Role of Nambu-Goldstone modes in the fermionic-superfluid point contact

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In fermionic superfluids that are charge neutral, Nambu-Goldstone (NG) modes, also known as Anderson-Bogoliubov modes, emerge as a result of spontaneous symmetry breaking. Here, we discuss DC transport properties of such NG modes through a quantum point contact. We show that contrary to a naive view that enhancement of the phase stiffness may suppress transport of the NG modes, there must be an anomalous contribution that survives at low temperature. This contribution originates from the conversion process between the condensate and the NG mode. We find that within the BCS regime the anomalous contribution is enhanced with increasing channel transmittance and attractive interaction, and leads to a temperature-dependent Lorenz number and absence of the bunching effect in current noise.

I. INTRODUCTION

In mesoscopic transport phenomena through small constrictions, quantum-mechanical effects are known to be directly reflected in transport coefficients. One of the best known is the Landauer formula in which the two-terminal conductance of normal metals is quantized [1]. This quantization originates from the disappearance of relevant length scales in constriction due to the ballistic condition and therefore is clearly seen in a quantum point contact where a constriction has a short one-dimensional structure [2,3].

In addition to shapes of constriction, states of matter in reservoirs play an important role in mesoscopic transport. The prototype example is a superconducting point contact where reservoirs consist of superconductors [4,5]. In this case, it is known that the direct current does not obey Ohm's law [4,5]. The key ingredient there is multiple Andreev reflections [6] where quasiparticles repeat Andreev reflections at the boundaries between superconductor and contact. As a result, the current-bias characteristics become highly nonlinear [7].

In contrast, each constituent particle in detail is expected to be irrelevant in mesoscopic transport. For instance, when electrons are replaced by other fermions, e.g., neutral atoms such as ^{6}Li and ^{40}K, essentially the same phenomena are observed as long as similar states of matter are prepared. This type of universality can nowadays be confirmed with ultracold atomic gases. Indeed, a two-terminal transport setup with a quantum point contact has been realized in experiments of ultracold Fermi gases [8], which observed the conductance quantization [9] and nonlinear current-bias characteristics [10].

It must be noted, however, that the presence or absence of charge may cause a difference in transport between elec-

tron and atomic systems. Specifically, this difference may qualitatively be important for systems with Bose-Einstein condensation of Cooper pairs where Nambu-Goldstone (NG) modes emerge due to spontaneous symmetry breaking [11]. In the case of electrons (superconductors), an effect of the Coulomb interaction is inevitable, which turns the NG modes into the gapped plasma modes [12]. In the case of atoms (superfluids), on the other hand, these NG modes, also known as Anderson-Bogoliubov modes, remain gapless [12,13], and the NG modes may play an important role in low-energy transport. At the same time, as the NG modes are a nonsuperfluid component, there is a view that the effects of such modes are negligible at low temperature. Since the NG modes in mesoscopic transport have yet to be incorporated in an explicit manner, it is not clear whether it is reasonable to neglect the effect of the gapless mode in experiments of ultracold atomic gases.

In this paper, we analyze direct currents pertinent to transport of the NG modes in the fermionic superfluid point contact, as illustrated in Fig. 1. On one hand, we demonstrate that the naive view mentioned above is partially correct in that the exchange process of the NG modes between reservoirs is indeed suppressed at low temperature. On the other hand, we uncover an anomalous process such that condensate elements are converted into the NG modes and vice versa. What is remarkable is that this process survives even at absolute zero. It is discussed that within the BCS regime the anomalous contribution is enhanced with increasing channel transmittance and attractive interaction of fermions, and is absent in the heat current, which causes a breakdown of the Wiedemann-Franz law.

This paper is organized as follows. Section II discusses an effective action of the NG modes in fermionic superfluids in terms of functional integrals. In Sec. III, we introduce the tunneling Hamiltonian approach to discuss transport of the NG modes in the superfluid point contact. In Sec. IV, the pair current expression including effects of the NG modes is obtained by using the analyses in Secs. II and III. Section V discusses several topics related to the results of Sec IV.
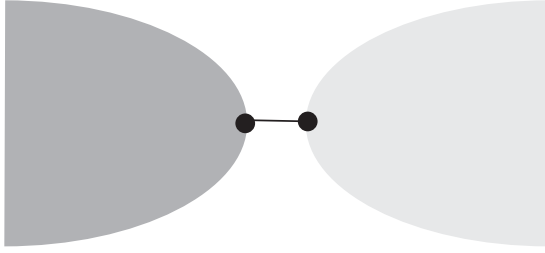

FIG. 1. Schematic illustration of the fermionic superfluid point contact, where two macroscopic reservoirs are fulfilled by *s*-wave Fermi superfluids. We analyze a DC current including transport of the Nambu-Goldstone modes in terms of the tunneling Hamiltonian, provided that the constriction length is shorter than the superfluid coherence length.

Section VI summarizes this paper and the calculation of correlation functions is shown in detail in the Appendix.

