectrum. Based on the above, we will be able to show that JT gravity is quantum chaotic. Our main aim is therefore to compute $Z^h_{\Gamma\backslash\mathbb{H}}$. Fortunately, this problem has been intensively investigated in both the mathematics and physics literature [49–55].

III. EVALUATION OF THE JT PARTITION FUNCTION BY THE SELBERG TRACE FORMULA

In this section we compute the partition function $Z^h_{\Gamma\backslash\mathbb{H}}$ associated with the Maass Laplacian [Eq. (4)] on the infinite-area hyperbolic Riemann surfaces of genus h. Although the spectral properties of the above surfaces are not the naive sum of the funnel and compact core contributions, we find it to be pedagogical, and useful in order to set notation and conventions, to briefly discuss separately the spectral features of the above subregions.

A. Spectral features of the isolated compact core and funnel

Let us start with the spectral features of the funnel. In simple terms, Selberg trace formulas relate classical and quantum properties of the system. They contain several contributions depending on the characteristics of the Riemann surfaces and the associated spectrum of the Laplacian. However, there is a universal term given by the length spectrum $\mathcal{L}_X$ of a given hyperbolic Riemann surface X which corresponds to the set of lengths $\ell(\gamma)$ of primitive periodic orbits γ, where on the other hand the latter is an element of the conjugacy classes of Γ. Since we are just interested in the chaotic features of the system, it will be enough to discuss this contribution to the trace formula. In the case of the funnel, a regularized trace formula for nonzero magnetic field can be easily obtained following the results in Ref. [44]. This is given as the logarithmic derivative of the Selberg zeta function $Z_F(s)$ involving the product of the length spectrum labeled by the only closed geodesic of the funnel, ℓ_F, together with unbounded multiplicities. Since the regularization of the resolvent kernel for the funnel involves a subtraction of the one corresponding to $\mathbb{H}$, one can prove that the resulting expression is not directly influenced by the nonzero magnetic field and takes the form

$$\Phi_{F_\ell}(s) = \frac{Z'_{F_\ell}}{Z_{F_\ell}}(s), \quad Z_{F_\ell} = e^{-s\ell_F/4}\prod_{k\geqslant 0}(1-e^{-(s+2k+1)\ell_F})^2, \tag{8}$$

where k denotes multiplicities. Even though the hyperbolic ends determine many features of the spectrum, they are not directly relevant in the study of quantum chaotic features of the system as there is only one short geodesic which is in general negligible to produce any substantial modification of short-range spectral correlations.

For the compact core Z_{CC} (a bordered Riemann surface) the exact analytical expression of the heat kernel, equivalent to the partition function, for nonzero magnetic field is given by the Selberg trace formula [48–54],

$$\begin{aligned} Z_{CC} \equiv \mathrm{Tr}\, e^{-\tau H} &= e^{-\tau/2(1/4+b^2)}\frac{V_h}{2\pi}\int_0^\infty dk\frac{k\,e^{-k^2\tau/2}\sinh 2\pi k}{\cos 2\pi b+\cosh 2\pi k} \\ &\quad + \sum_{m=1}^{\infty}\sum_{\ell\in\mathcal{L}(\gamma)} A_{\ell,m}\, g(m\ell(\gamma))e^{-\tau b^2/2}, \\ g(m\ell(\gamma)) &= \frac{1}{4\sqrt{\pi\tau}}e^{-\tau/8-[m\ell(\gamma)]^2/4\tau}, \\ A_{\ell,m} &= \frac{\ell}{2\sinh(m\ell/2)}, \end{aligned} \tag{9}$$

where H is the Maass Laplacian defined above, m corresponds to the multiplicities of the primitive periodic orbits, and $V_h = -2\pi\chi$ (in front of the induced continuous contribution to the heat kernel) is the area of the surface. This last dependence has to be removed regarding the interpretation of the partition function as that of a gravitational system, as pointed out in Ref. [38]. We notice that the explicit b dependence enters as a simple shift of the case without magnetic field. In addition, to simplify notation, we have not included explicitly the term involving the geodesic related to the border of the surface; see Ref. [51] for a full expression. From Eq. (9) we see that the trace formula depends on the corresponding Fuchsian group Γ associated with the quotient surface, which in turn depends on the parameters associated with the moduli of the hyperbolic Riemann surface. However, one of the strengths of the trace formula is that even though we may expect—on general grounds—a change in the length spectrum for different points in the moduli space, the partition function will still be expressed in a very compact form [Eq. (9)]. As was argued earlier, the detailed account of the moduli space is not important for our purposes since the quantum chaotic nature of the motion occurs for all Fuchsian groups associated with a given hyperbolic surface of genus h.

