PHYSICAL REVIEW PHYSICS EDUCATION RESEARCH **19,** 020136 (2023)

Instructional model for teaching blended math-science sensemaking in undergraduate science, technology, engineering, and math courses using computer simulations

Leonora Kaldaras[1,2,*,†] and Carl Wieman[1,‡]

[1]*Stanford Graduate School of Education, Stanford, CA (primary) and PhET Interactive Simulations, Department of Physics, University of Colorado Boulder, Boulder, Colorado 80309-0390, USA*
[2]*Stanfod Graduate School of Education and Department of Physics, Stanford University, Stanford, California 9435, USA*

(Received 23 May 2023; accepted 28 August 2023; published 29 September 2023)

The ability to express scientific concepts in mathematical terms and integrate scientific and mathematical reasoning about a phenomenon is a foundational cognitive process involved in scientific thinking. This process called "blended math-science sensemaking" (MSS) is a desired skill for all science, technology, engineering, and math (STEM) students, but few students are learning it, and there is little research on how to teach it. In this work we introduce the development and testing of a novel instructional method for teaching MSS that is suitable for use in STEM courses in undergraduate and K-12 educational settings. This study builds on our past work on developing and validating a framework for characterizing in detail the cognitive levels involved in such sensemaking. This work uses the unique power of interactive simulations for assessing and developing MSS. We designed instructional activities to help students use MSS in the contexts of heat capacity and Coulomb's law. The heat capacity activity was piloted in a freshmen chemistry course and the Coulomb's law activity was piloted in a freshmen physics course. The results indicate that for students who came in with no knowledge of the relevant equation the activity supported the development of both the equation, and their understanding of the mathematical relationships of the equation. These results indicate that the teaching approach helps students engage in MSS at higher levels of cognitive complexity.

DOI: 10.1103/PhysRevPhysEducRes.19.020136

I. INTRODUCTION

The ability to express scientific concepts in mathematical terms, and more generally integrate scientific and mathematical reasoning about a phenomenon, is a foundational cognitive process that is at the heart of scientific thinking [1–3]. This cognitive process called "blended math-science sensemaking" is a necessary skill for scientists, but it is equally necessary for all citizens to be able to appropriately use the results and reasoning of science to make better decisions in their jobs and in their personal lives. This blending is a vital part of scientific thinking. Although the value of such sensemaking is well recognized, few students are learning it from their science, technology, engineering, and math (STEM) courses [4–7], and there is little research on how to teach it effectively.

In this work we introduce the development and testing of a novel instructional model for teaching blended math-science sensemaking (MSS) that is suitable for use in STEM courses in undergraduate and K-12 educational settings. This study builds on our past work on developing and validating a framework for characterizing in detail the cognitive levels involved in such sensemaking [8]. It also employs the insights we gained as to the unique power of interactive simulations for assessing and developing MSS.

Sensemaking is a *dynamic process* of building or revising an explanation to ascertain the mechanism underlying a phenomenon in order to resolve a gap or inconsistency in one's understanding [9]. Blended MSS is reflected in deep conceptual understanding of quantitative relationships and scientific meaning of equations [2,3]. This type of sensemaking has been recognized to be a critical component of expertlike understanding and expert mental models in physics [10] as well as a critical component required for developing authentic problem-solving skills [11,12]. Various MSS aspects have been described for specific disciplines [5,7,13–16]. Further, different ways of engaging in MSS have been described

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[*] Corresponding author: kaldaras@stanford.edu
[†] Corresponding author: leka2293@colorado.edu
[‡] cwieman@stanford.edu

in literature focusing on a cognitive framework that outlines the process of MSS and relating it to basic cognition [17].