II. EFFECTIVE FIELD THEORY OF ANDERSON-BOGOLIUBOV MODES

Here, we sketch how the effective action of the Anderson-Bogoliubov modes in the bulk (each reservoir) is obtained from fermionic superfluids. To this end, we consider the following bulk Hamiltonian (we use units $\hbar = k_B = 1$):

$$H_0 = \int d^3x \, \mathcal{H}_0, \tag{1}$$

$$\mathcal{H}_0 = \sum_{\sigma=\uparrow,\downarrow} \psi_\sigma^\dagger \left(-\frac{\nabla^2}{2M} - \mu \right) \psi_\sigma + g \psi_\uparrow^\dagger \psi_\downarrow^\dagger \psi_\downarrow \psi_\uparrow, \tag{2}$$

where ψ_σ is the fermionic field with spin σ, M is the mass of fermions, and μ is the chemical potential. In addition, the coupling constant g is assumed to be negative to ensure an attractive interaction between fermions. We note that the above Hamiltonian is nothing but the celebrated BCS Hamiltonian and is also known to describe two-component Fermi gases interacting via broad Feshbach resonances [14].

We focus on the low-temperature regime where $\uparrow$ and $\downarrow$ fermions form Cooper pairs and Bose-Einstein condensation of Cooper pairs occurs. When the attractive interaction between fermions is weak (BCS regime), such condensation can be captured by the mean-field theory [15]. By adopting this theory, the original Hamiltonian reduces to

$$\mathcal{H}_0 \to \sum_\sigma \psi_\sigma^\dagger \left(-\frac{\nabla^2}{2M} - \mu \right) \psi_\sigma - \Delta \psi_\uparrow^\dagger \psi_\downarrow^\dagger - \bar{\Delta} \psi_\downarrow \psi_\uparrow. \tag{3}$$

Here, we introduce the gap parameter,

$$\Delta = -g \langle \psi_\downarrow \psi_\uparrow \rangle, \tag{4}$$

which is assumed to be a constant in space-time. Since the Hamiltonian above is quadratic in ψ, we can obtain a quasiparticle excitation by diagonalizing it. The resultant quasiparticle is called a Bogoliubov mode that is fermionic and has the energy gap $2|\Delta|$.

In addition to the fermionic quasiparticle excitation, there are bosonic collective excitations in fermionic superfluids. The NG mode is then the dominant excitation at low energy,

since it is gapless due to the NG theorem [11]. To see this gapless mode, we consider fluctuations from the mean field in the gap function. In general, the gap parameter is complex and therefore there are two directions of fluctuations, amplitude fluctuation and phase fluctuation, which are related to the Higgs mode and the NG mode, respectively. Since the former excitation has a gap $2|\Delta|$ [16], we can focus on the phase fluctuation in the low-frequency regime $|\omega| < |\Delta|$. In this case, the gap parameter with the phase fluctuation is expressed as follows:

$$\Delta(x) = |\Delta| e^{2i\phi(x)}, \tag{5}$$

where ϕ is a real bosonic field describing the phase fluctuation. To obtain an effective theory of the NG mode, it is convenient to adopt the functional integral formalism. By integrating out the fermionic fields in the mean-field Hamiltonian (3), the partition function can be expressed in terms of ϕ such that

$$Z = \int \mathcal{D}\phi \, e^{-S_{\text{eff}}(\phi)}. \tag{6}$$

Here, S_{eff} is the effective action of the NG mode, and is in general a complicated function in ϕ. By performing the leading-order analysis of the gradient approximation in ϕ, however, S_{eff} becomes the following quadratic form [11]:

$$S_{\text{eff}} \approx \frac{\rho_s}{2} \int_0^\beta d\tau \int d^3x \left[\frac{1}{v^2}(\partial_\tau \phi)^2 + (\nabla \phi)^2 \right], \tag{7}$$

where v is the speed of sound and ρ_s is the superfluid density. At absolute zero, they are explicitly determined as

$$v = \frac{v_F}{\sqrt{3}}, \tag{8}$$

$$\rho_s = \frac{n}{M}, \tag{9}$$

with the Fermi velocity v_F and number density n.

The procedure introduced above is called bosonization in the way that the effective theory of the bosonic collective mode is obtained from the fermionic action. In our derivation, the mean-field theory and gradient approximation are explicitly employed. Regardless of the strength of the attractive interaction, however, the superfluid phase with spontaneous breaking of U(1) symmetry is known to emerge in the low-temperature regime of the spin-balanced mixture [14]. This means that the NG modes with the linear gapless dispersion are always present in the superfluid phase [17]. Indeed, as far as the low-energy regime is concerned, the form of the effective action (7) is universal for the U(1) symmetry-breaking phase [18]. Then, effects on the interaction and temperature are reflected as renormalization of ρ_s and v.

III. TUNNELING HAMILTONIAN APPROACH

Now that the effective action of the NG mode in the bulk superfluid is obtained, we wish to discuss how this gapless mode affects two-terminal point contact transport, where two macroscopic reservoirs (L and R) are connected through a short one-dimensional wire. In particular, we focus on a regime where the length of the one-dimensional wire is shorter than the superfluid coherence length $v_F/(\pi|\Delta|)$. The constriction in detail is then known to be irrelevant [19] and

transport of such a system can be discussed with the following tunneling Hamiltonian [20–22]:

$$H = H_0 + H_T, \tag{10}$$

$$H_0 = H_L + H_R, \tag{11}$$

where $H_{L(R)}$ is the grand Hamiltonian of the left (right) reservoir, and

$$H_T = t \sum_\sigma \psi_{\sigma,R}^\dagger(\mathbf{0})\psi_{\sigma,L}(\mathbf{0}) + \text{H.c.} \tag{12}$$

is the tunneling term with the tunneling amplitude t. Since in this model, a particle exchange occurs at the single point $\mathbf{x} = \mathbf{0}$, it follows that the currents are expressed by the fields at $\mathbf{x} = \mathbf{0}$. Thus, in what follows, we omit the spatial index in fields for brevity. Based on this Hamiltonian, the particle current operator is calculated as

$$I = -\dot{N}_L = i[N_L, H_T] = -it \sum_\sigma \psi_{\sigma,R}^\dagger \psi_{\sigma,L} + \text{H.c.} \tag{13}$$

We note that the tunneling term above represents the single-particle tunneling between the reservoirs. Therefore, the direct current calculated with the mean-field Hamiltonian (3) turns out to be associated with quasiparticle processes including multiple Andreev reflections [20].