More importantly, unlike the Gutzwiller trace formula [56], broadly used to compute the spectral density of quantum billiards in flat space, which is only valid in the semiclassical limit, the Selberg trace formula is exact; namely, the exact quantum spectrum of the system is encoded in the primitive periodic orbits of the classical counterpart. Another remarkable feature is that the dependence on the magnetic field is just a prefactor, of not much physical relevance. This feature will greatly simplify the calculation of spectral correlations associated with $Z^h_{\Gamma\backslash\mathbb{H}}$ that we discuss now.

B. Spectral features associated with $Z^h_{\Gamma\backslash\mathbb{H}}$

Having discussed the spectral properties of each separate subregion of the hyperbolic Riemann surfaces, we now study them as a whole. For the case of a vanishing magnetic field, the spectrum of the total surface is relatively well understood [44].

Since the surfaces are noncompact of infinite area, it is more convenient to characterize the spectrum through resonances s which are the poles of the (meromorphically continued) resolvent $R(s) = \mathrm{Tr}[H - s(1-s)]^{-1}$. Even in this more general situation, it is possible to show [57,58] that the zeros of the Selberg zeta function, $Z_X(s) = \prod_{\mathcal{L}_X}\prod_{m=0}^{\infty}(1-e^{-(s+m)\ell})$, $\mathrm{Re}(s) > 1$, provide an exact description of the quantum spectrum or resonances [59].

More interestingly, for the purpose of the calculation of spectral correlations, the partition function, and therefore the spectral density, take the form of a regularized Selberg trace formula [60] which is given by the length spectrum $\mathcal{L}_X$ of the surface where the contribution of each primitive periodic orbit is still given by $A_{\ell,m}$, as in the compact case [Eq. (11)]. However, we note that this length spectrum is in principle completely different from the one corresponding to the compact core. For instance, we expect that in this case some geodesics originally in the compact core are now missing as the surface is now of infinite area and therefore corresponding to that of an open system, where it is possible for them to escape outside the compact core. Regarding the dependence on the magnetic field, we do not expect a qualitative dependence because the Selberg trace formula above is largely independent of it though no firm conclusion can be achieved until a full analysis of this dependence is carried out.

Even though the spectrum of the Maass-Laplace operator can in principle be computed from information of the length spectrum $\mathcal{L}_X$, explicit calculations of the latter for infinite-area surfaces are scarce; see Ref. [61] for a surface with $h = 1$. However, our main aim is not a detailed knowledge of the spectrum but rather to clarify whether the Maass-Laplace spectrum, or correspondingly the JT spectrum, is quantum chaotic with spectral correlations described by RMT.

An important quantity which helps in answering this question is the the Hausdorff dimension δ of the classical attractor (trapped set) [44,62–64] introduced in Eq. (7). This dimension is directly related to the escape rate $\upsilon = 1 - \delta$, the rate at which trajectories close to the trapped set escape to infinity in units of the inverse of the Heisenberg time T_H.

Crucial for the forthcoming spectral analysis is the fact that for all hyperbolic Riemann surfaces of infinite area, $\delta > 0$ [44,62]. Its specific value will be highly dependent on other parameters defining the surface, such as the genus number, but it is always positive.

The finiteness of δ is directly related to other relevant results:

(i) For $\delta > 0$, the number of closed geodesics of length less than t, for t sufficiently large, grows exponentially, $\sim e^{\delta t}/\delta t$ [46,47]. This is a generalized version for hyperbolic Riemann surfaces of infinite area of the so-called *prime geodesic theorem*.

(ii) The eigenvalues with the largest real part are δ, and there are no other resonances in the line $\text{Re}(s) = \delta$ [64,65].

(iii) The topological entropy S_T of the geodesic flow of the trapped set is positive, $S_T = \delta$ [66,67].

The finite topological entropy is a distinctive feature of classical chaos, while the exponential growth of long periodic orbits plays a key role in the demonstration of quantum chaotic features in the spectrum.

In summary, taking into account that the partition function is still given by just the length spectrum $\mathcal{L}_x$ [59], with amplitudes $A_{\ell,m}$ similar to the compact case [Eq. (11)], and that the classical dynamics is chaotic, we broadly expect that quantum JT gravity is also quantum chaotic, namely, its spectral correlations are well described by RMT. In the next section we provide evidence that this is the case.

