Rapid Communications

Nonlinear evolution and signaling

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We propose a condition, called convex quasilinearity, for deterministic nonlinear quantum evolutions. Evolutions satisfying this condition do not allow for arbitrary fast signaling, therefore, they cannot be ruled out by a standard argument. We also give an explicit example of a nonlinear qubit evolution satisfying quasilinearity.

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I. INTRODUCTION

For almost a century, quantum mechanics (QM), in a version based on the Hilbert space, was formulated as a linear theory preserving the superposition principle for pure states. However, many authors for different reasons undertook attempts to generalize this theory by also including nonlinear operations (see, e.g., Refs. [1–5] and references therein). In fact, a part of the QM formalism related to the measurement description uses nonlinear (stochastic) operations such as the selective projection postulate. Most of the proposed attempts lie in the replacement of the linear time evolution of quantum systems by a nonlinear one (see, e.g., Refs. [1,3]). It is widely believed that such deterministic nonlinear generalizations of the Schrödinger equation allow for signaling, i.e., allows one to send signals over arbitrarily large distances in a finite time (see, e.g., Ref. [6]). Arguments supporting this claim (in the context of the Weinberg model [3]) were clearly given by Gisin in Ref. [7] (compare also Refs. [8,9]). Gisin's arguments are based on the observation that deterministic nonlinear time evolution destroys the equivalence of quantum ensembles defining the same mixed state of the considered system. As a consequence, it creates the possibility of an instantaneous communication for spacelike separated observers with the help of systems of entangled particles. Evidently such a possibility is in an apparent conflict with the special relativity.

Let us mention here that some authors gave arguments that deterministic nonlinear dynamics in special circumstances does not allow for signaling [8,10–15]. However, these arguments seems to be insufficient (compare Refs. [16,17]). For example, the Czachor and Doebner [11] approach implies the modification of the state reduction postulate while Helou and Chen [15] postulate the extension of the Born rule. Our goal is different: We are looking for such an extension of deterministic linear dynamics that does not allow for faster than light signaling and at the same time does not require anything to be changed in the remaining part of the quantum formalism.

Notice also that there exist nonlinear stochastic evolution equations free of the problems with signaling [6]. Such models were proposed in various contexts, with one of the most important being collapse models (see Refs. [18,19] and references therein).

In this Rapid Communication we propose another condition for deterministic nonlinear quantum evolutions—quasilinearity [Eq. (7)]. This condition guarantees that evolution preserves the equivalence of quantum ensembles. Consequently, the Gisin argument [7] does not work in this case.

II. GISIN'S ARGUMENT

Following Gisin [7], let us assume that two distant observers, say, A and B, want to establish instantaneous communication. In the half of the distance between them there is a source emitting pairs of spin-1/2 particles. The initial spin state of the particle is the Bell state. Particles move along the z axis, one towards A and the second one towards B. Observer A performs a polarization procedure of the flux of particles directed to him. To this end, A measures without selection the projector $\pi_\varphi = \frac{1}{2}(I + \boldsymbol{\zeta}_\varphi \cdot \boldsymbol{\sigma}) \otimes I$ with a possibility of changing the polarization angle φ. Here, the polarization vector

$$\boldsymbol{\zeta}_\varphi = (\cos\varphi, \sin\varphi, 0). \tag{1}$$

As an effect, the Bell state converts into the mixture

$$\rho_\varphi = \pi_\varphi \rho_{\text{Bell}} \pi_\varphi + (I - \pi_\varphi)\rho_{\text{Bell}}(I - \pi_\varphi) \equiv \tfrac{1}{2}\rho_1 + \tfrac{1}{2}\rho_2. \tag{2}$$

Consequently, the reduced density matrix $\rho_B = \text{Tr}_A(\rho_\varphi)$, accessible to observer B, has the form of an appropriate ensemble,

$$\rho_B = \tfrac{1}{2}\rho_{B_\varphi^1} + \tfrac{1}{2}\rho_{B_\varphi^2} = \tfrac{1}{2}I, \tag{3}$$

with

$$\rho_{B_\varphi^1} = \tfrac{1}{2}[I - (\boldsymbol{\zeta}_\varphi \cdot \boldsymbol{\sigma})], \tag{4}$$

$$\rho_{B_\varphi^2} = \tfrac{1}{2}[I + (\boldsymbol{\zeta}_\varphi \cdot \boldsymbol{\sigma})]. \tag{5}$$

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The ensemble accessible to observer B is equivalent to the fully depolarized state. Thus, any quantum-mechanical measurement of observer B is incapable of distinguishing between different ensembles of this form for different values of the polarization angle φ. Consequently, arbitrary fast communication is excluded by the QM rules.