In addition to the quasiparticle tunneling, the tunneling Hamiltonian allows us to discuss the pair tunneling process related to transport of the NG modes. The key point is that in the tunneling Hamiltonian approach, the tunneling term is treated as perturbation and therefore the pair tunneling process is generated as a higher-order tunneling effect [23]. To see this in an explicit manner, we temporarily consider the zero-bias situation, where a contribution from left to right is balanced with one from right to left. By using the imaginary time formalism, a contribution related to the NG modes is extracted as

$$\begin{aligned}
\langle I_\uparrow \rangle &= -it \Bigg\langle \sum_{n=0}^\infty \frac{(-1)^{n+1}}{(n+1)!} \int_0^\beta d\tau_1 \cdots \int_0^\beta d\tau_{n+1} \\
&\quad \times T_\tau[\psi_{\uparrow,R}^\dagger \psi_{\uparrow,L} H_T(\tau_1) \cdots H_T(\tau_{n+1})] \Bigg\rangle_0 + \text{H.c.} \\
&\rightarrow \frac{i\alpha t^2 \Delta_L \Delta_R}{g^2} \langle e^{-2i\phi_R} e^{2i\phi_L} \rangle + \text{H.c.},
\end{aligned} \tag{14}$$

where $\langle \cdots \rangle_0$ means the average without the tunneling term, and in the last line of the equation, we extract the pair tunneling process and introduce the short timescale α for the extraction. Since a similar result is obtained for $\langle I_\downarrow \rangle$, the pair current operator related to transport of the NG modes is obtained as

$$I_p = -it_p e^{-2i\phi_R} e^{2i\phi_L} + \text{H.c.}, \tag{15}$$

with $t_p = -2\alpha t^2 \Delta_L \Delta_R/g^2$. In addition, H_T generates the pair tunneling term, showing up even order in t. Indeed, by using the similar trick used in Eq. (14), the imaginary time evolution

operator of even order is transformed into

$$\sum_{n=2,4,\cdots} \int_0^\beta d\tau_1 \cdots \int_0^{\tau_{n-1}} d\tau_n H_T(\tau_1) \cdots H_T(\tau_n)$$

$$\rightarrow \sum_{n=1}^\infty \int_0^\beta d\tau_1 \cdots \int_0^{\tau_{n-1}} d\tau_n H_p(\tau_1) \cdots H_p(\tau_n), \tag{16}$$

where

$$H_p = t_p e^{-2i\phi_R} e^{2i\phi_L} + \text{H.c.} \tag{17}$$

is the pair tunneling term. Thus, the discussions above imply that the pair tunneling contribution of the mass current is obtained as the average of Eq. (15) under the perturbation term H_p.

IV. PAIR CURRENT EXPRESSION

We now discuss a pair current expression in the presence of a chemical potential bias, $\Delta\mu = \mu_L - \mu_R$. In order to avoid a contribution of the Higgs modes, we postulate the condition $\Delta\mu, T < \Delta$.

We note that $\Delta\mu$ is the bias for each fermion. Therefore, when it comes to the pair tunneling, the bias between the reservoirs must be regarded as $2\Delta\mu$. We can also understand this result in terms of the gauge transformation technique conventionally utilized in the tunneling Hamiltonian approach, where the combination $\psi_{\sigma,R(L)}^\dagger(\tau)\psi_{\sigma,L(R)}(\tau)$ yields the factor $e^{-(+)i\Delta\mu\tau}$ [24]. Then, since the pair tunneling process is related to the combination $\psi_{\uparrow,R(L)}^\dagger(\tau)\psi_{\downarrow,R(L)}^\dagger(\tau)\psi_{\downarrow,L(R)}(\tau)\psi_{\uparrow,L(R)}(\tau)$, such a term gives rise to the factor $e^{-(+)2i\Delta\mu\tau}$, meaning that the chemical potential bias on the pair tunneling is $2\Delta\mu$ [22].

To obtain a pair current expression, we also note that the average of the current at real time τ can be expressed as

$$\langle I_p(\tau) \rangle = 2\text{Re}[t_p G_{LR}^<(\tau,\tau)], \tag{18}$$

with lesser Green's function

$$G_{LR}^<(\tau,\tau') = -i\langle e^{-2i\phi_R(\tau')} e^{2i\phi_L(\tau)} \rangle. \tag{19}$$

By using the expression above, we turn to calculate the current expression including arbitrary order in t_p. To this end, it is important to recall the following properties of uncoupled retarded and advanced Green's functions under the Gaussian action [23]:

$$g_{L(R)}^R(\tau) = -i\theta(\tau)\langle[e^{Ai\phi_{L(R)}(\tau)}, e^{Bi\phi_{L(R)}(0)}]\rangle_0 = 0, \tag{20}$$

$$g_{L(R)}^A(\tau) = i\theta(-\tau)\langle[e^{Ai\phi_{L(R)}(\tau)}, e^{Bi\phi_{L(R)}(0)}]\rangle_0 = 0, \tag{21}$$

unless $A + B = 0$. These properties on the average without H_p forbid emergence of the anomalous average contribution appearing in the quasiparticle contribution in superconducting systems [20], and render the current calculation simple.

