

## Effect of Berry phase on nonlinear response of two-dimensional fermions

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We develop a theory of nonlinear response to an electric field of two-dimensional (2D) fermions with topologically nontrivial wave functions characterized by the Berry phase  $\Phi_n = n\pi$ ,  $n = 1, 2, \dots$ . In particular, we find that owing to the suppression of backscattering at odd  $n$ , Hall field-induced resistance oscillations, which stem from elastic electron transitions between Hall field-tilted Landau levels, are qualitatively distinct from those at even  $n$ : Their amplitude decays with the electric field and their extrema are phase shifted by a quarter cycle. The theory unifies the cases of graphene ( $n = 1$ ) and graphite bilayer ( $n = 2$ ) with the case of conventional 2D electron gas ( $n = 0$ ) and suggests another method to probe backscattering in topological 2D systems.

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The topological property of two-dimensional (2D) massless Dirac fermions, expressed in terms of the Berry phase  $\Phi_1 = \pi$ , is responsible for peculiar Landau quantization manifesting itself in the phase-shifted Shubnikov–de Haas oscillations (SdHOs) and unconventional quantum Hall effect [1–4]. This makes the Dirac fermions in graphene fundamentally distinct from both the conventional 2D electron gas (2DEG) in quantum wells ( $\Phi_0 = 0$ ) and topologically nontrivial fermions in graphite bilayers ( $\Phi_2 = 2\pi$ ) [5,6]. Another immediate consequence of the topological nature of wave functions of massless Dirac fermions is the absence of elastic backscattering off scalar potentials, which has various manifestations in electronic properties. In particular, it lies at the origin of Klein tunneling [7] and its implications for the resistivity of graphene-based  $n$ - $p$ - $n$  junctions [8–10]. Furthermore, it removes a sharp cusp of static polarizability of degenerate carriers at doubled Fermi wave number (characteristic for conventional 2DEG and graphite bilayers) [11], leading to the enhanced spatial decay of Friedel oscillations [12].

Another well-known effect which crucially depends on backscattering is Hall field-induced resistance oscillations (HIROs) [13–15] which emerge in the differential resistivity  $r$  of a 2DEG subjected to an elevated current density  $j$  and perpendicular magnetic field  $B$ . These oscillations appear due to the property of enhanced phase space for elastic transitions in 2DEG in the vicinity of backscattering. In the presence of classically strong  $B$ , the backscattering means a spatial shift of the cyclotron orbit guiding center twice the cyclotron radius  $R_c$ , so the transition rate increases each instant when  $2R_c$  equals the integer multiple of spatial separation between Landau levels (LLs) tilted by the electric field  $E$ . Therefore,  $r$  os-

cillates with the dimensionless parameter  $\epsilon = 2R_c|e|E/\hbar\omega_c$ , where  $E \simeq E_H$ ,  $E_H$  is the Hall electric field, and  $\omega_c$  is the cyclotron frequency. As a result, HIROs are periodic in  $1/B$  with the frequency ( $\epsilon \equiv B_1/B$ )

$$B_1 = \frac{8\pi\hbar c}{ge^2v_F}j, \quad (1)$$

where  $v_F$  is the group velocity at the Fermi level and  $g$  is the degree of band degeneracy.

To date, HIROs have been observed only in topologically trivial ( $n = 0$ ) 2D systems based on GaAs/AlGaAs [13,14], Ge/SiGe [16], and ZnO/MgZnO [17] heterostructures. In overlapping LLs, they are described by quantum correction to the differential resistivity  $\delta r$  [15],

$$\frac{\delta r}{\rho_0} = 2\lambda^2\mathcal{H}_0(\epsilon), \quad \mathcal{H}_0(\epsilon) = \frac{8}{\pi} \frac{\tau}{\tau_\pi} \cos 2\pi\epsilon, \quad \pi\epsilon \gg 1, \quad (2)$$

where  $\rho_0$  is the resistivity at  $B = 0$ ,  $\tau$  is the transport scattering time,  $\tau_\pi$  is the backscattering time,  $\lambda = \exp(-\pi/\omega_c\tau_q)$  is the Dingle factor, and  $\tau_q$  is the quantum lifetime. As a result, at  $n = 0$  HIROs are described by  $\mathcal{H}_0(\epsilon)$  whose maxima occur at  $\epsilon = m = 1, 2, 3, \dots$  and whose amplitude is independent of  $\epsilon$ .