IV. SPECTRAL CORRELATIONS OF JT GRAVITY: SPECTRAL FORM FACTOR AND WIGNER TIME DELAY FLUCTUATIONS

Having investigated the features of the partition function of JT gravity, we now move to the calculation of spectral correlations in order to confirm agreement with the predictions of RMT. For that purpose, the first step is to relate the JT partition function to the spectral density: $\rho(E) = \sum_i \delta(E - E_i) = \bar{\rho} + \rho_{\text{osc}}$, where the first term stands for the monotonous part, whereas the second term stands for the oscillating part. We note that the spectrum is in general complex, so the spectral density is smoothed out with respect to a Dirac delta function. In general, only the latter enters into observables relevant to establish the quantum chaotic nature of the spectrum. Fortunately, the spectral density is nothing but the Laplace transform of the partition function $\rho(E) \sim \int_C d\tau\, Z(\tau)\, e^{E\tau}$, with C being a vertical path in the complex plane. Taking into account that the JT partition function—given by the trace formula—is also naturally split into a monotonous and an oscillating part, the spectral density can be expressed in terms of the length ℓ of periodic orbits

$$\rho_{\text{osc}} \sim \sum_{\mathcal{L}_x} A_\ell e^{ik\ell}, \tag{10}$$

where $E \sim k^2$ and

$$A_\ell \sim \frac{\ell}{2\sinh(\ell/2)}, \tag{11}$$

where for convenience we have not included multiplicities m as they lead to subleading corrections in the correlations. We note that different length spectra lead to different attractor dimensions $\delta > 0$. We recall also that the length spectrum for the whole surface is not in principle related to that of the compact core [Eq. (9)]. Importantly, we will define the spectral correlators not in terms of the partition function defined in the previous section but by the spectral density above. We do that for convenience as we shall use results from the quantum chaos literature where spectral correlations are defined through the density. We do not expect any major difference with an approach based from the beginning on the correlation of partition functions as in Ref. [29]. Likewise, the graphical depiction of these correlations as multiboundary spaces with correlations should be applicable in our case, though we do not further pursue this identification as it is evident that the information contained in the correlations of partition functions should be equivalent to that computed in this section.

In order to characterize the nature of the quantum dynamics we will investigate the spectral form factor and the Wigner time delay fluctuations. The latter is an observable employed to characterize the spectrum of open quantum chaotic systems. It is relevant in our case since $0 < \delta < 1$ and therefore the escape rate $\upsilon > 0$.

A. Spectral form factor

The simple form corresponding to the oscillating part of the spectral density—given in terms of a sum over primitive periodic orbits—makes possible the analytical calculation of spectral correlations by using semiclassical techniques. Our aim is to show explicitly that level statistics agree with the RMT prediction which is a signature of quantum chaos. If the escape rate υ is negligible, namely, many closed geodesics stay in the compact core, then the lowest part of the spectrum is discrete, and the system is effectively closed. In that case, the spectral form factor $K(\tau)$, the Fourier transform of the two-level correlation function, is a good indicator of quantum chaos. Despite the (imaginary) magnetic field, which breaks time-reversal invariance, spectral correlations seem to fall within the Gaussian orthogonal ensemble (GOE) universality class typical of systems with time-reversal invariance though we shall see that further research may be needed to confirm the universality class. What is beyond any reasonable doubt is that time-reversal symmetry is broken in our model.

The argument supporting GOE correlations despite the breaking of symmetry is well known [52]. In systems with time-reversal symmetry there are always two degenerate periodic orbits for a given length corresponding to a given orbit and the same orbit after time reversal. The particularity of our system is that the action of the periodic orbit in the presence of a magnetic field only differs from the action without a magnetic field by a constant shift. This means that despite the fact that the lengths of a given periodic orbit and the same periodic orbit after time reversal are different, the actions are the same (up to a constant shift), so level statistics are GOE, not Gaussian unitary ensemble (GUE). Therefore spectral correlators depend on the magnetic field as an irrelevant prefactor. For more details we refer the reader to Ref. [52].

Having said that, a word of caution is in order. As we said, the Selberg trace formula depends on the magnetic field as an irrelevant prefactor that is a function B^2, so it should not matter whether B is real or imaginary. However, without a full derivation from scratch of the Selberg trace formula for the case of an imaginary magnetic field, which is beyond the scope of this paper, we could not be sure that the only difference is the mentioned change in an irrelevant prefactor. In any case, the dynamics will be quantum chaotic in any circumstance with $K(\tau) = A\tau$ for $\tau \ll 1$ with $A = 1$ for GUE and $A = 2$ for GOE. We show next that the spectral form factor for JT gravity agrees with the RMT prediction in this limit and derive the extension of this result in the case of open systems with a finite escape ratio υ.