In order to obtain the current expression including an arbitrary order in t_p, we consider the Dyson equation in the real-time formalism [25]. By using the so-called Langreth rules [25], the Dyson equation of $G^<$ is obtained as

$$G^< = (1 + G^R V) \circ g^< \circ (1 + V G^A), \tag{22}$$

where $\circ$ denotes integration over the internal time variable from minus infinity to plus infinity, V is one-particle potential, $G^{R(A)}$ is the exact retarded (advanced) Green's function including the effect of V, and $g^<$ is the uncoupled lesser Green's function. By applying the Dyson equation above for the tunneling Hamiltonian where V is replaced by the tunneling amplitude, we obtain

$$t_p G_{LR}^< = g_L^R \circ \bar{T}_p^R \circ g_R^< \circ T_p^A + T_p^R \circ g_L^< \circ \bar{T}_p^A \circ g_R^A. \quad (23)$$

Here, we introduce the renormalized tunneling amplitudes

$$T_p^{R,A} = t_p + t_p g_L^{R,A} \circ \bar{t}_p g_R^{R,A} \circ T_p^{R,A}, \quad (24)$$

$$\bar{T}_p^{R,A} = \bar{t}_p + \bar{t}_p g_R^{R,A} \circ t_p g_L^{R,A} \circ \bar{T}_p^{R,A}. \quad (25)$$

By performing the Fourier transformation, the renormalized tunneling amplitudes can be solved as

$$T_p^{R,A}(\omega) = \frac{t_p}{1 - |t_p|^2 g_L^{R,A}(\omega - 2\Delta\mu) g_R^{R,A}(\omega)}, \quad (26)$$

$$\bar{T}_p^{R,A}(\omega) = \frac{\bar{t}_p}{1 - |t_p|^2 g_L^{R,A}(\omega - 2\Delta\mu) g_R^{R,A}(\omega)}. \quad (27)$$

To obtain the above, we use the fact that tunneling is an energy conserving process and the origins of frequencies between the reservoirs are different by $2\Delta\mu$. Thus, the DC pair current is obtained as

$$\langle I_p(\tau) \rangle = -\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{2|t_p|^2}{\left|1 - |t_p|^2 g_L^R(\omega - 2\Delta\mu) g_R^R(\omega)\right|^2}$$
$$\times \left(\mathrm{Im}[g_L^R(\omega - 2\Delta\mu)] \mathrm{Im}[g_R^<(\omega)] \right.$$
$$\left. + \mathrm{Im}[g_R^A(\omega)] \mathrm{Im}[g_L^<(\omega - 2\Delta\mu)] \right). \quad (28)$$

For the sake of qualitative discussions on the transport properties, we consider the small bias regime where $O(\Delta\mu^2)$ is negligible. By using the expressions of Green's functions obtained in the Appendix, we reach the following linear current-bias relation:

$$\langle I_p \rangle = (G_{an} + G_{NG})\Delta\mu. \quad (29)$$

Here, we classify the conductance into two contributions, since the lesser Green's function contains a part proportional to a condensate $\delta(\omega)$ and one proportional to the phonon distribution $n(\omega) = \frac{1}{e^{\omega/T} - 1}$. The former yields G_{an} given by

$$G_{an} = \mathcal{T}^2 \frac{2M^2 v^3}{\pi \rho_s}, \quad (30)$$

where

$$\mathcal{T}^2 = \frac{4|t_p|^2 / (M^2 v^4)}{\left|1 - |t_p|^2 g_L^R(0) g_R^R(0)\right|^2} \quad (31)$$

is the dimensionless parameter related to the channel transmittance. This contribution is anomalous in that condensation causes the direct current and does not vanish even at absolute zero. Physically, G_{an} is related to the conversion process between the condensate and NG mode, which also appears in the transport of bosonic systems [26,27]. On the other hand, the latter originates from normal tunneling of the NG modes between the reservoirs. At a low temperature, G_{NG} is reduced
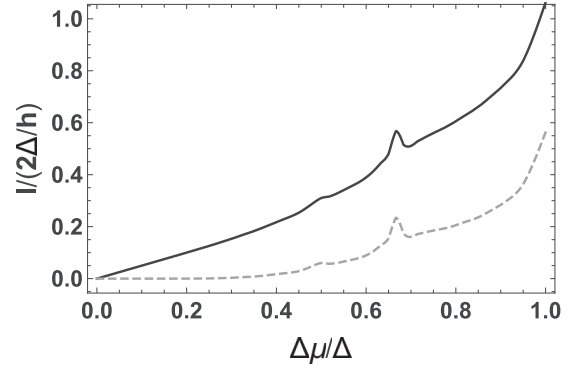