However, according to Gisin's gedanken prescription, a selective measurement by observer B can be prefaced by a nonlinear evolution (as the Weinberg nonlinear evolution [3]) applied to the reduced density ρ_B and to the related ensembles. Gisin shows that the Weinberg nonlinear time evolution does not respect the equivalence of ensembles so the evolved ensemble is polarization dependent ($\boldsymbol{\zeta}_\varphi$ dependent). Consequently, changes in the polarizations made by observer A can be registered by observer B, i.e., instantaneous communication is possible. Thus, to avoid contradiction with relativity, nonlinear evolution must be ruled out of the QM formalism.

We show that this argument does not work for nonlinear evolutions satisfying the quasilinearity condition.

III. THE CONVEX QUASILINEAR MAP

Let us consider ensembles of the form

$$\lambda\rho_a + (1-\lambda)\rho_b \equiv \rho, \tag{6}$$

where $0 \leqslant \lambda \leqslant 1$ and ρ_a, ρ_b belong to the convex set S of density matrices [$S \subset \mathrm{End}(\mathcal{H})$, $\dim \mathcal{H} = n < \infty$]. A trace-preserving map Φ in S is convex quasilinear if for each non-negative $\lambda \leqslant 1$ and for arbitrary density operators ρ_a, ρ_b there exists such $\bar{\lambda}$, $0 \leqslant \bar{\lambda} \leqslant 1$, that it holds,

$$\Phi[\rho] = \Phi[\lambda\rho_a + (1-\lambda)\rho_b] = \bar{\lambda}\Phi[\rho_a] + (1-\bar{\lambda})\Phi[\rho_b]. \tag{7}$$

This class of quantum operations also contains linear maps. The essence of this map is that it transforms convex combinations of density operators into convex combinations of their images so it preserves the convex structure of S. Moreover, it preserves the equivalence of ensembles related to each fixed density matrix. Indeed, let $\rho = \lambda\rho_a + (1-\lambda)\rho_b$ and $\rho = p\rho_A + (1-p)\rho_B$, where $0 \leqslant \lambda$ and $p \leqslant 1$ are two decompositions of the density matrix ρ. Now, applying a convex quasilinear map Φ, we obtain $\Phi[\rho] = \Phi[\lambda\rho_a + (1-\lambda)\rho_b] = \bar{\lambda}\Phi[\rho_a] + (1-\bar{\lambda})\Phi[\rho_b]$ and $\Phi[\rho] = \Phi[p\rho_A + (1-p)\rho_B] = \bar{p}\Phi[\rho_A] + (1-\bar{p})\Phi[\rho_B]$. But Φ is defined on the whole set S, consequently both of these expressions are well defined and are equal since equality is transitive. Therefore, Φ gives the same result when applied to these decompositions.

Usually, it is assumed that deterministic quantum evolution preserves mixtures; it is realized by the requirement of the linearity of Φ [13,20–23]. However, in our opinion, this assumption is too restrictive and can be generalized to condition (7). In fact, standard quantum-mechanical selective measurements belong to the class of quasilinear transformations. Indeed, let us define the trace-preserving nonlinear map corresponding to a selective measurement, $\Phi[\rho] = \frac{\Pi\rho\Pi}{\mathrm{Tr}(\Pi\rho\Pi)}$, where Π is a projector. Applying Φ to the density matrix (6), we get

$$\begin{aligned}\Phi[\rho] &= \frac{\lambda\Pi\rho_a\Pi + (1-\lambda)\Pi\rho_b\Pi}{\mathrm{Tr}(\Pi\rho)} \\ &= \lambda\frac{\mathrm{Tr}(\Pi\rho_a)}{\mathrm{Tr}(\Pi\rho)}\Phi[\rho_a] + (1-\lambda)\frac{\mathrm{Tr}(\Pi\rho_b)}{\mathrm{Tr}(\Pi\rho)}\Phi[\rho_b].\end{aligned} \tag{8}$$