In this Rapid Communication we demonstrate that the nonlinear response of topologically nontrivial ( $n \geq 1$ ) 2D systems crucially depends on the parity of  $n$ . For even  $n$ , backscattering is not suppressed and the behavior is qualitatively the same as in conventional 2DEGs. However, suppression of backscattering in massless Dirac systems, as well as in any other system with odd  $n$ , leads to a decay of the HIRO amplitude (as  $1/\epsilon$ ) and produces a characteristic quarter-cycle phase shift of the oscillations towards larger  $\epsilon$ . In particular, for Dirac fermions ( $n = 1$ ) we obtain [see the definition of  $\tau_\pi$  for this case after Eq. (32)]

$$\frac{\delta r}{\rho_0} = 2\lambda^2\mathcal{H}_1(\epsilon), \quad \mathcal{H}_1(\epsilon) = \frac{16}{\pi^2} \frac{\tau}{\tau_\pi} \frac{\sin 2\pi\epsilon}{\epsilon}, \quad \pi\epsilon \gg 1, \quad (3)$$

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which describes HIROs with the frequency given by Eq. (1) and the maxima near  $\epsilon = m + 1/4$ . We also find that the HIRO amplitude increases with  $n$  for odd  $n$ . Our conclusion that the absence of backscattering leads to a phase shift of nonlinear magnetoresistance oscillations can become a basis for another method to probe backscattering in topological 2D systems.

Our theory of nonlinear magnetotransport is developed for the regime of large occupation factors (high LLs), classically strong magnetic fields, and overlapping LLs, which is relevant for the observation of HIROs [18]. We consider spin-degenerate 2D systems described by the Hamiltonian

$$\hat{H} = \epsilon(k) \begin{pmatrix} 0 & e^{-in\varphi} \\ e^{in\varphi} & 0 \end{pmatrix}, \quad (4)$$

where  $\varphi$  is the angle of the wave vector  $\mathbf{k}$ , the energy spectrum  $\pm\epsilon(k)$  is isotropic, and the winding number  $n$  gives the Berry phase  $\Phi_n = n\pi$ . The Hamiltonian  $\hat{H}$  describes fermions in graphene ( $K$  valley) at  $n = 1$  [ $\epsilon(k) = \hbar v_F k$  with constant Fermi velocity  $v_F \simeq 10^8$  cm/s] and in a graphite bilayer at  $n = 2$  [ $\epsilon(k) = (\hbar k)^2/2m^*$  with a constant effective mass  $m^* \simeq 0.037$  of the free-electron mass]. For these particular systems,  $n$  can be viewed as the degree of chirality in the carbon sublattice space [6]. Even larger integers are realizable by the stacking of 3 or more carbon monolayers [19]. Though  $\hat{H}$  formally produces a two-band spectrum, we consider only intraband excitations and the topologically trivial case ( $n = 0$ ) can be equally applied to 2DEG in quantum wells.

To derive expressions for the resistivity, we use Eq. (4) and consider the elastic scattering of fermions by impurities. Adopting the methods developed for 2DEG with a parabolic spectrum [18,20], in particular, using the reference frame moving with the drift velocity [15,20], we can write the steady-state Boltzmann equation for the distribution function  $f_{\epsilon,\varphi}$  of a 2DEG placed in a perpendicular magnetic field  $\mathbf{B} = (0, 0, B)$  and an in-plane electric field  $\mathbf{E} = (E_x, E_y, 0)$  in the following form,

$$\omega_c \frac{\partial f_{\epsilon,\varphi}}{\partial \varphi} = \frac{2\pi}{\hbar} \int_0^{2\pi} \frac{d\varphi'}{2\pi} \int d\epsilon' \delta(\epsilon' - \epsilon - \gamma) \times w_{\mathbf{k}\mathbf{k}'} F_{\mathbf{k}\mathbf{k}'}^{(n)} v_{\epsilon'} (f_{\epsilon',\varphi'} - f_{\epsilon,\varphi}). \quad (5)$$