Despite a significant body of research on the subject, there is little on how to teach blended MSS effectively. Educational support is needed to help students develop MSS skills, which in turn is necessary for developing a deep understanding of science. Prior work has been conducted on creating instructional approaches that support authentic learning of physics focused on investigative exploration of natural phenomena through quantitative observational experiments [17–19]. This approach, however, does not explicitly support students in developing an understanding of why specific mathematical relationships are appropriate for describing a given natural phenomenon and relating all the aspects of the relationship to specific observations of the natural phenomenon described by it. This ability to provide a causal explanation for the equation structure in relation to the natural phenomenon it describes lies in the center of the MSS process [8] and is the focus of the instructional model described in this study. Moreover, the instructional model introduced in this study is grounded in previously validated cognitive framework for MSS [8], which provides a roadmap for developing learning sequences focused on engaging students in MSS at the increasing levels of sophistication and across various STEM disciplines. Grounding the learning sequence in a validated cognitive framework ensures that the learning sequence targets specific research-based skills and knowledge needed to develop higher proficiency in MSS. Furthermore, aligning learning sequences to specific levels of the validated cognitive framework allows for quick and efficient diagnosis of the types of MSS students are struggling with in a given context and provides information on the supports students need to overcome these challenges and develop MSS proficiency. The current study provides an instructional model for supporting MSS across various STEM disciplines, particularly physics and chemistry.

We believe that including supportive activities that target MSS in science courses will benefit all students and could remove barriers for underrepresented students: research has shown that such barriers might be tied to insufficient preparation in math [13,20,21]. In this study we are introducing an instructional model for designing instructional materials and assessments that don't just support the development of routine math application skill but foster the ability to interpret math in science for explaining phenomena and solving real-life problems.

Designing effective ways of teaching and assessing any construct requires understanding of how proficiency in that construct develops with time [22]. A construct refers to a coherent collection of concepts and skills that can be used to explain performance on a test for a given topic. Proficiency in a construct refers to describing what mastery in that construct looks like [22]. The understanding of how proficiency in a construct develops is essential for designing effective instructional and assessment strategies, empirical testing, and valid interpretation of assessment results and aligning curriculum, instruction, and assessment with the purpose of helping students achieve higher proficiency in a construct [22].

Generally, cognitive models, such as learning progressions (LPs), describe how students represent knowledge and develop proficiency in a construct [22]. Little work has been done on formulating and testing a theory of MSS that (i) outlines proficiency levels of this cognitive concept and (ii) applies across different scientific fields. Initial work was done by Zhao and Schuchardt [2] that proposed a theoretical cognitive model for independent math and science dimensions outlining increasingly sophisticated proficiency levels. Their model is grounded in a review of relevant literature across different fields (including math, physics, chemistry, biology) and represents mathematical and scientific sensemaking as separate dimensions. Building off this work, we developed and validated a cognitive framework for blended MSS proficiency levels [8]. We have found that student blended MSS proficiency is largely independent of the specific disciplinary context and functions as an LP [8].

In this study we are using our MSS framework as a guide to develop instructional sequences in physics and chemistry to support students in developing MSS cognitive skills. The instructional sequences we developed are based on PhET simulations (sims) [23]. PhET sims represent a uniquely suitable tool for helping build instructional sequences that provide an authentic learning environment for fostering MSS. Specifically, sensemaking is a dynamic process [9] that focuses on the interplay of action and interpretation of the results of the action [24]. Therefore, supporting the process of sensemaking, including MSS, calls for a dynamic and interactive learning environment that allows for continuous accumulation of new quantitative evidence and feedback associated with changing the parameters of the system in question in order to support revising the explanations and developing quantitative accounts of the phenomena. PhET sims possess all these features and offer great potential for creating effective learning environments for supporting MSS. PhET sims can be enhanced by coupling them with relevant instructional materials that include additional information (such as data, reading materials, etc.). PhET sims offer a wide range of disciplinary contexts. Over the years, the PhET sims project has developed numerous simulations across various fields of science; they are free and globally accessible and used nearly 1×10^6 times/day. All these features make PhET sims a uniquely useful tool for designing instructional sequences that can effectively support MSS.

We believe that the instructional model introduced in this paper will help develop MSS proficiency in the classroom. This work presents the first attempt to develop research-based instructional materials to support MSS proficiency development grounded in the validated cognitive framework. The two research questions (RQ) addressed in this paper are as follows:

RQ1: Are the instructional sequences developed following the instructional model grounded in the cognitive framework effective in helping students develop higher MSS proficiency?

RQ2: What instructional elements are helpful in supporting students' progress along the levels of the cognitive framework for MSS?

We will discuss the implementation and results of using these sequences in introductory undergraduate chemistry and physics classrooms. We will focus specifically on discussing student learning outcomes in terms of student progress along the levels of the cognitive framework, which will help answer RQ 1. We will also discuss specific instructional scaffolds and tasks that supported students' progress towards higher levels of the cognitive framework, which will answer RQ 2.