The spectral form factor is explicitly given by

$$K(\tau) = \left\langle \int_{-\infty}^{\infty} \frac{d\eta}{\tilde{\rho}(E)} \rho_{\rm osc}(E+\eta/2)\rho_{\rm osc}(E-\eta/2) e^{2\pi i \eta \tilde{\rho}(E)\tau} \right\rangle_E . \tag{12}$$

where the average is over an energy interval ΔE, around E of typical width larger than the mean level spacing but much smaller than other dynamical scales of the problem. Inserting the analytical expression for the density [Eqs. (10) and (11)] results in [11]

$$K(\tau) = \frac{1}{T_H} \left\langle \sum_{\ell,\ell' \in \mathcal{L}_X} A_\ell A_{\ell'} e^{i(S_\ell - S'_\ell)/\hbar} \delta\left(T - \frac{\ell + \ell'}{2}\right) \right\rangle_E , \tag{13}$$

where $S_\ell \sim k\ell$ is the classical action related to the geodesic ℓ and momentum k and $T_H = 2\pi\hbar\tilde{\rho}(E)$ is the Heisenberg time, the time scale related to the mean level spacing. Off-diagonal terms $\ell \neq \ell'$ are suppressed due to both the sum over quasirandom phases and the averaging procedure, so we only consider primitive periodic orbits without repetition. With these simplifications, the spectral form factor is given by

$$K(\tau) \sim \frac{1}{T_H} \sum_{\ell \in \mathcal{L}_X} A_\ell^2 \delta(T - \ell). \tag{14}$$

In the semiclassical limit of interest, the integral is dominated by long periodic orbits, so, using Eq. (11), $A_\ell \approx \ell/e^{\ell/2}$. The sum is then replaced by an integral where the density of periodic orbits is the derivative of the number of periodic orbits of length less than L', which according to the *prime geodesic theorem* [46,47] is $\sim e^{\delta L'}/L'$. Therefore the integral simplifies to

$$\begin{aligned} K(\tau) &\sim \frac{1}{T_H} \int_{-\infty}^{\infty} dL' L' e^{-(1-\delta)L'} \delta(T - L') \\ &\sim \frac{T}{T_H} e^{-(1-\delta)T} \sim \tau e^{-\upsilon\tau}, \end{aligned} \tag{15}$$

where $T/T_H = \tau$ is the time measured in units of the Heisenberg time and $\upsilon = (1-\delta)T_H$ is the rate escape, which agrees with the random matrix result in the limit $\delta \to 1$ corresponding to a closed quantum chaotic system. For $\delta \neq 1$, it also agrees with the prediction for an open quantum chaotic system in the semiclassical limit [68–70] or a random scattering matrix [71]. We note that since δ is the lowest eigenvalue or resonance, $T_H \sim f(\delta)$ with $T_H \to \infty$ for $\delta \to 0$ close to the ground state.

The calculation of higher-order terms in the τ expansion is feasible though rather cumbersome. In the limit $\delta \to 1$, it has been carried out in Ref. [48], and it agrees with the RMT prediction. We expect that for $\delta \neq 1$, it would also agree with previous results from RMT and semiclassical open quantum chaotic systems [70].

The variable τ is measured in units of the Heisenberg time, so even though it is an expansion in small τ, it describes the time evolution of the system for long times of the order of, but smaller than, the Heisenberg time.

In summary, up to the moduli space problem [72], the spectral form factor of JT gravity agrees with the random matrix theory prediction that indicates that it is quantum chaotic for sufficiently long times.

B. Wigner time delay fluctuations

Since our system is open and we do not know the precise value of δ in our case, it is in principle not clear to what extent it is possible to demonstrate the existence of quantum chaos from the spectral fluctuations for sufficiently small δ where it may not be possible to distinguish single eigenvalues. We note, however, that there are observables that signal quantum chaotic features in open chaotic systems. One of the most popular is the Wigner time delay τ_W [73,74], defined as the extra time a scattering process takes with respect to free motion. Alternatively, it can also be defined as the difference between the spectral density of the open scattering system, which includes resonances (poles of the resolvent), and a free system. Using the expression for the spectral density [Eq. (10)], it is given by

$$\tau_W \sim \bar{\tau}_W + \frac{1}{\upsilon T_H} {\rm Re} \sum_{m=0} \sum_{\ell \in \mathcal{L}_X} A_{\ell,m} e^{\frac{i}{\hbar} m S_\ell}, \tag{16}$$

where $\bar{\tau}_W$ is the smooth part of the time delay that does not enter into the calculation of fluctuations, $A_{\ell,m}$ was defined in Eq. (11), $S_\ell \propto \ell$ is the classical action related to the primitive geodesic of length ℓ, and υ is the escape rate defined above. A RMT prediction, based on the modeling of the scattering matrix as a random matrix, for the variance and, among others, energy fluctuations of τ_W is available [71,75]. Assuming that the time scale related to the shortest periodic orbit is the smallest length scale in the problem, it agrees with that of deterministic quantum chaotic systems by using the trace formula [69,70,76,77]. Interestingly, the same results are obtained by using only the periodic orbits inside the scattering region or including the full orbits that eventually escape from it (see Ref. [78] and references therein).