The right-hand side is the collision integral describing elastic scattering of electrons by impurities within a single valley. Here,  $\gamma = \hbar \mathbf{v}_D \cdot \mathbf{q}$  is the work of the electric force  $|e|\mathbf{E}$  during a scattering event,  $\mathbf{v}_D = c[\mathbf{E} \times \mathbf{B}]/B^2$  is the drift velocity,  $\mathbf{q} = \mathbf{k} - \mathbf{k}'$  is the wave vector transmitted in scattering,  $v_\epsilon$  is the density of states per spin and valley, and  $w_{\mathbf{k}\mathbf{k}'}$  is the Fourier transform of the correlation function of the impurity potential. It is assumed below that  $\gamma \ll \epsilon$  so that  $|\mathbf{k}'| \simeq |\mathbf{k}| \equiv k_\epsilon$ . The cyclotron frequency  $\omega_c = |e|B/m^*c$  is determined by the effective (cyclotron) mass  $m^* = \hbar k_\epsilon/v_F$ , which, in general, is energy dependent (for graphene,  $k_\epsilon = \epsilon/\hbar v_F$ ). The function

$$F_{\mathbf{k}\mathbf{k}'}^{(n)} = F_\theta^{(n)} = (1 + \cos n\theta)/2 \quad (6)$$

is the squared overlap integral of columnar eigenstates of  $\hat{H}$  with wave vectors  $\mathbf{k}$  and  $\mathbf{k}'$ , and  $\theta = \varphi - \varphi'$  is the scattering angle. This function is not sensitive to the sign of  $n$  so that it is the same for different valleys ( $K$  and  $K'$  in graphene), but it is sensitive to the parity of  $n$ . It is seen directly that

backscattering ( $\theta = \pi$ ) is suppressed for odd  $n$  but not for even  $n$ .

For overlapping LLs, the oscillatory density of states at  $\epsilon \gg \hbar\omega_c$  follows from the Bohr-Sommerfeld quantization rule corrected by the Berry phase [21],

$$v_\epsilon \simeq \bar{v}_\epsilon [1 - 2\lambda_\epsilon \cos(\pi k_\epsilon^2 l_B^2 + \Phi_n)], \quad (7)$$

where  $\bar{v}_\epsilon$  is the density of states at  $B = 0$  (for graphene,  $\bar{v}_\epsilon = \epsilon/2\pi \hbar^2 v_F^2$ ),  $l_B = \sqrt{\hbar c/|e|B}$  is the magnetic length, and  $\lambda_\epsilon$  is the Dingle factor at energy  $\epsilon$ . The quantity  $k_\epsilon^2 l_B^2/2$  is the number of magnetic flux quanta inside the cyclotron orbit of electron with energy  $\epsilon$ . Equation (7) can also be derived from the self-consistent Born approximation (for graphene, see, e.g., Ref. [22]). The density of states has maxima at the Landau quantization energies,  $\epsilon_N = \hbar v_F l_B^{-1} \sqrt{2N}$  ( $N = 0, 1, 2, \dots$ ) for graphene and  $\epsilon_N = \hbar\omega_c(N - 1/2)$  for a graphite bilayer. In the latter case,  $\epsilon_N$  approximates the exact spectrum  $\epsilon_N = \hbar\omega_c \sqrt{N(N-1)}$  [6] at  $N \gg 1$ .