II. THEORETICAL FRAMEWORK

A. Elements of an effective learning system

1. Cognitive framework for blended math-science sensemaking

Effective learning systems support development of higher proficiency in a topic by offering cognitively appropriate learning supports. Prior research suggests using validated cognitive frameworks, such as LPs, as roadmaps for building such learning systems [25–28]. LPs describe how proficiency in a domain develops by outlining increasingly sophisticated ways of thinking about a construct [22]. Aligning learning systems to validated LPs ensures that students are provided with research-based scaffolding aimed at helping them build deeper understanding of a construct coherently over time as they progress through the learning system [25,29].

The learning sequences developed in this study are aligned with our previously validated cognitive framework for MSS [8] that reflects increasingly sophisticated ways of blending math and science cognitive dimensions to make sense of scientific phenomena mathematically. The cognitive framework has been shown to function as an LP and is shown in Table I. The framework describes increasingly sophisticated ways of engaging in MSS as students are working towards developing a mathematical formula describing the scientific phenomenon in question or building a deeper understanding of the known formula. The framework consists of three broad levels: qualitative (level 1), quantitative (level 2), and conceptual (level 3). Each broad level consists of three sublevels: "description," "pattern," and "mechanism." At the qualitative level students cannot develop the exact mathematical relationship describing the scientific phenomenon in question, but they can identify the relevant variables (description), qualitative patterns among the variables (pattern), and describe a qualitative causal mechanism of the phenomenon (mechanism). At the quantitative level students can identify numerical values of the relevant variables (description), quantitative patterns among the variables (pattern), and develop a mathematical relationship describing the phenomenon and justify the equation using numerical values of the variables (mechanism). At the conceptual level students justify the scientific need to include all unobservable variables and constants into the mathematical relationship (description), justify the mathematical relationship by relating the observed quantitative patterns to specific mathematical operations (pattern), and describe the causal mechanism of the phenomenon reflected in the equation structure (mechanism).

The cognitive framework shown in Table I is the basis of the instructional model for teaching MSS discussed further below and was used as a guide in the current study to develop the instructional sequences. All the tasks in the learning sequences were designed to align with specific levels of the framework shown in Table I. In the two instructional sequences designed for this study students were working towards developing a mathematical relationship for a scientific phenomenon in question. The

TABLE I. Theoretical blended math-sci sensemaking framework [8]. Note that the examples provided in the table assume students are working towards developing a mathematical relationship describing the scientific phenomenon in question.

1 Qualitative	Description	Students can use observations to identify which measurable quantities (variables) contribute to the phenomenon. *Example: force and mass make a difference in the speed of a car.*
	Pattern	Students recognize patterns among the variables identified using observations and can explain *qualitatively* how the change in one variable affects other variables, and how these changes relate to the scientific phenomenon in question. *Example: the smaller car speeds up more than the big car when the same force is exerted on both.*
	Mechanism	Students demonstrate *qualitative* understanding of the underlying causal scientific mechanism (cause-effect relationships) behind the phenomenon based on the observations but can't define the exact mathematical relationship. *Example: it is easier to move lighter objects than heavy objects, so exerting the same force on a lighter car as on a heavy car will cause the lighter car to speed up faster.*

(Table continued)

