

# Substance-based and sequential reasoning about current: An example from a bulb-ranking task using a resources theoretical lens

Lauren C. Bauman<sup>1</sup>, Trà Huỳnh<sup>2</sup>, and Amy D. Robertson<sup>3</sup>

<sup>1</sup>*Department of Physics, University of Washington, Box 351560, Seattle, Washington 98195, USA*

<sup>2</sup>*Department of Physics and Astronomy, Western Washington University, Bellingham, Washington 98225, USA*

<sup>3</sup>*Department of Physics, Seattle Pacific University, Seattle, Washington 98119, USA*



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Literature on student ideas about circuits largely focuses on misunderstandings and difficulties, with seminal papers framing student thinking as stable, difficult to change, and connected to incorrect ontological categorizations of current as a thing rather than a process. In this paper, we analyzed 417 student responses to a conceptual question about electric circuits using a lens consistent with resources theory. We found that though indicators of substance-based reasoning about current are common in student responses, this reasoning is not predictive of other difficulties reported in the literature, such as “current is consumed” or “the battery is a constant source of current.” We also found that students use substance-based reasoning in resourceful ways, suggesting that substance-based reasoning may in fact be a productive starting place for instruction on circuits.

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## I. INTRODUCTION

One of the earliest and most enduring strands of physics education research has been to identify student ideas about particular physics topics, from forces to energy to circuits and mechanical waves [1,2]. Characterizations of student thinking have then guided curriculum development, and, importantly, informed physics instructors’ knowledge of student ideas (KSI) [3], which shapes instructional decision making [4,5].

Student ideas about electric circuits are among the most well-documented of all introductory physics topics. Much of this literature focuses on students’ (mis)understanding of current [6–18], citing in particular that students often treat the battery as a constant source of current [6,7,9,13,14,16], or seem to believe that current is used up as it moves through circuit elements [8,9,13,15,18].<sup>1</sup>

<sup>1</sup>Among research on student thinking about circuits, there are a few investigations that frame student ideas about circuits as resources, or as productive beginnings of canonical understandings. For example, Odden and Russ [19] and Cosgrove [20] use case studies to illustrate students engaged in productive sense-making and/or analogy making in the context of circuits, and Burde and Wilhelm use knowledge in pieces to develop curriculum that leverages students’ intuitions and sensory experiences to develop curriculum [21].

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A subset of this literature on student ideas about circuits represents students as having a relatively coherent, incorrect model of simple circuits: “There is a source, called the battery, of some substance, called the current, which is consumed by the device (e.g., the lamp) in the circuit” [22]. A particularly well-cited example of this is Shipstone’s cookie monster model, where current is represented by cookies that are eaten by circuit elements (Fig. 1). Shipstone’s model illustrates one possible consequence of substancelike thinking about current: in tracing current flow through the wires, students may reason that current is influenced *as it encounters each element*. “Sequential reasoning” [10,23], as this is called in the literature, is broadly characterized as the belief that the information about a change in the circuit is only transmitted in the direction the current is flowing [15], engendering a “local” point of view. Some researchers suggest that difficulties like these could be overcome by having instructors address students’ “materialistic commitment,” which may involve replacing or refining students’ intuitive idea of substancelike current flow with a more correct or complex model [22].

In fact, Chi and colleagues contend that it is students’ incorrect categorization of current as a *thing* rather than a *process* that causes this cluster of difficulties in circuits to be common, robust, stable, and highly resistant to change [24–29]. Chi and colleagues’ work focuses on students’ ontological categorization of physics and other science concepts—e.g., what students think current *is* (a substance, a process, etc.). According to their research, conceptual change is difficult because students have little intuition about the *process* ontology, but they have a robust intuitive

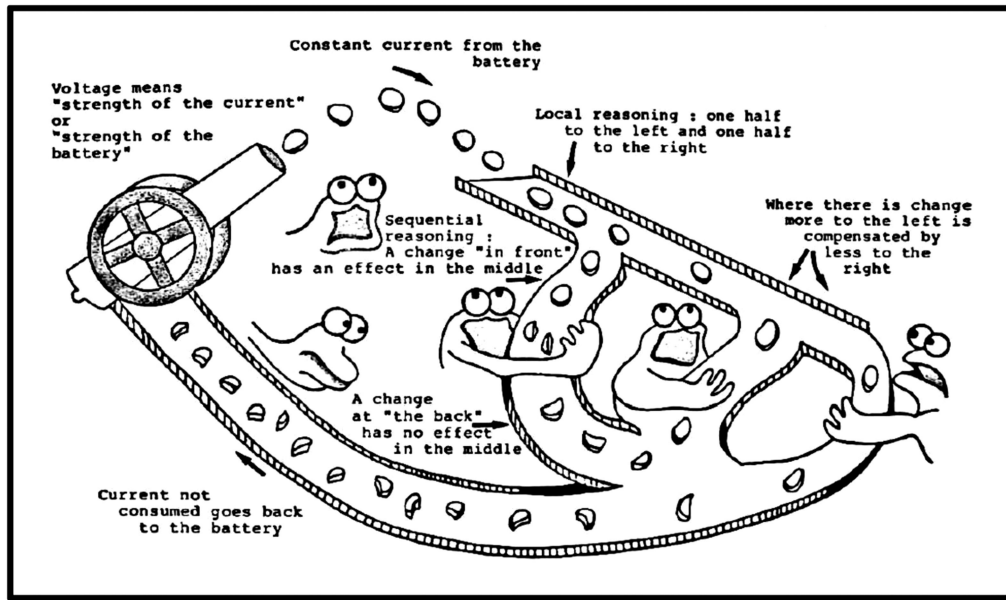


FIG. 1. The cookie monster model reprinted from D. M. Shipstone *et al.*, A study of students' understanding of electricity in five European countries, *Int. J. Sci. Educ.* 10, 303 (1988), with permission of the publisher, Taylor & Francis Ltd.

understanding of the attributes of *things* (or material substances), informed by their everyday experiences [22]. Researchers who seek to advance this theory suggest that instructors should avoid reinforcing a novice's existing (incorrect) ontological commitments about current by redirecting students towards the correct categorization and "shunning any language that uses the ontological attributes of material substances" [22].

Other researchers challenge a "stable view of ontological structure" [30], instead posing that ontologies are flexible and dynamic, where students can move between and merge ontologies as they reason about physics [30,31]. In this view, ontologies that are not technically correct can serve as *resources* for students' learning, and in fact have affordances for learning about particular physics topics [32,33]. For example, Scherr *et al.* [33] highlight that a substance metaphor for energy affords a view of energy as conserved, localized, and able to transfer between objects. Brookes and Etkina [34] advocate for framing students' use of technically incorrect ontologies as evidence of their engagement in "ontological disambiguation," an important part of the negotiation of the meaning of language in physics. Many of these papers point to physicists' (flexible) use of substance-based ontologies for different topics—e.g., the use of Kirchhoff's loop rules often implicitly treats current as a substance.