FIG. 2. Typical behavior of the current-bias characteristics in the fermionic superfluid point contact at $T = 0$. The dashed curve represents the quasiparticle current in which the transmittance is given as 0.5 [20].[1] On the other hand, the total contribution being the sum of the quasiparticle and pair currents is plotted in the solid curve, where $G_{an} = 1/h$.

to the following simple form:

$$G_{NG} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{8|t_p|^2 \mathrm{Im}[g^R(\omega)]^2}{\left|1 - |t_p|^2 g_L^R(\omega) g_R^R(\omega)\right|^2} \left(-\frac{\partial n(\omega)}{\partial \omega} \right)$$
$$\approx \mathcal{T}^2 \frac{2M^2 v^2 T^2}{3\pi \rho_s^2}, \quad (32)$$

where we use

$$-\frac{\partial n(\omega)}{\partial \omega} = \frac{1}{4T \sinh^2[\omega/(2T)]}. \quad (33)$$

In contrast to the anomalous contribution, the contribution above explicitly depends on temperature in such a way that it vanishes at zero temperature. This is due to the fact that the NG modes that are the nonsuperfluid components must be suppressed at a low temperature.

V. DISCUSSION

Here, we discuss some implications based on the results obtained in the previous sections.

First, it turns out that the contributions related to the NG modes obey Ohm's law at small biases, in contrast to the quasiparticle contribution, which is essentially nonlinear in $\Delta\mu$. As can be seen from the definition of t_p and Eq. (31), the leading order term in the pair current is proportional to t^4. In order to measure the NG mode contribution, we therefore look at the high transmittance regime beyond the tunneling limit proportional to t^2.

As shown in the dashed line of Fig. 2, in the intermediate transmittance regime, the quasiparticle current shows the subharmonic gap structure at $\Delta\mu/\Delta = 2/n$ ($n = 1, 2, \ldots$) due to the multiple Andreev reflections and is still negligible at the small bias. On the other hand, in the BCS limit at

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[1]In the tunneling Hamiltonian approach, the channel transmittance of each fermion is expressed as $\frac{4t^2/[\pi\rho_q(\mu_F)]^2}{\{1 + t^2/[\pi\rho_q(\mu_F)]^2\}^2}$ with the single-particle density of states at the Fermi level $\rho_q(\mu_F)$.

zero temperature, we obtain $G_{\mathrm{an}}/\mathcal{T}^2 \sim 4\pi^2/(\sqrt{3}h)$. Since we apply the effective field theory approach, which inevitably contains the short-range cutoff, it is difficult to *a priori* obtain the quantitative value of the conductance. At the same time, this estimation implies that the bias-independent conductance of the order of $1/h$ arises from the anomalous contribution. In this regime at small bias, it is thus expected that the mass current is dominated by the Ohmic signal originating from the NG modes (see the solid line in Fig. 2).

Next, we note that t_p depends on the interatomic interaction g. By using the gap formula in the BCS theory $\Delta \sim e^{1/[gN(0)]}$ with the density of state at the Fermi level $N(0)$, the pair tunneling amplitude is found to behave as $t_p \sim \frac{e^{2/[gN(0)]}}{g^2}$. This coupling dependence in t_p shows that when increasing the strength of the coupling constant $|g|$, the pair tunneling effect is enhanced at least within the BCS regime. Physically, this means that with increasing $|g|$, the size of the Cooper pairs decreases and the pair tunneling event through the point contact tends to occur. In addition, at the zero-temperature limit where $\rho_s = \rho$, the conductance turns out to be inversely proportional to the speed of sound v. A recent experiment on the Bragg spectroscopy confirms that v is decreased with increasing strength of the coupling $|g|$ [28]. Therefore, as far as the BCS regime is concerned, the anomalous contribution is expected to be enhanced with increasing $|g|$.

In what follows, we address the effects of temperature bias and electric charge.

A. Effect of a temperature bias

We now consider a situation without chemical potential bias but with temperature bias $\Delta T = T_L - T_R$. As in the case of the chemical potential bias, we focus on the regime $\Delta T, T < \Delta$.

First, we discuss the effect of ΔT on the mass current. To this end, we note that the DC pair current expression (28) is available even in the presence of ΔT. This is because in the two-terminal system a temperature is fixed in each reservoir, and the calculations of the correlation functions without the tunneling term discussed in the Appendix are available regardless of the presence or absence of ΔT. Then, up to linear order in ΔT, the pair current can be expressed as

$$\langle I_p \rangle = (L_{12,\mathrm{an}} + L_{12,\mathrm{NG}})\Delta T, \tag{34}$$

where $L_{12,\mathrm{an}}$ and $L_{12,\mathrm{NG}}$ are, respectively, anomalous and normal NG mode contributions similar to Eq. (29). By using the correlation functions obtained in the Appendix, we find

$$L_{12,\mathrm{an}} \propto \int d\omega \, \omega \delta(\omega) = 0. \tag{35}$$

Thus, the anomalous contribution is not induced by ΔT. We note that this is consistent with the result found in the bosonic superfluid point contact [27]. Similarly, normal contribution of the NG modes at a low temperature is obtained as

$$L_{12,\mathrm{NG}} \propto \int d\omega \frac{\omega^3}{\sinh^2(\omega/2T)} \to 0, \tag{36}$$

where we use

$$\frac{\partial n(\omega)}{\partial T} = \frac{\omega}{4T^2 \sinh^2[\omega/(2T)]}. \tag{37}$$

Here, we point out that the absence of $L_{12,\mathrm{NG}}$ is a consequence of the quadratic action (7), which predicts $\mathrm{Im}[g^R(\omega)] \propto \omega$. In general, there must be effects beyond quadratic theory such as interactions between the NG modes leading to the nonzero value of $L_{12,\mathrm{NG}}$. In a low enough temperature where the NG modes are rarely excited, however, such effects are negligible, and the Seebeck effect related to the NG modes is absent.