The length of  $\mathbf{q}$  and its angle  $\varphi_q$  can be expressed as

$$q = 2k_\epsilon \sin \frac{\theta}{2} \equiv q_{\epsilon,\theta}, \quad \varphi_q = \phi + \pi/2, \quad \phi \equiv \frac{\varphi + \varphi'}{2}, \quad (8)$$

and the work  $\gamma$  can be conveniently rewritten as

$$\gamma = -2\hbar k_\epsilon v_D \cos \phi' \sin \frac{\theta}{2}, \quad \phi' \equiv \phi - \chi, \quad (9)$$

where  $\chi$  is the angle of the electric field  $\mathbf{E}$ . The assumed strong inequality  $\gamma \ll \epsilon$  always holds at  $v_D \ll v_F$ .

Equation (5) is easily solved in the regime of classically strong magnetic fields ( $v_F \tau \gg k_\epsilon l_B^2$ ), when one can replace the distribution functions under the integral by an isotropic distribution  $f_\epsilon$ . Substituting such a solution into the expression for the current density,

$$\mathbf{j} = \mathbf{j}^{(d)} + en_s \mathbf{v}_D, \quad \mathbf{j}^{(d)} = ge \int d\epsilon v_\epsilon \int_0^{2\pi} \frac{d\varphi}{2\pi} \mathbf{v}_F f_{\epsilon,\varphi}, \quad (10)$$

where  $n_s$  is the carrier density, and taking into account that the impurity potential correlator depends only on the absolute value of  $\mathbf{q}$ ,  $w_{\mathbf{k}\mathbf{k}'} = w(q)$ , we obtain the dissipative conductivity ( $\mathbf{j}^{(d)} = \sigma_d \mathbf{E}$ ),

$$\sigma_d = \frac{g\pi \hbar c^2}{B^2} \int d\epsilon k_\epsilon^2 \int_0^{2\pi} \frac{d\varphi'}{2\pi} \int_0^{2\pi} \frac{d\theta}{2\pi} w(q_{\epsilon,\theta}) F_\theta^{(n)} \gamma^{-1} \times (1 + \cos 2\phi')(1 - \cos \theta) v_\epsilon v_{\epsilon+\gamma} (f_\epsilon - f_{\epsilon+\gamma}). \quad (11)$$

The isotropic part of the distribution function standing in Eq. (11) can be represented as a sum of quasiequilibrium Fermi distribution  $f_\epsilon^{(0)} = \{\exp[(\epsilon - \epsilon_F)/k_B T_e] + 1\}^{-1}$ , where  $T_e$  is the temperature of carriers and  $k_B$  is the Boltzmann constant, and a small nonequilibrium contribution  $\delta f_\epsilon$  caused by the field-induced redistribution of electrons in the energy domain in the presence of Landau quantization [15,23]. The function  $\delta f_\epsilon$  shows rapid oscillations similar to those in  $v_\epsilon$ , Eq. (7), and can be found from Eq. (5) averaged over  $\varphi$ . To describe relaxation of the isotropic distribution, the inelastic relaxation term  $-\delta f_\epsilon/\tau_{in}$  ( $\tau_{in}$  is the inelastic relaxation time), approximating the linearized collision integral for electron-electron scattering, should be added to the right-hand side of

Eq. (5). To the first order in  $\lambda_\varepsilon$ ,

$$\delta f_\varepsilon \simeq -\frac{2\lambda_\varepsilon \hbar k_F v_D [\partial \eta_n(\zeta)/\partial \zeta]}{\tau_{in}^{-1} + \tau_q^{-1} - \eta_n(\zeta)} \frac{\partial f_\varepsilon^{(0)}}{\partial \varepsilon} \sin(\pi k_\varepsilon^2 l_B^2 + \Phi_n), \quad (12)$$

where  $k_F = \sqrt{4\pi n_s/g}$  is the Fermi wave number. The quantum and the transport scattering rates are given by

$$\frac{1}{\tau_q} = \int_0^\pi \frac{d\theta}{\pi} \frac{1}{\tau_n(\theta)}, \quad \frac{1}{\tau} = \int_0^\pi \frac{d\theta}{\pi} \frac{1 - \cos \theta}{\tau_n(\theta)}, \quad (13)$$

respectively, with

$$\frac{1}{\tau_n(\theta)} = \frac{2\pi}{\hbar} \frac{1}{v_{\varepsilon_F}} w(q_{\varepsilon_F, \theta}) F_\theta^{(n)}. \quad (14)$$