In this paper, we analyze 417 introductory students' responses to a written conceptual question about physics, showing that though students frequently use substancelike and sequential reasoning about current, students who did so were no more likely to exhibit the misconceptions reported by the literature than students who did not. That is, though they use language consistent with a substance ontology for

current, they are not more likely to treat the battery as a constant source of current, and rarely answer as though current is consumed. This claim suggests that substance-based reasoning is not necessarily predictive of canonically incorrect answers; in fact, we show that students deploy substance-based reasoning in productive ways, adding to literature that frames substance ontologies as *resources* for students' learning.

Why is this important? Our work with pre- and in-service physics teachers suggests to us that instructors often experience the presence of incorrect answers—including and especially ideas reported as misconceptions by the literature—as *threats* to students' academic and professional success [35]. Indeed, in an ongoing study on physics faculty ideas about student thinking, we have noticed that it is very difficult for physics faculty to focus on the generativity of student ideas when they perceive students to have misconceptions. In framing substance-based ontologies as the *source* of canonically incorrect answers, then, seminal literature on students' ideas about circuits has the potential to shape instructional decision-making, toward framing substance-based ontologies for current as problematic and in need of correction, and away from framings of substance-based reasoning as a *resource* for learning about current, albeit with limitations. Our goal in this paper is to offer an analysis that can support the latter framing, in service of advancing a resources-oriented instructional approach in introductory physics. Such a framing would treat students' ideas as potentially productive, even when canonically incorrect, and would seek to build from students' ideas in instruction [36–38].

In the remainder of the paper, we provide details about our research context and analytic approach (Secs. II and III,

respectively), then offer support for our claim (Sec. IV) before discussing some implications of our work (Sec. V).

## II. RESEARCH CONTEXT

For this paper, we analyzed 417 introductory physics students' written responses to two different adaptations of the rank-the-bulbs question (Fig. 2), originally published in [9], as part of a broader study to identify student conceptual resources about circuits. One version of the rank-the-bulbs question asked students to predict the ranking of the brightness of the bulbs before and after a switch was opened, and the other asked students to explain a ranking that we provided. The latter adaptation of the original question—the “explain” version—was based on previous research that suggests that students sometimes offer more mechanistic reasoning when asked to explain (rather than predict) a phenomenon [39].

In the study in which the question originally appeared [9], the authors write that the idea that the battery is a constant current source is “perhaps the most pervasive and persistent difficulty that students have with dc circuits.” Engelhardt and Beichner [7], who used a similar version of this question on the DIRECT circuits concept inventory, likewise found that this question frequently elicited reasoning about the battery as a constant source of current, as well as sequential reasoning. For example, students using sequential reasoning may not recognize that the brightness of bulb *A* is affected by the switch because the change (opening the switch) occurs after the current has “passed” bulb *A*. We did not design this study with the intent of exploring substance-based or sequential reasoning; we meant to be identifying conceptual resources for understanding circuits. However, this question did provide an appropriate context for this study, prompted by our noticing

the rarity of specific difficulties reported in the literature (including in questions for which the difficulties had been reported as common, such as the rank-the-bulbs question).

A canonically correct ranking of the bulbs in the rank-the-bulbs question can be arrived at using Kirchoff's junction rule, where current splits and combines at junctions. In this case, all of the current from the battery goes through *A*, splits in half at the junction (because *B* and *C* are identical bulbs), and rejoins to go through bulb *D*. (We would arrive at the same conclusion if we traced the current in the opposite direction through the circuit.) Thus, the current through *A* and *D* are the same; the current through *B* and *C* are the same; and the current through *B* and *C* is less than that through *A* and *D*. For part (b), opening the switch removes the parallel component of the circuit. Therefore, the total resistance increases and in turn bulb *A* and *D*, which are directly connected to the battery flow and receive all of the current in the circuit, dim. Bulb *C* goes out because there is no longer any current moving through that branch of the circuit. If we treat the circuit elements as ideal resistors (as is often done in conceptual activities in introductory physics), we can reasonably approximate the current through the bulbs using Ohm's law. In the original circuit, bulb *B* received half of the current in the main branch of the circuit; bulb *B* will increase in brightness if the current in the main circuit is more than half of what it was before the switch was opened. Because the battery remains the same,  $V$  in Ohm's law remains the same. This means that as long as the equivalent resistance of the (new) series circuit is less than twice as much as the equivalent resistance of the (old) circuit, bulb *B* will be brighter. The equivalent resistance of the original circuit is  $2.5 \times R$ . The equivalent resistance of the new circuit is  $3R$ , which is less than  $5R$ . Hence, bulb *B* will brighten.

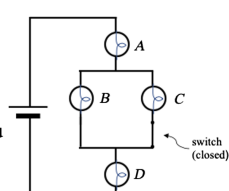
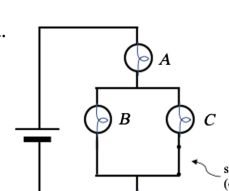
<p><i>Rank-the-bulbs explain</i></p> <p>The switch in the circuit shown is originally closed.</p>  <p>a. When the switch is closed, the ranking of the brightness of the bulbs is <math>A = D &gt; B = C</math>. How do you make sense of this ranking? (We really want to know what makes sense to you, so if this doesn't make sense, say why not or what you expected differently.)</p> <p>b. When the switch is opened, bulbs <i>A</i> and <i>D</i> dim, bulb <i>B</i> gets brighter, and bulb <i>C</i> goes out (does not light). In this scenario, bulbs <i>A</i>, <i>B</i>, and <i>D</i> are equally bright. How do you make sense of this? (Again, if it doesn't make sense, please say so, and why not.)</p>	<p><i>Rank-the-bulbs predict</i></p> <p>The switch in the circuit shown is originally closed.</p>  <p>a. Rank the brightness of bulbs <i>A</i> through <i>D</i>. Say why your answer makes sense to you.</p> <p>b. Predict how opening the switch will affect the brightness of each bulb. Say why your answer makes sense to you.</p>
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FIG. 2. Rank-the-bulbs explain and predict questions.

TABLE I. Information on universities and instructional context as reported by instructors.