TABLE I. *(Continued)*

2 Quantitative	Description	Students recognize that the variables identified using the observations provide measures of scientific characteristics and can explain *quantitatively* how the change in one variable affects other variables (but not recognizing the quantitative patterns yet), and how this change relates to the phenomenon. Students not yet able to express the phenomenon as an equation. *Example: recognizing that as variable A changes by 1-unit, variable B changes by 2 units.*
	Pattern	Students *recognize quantitative patterns* among variables and explain *quantitatively (in terms of an equation or formula)* how the change in one parameter affects other parameters, and how these changes relate to the phenomenon in question. Students not yet able to relate the observed patterns to the operations in a mathematical equation and cannot develop the exact mathematical relationship yet. *Example: recognizing mathematical relationships such is direct linear and inverse linear among others*
	Mechanism	Students can explain *quantitatively* (express relationship as an equation) for how the change in one variable affects other variables based on the quantitative patterns derived from observations. Students include the relevant variables that are not obvious or directly observable. Students not yet able to explain conceptually why each variable should be in the equation beyond noting that the specific numerical values of variables and observed quantities match with this equation. Students cannot explain how the mathematical operations used in the equation relate to the phenomenon, and why a certain mathematical operation was used. Students can provide causal account for the phenomenon. *Example: In* $F_{\text{net}} = ma$*, multiplication makes sense because when applied force on the mass of 50 kg increases from 10 to 20 N, acceleration increases by 2. That only makes sense for a multiplication operation.*
3 Conceptual	Description	Students can describe the observed phenomenon in terms of an equation, and they can explain why all variables or constants (including unobservable or not directly obvious ones) should be included in the equation. Students not yet able to explain how the mathematical operations used in the formula relate to the phenomenon. *Example: In* $F = ma$*, the* F *is always less than applied force by specific number, so there must be another variable subtracted from* F_{applied} *to make the equation work. The variable involves the properties of the surface. So, the equation should be modified:* $F_{\text{applied}}\text{-(variable)} = ma$
	Pattern	Students can describe the observed phenomenon in terms of an equation, and they can explain why all variables or constants (including unobservable or not directly obvious ones) should be included in the equation. Students not yet able to provide a causal explanation of the equation structure. *Example: In* $F_{\text{net}} = ma$*, multiplication makes sense because as applied force on the same mass increases, acceleration increases linearly, which suggests multiplication.*
	Mechanism	Students can describe the observed phenomenon in terms of an equation, and they can explain why all variables or constants (including unobservable or not directly obvious ones) should be included in the equation. Students can fully explain how the mathematical operations used in the equation relate to the phenomenon in questions and therefore provide causal explanation of the equation structure, that is how the equation (the variables and the mathematical operations among the variables) is describing the causal mechanism of the scientific phenomenon. *Example: since greater acceleration is caused by applying a larger net force to a given mass, this shows a positive linear relationship between a and* F_{net}*, which implies multiplication between m and a in the equation, or* $F_{\text{net}} = ma$*.*

tasks were designed to engage students in MSS at all the sublevels of the framework starting from the lowest and progressing to the highest level at the end of the activity.

2. Using computer simulations to support engagement in blended math-science sensemaking

Effective engagement in MSS calls for interactive systems that enable dynamic acquisition and analysis of quantitative data that support building a deeper understanding of quantitative relationships describing phenomena. PhET sims represent a suitable system for designing such learning environments. The design features embedded in PhET sims allow one to change the numerical values of the parameters and observe how the imposed changes affect the numerical values of other parameters and the phenomenon in the simulation. PhET sims are easily supplemented with numerical data if needed either by collecting the data from the simulation directly, or by using other numerical data that aligns with the sim. The instructional model presented here leveraged the capabilities of PhET sims to create learning environments that support authentic engagement in the MSS process at various levels of sophistication as reflected in the cognitive framework shown in Table I.

B. Instructional model for blended math-science sensemaking

Figure 1 reflects all the components of the instructional model for MSS introduced in this study and relationships between them. The two components include the cognitive framework (Table I) and PhET sims. Specifically, as stated above, the instructional model is grounded in the previously validated cognitive framework for MSS (Table I). The learning activity is a series of tasks that are aligned to a specific sublevel of the cognitive framework and support students' engagement in MSS at that sublevel. The dotted arrows indicate the alignment between the activity tasks and the framework sublevels. The sublevels of the framework serve as learning performances targeted by the specific activity tasks. Successful completion of the task serves as evidence of engagement in MSS at that sublevel. There might be multiple tasks designed to support one sublevel. Similarly, one task can allow for MSS engagement at different sublevels of the framework as will be demonstrated further below. Notice that this instructional model incorporates elements of backwards design [29] by targeting desired learning outcomes based on sublevel of the framework shown in Table I and aligning tasks and activities to meet those learning outcomes as shown in Fig. 1. The MSS framework in this context serves as a guide to acceptable evidence for whether students have met the learning outcomes for specific framework levels.

Finally, to ensure authentic engagement in the MSS process at all levels of the framework, tasks are designed to be completed using PhET interactive computer simulations. PhET simulations go beyond static scenarios and support sensemaking in its authentic *dynamic* state reflected in "building and revising an explanation in order to "figure something out"—to ascertain the mechanism underlying the phenomenon in order to resolve a gap or inconsistency in one's understanding" [9]. PhET simulations support that *dynamic* state of sensemaking, therefore providing an environment for engaging in and developing higher proficiency in MSS. This makes interactive simulations another important component of the instructional model.