Now, Eq. (8) has the form (7) provided that

$$1 - \lambda\frac{\mathrm{Tr}(\Pi\rho_a)}{\mathrm{Tr}(\Pi\rho)} = (1-\lambda)\frac{\mathrm{Tr}(\Pi\rho_b)}{\mathrm{Tr}(\Pi\rho)}. \tag{9}$$

But using (6) one can easily verify that (9) really holds. Therefore, Eq. (8) can be cast in the form (7) with

$$\bar{\lambda} = \lambda\frac{\mathrm{Tr}(\Pi\rho_a)}{\mathrm{Tr}(\Pi\rho)} \tag{10}$$

and $0 \leqslant \bar{\lambda} \leqslant 1$. From a technical point of view there is a problem with the definition of Φ in the case when $\mathrm{Tr}(\Pi\rho) = 0$. In standard discussions of the state reduction postulate this is not a problem because the state $\Phi[\rho]$ after the measurement is realized with the probability $p = \mathrm{Tr}(\Pi\rho)$. Consequently, when $\mathrm{Tr}(\Pi\rho) = 0$, then the state $\frac{\Pi\rho\Pi}{\mathrm{Tr}(\Pi\rho\Pi)}$ is not realized. In our example we want to have Φ defined on the whole space of states, thus it should be defined also in the case when $\mathrm{Tr}(\Pi\rho) = 0$. In such a case we set $\Phi[\rho] = \frac{\Pi}{\mathrm{Tr}\,\Pi}$. Equations (8)–(10) are consistent with this definition.

Notice that, in a special case when Π is a rank one projector, then $\Phi[\rho] = \Pi$ for all ρ. In such a situation Φ is convex linear but in a somewhat trivial way: $\Phi[\rho] = \lambda\Phi[\rho_a] + (1-\lambda)\Phi[\rho_b] = \lambda\Pi + (1-\lambda)\Pi = \Pi$, for all λ, ρ_a and ρ_b, i.e., the result of $\Phi[\rho]$ has no dependence on ρ. However, in a general case, $\Phi[\rho] = \frac{\Pi\rho\Pi}{\mathrm{Tr}(\Pi\rho\Pi)}$ is nonlinear but convex quasilinear.

The convex quasilinearity of some classes of quantum operations was noted by Kraus in Ref. [24] (although Kraus did not use this name). In fact, the applicability of the selective measurements to the quantum mechanics is strongly related to the quasilinearity property of this stochastic operation. Indeed, it implies that the set of ensembles representing a fixed density matrix ρ is mapped on the set of ensembles representing $\Phi(\rho)$.

IV. NONLINEAR EVOLUTION SATISFYING THE QUASILINEARITY CONDITION

In view of the above discussion it seems that there are no objections to consider deterministic convex quasilinear evolutions. Such an evolution map $\rho(t) = \Phi_t[\rho_0]$, with the initial condition $\rho(0) = \rho_0$, should form a semigroup satisfying relation (7) for each value of the time parameter t; namely, if

$$\lambda\rho_{a0} + (1-\lambda)\rho_{b0} = \rho_0, \tag{11}$$

then

$$\Phi_t[\lambda\rho_{a0} + (1-\lambda)\rho_{b0}] = \lambda(t)\Phi_t[\rho_{a0}] + [1-\lambda(t)]\Phi_t[\rho_{b0}], \tag{12}$$

with the conditions $\lambda(0) = \lambda$ and $0 \leqslant \lambda(t) \leqslant 1$. To show that the set of deterministic convex quasilinear evolutions is nonempty, we will construct a simple model of a qubit evolution satisfying (12).

To define the evolution of a qubit, we should determine the nonlinear evolution of its Bloch vector. In the construction of our model we use the well-known transformation rule for a three-velocity $\mathbf{v}$ under Lorentz boosts in a given direction, say, $\mathbf{e}$ ($|\mathbf{e}| = 1$),

$$\mathbf{v}' = \frac{\mathbf{v} + \mathbf{e}[\sinh\eta + (\cosh\eta - 1)(\mathbf{e}\cdot\mathbf{v})]}{\cosh\eta + (\mathbf{e}\cdot\mathbf{v})\sinh\eta}, \tag{13}$$

where η is the rapidity (we work in natural units with $\hbar = c = 1$). The above transformation (13) is one of the standard examples of nonlinear transformations appearing in physics.