University	Students	Context (where or when questions were given)	Key components of course
U1	Intro, calc.-based physics class for science and engineering majors.	Homework during instruction	Lecture, lab, and tutorials [40]
U2	Intro, calc.-based physics class for science and engineering majors.	Homework during instruction	Combined lecture or discussion section (including clickers), group problem solving, use of tutorials and lab
U3	Intro, calc.-based physics class for science and engineering majors.	Exam after instruction	Lecture (including clickers) and tutorials [40]
U4	Intro, algebra-based physics class for biology, science, and healthcare majors.	Homework during instruction	Flipped class, SCALE-UP classroom environment [41]
U5	Intro, calc.-based physics class for science and engineering majors.	Quiz after instruction	Active learning environment moved online during the pandemic, clicker questions

We analyzed a total of 417 written responses from introductory physics students across 5 colleges and universities. U1 is a large public research university located in the Northwest United States. U2 is a small public community college located in the Northeast. U3 is a midsize private, four-year university located in the South. U4 is a large public research university located in the Northeast. U5 is a large private four-year university located in the Southeast.

The questions were integrated into regular classroom activities on homework, exams, and quizzes that were given during and after instruction (depending on the sample). The course response rates for our questions were 79%–89% for

U1, 65% for U2, 90%–100% for U3, 46% for U4, and 48% for U5. (Ranges signify that there were two different samples from the same university.) The response rates depend on the number of students who completed the assignment and whether the student consented to allow their response to be used in this research. Additional information about our sample including classroom context and the context in which the questions were administered is given in Table I.

The racial and/or ethnic demographics for the colleges or universities in our study versus all college or university students are shown in Fig. 3. Our choice to report

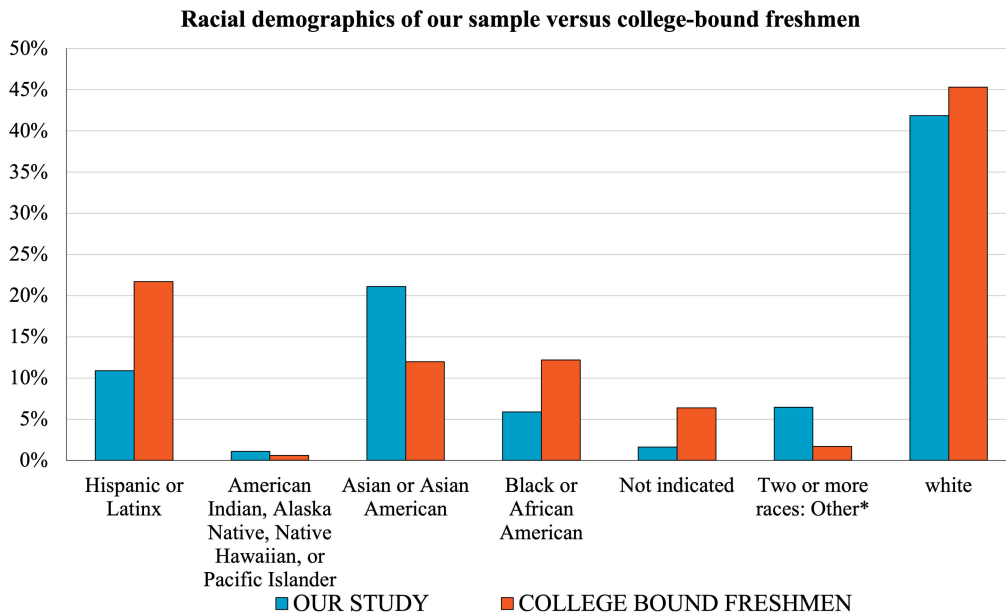


FIG. 3. Racial and/or ethnic demographics of institutions in our sample (blue, left) versus all college-bound freshmen (orange, right). Blue bars were constructed using demographic data provided by offices of institutional research or institutional websites, weighted by sample size. Orange bars were constructed using data from Kanim and Cid [44].

demographic information at the university level rather than at the classroom level is informed by the lack of clarity about what constitutes a representative sample of introductory physics students [42]. That is, we do not have large-scale demographic information about who enrolls in introductory physics courses [43] and so it is not possible to compare our classroom-level sample to a national sample of introductory physics students in order to make claims of representativeness. Albeit a limitation, it is, however, possible to compare a university-level sample to a national sample of university data to make claims about the representativeness of the *institutions* from which we sampled. Preliminary work from our team suggests that the racial, gender, and wealth demographics of introductory physics courses are not representative of these demographics at the university, and thus more work is needed to establish a representative sample of introductory physics students in order for course-level comparisons to be made [42].

Figure 3 suggests that our study includes institutions with a higher fraction of Asian and Asian-American students and students from two or more races than the fractions of these groups among all college-bound freshmen, and a lower fraction of Latinx and Hispanic students and Black and African American students than for all college-bound freshmen. In addition, the median parental income of the students at colleges or universities in our study is higher than the national average. This sampling limits the generalizability of our results [44].

### III. ANALYSIS

Our analysis explores the extent to which (i) substance-based language about current co-occurs with (ii) commonly reported misunderstandings about current that are thought to be rooted in a model of current as a substance. To explore this, we looked for markers of substance-based and sequential reasoning, and for evidence of the particular difficulties that current is consumed and that the battery is a constant source of current. We constructed a coding scheme using a combination of inductive and deductive methods; in particular, we took the literature's depiction of substance-based and sequential reasoning as a first-pass coding scheme and refined it in conversation with our data. For example, we included markers of substance-based reasoning described by Slotta, Chi, and colleagues [28,29] and markers of substance-based reasoning that emerged in our dataset; one or the other on their own would not have allowed us to fully characterize substance-based reasoning in our sample. We describe the construction of our coding scheme in more detail in the remainder of this section.

*Use of current reasoning.*—Because we are focused on student reasoning about *current*, our first code—“use of current reasoning”—served as a first filter. If a response used the words “current” or “electrons,” it was automatically included. If a response did not include any ideas about current, it was not included in the subsequent analysis.

Responses that were filtered out either gave a ranking with no explanation, used ideas exclusively about voltage and/or resistance, focused only on how the elements were arranged (e.g., “*A, B, and D are connected in series, so they'll be the same brightness*”), or used language that was too vague to interpret as current (e.g., “*A and D receive the same input.*”). Table II shows the sample sizes from each university and the percentage of responses that used “current reasoning.”

*Indicators of sequential and substancelike reasoning.*—As we summarized above, existing research suggests that difficulties like current is consumed, current is constant, and sequential reasoning are rooted in students' commitment to a substance-based ontology. The task of determining ontological beliefs about current on the basis of their response to a single question—such as in our study—is impossible, particularly if one assumes that ontological beliefs are not stable and coherent. Therefore, we coded for *indicators* of substancelike and sequential reasoning, which are two manifestations of a (dynamic) substance-based ontology. Substancelike reasoning is consistent with Shipstone's cookie monster model in which current is characterized as cookies (physical substance that moves, can be used up, etc.). Sequential reasoning is also part of Shipstone's model in which changes in the circuit will be temporally and spatially dependent.