The following sections describe the application of the instructional model shown in Fig. 1 to designing instructional activities for supporting MSS in introductory chemistry and physics courses. We will discuss the implementation results of these activities and implications for research and instruction.

III. METHODS

A. Choosing science topics

We chose the topics of Coulomb's law for physics and heat capacity for chemistry classes after discussion with the course instructors who implemented the activities in their corresponding classrooms. These topics were chosen because they were part of the corresponding course curriculum, allowed for meaningful engagement in MSS at all the levels of the cognitive framework shown in Table I, and had an associated PhET sim available that could be used in the learning activity. The screenshots of the simulations are shown in Figs. 2 and 3, respectively.

The Coulomb's law simulation allows students to explore the relationship between the amount and the type of charge, the distance between charges and the associated attractive or repulsive electric force. The target formula for the Coulomb's law activity was [$F = (Q_1 \times Q_2/\text{distance}^2) \times$ Coulomb's constant], where "F" is electric force exerted by one charge on the other, "Q_1" and "Q_2" are the magnitudes of each charge, "distance" is the distance between charges and "Coulomb's constant" is a proportionality constant

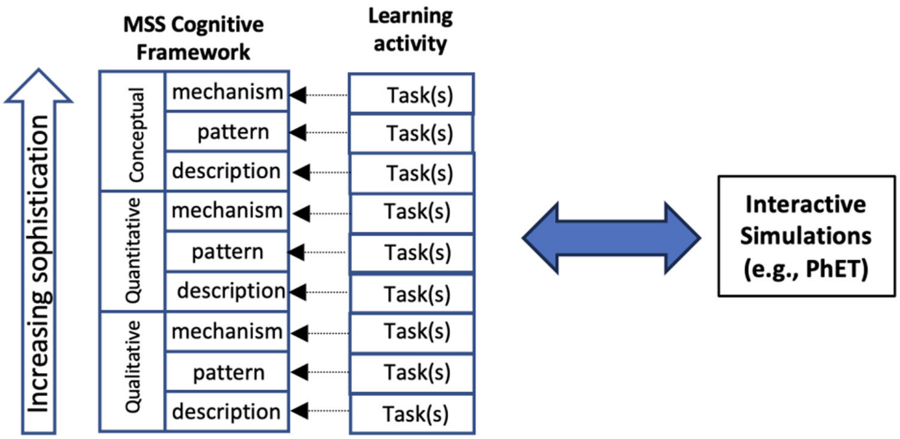

FIG. 1. Instructional model for blended math-science sensemaking.

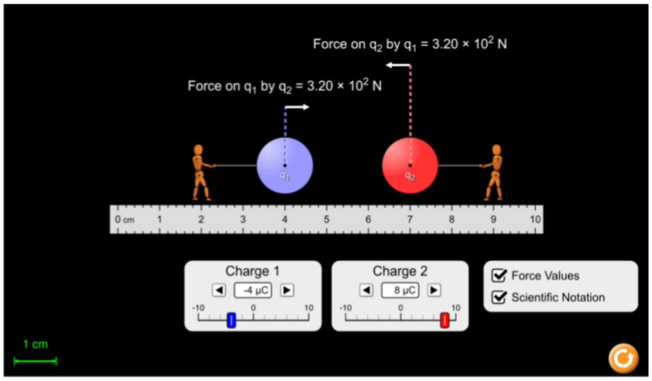

FIG. 2. Screenshot of Coulomb's law simulation.

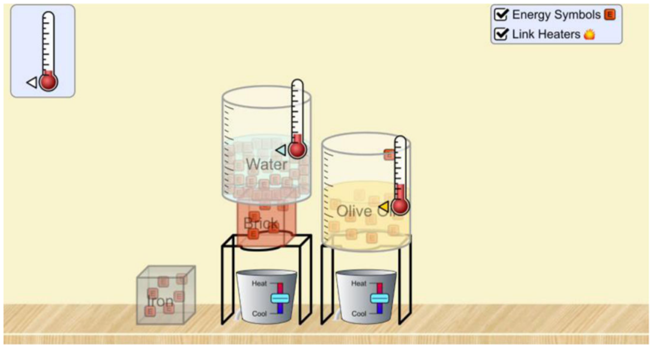

FIG. 3. Screenshot of Energy Forms and Changes simulation.

specific to the medium in which the two charges are interacting.