Let us notice two obvious facts: (i) Boosts in a given direction form a one-parameter subgroup of the Lorentz group, and (ii) the length of a three-velocity is always less than or equal to 1.

Observation (i) allows us to treat the transformation rule (13) as an equation defining the time evolution of a three-vector. Namely, we can rewrite Eq. (13) as

$$\mathbf{n}(t) = \frac{\boldsymbol{\xi} + \mathbf{e}[\sinh(gt) + [\cosh(gt) - 1](\mathbf{e}\cdot\boldsymbol{\xi})]}{\cosh(gt) + (\mathbf{e}\cdot\boldsymbol{\xi})\sinh(gt)}, \tag{14}$$

where the constant g has been introduced for dimensional reasons and $\mathbf{n}(0) \equiv \boldsymbol{\xi}$ and $\boldsymbol{\xi}^2 \leqslant 1$. Now, point (i) implies that if we write $\mathbf{n}(t) = f_t(\boldsymbol{\xi})$, then $f_{t_2} \circ f_{t_1} = f_{t_1+t_2}$, therefore, (14) is a nonlinear time evolution of $\mathbf{n}(t)$.

Next, point (ii) gives us the possibility of identifying the vector $\mathbf{n}(t)$ with the Bloch vector defining a qubit density matrix $\rho(t)$,

$$\rho(t) = \tfrac{1}{2}[I + \mathbf{n}(t)\cdot\boldsymbol{\sigma}]. \tag{15}$$

It means that the evolution f_t corresponds to a nonlinear evolution of a qubit density matrix,

$$\rho(t) = \Phi_t[\rho_0], \tag{16}$$

where $\rho_0 = \frac{1}{2}(I + \boldsymbol{\xi}\cdot\boldsymbol{\sigma})$. Point (ii) implies that the condition $\mathbf{n}(t)^2 \leqslant 1$ is preserved for all t. Because the magnitude of a unit vector does not change under the evolution (14), the subset of pure states is invariant under the evolution Φ_t.

Moreover, we can easily check that $\mathbf{n}(t) \to \mathbf{e}$ in the limit $t \to \infty$. Therefore, under the evolution (14) mixed states evolve into pure states. In such cases the von Neumann entropy of the state (15) decreases during the evolution. This observation, together with the second law of thermodynamics, suggest that a carrier physical system (e.g., a spin-1/2 particle or a two-level atom) of the qubit evolving according to Eq. (14) cannot be isolated. Instead, it should be treated as an open quantum system interacting with an environment. Of course, it does not exclude the use of the evolution (14) in the considered Gisin gedanken experiment.

Let us notice that the Bloch vector (14) is a solution of the following nonlinear differential equation,

$$\dot{\mathbf{n}} = g(\mathbf{e} - \mathbf{n}(\mathbf{e}\cdot\mathbf{n})), \tag{17}$$

under the initial condition $\mathbf{n}(0) = \boldsymbol{\xi}$.

Now, using the evolution (14), we can show that if Eq. (11) holds, then

$$\rho(t) = \lambda(t)\rho_a(t) + [1 - \lambda(t)]\rho_b(t), \tag{18}$$

where

$$\rho_a(t) = \tfrac{1}{2}[I + \mathbf{n}_a(t)\cdot\boldsymbol{\sigma}], \quad \rho_b(t) = \tfrac{1}{2}[I + \mathbf{n}_b(t)\cdot\boldsymbol{\sigma}], \tag{19}$$

and both Bloch vectors $\mathbf{n}_a$ and $\mathbf{n}_b$ evolve according to the nonlinear law (14) under the replacement $\boldsymbol{\xi} \to \boldsymbol{\xi}_a$ or $\boldsymbol{\xi}_b$, respectively. Using Eqs. (19), (18), and (14) we can find the coefficient $\lambda(t)$. It is given by

$$\lambda(t) = \frac{1 + (\mathbf{e}\cdot\boldsymbol{\xi}_a)\tanh(gt)}{1 + (\mathbf{e}\cdot\boldsymbol{\xi})\tanh(gt)}\lambda. \tag{20}$$

Therefore, we can conclude that each ensemble (11) equivalent to ρ_0 evolves under prescription (14) into ensemble (18) equivalent to $\rho(t)$. We see that the coefficient $\lambda(t)$ explicitly depends on time. However, in view of our previous remark that a qubit evolving according to Eq. (14) cannot be treated as an isolated system, the dependence of λ on time is rather expected.