We developed a code, “describes current as physical and quantifiable entity,” to characterize substancelike reasoning, and a code, “describes current as moving through the circuit in a this-then-this manner,” to characterize sequential reasoning in our context. For a response to receive the substancelike code, it must have contained a statement in which current is treated as a “thing.” For example, students often used verbs such as “moves,” “splits,” “rejoins,” and “travels” to describe current's substancelike behaviors. Slotta, Chi, and colleagues [28,29] defined several “substance predicates for coding material substance conceptions” which informed our codes. Student responses rarely included drawings or diagrams; when they did, arrows that split at junctions, for example, were considered alongside text.

TABLE II. Percentage of responses from each sample that contained “current reasoning” and were therefore used in our analysis.

Question version	Sample	Current reasoning
Explain version	U1 ( $N = 123$ )	93%
	U2 ( $N = 24$ )	92%
	U3 ( $N = 55$ )	87%
Predict version	U1 ( $N = 130$ )	91%
	U3 ( $N = 31$ )	71%
	U4 ( $N = 28$ )	71%
	U5 ( $N = 12$ )	100%

Sequential reasoning often took the form of tracing current through the circuit in a “this-then-this” manner, where the response “followed” the current through the circuit as though it reaches different circuit elements at different times. For example, many responses used reasoning such as, “*The current travels through bulb A, then splits at the first junction before rejoining at the second junction just before D.*”

Although substancelike and sequential reasoning are not the same, there is substantial overlap, empirically, in students’ use of these kinds of reasoning. For example, many responses used the general line of reasoning that the current “*leaves the battery, and then travels through A before splitting at the junction [...]*” This response has indicators of sequential reasoning (“*then travels through A, before splitting*”), but it also has indicators of substancelike reasoning (“*current leaves [...]* then *travels through [...]* before *splitting [...]*”). In this sense, even a single word or phrase in a student’s response could receive both substancelike and sequential reasoning codes.

*Current is consumed.*—The misconception that “current is used up” is reported in much of the early literature on student ideas about circuits [9,13,15]. In the context of the rank-the-bulbs question, McDermott and Shaffer found that the incorrect ranking  $A > B = C > D$  occurred in about 35% of responses [9]. In half of those responses, students used reasoning that the current was “used up” by  $A$ ,  $B$ , and  $C$  so there was less left for  $D$ . This reasoning is consistent with Shipstone’s cookie monster model where the current is “eaten up,” in order, by the circuit elements, and the “leftover” current returns to the battery [15].

In our preliminary analysis of a subset of responses in our study, we did not see any evidence of students using this idea. We did, however, see evidence of students explicitly conserving current. For example, one student wrote: “*All current through B & C flows through A & D. This means A & D have equal current & brightness.*” We wanted to track the frequency of these ideas, so we defined two codes: (1) “evidence that current is consumed” and (2) “lack of evidence that current is consumed.” “Lack of evidence that current is consumed” need not mean that current is conserved, or at least need not be explicitly stated as such; students need only *not* to imply (or state directly) that current *is* consumed.

There were not many instances of the current *is* consumed idea in our dataset, and because we aim to show that this idea is *not* common, we applied the code liberally, marking when there was *any* inkling of the idea current is consumed to err on the side of reporting false positives as opposed to false negatives. For example, one student wrote: “*Opening the switch will make the A bulb the brightest and the last bulb will be the dimmest because bulb A gets the most current and bulb D gets the least.*” It is possible this student did not conceive of current as being consumed; they never overtly said current is “used” or “consumed,” and we

could imagine a model where current is conserved in some way, but more or less goes to different parts of the circuit. However, we chose to code this response as “evidence for current is consumed” because looking at the circuit diagram, we can also imagine a scenario where the student viewed the current as being used or consumed when they described “*A gets the most*” and “*D gets the least.*”

*Battery is a constant source of current.*—Engelhardt and Beichner [7] asked a version of the rank-the-bulbs question and found that students often predicted that the brightness of bulb  $A$  would not change when the switch is opened (part b of our questions) because the battery is a constant source of current. This idea is consistent with McDermott and Shaffer’s findings [9], and with Shipstone’s cookie monster model in which there is a “set amount of cookies” (current) provided by the battery [15].

In our dataset, our best approximation of evidence for the idea of “battery as a constant source of current” is when students did not acknowledge a *change* in the current through the circuit or a *change* in the brightness of bulbs  $A$  and  $D$  when the switch was opened (instead treating the current as constant). We defined two codes: (1) “explicitly attends to the change in total current after the switch is opened” and (2) “does not explicitly attend to the change in total current after the switch is opened.” For a response to be coded as (1), the student must have stated in part b that current changes after the switch is opened. If the student did not acknowledge this point, it was coded as (2).

Importantly, receiving the code “does not explicitly attend to the change in total current” does not necessarily mean the student thinks the battery is a constant source of current. For example, many students responded to part b along the lines of: “*Now the bulbs are all in series so they will all have the same brightness.*” This statement is accurate but is an incomplete answer to the question; the student ranked the brightness of the bulbs but did not attend to the part of the question that asked them to explain the change in brightness of each bulb. In this case, the student does not say anything about the change in current (but also does not say the current is unchanged); we simply do not know what they think about the current through the battery. Other responses explicitly stated, “*the current remains constant.*” Both examples would receive the code “does not explicitly attend to the change in total current,” and thus the frequency of responses we report for this code is likely an over estimation of the “battery as a constant source of current” misunderstanding.

*Coding.*—Table III depicts our final coding scheme, which was developed and refined through collaborative conversations between all of the authors. Once the original coding scheme was developed, authors L. C. B. and T. H. independently coded 10% of the total data, comparing codes and further refining the scheme. After coming to consensus, L. C. B. and T. H. coded an additional 20% of responses independently. Standard statistical measures of

TABLE III. Final coding scheme.