Next,

$$\eta_n(\zeta) = \int_0^\pi \frac{d\theta}{\pi} \frac{1}{\tau_n(\theta)} J_0 \left( 2\zeta \sin \frac{\theta}{2} \right), \quad (15)$$

where  $J_\alpha$  denotes a Bessel function of the first kind, and

$$\zeta = \pi \varepsilon, \quad \varepsilon = 2k_F^2 l_B^2 \frac{v_D}{v_F} \simeq \frac{8\pi \hbar c}{g e^2 v_F} \frac{j}{B}. \quad (16)$$

Assuming degenerate carriers, we have set  $\varepsilon = \varepsilon_F$ ,  $k_\varepsilon = k_F$  in all quantities whose energy dependence is weak.

Substituting  $f_\varepsilon = f_\varepsilon^{(0)} + \delta f_\varepsilon$  into Eq. (11) and calculating the integrals over  $\varepsilon$  and  $\phi'$  analytically, we represent the result as an expansion in powers of the Dingle factor,

$$\sigma_d = \sigma_d^{(0)} + \sigma_d^{(1)} + \sigma_d^{(2)}, \quad (17)$$

where  $\sigma_d^{(0)} = n_s \hbar k_F c^2 / v_F \tau B^2$  is the Drude conductivity and

$$\sigma_d^{(1)} = -\sigma_d^{(0)} 4\lambda \mathcal{D}_T \cos(\pi k_F^2 l_B^2 + \Phi_n) s_n(\zeta) \quad (18)$$

is the term describing SdHOs, where  $\mathcal{D}_T = X_T / \sinh X_T$ , with  $X_T = 2\pi^2 k_B T_e k_F l_B^2 / \hbar v_F$ . The influence of the electric field on SdHOs is described by

$$s_n(\zeta) = -\frac{\tau}{\zeta} \frac{\partial \eta_n(\zeta)}{\partial \zeta}, \quad (19)$$

as  $\delta f_\varepsilon$  does not contribute to  $\sigma_d^{(1)}$  [24]. SdHOs are strongly suppressed by temperature at  $X_T \gg 1$ , owing to rapid oscillations of  $v_\varepsilon$ . In the last term,  $\sigma_d^{(2)}$ , we retain only the contributions that survive at  $X_T \gg 1$ . We find

$$\sigma_d^{(2)} = \sigma_d^{(0)} 2\lambda^2 [h_n(\zeta) + g_n(\zeta)], \quad (20)$$

where the parts

$$h_n(\zeta) = -\tau \frac{\partial^2 \eta_n(\zeta)}{\partial \zeta^2} \quad (21)$$

and

$$g_n(\zeta) = -\frac{2\tau [\partial \eta_n(\zeta)/\partial \zeta]^2}{\tau_{in}^{-1} + \tau_q^{-1} - \eta_n(\zeta)}, \quad (22)$$

respectively, come from substitution of  $f_\varepsilon^{(0)}$  and  $\delta f_\varepsilon$  into Eq. (11) and are often referred to as the displacement and the inelastic contributions.

For  $n = 0$ , the results given by Eqs. (15)–(22) reproduce those obtained previously for the 2DEG with parabolic spectrum [15,24]. However, they now carry important topological distinction between even and odd  $n$  because the field-dependent quantities  $\eta$ ,  $s$ ,  $h$ , and  $g$  depend on the Berry phase

through the overlap factor  $F_\theta^{(n)}$  in Eq. (14). To demonstrate this distinction, we consider the case of a sharp ( $\delta$ -correlated) scattering potential, for which  $w(q_{\varepsilon_F, \theta})$  is constant and  $\tau/\tau_q = 1 + \delta_{n,1}$ , and the integral in Eq. (15) can be evaluated analytically [below,  $J_\alpha \equiv J_\alpha(\zeta)$ ],

$$\eta_n(\zeta) = (J_0^2 + J_n^2)/\tau \xi_n, \quad \xi_n = 1 + \delta_{n,0} - \delta_{n,1}/2. \quad (23)$$