A useful indicator of quantum chaos is the Wigner time delay variance $\mathrm{var}(\tau_W)$, given by [70,76,78]

$$\mathrm{var}(\tau_W) \sim \frac{1}{T_H^2} {\rm Re} \left\langle \sum_{\ell,\ell' \in \mathcal{L}_X} A_\ell A^*_{\ell'} e^{i(S_\ell - S_{\ell'})/\hbar} \right\rangle, \tag{17}$$

where only $m = 1$ terms, neglecting repetitions, are considered because higher-m contributions are exponentially smaller.

As in the calculation of the spectral form factor, the leading term in the semiclassical approximation corresponds to the diagonal approximation $\ell = \ell'$. The resulting single sum can be efficiently evaluated by using the prime geodesic theorem [46,47] and the expression for A_ℓ [Eq. (11)] in the limit of large ℓ. That results in

$$\mathrm{var}(\tau_W) \sim \frac{1}{T_H^2 \upsilon^2} + \cdots. \tag{18}$$

This in agreement with the RMT prediction. Indeed, assuming time-reversal invariance, agreement has been found up to eighth order in the expansion parameter $1/T_H \upsilon$ [70]. We note that this expansion parameter is a sensible choice because if $\upsilon T_H \geqslant 1$, the spectrum could at all effects be considered discrete, as in a closed system, where observables such as the spectral form factor are more suitable to describe the quantum motion.

Agreement with RMT predictions is also found for other observables such as energy correlation of the time delay or other correlations involving the scattering matrix [78,79].

The main goal of the paper was to show that the result of a quantization of JT gravity, whenever topological fluctuations are allowed, is that the dynamics is quantum chaotic. Namely, spectral correlations are given by RMT for long time scales of the order of the Heisenberg time. The results of this section strongly suggest that this is the case. We stress that the key feature of this finding is that for any $\delta > 0$ there is an asymptotic exponential growth [46,47] of primitive periodic orbits leading to a finite topological entropy. This property guarantees the analytical calculation of level statistics by using semiclassical techniques. We therefore expect that the obtained random matrix correlations are a robust feature of the spectrum of the Maass Laplacian on any of the hyperbolic Riemann surfaces of infinite area in the moduli space of the theory because in all of them $\delta > 0$.

V. DISCUSSION AND CONCLUSION

In this exploratory study, we have studied the quantization of JT gravity whenever topological fluctuations are allowed. We have shown that the quantum gravity problem is mapped onto the calculation of the spectrum of a certain Maass-Laplace differential operator on noncompact Riemann surfaces of infinite area and genus $h \geqslant 1$. Remarkably, the spectrum of this open chaotic system is semiclassically exact. The spectrum and resonances of this operator are written explicitly by a Selberg trace formula, namely, a sum over geodesics of the classical counterpart. Resonances, corresponding to classical trajectories escaping to infinity, are zeros of the associated Selberg theta function. The spectral form factor as well as the variance of the Wigner time delay agree with the RMT prediction. This is an indication that full quantum ergodicity is a distinctive feature of quantum JT gravity. We stress that a requirement to observe quantum chaos is that the time scale related to the shortest closed geodesic, a sort of Thouless time, must be much shorter than the inverse of the classical escape rate $\gamma = 1 - \delta$ with $\delta > 0$. These quantities will depend among others things on the dilaton boundary value, which is proportional to the *magnetic* field, and the genus $h \geqslant 1$ of the surface. It would be interesting to carry out an explicit calculation of δ and γ to fully confirm the quantum chaotic nature of quantum JT gravity. Moreover, the analogy between quantum JT gravity and the charged particle picture is only fully justified [37,38] in the Schwarzian limit corresponding to a large and imaginary magnetic field. Therefore this is another condition for our results to hold.

Other topics that deserve further attention are the generalization to higher spatial dimensions, the calculation of nonuniversal corrections to random matrix results for sufficiently short times, and, following the ideas of Ref. [33], the generalization of the results presented here to the supersymmetric case.

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[1] A. I. Larkin and Yu. N. Ovchinnikov, Quasiclassical method in the theory of superconductivity, Zh. Eksp. Teor. Fiz. **55**, 2262 (1968) [Sov. Phys. JETP **28**, 1200 (1969)].

[2] G. P. Berman and G. M. Zaslavsky, Condition of stochasticity in quantum nonlinear systems, Physica A (Amsterdam, Neth.) **91**, 450 (1978).