Code	Example
Current consumed: “Evidence for current is consumed”	“A will be the brightest <i>since the current reaches it first</i> . B and C are less then A and equally bright since they are in parallel [...] <i>D should be less bright than A,B,C since it is the last bulb in [the] series circuit.</i> ” [rank-the-bulbs explain]
“No evidence for current is consumed”	“ <i>The same current</i> would be going through A, B, and D so they would all be the same brightness.” [rank-the-bulbs predict]
Battery constant source of current: “Response does <i>not</i> explicitly attend to the change (decrease) in total current after the switch is opened.”	“ <i>A = B = D. The current through A, B and D will be same,</i> so the brightness will also be the same.” [rank-the-bulbs predict]
“Response explicitly attends to the change (decrease) in total current after the switch is opened.”	“[...] A and D dim when the switch is opened is because <i>there is less current leaving the battery</i> . When the two bulbs B and C were connected in series and the switch was closed (“current 1”) the battery’s output rate was larger. When one of the bulbs in series was removed by opening the switch (“current 2”), <i>the battery’s output rate decreased.</i> ” [rank-the-bulbs explain]
Indicators of sequential and substancelike reasoning: “Substancelike reasoning: Describes current as physical and quantifiable entity.”	“Bulb A and bulb D have the <i>same amount</i> of current [...] while bulb B and C have the current <i>split between</i> them.” [rank-the-bulbs predict]
“Sequential reasoning: Describes current as moving through the circuit in a this-then-this manner.”	“Current will move <i>from A, split at the junction, move through C &amp; B, rejoin at the lower junction, then move back through D</i> ” [rank-the-bulbs explain]

interrater agreement, such as Cohen’s kappa, are suitable for codes that are independent or mutually exclusive [45]; we do not make this assumption about our coding scheme overall, so to apply Cohen’s kappa we must consider each code independently. The Cohen’s kappa values ranged from 0.76 to 1.00, all above 0.75, which indicates excellent agreement beyond chance [45]. Disagreements were largely superficial in nature; for example, one coder or the other missed a relevant phrase that pointed to substance-based or sequential reasoning. After discussion, there were no disagreements about the way the coding scheme should be applied. L. C. B. then coded the remaining data independently.

IV. RESULTS

In this section, we substantiate our claim that though students frequently used substancelike and sequential reasoning about current, students who did so were no more likely to exhibit the misconceptions reported by the literature than students who did not. We start by showing that indicators of substance-based and sequential reasoning are common in student responses to the rank-the-bulbs question. We then give evidence that responses that use language consistent with a substance ontology for current do not more frequently evidence the misunderstandings that current is consumed or that the battery is a constant source of

current than responses that use neither substance nor sequential reasoning. Finally, we provide some evidence that students use substance-based reasoning in resourceful ways.

A. Indicators of substance-based and sequential reasoning are common in student responses to the rank-the-bulbs question

In response to the rank-the-bulbs question, many student responses included language consistent with substancelike reasoning and/or sequential reasoning. As summarized in Table IV, for the predict version of the rank-the-bulbs question, 69% of student responses included descriptors of current as a physical and quantifiable entity; for the explain version of the question, 76% of responses included such descriptors. For example, in response to part (a), students wrote

Bulb A and bulb D have the same amount of current [...] while bulb B and C have the current split between them.

[...] because B and C have equal resistance, the current will split equally between B and C.

A and D have the same amount of current going through [...] C and B split the current due to parallel assembly.

Bulbs *A* & *D* get all the current. *B* & *C* get part of the current.

Words like *same amount*, *divide*, *split*, *add*, *join*, *total*, *half*, *some*, *less*, *more*, and *fraction* were typical of these responses and are references to quantifiable entities which is a characteristic of substances. Often, students justified their ranking of the brightness of the bulbs by citing the *amount* of current each bulb *receives*.

Indicators of sequential reasoning appeared less often but were still common: 39% of student responses to the predict question and 34% of student responses to the explain question used descriptors of current as moving through the circuit in a “this-then-this” manner. For example,

*B* and *C* are in parallel, so we can conceptualize this as the current splitting between them and joining up again before *D* [...]

*A* will have the same current through it as *D* *b/c* the current is added again after going through *B* and *C* and *B* and *C* are less because the current is divided equally between them which is also why they are equal.

The current that goes through *A* will divide equally between *B* and *C* and then the current from *B* and *C* will add up and pass through *D*.

These responses described current as *traveling* through wires in a particular sequence using words like *joining up again before...*, *...after going through B and C...*, *then*.

Substancelike and sequential reasoning often showed up together: 34% of responses to the rank-the-bulb explain question and 36% of responses to rank-the-bulb predict question were coded as containing both the “current as physical and quantifiable entity” and “describes current as moving through the circuit in a this-then-this manner” codes. Almost *all* the responses that received the “this-then-this” (sequential) code also received the “quantifiable/physical entity” (substance) code. This observation is unsurprising as using this-then-this reasoning almost always requires conceptualizing current as a thing and is consistent with the literature’s characterization of sequential reasoning as part of a substance model for current.

### **B. Responses that use language consistent with substance ontologies and sequential reasoning are not more likely to evidence misunderstandings such as current is consumed and the battery is a constant source of current than responses that do not**

In the previous section, we showed that a significant fraction of responses in our sample used language that was consistent with substance and/or sequential reasoning. In many cases, the literature positions other common misunderstandings—for example, current is consumed and current is constant (i.e., the battery is a constant source of current)—as being characteristic (and for some, even deterministic) of a coherent substance-based ontology. However, in our analysis, we found little evidence that substance-based and sequential reasoning are connected to—let alone deterministic of—the use of the ideas that current is consumed or that current is constant (Table V).

Table V is a contingency table that shows the percentage of responses that used substance and/or sequential reasoning, separated by whether the responses explicitly described current as changing when the switch is opened versus those who did not attend to the change in current. What Table V suggests is that there is little difference in the distribution of these responses among students who (i) did and (ii) did not use substance-based and/or sequential reasoning: 55% of students who used substance-based and/or sequential reasoning did not explicitly attend to the change in current when the switch is opened, and 56% of those who used neither substance-based nor sequential reasoning did not explicitly attend to this change. That is, for our sample, it does not seem to be the case that using substance-based and/or sequential reasoning makes it more likely for students to use the “constant current” idea.

The “current is consumed” idea was nearly absent in our data: 3 out of 359 student responses total evidenced this misunderstanding, with 1% of the responses that used substance-based and/or sequential reasoning evidencing this idea, and 1% of the responses that used neither substance-based nor sequential reasoning using this idea. As with the “constant current” idea, there is no obvious correlation between substance-based and sequential reasoning and the “current is consumed” idea.

In fact, as shown in Table IV, about a third of students answered the rank-the-bulbs question using reasoning about current that is consistent with substancelike and sequential reasoning. In 94% of these cases, students

TABLE IV. Summary of findings presented in Sec. IV A.

Code	Explain	Predict	Overall
“Substancelike reasoning: Describes current as a physical and quantifiable entity”	76%	68%	72%
“Sequential reasoning: Describes current as moving through the circuit in a this-then-this manner”	34%	40%	37%
Both substancelike and sequential reasoning	33%	37%	35%

TABLE V. Percentage of responses that were coded as explicitly or not explicitly attending to the change in current, by indicator of sequential and/or substancelike reasoning.

	Substance and/or sequential reasoning ( $N = 266$ )	Neither substance nor sequential reasoning ( $N = 93$ )
Does NOT explicitly attend to the change in current ( $N = 199$ )	147	52
Explicitly attends to the change in current ( $N = 160$ )	119	41

applied these ideas productively, reasoning with loop laws to answer the question correctly.