Next, we discuss the heat current induced by ΔT. In the tunneling Hamiltonian, the heat current operator is calculated as

$$I_Q = -\dot{H}_L = t \sum_\sigma \psi_{\sigma,R}^\dagger \frac{d}{d\tau}\psi_{\sigma,L} + \mathrm{H.c.}, \tag{38}$$

where we use the Heisenberg equation of motion for $\psi_{\sigma,L}$. By using the similar trick used in the mass current, the heat pair current may be obtained as [27]

$$\langle I_{p,Q}(\tau) \rangle = 2 \lim_{\tau' \to \tau} \mathrm{Re}\left[it_p \frac{d}{d\tau} G_{LR}^<(\tau, \tau') \right]. \tag{39}$$

In the frequency space, the expectation above is simplified as

$$\langle I_{p,Q} \rangle = -\int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{2\omega|t_p|^2}{\left|1 - |t_p|^2 g_L^R(\omega)g_R^R(\omega)\right|^2}$$
$$\times \left(\mathrm{Im}[g_L^R(\omega)]\mathrm{Im}[g_R^<(\omega)] + \mathrm{Im}[g_R^A(\omega)]\mathrm{Im}[g_L^<(\omega)] \right). \tag{40}$$

We now explicitly consider the linear regime in ΔT. Then, the heat pair current can be expressed as

$$\langle I_{p,Q} \rangle = (L_{22,\mathrm{an}} + L_{22,\mathrm{NG}})\Delta T. \tag{41}$$

The anomalous contribution of the heat current is shown to vanish, since

$$L_{22,\mathrm{an}} \propto \int d\omega \, \omega^2 \delta(\omega) = 0. \tag{42}$$

On the other hand, the normal contribution of the heat current is given by

$$L_{22,\mathrm{NG}} = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{4\omega|t_p|^2 \mathrm{Im}[g^R(\omega)]^2}{\left|1 - |t_p|^2 g_L^R(\omega)g_R^R(\omega)\right|^2} \left(\frac{\partial n(\omega)}{\partial T} \right)$$
$$\approx \mathcal{T}^2 \frac{4\pi M^2 v^2 T^3}{15\rho_s^2}. \tag{43}$$

The discussions above mean that the thermal conductance induced by the NG modes reduces to $L_{22,\mathrm{NG}}$. Moreover, the corresponding Lorenz number is obtained as

$$L_p \approx \frac{L_{22,\mathrm{NG}}}{T G_{\mathrm{an}}} \approx \frac{2\pi^2 T^2}{15 v \rho_s}. \tag{44}$$

It is notable that the Lorenz number depends on thermodynamic quantities such as temperature, v, and ρ_s. Thus, this is quite different from the Wiedemann-Franz law where the Lorenz number is expressed in terms of fundamental constants in physics such as the Boltzmann constant. As in the case of the bosonic superfluid case, the breakdown of

the Wiedemann-Franz law is attributed to the presence of the anomalous contribution of the mass current that cannot be interpreted as simple quasiparticle tunneling between the reservoirs [27]. Moreover, the expression above shows that the Lorenz number becomes small at a low temperature, which is consistent with the recent experimental observation [29].

B. Current noise

It is interesting to look at the current noise behavior induced by the pair current [30]. The current noise can be introduced as

$$
S_p(\omega) = \int_{-\infty}^{\infty} d\tau \, e^{i\omega\tau} \langle \Delta I_p(\tau) \Delta I_p(0) + \Delta I_p(0) \Delta I_p(\tau) \rangle,
\tag{45}
$$

where $\Delta I_p(\tau) = I_p(\tau) - \langle I_p(\tau) \rangle$. At the level of the approximation used in this work, the current noise is expressed in terms of lesser and greater Green's functions as follows [31]:

$$
S_p(\omega) = \int d\tau \, e^{i\omega\tau} \big[t_p^2 G_{LR}^<(-\tau) G_{LR}^>(\tau) + \bar{t}_p^2 G_{RL}^<(-\tau) G_{RL}^>(\tau)
$$
$$
- |t_p|^2 G_{RR}^<(-\tau) G_{LL}^>(\tau) - |t_p|^2 G_{LL}^<(-\tau) G_{RR}^>(\tau)
$$
$$
+ (\tau \to -\tau) \big].
\tag{46}
$$

We now focus on the white noise limit $\omega = 0$, where a nontrivial relation between the current noise and DC current is expected. Moreover, we can neglect the contribution of the normal tunneling of the NG modes by focusing on the low-temperature regime and keeping only the contribution from the anomalous process. Then, the current noise is reduced to the following simple expression:

$$
S_p(0) \approx 4\mathcal{T}^2 G_{\text{an}} \Delta\mu,
\tag{47}
$$

where we use $\coth(\beta\Delta\mu) \to 1$. Here we note that in the above expression, a contribution proportional to $\mathcal{T}^4$ is absent, which is correct up to $\Delta\mu$. This is in sharp contrast to the shot noise of noninteracting particles where $\mathcal{T}^4$ contributions bringing bunching (antibunching) effects in bosons (fermions) is present.