We further find

$$s_n(\zeta) = [2J_0 J_1 + (J_{n+1} - J_{n-1}) J_n] / \zeta \xi_n, \quad (24)$$

$$h_n(\zeta) = [J_0^2 - 2J_1^2 - J_0 J_2 - (J_{n+1} - J_{n-1})^2 / 2 - J_n (J_{n-2} - 2J_n + J_{n+2}) / 2] / \xi_n, \quad (25)$$

and

$$g_n(\zeta) = -\frac{2[\zeta s_n(\zeta)]^2}{\tau/\tau_{in} + 1 + \delta_{n,1} - (J_0^2 + J_n^2)/\xi_n}. \quad (26)$$

In the weak-field limit,  $\zeta \ll 1$ , the above expressions describe quadratic in  $E$  corrections to the conductivity. In particular,  $s_n = 1 - \mu_n \zeta^2$  and  $h_n = 1 - 3\mu_n \zeta^2$ , where  $\mu_0 = 3/8$ ,  $\mu_1 = 1/4$ ,  $\mu_2 = 7/16$ , and  $\mu_n = \mu_0$  for  $n > 2$ . Remarkably, the function  $g_n$  in this limit does not depend on  $n$ :  $g_n = -2\zeta^2/(\tau/\tau_{in} + \zeta^2/2)$ . If the inelastic relaxation is slow,  $\tau/\tau_{in} \ll 1$ , this function gives the main contribution to the weak-field nonlinear response in  $\sigma^{(2)}$ .

In the strong-field limit,  $\zeta \gg 1$ , Eqs. (24)–(26) describe field-induced oscillations of conductivity caused by transitions of carriers between different LLs. The result depends on the parity of  $n$  in an essential way because the influence of backscattering on the conductivity becomes important. For even  $n$ ,

$$s_n(\zeta) = -\frac{4 \cos 2\zeta}{\pi \zeta^2 (1 + \delta_{n,0})}, \quad h_n(\zeta) = \frac{8 \sin 2\zeta}{\pi \zeta (1 + \delta_{n,0})},$$

$$g_n(\zeta) = -\frac{16(1 + \cos 4\zeta)}{\pi^2 \zeta^2 (\tau/\tau_{in} + 1)(1 + \delta_{n,0})^2}, \quad (27)$$

whereas for odd  $n$ ,

$$s_n(\zeta) = \frac{2(1 - n^2 \sin 2\zeta)}{\pi \zeta^3 (1 - \delta_{n,1}/2)}, \quad h_n(\zeta) = -\frac{4n^2 \cos 2\zeta}{\pi \zeta^2 (1 - \delta_{n,1}/2)},$$

$$g_n(\zeta) = -\frac{8(1 + \delta_{n,1})^2 (1 - n^2 \sin 2\zeta)^2}{\pi^2 \zeta^4 (\tau/\tau_{in} + 1 + \delta_{n,1})}. \quad (28)$$

For odd  $n$ , all these quantities decrease with  $\zeta$  faster than for even  $n$  and the oscillations gain in amplitude with increasing  $n$ . Regardless of the parity of  $n$ , the main contribution to the strong-field response comes from  $h_n$ . This contribution describes HIROs, whose behavior in conventional 2DEG and in a graphite bilayer ( $n = 2$ ) is predicted to be qualitatively the same, while in graphene ( $n = 1$ ) their behavior is essentially different. Application of the theory to carbon multilayers with  $n > 2$ , however, is limited because Eq. (4) will be valid for an ever reduced range of energies as  $n$  increases. Moreover, any perturbation that involves a potential difference between the layers will renormalize the eigenstates, thereby leading to the case when the Berry phase is no longer an integer multiple of  $\pi$ . Consideration of this case is beyond the scope of this Rapid Communication.