[3] E. P. Wigner, On the statistical distribution of the widths and spacings of nuclear resonance levels, Math. Proc. Cambridge Philos. Soc. **49**, 790 (1951).

[4] F. J. Dyson, Statistical theory of the energy levels of complex systems. I, J. Math. Phys. (Melville, NY, U. S.) **3**, 140 (1962).

[5] F. J. Dyson, Statistical theory of the energy levels of complex systems. II, J. Math. Phys. (Melville, NY, U. S.) **3**, 157 (1962).

[6] F. J. Dyson, Statistical theory of the energy levels of complex systems. III, J. Math. Phys. (Melville, NY, U. S.) **3**, 166 (1962).

[7] F. J. Dyson, A Brownian-motion model for the eigenvalues of a random matrix, J. Math. Phys. (Melville, NY, U. S.) **3**, 1191 (1962).

[8] O. Bohigas and J. Flores, Two-body random Hamiltonian and level density, Phys. Lett. B **34**, 261 (1971).

[9] K. B. Efetov, Supersymmetry and theory of disordered metals, Adv. Phys. **32**, 53 (1983).

[10] O. Bohigas, M. J. Giannoni, and C. Schmit, Characterization of Chaotic Quantum Spectra and Universality of Level Fluctuation Laws, Phys. Rev. Lett. **52**, 1 (1984).

[11] M. Sieber and K. Richter, Correlations between periodic orbits and their role in spectral statistics, Phys. Scr., T **T90**, 128 (2001).

[12] S. Müller, S. Heusler, P. Braun, F. Haake, and A. Altland, Semiclassical Foundation of Universality in Quantum Chaos, Phys. Rev. Lett. **93**, 014103 (2004).

[13] J. J. M. Verbaarschot and I. Zahed, Spectral Density of the QCD Dirac Operator Near Zero Virtuality, Phys. Rev. Lett. **70**, 3852 (1993).

[14] J. Maldacena, S. H. Shenker, and D. Stanford, A bound on chaos, J. High Energy Phys. 08 (2016) 106.

[15] A. Kitaev, A simple model of quantum holography, in KITP Strings Seminar and Entanglement 2015 Program, 12 February, 7 April and 27 May 2015, http://online.kitp.ucsb.edu/online/entangled15/.

[16] J. Maldacena and D. Stanford, Remarks on the Sachdev-Ye-Kitaev model, Phys. Rev. D **94**, 106002 (2016).

[17] K. Jensen, Chaos in AdS_2 Holography, Phys. Rev. Lett. **117**, 111601 (2016).

[18] J. B. French and S. S. M. Wong, Validity of random matrix theories for many-particle systems, Phys. Lett. B **33**, 449 (1970).

[19] K. K. Mon and J. B. French, Statistical properties of many-particle spectra, Ann. Phys. (Amsterdam, Neth.) **95**, 90 (1975).

[20] L. Benet and H. A. Weidenmüller, Review of the k-body embedded ensembles of Gaussian random matrices, J. Phys. A: Math. Gen. **36**, 3569 (2003).

[21] S. Sachdev and J. Ye, Gapless Spin-Fluid Ground State in a Random Quantum Heisenberg Magnet, Phys. Rev. Lett. **70**, 3339 (1993).

[22] S. Sachdev, Holographic Metals and the Fractionalized Fermi Liquid, Phys. Rev. Lett. **105**, 151602 (2010).

[23] J. Maldacena, D. Stanford, and Z. Yang, Conformal symmetry and its breaking in two-dimensional nearly anti-de Sitter space, Prog. Theor. Exp. Phys. **2016**, 12C104 (2016).

[24] A. Almheiri and J. Polchinski, Models of AdS_2 backreaction and holography, J. High Energy Phys. 11 (2015) 014.

[25] J. Engelsöy, T. G. Mertens, and H. Verlinde, An investigation of AdS_2 backreaction and holography, J. High Energy Phys. 07 (2016) 139.

[26] A. M. García-García and J. J. M. Verbaarschot, Spectral and thermodynamic properties of the Sachdev-Ye-Kitaev model, Phys. Rev. D **94**, 126010 (2016).

[27] J. S. Cotler, G. Gur-Ari, M. Hanada, J. Polchinski, P. Saad, S. H. Shenker, D. Stanford, A. Streicher, and M. Tezuka, Black holes and random matrices, J. High Energy Phys. 05 (2017) 118; Erratum: 09 (2018) 002.

[28] A. M. García-García and J. J. M. Verbaarschot, Analytical spectral density of the Sachdev-Ye-Kitaev model at finite n, Phys. Rev. D **96**, 066012 (2017).

[29] P. Saad, S. H. Shenker, and D. Stanford, JT gravity as a matrix integral, arXiv:1903.11115.