C. Absence of transport of the Nambu-Goldstone modes in charged superconductors

A fact that the current contributions discussed in this work are negligible in charged superconductors can explicitly be shown as follows. In the presence of the Coulomb interaction, the effective action of the NG modes is modified as [11]

$$
S_{\text{eff}} = \frac{\rho_s}{2\beta\Omega} \sum_k k^2 \left[\frac{\omega_n^2}{v^2 k^2 + \omega_p^2} + 1 \right] \phi(-q) \phi(q),
\tag{48}
$$

where $\sum_k \equiv \sum_{i\omega_n} \sum_{\mathbf{k}}$, $\omega_n = 2\pi n T$ with an integer n, and

$$
\omega_p^2 = \frac{4\pi e^2 n}{M}
\tag{49}
$$

is the plasma frequency. By using the action above, the imaginary time Green's function of the NG mode is obtained as

$$
\mathcal{G}(\omega_n) \equiv -4 \int_0^\beta d\tau \, e^{i\omega_n\tau} \langle T_\tau[\phi(\tau)\phi(0)] \rangle_0
$$
$$
= -\frac{4}{\rho_s \Omega} \sum_{\mathbf{k}} \frac{\omega_p^2 + v^2 k^2}{k^2(\omega_n^2 + \omega_p^2 + v^2 k^2)}.
\tag{50}
$$

As before, by using analytic continuation $i\omega_n \to \omega + i\eta$ with an infinitesimal positive parameter η, the retarded Green's function is obtained as

$$
g^R(\omega) = -\sum_{\mathbf{k}} \frac{2\sqrt{\omega_p^2 + v^2 k^2}}{\rho_s \Omega k^2}
$$
$$
\times \left[\frac{1}{\omega + \sqrt{\omega_p^2 + v^2 k^2} + i\eta} - \frac{1}{\omega - \sqrt{\omega_p^2 + v^2 k^2} + i\eta} \right].
\tag{51}
$$

Therefore, its imaginary term is given by

$$
\text{Im}[g^R(\omega)] = \sum_{\mathbf{k}} \frac{2\pi \sqrt{\omega_p^2 + v^2 k^2}}{\rho_s \Omega k^2}
$$
$$
\times \left[\delta(\omega + \sqrt{\omega_p^2 + v^2 k^2}) - \delta(\omega - \sqrt{\omega_p^2 + v^2 k^2}) \right].
\tag{52}
$$

The expression above implies that the imaginary term vanishes unless $|\omega| \gtrsim \omega_p$. In contrast, the frequency $|\omega| \lesssim \Delta\mu, T$ contributes to the mesoscopic current (28). Since typically $\omega_p \gg \Delta > \Delta\mu, T$, we conclude that the current contributions related to the NG modes are absent in charged superconductors.

VI. SUMMARY

By using the effective theory and tunneling Hamiltonian, we have discussed DC transport of the NG modes in the fermionic superfluid point contact. We have focused on the BCS regime and revealed the anomalous contribution in mass transport, which is the conversion process between the condensate and the NG mode. We also discussed that the anomalous contribution is not present in heat transport, which gives rise to a breakdown of the Wiedemann-Franz law and the absence of the bunching effect in current noise.

In addition to the BCS regime discussed in this work, it was recently shown that the anomalous contribution of the NG modes appears in the Bose-Einstein condensation regime [27]. Therefore, it is reasonable to expect that this contribution exists in the whole range of the BCS-Bose-Einstein condensation crossover. Indeed, the recent experiment on the current-biased Josephson junction implies the occurrence of a non-negligible bias-independent DC conductance at unitarity [32,33]. At the same time, the recent ETH experiment observes a non-negligible Seebeck coefficient [29], which cannot be explained with the processes discussed in this work. In order to address such a regime, therefore, a direct many-body calculation, which includes renormalization of Green's functions and vertex corrections rather than the effective field theory approach may be required.

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APPENDIX: CORRELATION FUNCTIONS

Here, we calculate the correlation functions in the bulk. In the current calculation, the following retarded Green's function is relevant:

$$g^R(\tau) = -i\theta(\tau)\langle [e^{2i\phi(\tau)}, e^{-2i\phi(0)}]\rangle_0. \quad (A1)$$

As far as three-dimensional configurations where phase fluctuations are small are concerned, the correlation function above is approximated as

$$g^R(\tau) \approx -4i\theta(\tau)\langle [\phi(\tau), \phi(0)]\rangle_0. \quad (A2)$$

Similarly, the lesser Green's function that is another relevant quantity in the current calculation is obtained as

$$\begin{aligned} g^<(\tau) &= -i\langle e^{-2i\phi(0)} e^{2i\phi(\tau)}\rangle_0 \\ &\approx -i - 4i\langle\phi(0)\phi(\tau)\rangle_0. \end{aligned} \quad (A3)$$