Returning to the Gisin gedanken experiment, the ensemble $\rho_B = \mathrm{Tr}_A(\rho_\varphi)$, accessible to observer B [Eq. (3)], evolves under (14) as follows,

$$\rho_B(t) = \tfrac{1}{2}[I + (\mathbf{e}\cdot\boldsymbol{\sigma})\tanh(gt)], \tag{21}$$

and

$$\rho_{B^1_\varphi}(t) = \frac{1}{2}\left(I + \frac{-\boldsymbol{\zeta}_\varphi + [\sinh(gt) - (\mathbf{e}\cdot\boldsymbol{\zeta}_\varphi)[\cosh(gt) - 1]]\mathbf{e}}{\cosh(gt) - (\mathbf{e}\cdot\boldsymbol{\zeta}_\varphi)\sinh(gt)}\cdot\boldsymbol{\sigma}\right), \tag{22}$$

$$\rho_{B^2_\varphi}(t) = \frac{1}{2}\left(I + \frac{\boldsymbol{\zeta}_\varphi + [\sinh(gt) + (\mathbf{e}\cdot\boldsymbol{\zeta}_\varphi)[\cosh(gt) - 1]]\mathbf{e}}{\cosh(gt) + (\mathbf{e}\cdot\boldsymbol{\zeta}_\varphi)\sinh(gt)}\cdot\boldsymbol{\sigma}\right), \tag{23}$$

where $\boldsymbol{\zeta}_\varphi$ is given in Eq. (1). So, using (20), we find that in this case

$$\lambda(t) = \tfrac{1}{2}[1 - (\mathbf{e}\cdot\boldsymbol{\zeta}_\varphi)\tanh(gt)]. \tag{24}$$

Therefore, finally, it really holds that

$$\lambda(t)\rho_{B^1_\varphi}(t) + [1 - \lambda(t)]\rho_{B^2_\varphi}(t) = \tfrac{1}{2}[I + (\mathbf{e}\cdot\boldsymbol{\sigma})\tanh(gt)] = \rho_B(t). \tag{25}$$

Thus, observer B cannot register any change of the polarization by observer A, exactly as in the standard case.

We can notice that the differential equation for the Bloch vector (17) resembles the equation for the Bloch vector in the simplified Weinberg model, discussed in Ref. [7], adapted to the qubit case,

$$\dot{\mathbf{n}} = g(\mathbf{e} \times \mathbf{n}(\mathbf{e}\cdot\mathbf{n})). \tag{26}$$

The solution of the last equation reads

$$\mathbf{n}(t) = \boldsymbol{\xi}\cos\theta(t) + (\mathbf{e}\times\boldsymbol{\xi})\sin\theta(t) + \mathbf{e}(\mathbf{e}\cdot\boldsymbol{\xi})[1 - \cos\theta(t)], \tag{27}$$

where $\theta(t) = gt(\mathbf{e}\cdot\boldsymbol{\xi})$ and $\mathbf{n}(0) = \boldsymbol{\xi}$. However, it can be explicitly shown that in general solution (27) does not satisfy the quasilinearity property (18). Consequently, evolution in the Weinberg model does not preserve the equivalence of ensembles and allows for arbitrary fast signaling.

V. CONCLUSIONS

We have shown that time evolutions satisfying the quasilinearity property (12) are admissible in the convex set of density operators even if they are nonlinear. As an example, we discussed the nonlinear time evolution of a qubit explicitly satisfying this property and we applied it to the famous gedanken nonlocal correlation experiment by Gisin [7]. We explicitly showed that this evolution does not allow for arbitrary fast signaling. The reason is that the equivalence of ensembles is preserved during the evolution. It is a general property of convex quasilinear evolutions. Therefore, such evolutions are not in contradiction with the special relativity at this level.

It remains an open question how big is the class of convex quasilinear evolutions. In Ref. [25] we have shown that each linear non-trace-preserving quantum operation generates a convex quasilinear operation. Thus, besides the example we have presented in this Rapid Communication, there exists a wide class of convex quasilinear operations. However, the question regarding the form of the most general convex quasilinear operation still remains open.

It is also interesting that, in the considered example of convex quasilinear evolution, mixed states evolve into pure states. It suggests that this kind of evolution might have potential applications in collapse models.

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