So far, we have shown (a) that many students described current as a substancelike thing, and a significant percentage of these students *also* described current as traveling through the circuit in a sequential, this-then-this manner. We have also shown that (b) the use of substancelike and/or sequential reasoning does not appear to predict the use of other incorrect pieces of the cookie monster model, as some of the literature about students' ideas may suggest. Next, we will take this argument a step further, showing that students in our sample used substance-based and sequential reasoning in resourceful ways. If generalizable, our results have the potential to mitigate concern about the use of substance-based and sequential reasoning and reframe substance-based ontologies for current as productive building blocks for instruction.

### C. Students use substance-based and sequential reasoning in resourceful ways

In this section, we use examples from our data to illustrate two ways in which students use substance-based and sequential reasoning that we consider to be resourceful. In the first set of examples, we show both that (i) correct answers to the rank-the-bulbs question can *depend on* substance-based and sequential reasoning, and that (ii) students use substance-based reasoning in *both* canonically correct and technically incorrect ways (i.e., similar to results in Sec. IV B, the use of substance-based reasoning in and of itself does not determine the correctness of students' answers). In the second set of examples, we show that students use substance-based reasoning about current in ontologically flexible ways, choosing among the features of substances that are relevant to the rank-the-bulbs question. This ontological flexibility is considered by the literature as a marker of expertise [30,31], as physicists and physics instructors also selectively use substance-based and sequential reasoning to analyze circuits (e.g., when using Kirchoff's loop rules). Together, these sets of examples suggest that substance-based reasoning can be both productive and flexibly deployed.

*Example 1: Correct reasoning that depends on a substance-based ontology for current.*—In response to part (a) of the rank-the-bulbs question, one student wrote

$(A = D) > (B = C)$  current through  $A$  passes through and splits to  $B$  and  $C$  (equal current through  $B$  and  $C$ ). Then the current comes back together and passes through  $D$ , ending up with the same amount as started through  $A$ .

This student's canonically correct response included indicators of both substancelike and sequential reasoning. They used language like *passes through*, *splits*, and *comes back together*, and they traced the current through the circuit in a this-then-this manner: "Current through  $A$  *passes through and splits to  $B$  and  $C$*  [...] then the current comes back together and passes through  $D$ ." This substance-based and sequential reasoning was central to their use of junction rules, supporting them in correctly identifying the relative brightness of the bulbs in the circuit. In response to part (b), the same student continued

Opening the switch will cause [bulb]  $C$  to lose brightness, because the current will not be able to reach it. However,  $A$ ,  $B$ , and  $D$  will all have equal brightness. The current passing through will no longer split to  $B$  and  $C$  as  $C$  is not a part of the circuit, so  $B$  will receive all the current.

Although this response accurately ranked the bulbs as equally bright, it does not draw attention to the change in current through the circuit after bulb  $C$  is removed. The student employed similar reasoning as in part (a), tracing the current sequentially<sup>2</sup> through the circuit, however now the current *passing through no longer has to split* so  $B$  will *receive all the current*. This student's ranking was not incorrect ( $A$ ,  $B$ , and  $D$  are equally bright), but it was also not complete, and it does not challenge the belief that the battery is a constant source of current because it did not explicitly state that  $A$  and  $D$  will decrease in brightness.

<sup>2</sup>Although this student's language strongly suggests sequential "tracing" of current, the student does not fully deploy the cookie monster model described by Shipstone. In particular, the switch is located spatially "downstream" of bulb  $C$  if you think—as this student does—about conventional current, and thus the cookie monster model would predict that bulb  $C$  would light (because the current would only "know" it cannot go through the branch after it reaches the open switch).

We would argue that this pair of responses makes apparent some of the affordances and limitations of a substance model for current. In particular, this student used substance-based and sequential reasoning in relatively consistent ways across the two parts of the rank-the-bulbs question. In the first part, this reasoning supported them in using junction rules to correctly rank the bulbs. In the second part, substance-based and sequential reasoning supported them in ranking the bulbs correctly within the series circuit, but did not prompt an analysis of current as variable, or as dependent on the resistance of the circuit. That the latter is true does not, however, in our view, make substance-based and sequential reasoning *unproductive*; it makes it productive for some purposes and less so for others. Instruction thus need not *replace* substance-based or sequential reasoning about current; it can support students in building from and understanding the limitations of this reasoning.

*Example 2: Ontologically flexible use of substance-based reasoning.*—In response to part (a) of the predict version of the rank-the-bulbs question, one student wrote

$A = D > B = C$ . The current goes through A then splits evenly and goes through B and C, so  $IB + IC = IA$ . Then the current comes back together and goes through D.

This student accurately ranked the brightness of the bulbs when the switch is closed, and their response has many indicators of both substancelike and sequential reasoning: “The current goes through A then splits evenly and goes through B and C, so  $IB + IC = IA$ . Then the current comes back together and goes through D.”

In response to part (b)—which asked students to rank the bulbs after the switch is opened (Fig. 2)—the same student continued

When the switch is open, bulb C will no longer be a path for the current to flow through. Bulb C does not light, and the overall resistance increases since a possible path for the current is removed. So, the current that flows through the circuit decreases and bulb B will increase in brightness because it no longer has to share the current with another bulb.

Here, the student accurately described the change in brightness of the bulbs: bulb C no longer lights, and bulb B brightens. Interestingly, they maintained some substance-like descriptions of current: “[...] no longer be a path for current to flow through [...] current that flows through the circuit [...] B [...] no longer has to share the current with another bulb.” However, this response does not fully align with a substance-based and sequential view of current. For example, the student recognized that bulb C does not light

when the switch is open because “*there’s no longer a path*” even though the switch is located *after* bulb C when thinking about conventional current “flow,” and they treat current as responsive to changes in the circuit. Thus, for this student, in this response, current is *both* a “thing” that flows through the circuit and passes through elements in a particular order *and* a thing that “knows” it cannot travel down a particular path even before it encounters the open switch. In short, this student used the substance-based model flexibly [30], drawing from the parts that supported them in modeling current as a substance that moves through the wires to light the bulbs, but also drawing from elements of another ontology about current as a thing that knows what to do before it arrives at a junction.

## V. DISCUSSION AND CONCLUSION

Historically, research on student thinking about circuits has largely focused on misunderstandings, difficulties, and misconceptions, particularly around students’ models of simple circuits and their conceptualizations of current. A few authors have offered hypotheses for why these ideas could be common, stable, and resistant to change, in particular, that students’ ontological commitment to current as a substancelike entity results in a coherent but incorrect model (e.g., Shipstone’s cookie monster model). Authors advancing these hypotheses suggest that in order to learn the correct model for current, students must make an ontological shift that requires instructionally addressing and uprooting ideas that align with current as a substance, and sometimes replacing that conceptualization with a more correct one. For example, in [22], Reiner *et al.* say

Naive conceptions of these topics are so robust because of a resistance or inability to change these ontological categorizations. A possible implication for physics instruction is that materialistic models should be avoided altogether in teaching such concepts. In these cases, instruction should attempt to introduce a new language of processes while shunning any language that uses the ontological attributes of material substances.