[30] P. Saad, Late time correlation functions, baby universes, and ETH in JT gravity, arXiv:1910.10311.

[31] I. Aref'eva and I. Volovich, Gas of baby universes in JT gravity and matrix models, Symmetry **12**, 975 (2020).

[32] K. Okuyama and K. Sakai, JT gravity, KdV equations and macroscopic loop operators, J. High Energy Phys. 01 (2020) 156.

[33] D. Stanford and E. Witten, JT gravity and the ensembles of random matrix theory, arXiv:1907.03363.

[34] M. Mirzakhani, Weil-Petersson volumes and intersection theory on the moduli space of curves, J. Am. Math. Soc. **20**, 1 (2007).

[35] M. Mirzakhani, Simple geodesics and Weil-Petersson volumes of moduli spaces of bordered Riemann surfaces, Invent. Math. **167**, 179 (2007).

[36] B. Eynard and N. Orantin, Weil-Petersson volume of moduli spaces, Mirzakhani's recursion and matrix models, arXiv:0705.3600.

[37] Z. Yang, The quantum gravity dynamics of near extremal black holes, J. High Energy Phys. 05 (2019) 205.

[38] A. Kitaev and S. J. Suh, Statistical mechanics of a two-dimensional black hole, J. High Energy Phys. 05 (2019) 198.

[39] A. Comtet, On the Landau levels on the hyperbolic plane, Ann. Phys. (Amsterdam, Neth.) **173**, 185 (1987).

[40] A. M. Polyakov, *Gauge Fields and Strings*, Contemporary Concepts in Phys. Vol. 3 (Harwood, Academic, Chur, Switzerland, 1987).

[41] A. M. García-García, Y. Jia, and J. J. M. Verbaarschot, Universality and Thouless energy in the supersymmetric Sachdev-Ye-Kitaev model, Phys. Rev. D **97**, 106003 (2018).

[42] D. Stanford and E. Witten, Fermionic localization of the Schwarzian theory, J. High Energy Phys. 10 (2017) 008.

[43] We will consider throughout the case of one asymptotic boundary.

[44] D. Borthwick, *Spectral Theory of Infinite-Area Hyperbolic Surfaces*, Progress in Mathematics Vol. 256 (Birkhäuser, Boston, 2007).

[45] Notice that the method used in Ref. [29], based on a decomposition of the surface like that shown in Fig. 1, accounts for the moduli using Mirzakhani's recursive method [34,35], which involves integrating functions which depend on the length of primitive *simple* closed geodesics instead.

[46] F. Naud, Expanding maps on Cantor sets and analytic continuation of zeta functions, Ann. Sci. Ec. Norm. Sup. Ser. 4 **38**, 116 (2005).

[47] L. Guillope, Sur la distribution des longueurs des geodesiques fermees d'une surface compacte a bord totalement geodesique, Duke Math. J. **53**, 827 (1986).

[48] M. Sieber, *Hyperbolic Geometry and Applications in Quantum Chaos and Cosmology*, London Mathematical Society Lecture Note Series Vol. 397 (Cambridge University Press, Cambridge, 2011).

[49] A. Selberg, Harmonic analysis and discontinuous groups in weakly symmetric Riemannian spaces with applications to Dirichlet series, J. Indian Math. Soc. **20**, 47 (1956).

[50] H. P. McKean, Selberg's trace formula as applied to a compact Riemann surface, Commun. Pure Appl. Math. **25**, 225 (1972).

[51] J. Bolte and F. Steiner, The Selberg trace formula for bordered Riemann surfaces, Commun. Math. Phys. **156**, 1 (1993).

[52] A. Comtet, B. Georgeot, and S. Ouvry, Trace Formula for Riemann Surfaces with Magnetic Field, Phys. Rev. Lett. **71**, 3786 (1993).

[53] E. D'Hoker and D. H. Phong, On determinants of Laplacians on Riemann surfaces, Commun. Math. Phys. **104**, 537 (1986).

[54] J. Bolte and C. Grosche, Selberg trace formula for bordered Riemann surfaces: Hyperbolic, elliptic and parabolic conjugacy classes, and determinants of Maass-Laplacians, Commun. Math. Phys. **163**, 217 (1994).

[55] D. A. Hejhal, *The Selberg Trace Formula for PSL (2, R)* (Springer, Berlin, 2006), Vol. 1.

[56] M. C. Gutzwiller, *Chaos in Classical and Quantum Mechanics* (Springer, New York, 2013), Vol. 1.

[57] D. Borthwick, C. Judge, and P. A. Perry, Determinants of Laplacians and isopolar metrics on surfaces of infinite area, Duke Math. J. **118**, 61 (2003).