Thus, up to this order of approximation, what we need to calculate is the correlation functions like $\langle\phi(\pm\tau)\phi(0)\rangle_0$. In order to evaluate these correlations, it is convenient to use the analytic continuation technique of imaginary time Green's functions. To see this, we consider the following imaginary time Green's function:

$$\begin{aligned} \mathcal{G}(\tau) &= -4\langle T_\tau[\phi(\tau)\phi(0)]\rangle_0 \\ &= -\frac{4}{(\beta\Omega)^2 Z}\sum_{k_1,k_2} e^{-i\omega_n\tau}\int \mathcal{D}\phi \ \phi(k_1)\phi(k_2)e^{-S_{\text{eff}}} \\ &= -\frac{4v^2}{\rho_s\beta\Omega}\sum_k \frac{e^{-i\omega_n\tau}}{\omega_n^2 + v^2 k^2}, \end{aligned} \quad (A4)$$

where Z is the partition function, T_τ is the imaginary time-ordering operator [23], and in the second line of equality we use the functional integrals in which time-ordered products are automatically ensured. In the frequency space, the above correlation function is expressed as

$$\mathcal{G}(\omega_n) = \int_0^\beta d\tau \, e^{i\omega_n\tau}\mathcal{G}(\tau) = -\frac{4v^2}{\rho_s\Omega}\sum_{\mathbf{k}}\frac{1}{\omega_n^2 + v^2 k^2}. \quad (A5)$$

Then, the retarded Green's function with real frequency ω can be obtained from analytic continuation $i\omega_n \to \omega + i\eta$ with an infinitesimal positive parameter η. By doing this, we obtain

$$\begin{aligned} g^R(\omega) &= -\frac{4v^2}{\rho_s\Omega}\sum_{\mathbf{k}}\frac{1}{2vk}\left(\frac{1}{vk+\omega+i\eta} + \frac{1}{vk-\omega-i\eta}\right) \\ &= -\frac{1}{v\pi^2\rho_s}\int_0^{v\Lambda} d(vk)\left[\frac{vk}{vk+\omega+i\eta} + \frac{vk}{vk-\omega-i\eta}\right], \end{aligned} \quad (A6)$$

where we introduce the momentum cutoff Λ to restrict the applicability of the tunneling Hamiltonian. By

using

$$\int dx\frac{x}{x\pm A} = x \mp A\ln|x\pm A|, \quad (A7)$$

with a real constant A, we reach

$$g^R(\omega) \approx -\frac{2\Lambda}{\pi^2\rho_s} - \frac{i\omega}{v\pi\rho_s}, \quad (A8)$$

where we neglect $O(\omega^2)$ terms. Similarly, by using $i\omega_n \to \omega - i\eta$, the advanced Green's function is obtained as

$$g^A(\omega) \approx -\frac{2\Lambda}{\pi^2\rho_s} + \frac{i\omega}{v\pi\rho_s}. \quad (A9)$$

We finally calculate the lesser Green's function at real frequency,

$$g^<(\omega) = -2\pi i\delta(\omega) - 4i\int_{-\infty}^\infty d\tau \, e^{i\omega\tau}\langle\phi(0)\phi(\tau)\rangle_0. \quad (A10)$$

In order to obtain a useful expression of the second term in the right-hand side, we consider the spectral representation of the Green's function,

$$\begin{aligned} &-4i\int_{-\infty}^\infty d\tau \, e^{i\omega\tau}\langle\phi(0)\phi(\tau)\rangle_0 \\ &= -\frac{8\pi i}{Z}\sum_{n,m} e^{-\beta E_m}\langle n|\phi|m\rangle\langle m|\phi|n\rangle\delta[\omega + (E_n - E_m)], \end{aligned} \quad (A11)$$

where $\{|n\rangle\}$ or $\{|m\rangle\}$ is the set of the energy eigenstates. Similarly, since

$$\begin{aligned} &-4i\theta(\tau)\langle[\phi(\tau), \phi(0)]\rangle_0 \\ &= -\frac{4i\theta(\tau)}{Z}\sum_{n,m} e^{i(E_n-E_m)\tau}\langle n|\phi|m\rangle\langle m|\phi|n\rangle(e^{-\beta E_n} - e^{-\beta E_m}), \end{aligned} \quad (A12)$$

the spectral representation of the retarded Green's function is given by

$$g^R(\omega) = \frac{4}{Z}\sum_{n,m}\frac{\langle n|\phi|m\rangle\langle m|\phi|n\rangle(e^{-\beta E_n} - e^{-\beta E_m})}{\omega + (E_n - E_m) + i\eta}. \quad (A13)$$

Since its imaginary term is given by

$$\begin{aligned} \text{Im}[g^R(\omega)] = &-\frac{4\pi}{Z}\sum_{n,m} e^{-\beta E_m}\langle n|\phi|m\rangle\langle m|\phi|n\rangle(e^{\beta\omega} - 1) \\ &\times \delta[\omega + (E_n - E_m)], \end{aligned} \quad (A14)$$

we obtain

$$-4i\int_{-\infty}^\infty d\tau \, e^{i\omega\tau}\langle\phi(0)\phi(\tau)\rangle_0 = 2i\,\text{Im}[g^R(\omega)]n(\omega). \quad (A15)$$

In total, the lesser Green's function is obtained as

$$g^<(\omega) = -2\pi i\delta(\omega) - \frac{2i\omega}{v\pi\rho_s}n(\omega). \quad (A16)$$

In a similar manner, the greater Green's function is obtained as

$$g^>(\omega) = -2\pi i\delta(\omega) - \frac{2i\omega}{v\pi\rho_s}[1 + n(\omega)]. \quad (A17)$$

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