This literature—for example, the excerpt above—paints substancelike reasoning about current (often evidenced by indicators of substance-based ideas: quantifiability, movement, this-then-this reasoning) as an insurmountable barrier to a canonically correct understanding of physics. Instructors who read the existing literature about student ideas about current may—sensibly so—then go on to plan instruction and/or use instructional sequences [28] that treats students’ existing, substancelike and sequential reasoning as barriers to learning, turning their students away from what they think and toward something else. Indeed, Slotta and Chi [28] offer “ontology training” modules that support instructors in “providing [students]

with some training about the target ontology” for electric current (an emergent processes ontology), “followed by direct instruction about electricity that avoids any use of terms or analogies that might promote the *material substance* ontology (e.g., the water flow analogy).”

Our analysis builds from resources-oriented framings of substance ontologies [30,32–34], showing that student use of substancelike and sequential reasoning is not more likely to co-occur with other difficulties or misconceptions in the cookie monster model, such as the ideas that current is constant or current is consumed. This is, we think, a *functional* result—showing that substance ontologies for current need not functionally produce incorrect answers—which supplements existing theoretical pieces like that from Gupta and colleagues. Gupta *et al.* [31] challenge Reiner *et al.*’s [22] conclusion by first showing how physics experts often straddle ontological categories—including current as a substance—in their discourse, and then presenting a case of one student, Kimberly, who similar to experts switches between ontological categories fluidly. Importantly, they identify ways in which a substance ontology supports Kimberly’s correct understandings of physics in certain contexts.

Further, we found that students often use substancelike and sequential reasoning in productive and ontologically flexible ways, in service of correct answers. Our work emphasizes the variability of student thinking: While misconceptions research has often historically connected multiple misconceptions into a coherent, incorrect, stable model [46,47], resources theory assumes that students will draw on multiple resources that are not necessarily consistent [37,48]. Our analysis emphasizes that these ideas are not packaged together into all-or-nothing groups: substance-based and sequential ideas about current are not necessarily predictive of other misconceptions; a student could use ideas aligned with sequential reasoning in one context and then use ideas that are in conflict with a sequential view in the next. Because our goal in this paper has been to challenge a stable view of substance-based and sequential reasoning as *necessarily* causing the use of misconceptions in the literature, we have not emphasized the context-dependence of substance-based and sequential reasoning. However, our work hints at the possibility of context-dependence—e.g., as summarized in Table IV, the prevalence of substance-based and sequential reasoning was different for the explain and predict versions of the rank-the-bulbs questions. Whether this variability is attributable to the question or to differences in samples, we do not know. Future work could explore questions about context-dependence, these and others.

Though our work lends empirical support both to work on ontological flexibility and to resources theory, it is worth noting that we did not set out to conduct a research study about ontologies or about the coherence of students’ models; if we had done so, we would have designed a different study. Instead, this study was inspired by an original noticing that “the current is consumed” idea was very uncommon in our dataset, in contrast to the commonality of this idea as reported in the literature. Our foregrounding of the ontological markers in student responses is our effort to be in conversation with the literature on student thinking, and to test how a resources-oriented analysis would speak to existing misconceptions- and difficulties-oriented analyses.

Our aim has been pragmatic—to inform instruction. If we assume that students’ have the ability to hold multiple, contradictory, and often incoherent ideas at once, as is suggested by our analysis, instructors need not necessarily worry about sequential and substance-based reasoning. In fact, on the basis of how students *use* these ideas in our dataset, perhaps they should be encouraged.