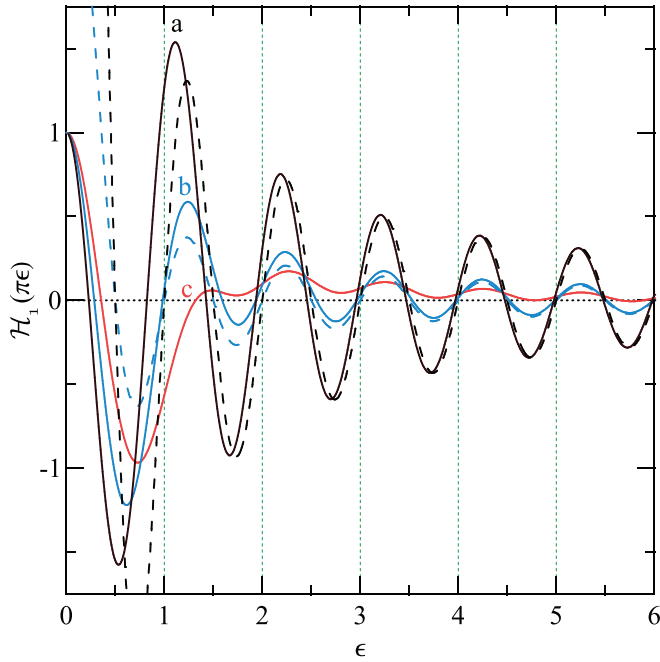


FIG. 1. Function  $\mathcal{H}_1(\pi\epsilon)$  for a sharp scattering potential (solid curve a), and for a smooth scattering potential modeled by  $w(q) \propto e^{-\alpha q}$ , where  $\alpha$  is chosen to provide  $\tau/\tau_q = 5$  (solid curve b) and  $\tau/\tau_q = 10$  (solid curve c). The dashed lines represent the limit  $\pi\epsilon \gg 1$  for curves a and b.

We next focus on Dirac fermions,  $n = 1$ . Using Eq. (17), we obtain the longitudinal resistivity,

$$\frac{\rho}{\rho_0} = 1 + 4\lambda \mathcal{D}_T \cos(\pi k_F^2 l_B^2) s_1(\zeta) + 2\lambda^2 [h_1(\zeta) + g_1(\zeta)],$$

$$\rho_0 = \frac{\hbar k_F}{e^2 n_s v_F \tau}. \quad (29)$$

In experiments, one usually measures the differential resistivity  $r \equiv \partial(j\rho)/\partial j$ . Then,  $r = \rho_0 + \delta r$ , where

$$\frac{\delta r}{\rho_0} = 4\lambda \cos(\pi k_F^2 l_B^2) \{[\partial(j\mathcal{D}_T)/\partial j] s_1(\zeta) + \mathcal{D}_T \mathcal{S}_1(\zeta)\} + 2\lambda^2 [\mathcal{H}_1(\zeta) + \mathcal{G}_1(\zeta)], \quad (30)$$

with  $\mathcal{S}_1(\zeta) \equiv \partial[\zeta s_1(\zeta)]/\partial \zeta$ ,  $\mathcal{H}_1(\zeta) \equiv \partial[\zeta h_1(\zeta)]/\partial \zeta$ ,  $\mathcal{G}_1(\zeta) \equiv \partial[\zeta g_1(\zeta)]/\partial \zeta$ . In the limiting case of a sharp scattering potential, we find  $s_1(\zeta) = [J_0 + J_2]^2$  and

$$\mathcal{S}_1(\zeta) = [J_0 + J_2][J_0 - 3J_2],$$

$$\mathcal{H}_1(\zeta) = J_0^2 - 6J_1^2 + 5J_2^2 - 2J_0J_2 + 2J_1J_3. \quad (31)$$