[58] S. J. Patterson and P. A. Perry, The divisor of Selberg's zeta function for Kleinian groups, Duke Math. J. **106**, 321 (2001).

[59] D. Borthwick, C. Judge, and P. A. Perry, Selberg's zeta function and the spectral geometry of geometrically finite hyperbolic surfaces, Comment. Math. Helv. **80**, 483 (2005).

[60] D. Borthwick and P. A. Perry, Inverse scattering results for manifolds hyperbolic near infinity, J. Geom. Anal. **21**, 305 (2011).

[61] D. Borthwick, Distribution of resonances for hyperbolic surfaces, Exp. Math. **23**, 25 (2014).

[62] A. F. Beardon, The exponent of convergence of Poincaré series, Proc. London Math. Soc. **s3-18**, 461 (1968).

[63] D. Sullivan, Entropy, Hausdorff measures old and new, and limit sets of geometrically finite Kleinian groups, Acta Math. **153**, 259 (1984).

[64] S. J. Patterson, The exponent of convergence of Poincaré series, Monatsh. Math. **82**, 297 (1976).

[65] S. J. Patterson, On a lattice-point problem in hyperbolic space and related questions in spectral theory, Ark. Mat. **26**, 167 (1988).

[66] S. J. Patterson, Lectures on measures on limit sets of Kleinian groups, in *Fundamentals of Hyperbolic Manifolds*, London Mathematical Society Lecture Note Series Vol. 328 (Cambridge University Press, Cambridge, 2006), pp. 291–292.

[67] M. S. Keane, T. Bedford, and C. Series, *Ergodic Theory, Symbolic Dynamics, and Hyperbolic Spaces* (Oxford University Press, Oxford, 1991).

[68] R. O. Vallejos, A. M. Ozorio de Almeida, and C. H. Lewenkopf, Quantum time delay in chaotic scattering: A semiclassical approach, J. Phys. A: Math. Gen. **31**, 4885 (1998).

[69] B. Eckhardt, Correlations in quantum time delay, Chaos **3**, 613 (1993).

[70] J. Kuipers and M. Sieber, Semiclassical expansion of parametric correlation functions of the quantum time delay, Nonlinearity **20**, 909 (2007).

[71] N. Lehmann, D. V. Savin, V. V. Sokolov, and H.-J. Sommers, Time delay correlations in chaotic scattering: Random matrix approach, Phys. D (Amsterdam, Neth.) **86**, 572 (1995).

[72] As was mentioned earlier, a fully rigorous calculation of level statistics requires knowledge of the Weil-Petersson volumes that enter into the evaluation of the moduli space. Unfortunately, the Selberg trace formula is expressed in terms of the action of all primitive closed geodesics of the geometry, while the Weil-Petersson volumes, which are necessary for the evaluation of the moduli space, require as an input the length of primitive *simple* closed geodesics, which is a much smaller subset. Therefore we could only evaluate the moduli space for such a subset, which is in principle irrelevant for the computation of spectral correlators. The evaluation of the form factor in this section does not alter this conclusion. However, the form factor in Eq. (15), after using the prime geodesic theorem, depends only on the length of the primitive closed geodesic weighted by a simple exponential function. The moduli space could be naively evaluated by using the expression for the Weil-Petersson volumes for all primitive geodesics, which would lead to a modified weight function. Although this is not in principle correct because of the arguments above, it is reassuring that, to leading order, results for the form factor would not be modified.

[73] E. P. Wigner, Lower limit for the energy derivative of the scattering phase shift, Phys. Rev. **98**, 145 (1955).

[74] F. T. Smith, Lifetime matrix in collision theory, Phys. Rev. **118**, 349 (1960).

[75] Y. V. Fyodorov and H.-J. Sommers, Statistics of resonance poles, phase shifts and time delays in quantum chaotic scattering: Random matrix approach for systems with broken time-reversal invariance, J. Math. Phys. (Melville, NY, U. S.) **38**, 1918 (1997).

[76] J. Kuipers and M. Sieber, Semiclassical relation between open trajectories and periodic orbits for the Wigner time delay, Phys. Rev. E **77**, 046219 (2008).

[77] P. W. Brouwer, Semiclassical theory of the Ehrenfest-time dependence of quantum transport, Phys. Rev. B **76**, 165313 (2007).

[78] J. Kuipers, D. V. Savin, and M. Sieber, Efficient semiclassical approach for time delays, New J. Phys. **16**, 123018 (2014).

[79] Y. V. Fyodorov and D. V. Savin, Resonance scattering of waves in chaotic systems, in *The Oxford Handbook of Random Matrix Theory*, edited by G. Akemann, J. Baik, P. Di Francesco (Oxford University Press, Oxford, 2011), pp. 703–722.