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- [1] L. C. McDermott and E. F. Redish, Resource letter: PER-1: Physics education research, *Am. J. Phys.* **67**, 755 (1999).
- [2] J. L. Docktor and J. P. Mestre, Synthesis of discipline-based education research in physics, *Phys. Rev. ST Phys. Educ. Res.* **10**, 020119 (2014).
- [3] L. Shulman, Knowledge and teaching: Foundations of the new reform, *Harv. Educ. Rev.* **57**, 1 (1987).
- [4] L. C. McDermott, Millikan lecture 1990: What we teach and what is learned—closing the gap, *Am. J. Phys.* **59**, 301 (1991).
- [5] L. C. McDermott, Oersted medal lecture 2001: “Physics education research—the key to student learning”, *Am. J. Phys.* **69**, 1127 (2001).
- [6] R. Cohen, B. Eylon, and U. Ganiel, Potential difference and current in simple electric circuits: A study of students’ concepts, *Am. J. Phys.* **51**, 407 (1983).
- [7] P. V. Engelhardt and R. J. Beichner, Students’ understanding of direct current resistive electrical circuits, *Am. J. Phys.* **72**, 98 (2003).
- [8] T. V. Goris, Common misunderstandings of electricity: Analysis of interview-responses of electrical engineering technology students, *Int. J. Eng. Pedagogy IJEP* **6**, 4 (2016).
- [9] L. C. McDermott and P. S. Shaffer, Research as a guide for curriculum development: An example from introductory electricity. Part I: Investigation of student understanding, *Am. J. Phys.* **60**, 994 (1992).
- [10] D. M. Shipstone, A study of children’s understanding of electricity in simple dc circuits, *Eur. J. Sci. Educ.* **6**, 185 (1984).
- [11] S. Cepni and E. Keleş, Turkish students’ conceptions about the simple electric circuits, *Int. J. Sci. Math. Educ.* **4**, 269 (2006).
- [12] A. T. Borges and J. K. Gilbert, Mental models of electricity, *Int. J. Sci. Educ.* **21**, 95 (1999).
- [13] P. M. Heller and F. N. Finley, Variable uses of alternative conceptions: A case study in current electricity, *J. Res. Sci. Teach.* **29**, 259 (1992).
- [14] A. Leniz, K. Zuza, and J. Guisasola, Difficulties understanding the explicative model of simple dc circuits in introductory physics courses, presented at PER Conf. 2014, Minneapolis, MN, [10.1119/perc.2014.pr.034](https://doi.org/10.1119/perc.2014.pr.034).
- [15] D. M. Shipstone, C. v Rhöneck, W. Jung, C. Kärrqvist, J.-J. Dupin, S. Johsua, and P. Licht, A study of students’ understanding of electricity in five European countries, *Int. J. Sci. Educ.* **10**, 303 (1988).
- [16] D. Smith and P. van Kampen, Teaching electric circuits with multiple batteries: A qualitative approach, *Phys. Rev. ST Phys. Educ. Res.* **7**, 020115 (2011).
- [17] S. Bull, T. J. Jackson, and M. J. Lancaster, Students’ interest in their misconceptions in first-year electrical circuits and mathematics courses, *Int. J. Electr. Eng. Educ.* **47**, 307 (2010).
- [18] C.-H. Tsai, H.-Y. Chen, C.-Y. Chou, and K.-D. Lain, Current as the key concept of Taiwanese students’ understandings of electric circuits, *Int. J. Sci. Educ.* **29**, 483 (2007).
- [19] T. O. B. Odden and R. S. Russ, “Charges are everywhere”: A case of student sensemaking about electric current, presented at PER Conf. 2017, Cincinnati, OH, [10.1119/perc.2017.pr.065](https://doi.org/10.1119/perc.2017.pr.065).
- [20] M. Cosgrove, A study of science-in-the making as students generate an analogy for electricity, *Int. J. Sci. Educ.* **17**, 295 (1995).
- [21] J.-P. Burde and T. Wilhelm, Teaching electric circuits with a focus on potential differences, *Phys. Rev. Phys. Educ. Res.* **16**, 020153 (2020).
- [22] M. Reiner, J. D. Slotta, M. T. H. Chi, and L. B. Resnick, Naive physics reasoning: A commitment to substance-based conceptions, *Cognit. Instr.* **18**, 1 (2000).
- [23] J.-L. Closset, Sequential reasoning in electricity, in *Proceedings of the First International Workshop* (Editions du Centre National de Recherche Scientifique, Paris, 1983).
- [24] M. T. H. Chi, Commonsense conceptions of emergent processes: Why some misconceptions are robust, *J. Learn. Sci.* **14**, 161 (2005).
- [25] M. Chi, Conceptual change within, and across ontological categories: Examples from learning, and discovery in science, in *Cognitive Models of Science*, edited by R. Giere and H. Feigl (University of Minnesota Press, Minneapolis, MN, 1992), Vol. 15, pp. 129–186.
- [26] M. T. H. Chi and J. D. Slotta, The ontological coherence of intuitive physics, *Cognit. Instr.* **10**, 249 (1993).
- [27] M. T. H. Chi, J. D. Slotta, and N. de Leeuw, From things to processes: A theory of conceptual change for learning science concepts, *Learn. Instr.* **4**, 27 (1994).
- [28] J. D. Slotta and M. T. H. Chi, Helping students understand challenging topics in science through ontology training, *Cognit. Instr.* **24**, 261 (2006).
- [29] J. D. Slotta, M. T. H. Chi, and E. Joram, Assessing students’ misclassifications of physics concepts: An ontological basis for conceptual change, *Cognit. Instr.* **13**, 373 (1995).
- [30] J. R. Hoehn and N. D. Finkelstein, Students’ flexible use of ontologies and the value of tentative reasoning: Examples of conceptual understanding in three canonical topics of quantum mechanics, *Phys. Rev. Phys. Educ. Res.* **14**, 010122 (2018).
- [31] A. Gupta, D. Hammer, and E. F. Redish, The case for dynamic models of learners’ ontologies in physics, *J. Learn. Sci.* **19**, 285 (2010).
- [32] A. Gupta, A. Elby, and L. D. Conlin, How substance-based ontologies for gravity can be productive: A case study, *Phys. Rev. ST Phys. Educ. Res.* **10**, 010113 (2014).
- [33] R. E. Scherr, H. G. Close, S. B. McKagan, and S. Vokos, Representing energy. I. Representing a substance ontology for energy, *Phys. Rev. ST Phys. Educ. Res.* **8**, 020114 (2012).
- [34] D. T. Brookes and E. Etkina, “Force,” ontology, and language, *Phys. Rev. ST Phys. Educ. Res.* **5**, 010110 (2009).
- [35] A. D. Robertson and L. J. Atkins Elliott, Truth, success, and faith: Novice teachers’ perceptions of what’s at risk in responsive teaching in science, *Sci. Educ.* **104**, 736 (2020).
- [36] D. Hammer, Student resources for learning introductory physics, *Am. J. Phys.* **68**, S52 (2000).
- [37] D. Hammer, A. Elby, R. E. Scherr, and E. F. Redish, Resources, framing, and transfer, in *Transfer of Learning from a Modern Multidisciplinary Perspective* (IAP, Greenwich, CT, 2005), pp. 89–119.

- [38] J. P. Smith, A. A. diSessa, and J. Roschelle, Misconceptions reconceived: A constructivist analysis of knowledge in transition, *J. Learn. Sci.* **3**, 115 (1993).
- [39] L. M. Goodhew, A. D. Robertson, P. R. L. Heron, and R. E. Scherr, Students' context-sensitive use of conceptual resources: A pattern across different styles of question about mechanical waves, *Phys. Rev. Phys. Educ. Res.* **17**, 010137 (2021).
- [40] Lillian C. McDermott, Peter S. Shaffer, and Physics Education Group, *Tutorials in Introductory Physics* (Pearson College Div., Upper Saddle River, NJ, 2002).
- [41] R. J. Beichner, SCALE-UP, [https://www.physport.org/methods/method.cfm?G=SCALE\\_UP](https://www.physport.org/methods/method.cfm?G=SCALE_UP).
- [42] R. Mondesir and A. D. Robertson, Toward characterizing the demographics of introductory physics courses, presented at PER Conf. 2020, virtual conference, 10.1119/perc.2020.pr.Mondesir.
- [43] P. Mulvey (private communication).
- [44] S. Kanim and X. C. Cid, Demographics of physics education research, *Phys. Rev. Phys. Educ. Res.* **16**, 020106 (2020).
- [45] M. Banerjee, M. Capozzoli, L. McSweeney, and D. Sinha, Beyond kappa: A review of interrater agreement measures, *Can. J. Stat. Rev. Can. Stat.* **27**, 3 (1999).
- [46] G. J. Posner, K. A. Strike, P. W. Hewson, and W. A. Gertzog, Accommodation of a scientific conception: Toward a theory of conceptual change, *Sci. Educ.* **66**, 211 (1982).
- [47] M. McCloskey, Intuitive physics, *Sci. Am.* **248**, No. 4, 122 (1983).
- [48] R. E. Scherr, Modeling student thinking: An example from special relativity, *Am. J. Phys.* **75**, 272 (2007).
- [49] Native Life & Tribal Relations, <https://www.washington.edu/diversity/tribal-relations/>.
- [50] Tribal Lands Statement, <https://www.wvu.edu/tribal-lands-statement>.
- [51] Acknowledging the Coast Salish Peoples, <https://spu.edu/diversity/spu-diversity-commitment/oie-x-land-page>.