If  $\zeta \gg 1$ , analytical results are obtained for an arbitrary correlator of the impurity potential,

$$\mathcal{S}_1(\zeta) \approx -\frac{\tau}{\tau_\pi} \frac{8 \cos 2\zeta}{\pi \zeta^2}, \quad \mathcal{H}_1(\zeta) \approx \frac{\tau}{\tau_\pi} \frac{16 \sin 2\zeta}{\pi \zeta}, \quad (32)$$

where  $\tau_\pi$  differs from  $\tau$  by the substitution  $w(q_{\epsilon_F, \theta}) \rightarrow w(q_{\epsilon_F, \pi})$  in Eq. (14). The function  $\mathcal{H}_1(\zeta)$  and its high-field asymptote, given by Eq. (32), are shown in Fig. 1 by solid and dashed curves “a,” respectively. Notice that for the sharp

scattering potential ( $\tau/\tau_\pi = 1$ ) Eq. (32) is an excellent approximation already at  $\epsilon \gtrsim 1$ . For comparison we also include curves “b” and “c,” which represent different cases of smooth disorder (see the figure caption).

The oscillations of  $\mathcal{H}_1(\zeta)$  describe HIROs for massless Dirac fermions, Eq. (3), which differ from HIROs for conventional 2DEG, Eq. (2), by a phase shift and by a decay  $1/\epsilon$ . These changes are the consequences of modification of the angular dependence of scattering probability and of its considerable reduction in the vicinity of backscattering ( $\theta \simeq \pi$ ), owing to the factor  $F_\theta^{(1)}$ . A similar phase shift and a stronger decay, for the same reasons, are also present in  $\mathcal{S}_1(\zeta)$  describing the field effect on SdHOs. It is unlikely, however, that  $\mathcal{S}_1(\zeta)$  can be accessed experimentally because heating of electrons by current, leading to a dependence of  $T_e$  on  $j$ , becomes important already at  $\zeta \ll 1$  and the nonlinearity associated with the first term in braces of Eq. (30) prevails [25]. We note that a decay of HIROs obtained in earlier numerical calculations of nonlinear resistivity in graphene [26,27] has also been attributed to electron heating, whereas the topological relevance of this effect has been overlooked.

In contrast to HIROs, the weak-field ( $\zeta \ll 1$ ) nonlinear response of Dirac fermions, dominated (in the absence of SdHOs) by  $\mathcal{G}_1(\zeta)$ , is very similar to the one for conventional 2DEG. This response is rather insensitive to the smoothness of the disorder, but sensitive to the ratio  $\tau/\tau_{\text{in}}$ . At  $\epsilon > 1$ , the function  $\mathcal{G}_1(\zeta)$  rapidly decreases as  $\epsilon^{-3}$ , much faster than  $\epsilon^{-1}$  in conventional 2DEG.

It is worth noting that there exists another phenomenon which crucially depends on backscattering, the magnetophonon oscillations of linear resistance due to the interaction of 2DEG with acoustic phonons [also known as the phonon-induced resistance oscillations (PIROs)] [28]. Recent observations of this phenomenon in graphene [29,30] show that PIROs *do not* shift their phase and behave just as those in conventional 2DEG with a parabolic band. This happens because the electron-phonon interaction in graphene is dominated by the gauge-field mechanism for which the interaction potential is not a scalar in the sublattice space and, as a result, backscattering is not suppressed. The microwave-induced resistance oscillations [31], which have not yet been observed in graphene, are not expected to change their phase either, as they are not sensitive to backscattering. Therefore, among the magneto-oscillatory phenomena specific for 2DEG in the regime of high LLs [18] only HIROs are expected to show profound changes in graphene, which makes them a special and promising tool for the experimental probing of backscattering.

In summary, we have developed a theory of nonlinear magnetoresistance for 2D fermions with a Berry phase  $\Phi_n = n\pi$ , based on a model unifying the conventional 2DEG, massless Dirac fermions in graphene, and fermions in a graphite bilayer. We have shown that the amplitude and the phase of nonlinear magnetoresistance oscillations of degenerate 2D fermion gas crucially depends on the parity of  $n$ . Such a distinction is a consequence of the suppression of backscattering off an impurity potential in the case of odd  $n$ . We believe that our results will stimulate nonlinear magnetotransport experiments in graphene and in other topological materials.

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