The energy forms and changes simulation allows one to explore the relationship between the amount of energy needed to raise the temperature of various substances. The target formula for the heat capacity activity was [energy required to change the T of a given amount of substance $= C \times \Delta T \times m$], where "$C$" is specific heat capacity of a substance, "ΔT" is the change in temperature, and "m" is mass of the substance.

B. Learning activity design

The goal of both learning activities was to have students develop a mathematical relationship for the topic of interest: Coulomb's law for physics and heat capacity for Chemistry. Table II shows the general activity structure that we followed in designing both activities. The full activities for physics and chemistry are available in the Supplemental Material [30]. This structure is widely applicable for developing MSS learning activities across STEM disciplines at the K-12 and undergraduate levels.

Note that the activity structure is based on the instructional model shown in Fig. 1 as reflected in each activity task being aligned to a specific cognitive framework level as shown in Table II and using PhET simulations. This ensures that the learning sequences follow a developmental

TABLE II. Learning activity structure for supporting blended math-sci sensemaking.

Activity Progress & increasing sophistication

Activity task[1](brief description)	Cognitive framework alignment
Preassessment (CR[2])	Assess the starting level before the activity
Identify the variables relevant for characterizing the phenomenon mathematically (MC[3])	Level 1 (description)
Identify qualitative patterns among the variables (MC)	Level 1 (pattern)
Identify quantitative patterns among the variables using the simulation and the data when applicable (MC)	Level 2 (pattern)
Midassessment: suggest and justify a mathematical relationship for the observed phenomenon using what you have learned so far in the activity (CR)	Assess the level upon exploring quantitative patterns
Postassessment: pick the most likely mathematical relationship for the observed phenomenon (MC) and justify your choice using what you have learned in this activity (CR)	Assess the level upon exploring quantitative patterns
Suggest how to calculate the value of the relevant constants (if applicable) and explain the scientific meaning of the constants (CR)[4]	Level 3 (description)
Describe the causal mechanism for the phenomenon reflected in the mathematical relationship that you proposed	Level 3 (mechanism)
Application questions for a given topic (if needed)	Assess whether level placement is consistent across contexts (transfer)

[1] Students are engaging with each task as they are interacting with the simulation.
[2] Constructed response.
[3] Multiple choice.
[4] Coulomb's law constant and specific heat constant for physics and chemistry activities, respectively.

approach and support student engagement in MSS according to the increasing levels of sophistication in the cognitive framework. The PhET simulation design supports student MSS at various levels of sophistication. For example, we ask students to use the simulation to identify the relevant variables (level 1 description), qualitative and quantitative patterns among the variables (level 1 pattern and level 2 pattern, respectively). Both simulations support engagement in MSS with these tasks by allowing students to change various conditions of the system and observe how the imposed changes affected the relevant variables. For example, in Coulomb's law sim the students can change the magnitude and the type of charges and the distance between them and observe how the magnitude of the associated attractive or repulsive electric force changes. Similarly, in Energy Forms and Changes sim the students can heat up various substances such as water, brick iron, and oil and observe how much energy shown in the form of energy cubes it takes to raise the temperature of various substances by a specific number of degrees.

Finally, both activities include three formative assessment tasks probing whether students develop the mathematical relationship of interest at various points in the activity: (1) preassessment before engaging in the activity, (2) midactivity assessment after exploring quantitative patterns using the simulation and the data (when applicable), (3) postactivity assessment. Importantly, all three assessment tasks distinguish between levels 1, 2, and 3 mechanism and level 3 pattern types of MSS. Specifically, for each of these assessments students might provide justification consistent with level 1 mechanism (qualitative causal mechanism only), level 2 mechanism (correct formula and qualitative justification), level 3 pattern (correct formula and justification relating quantitative patterns to mathematical operations in the proposed equation, but no explanation of the causal explanation of the equation structure), and level 3 mechanism (same as level 3 pattern including causal explanation of the equation structure). See Table I for sample responses for each of these sublevels.

C. Physics and chemistry activity design differences

Both chemistry and physics activity follow a similar design structure discussed in Sec. III B and shown in Table II. However, there are several design features that are different between the two activities. These differences are summarized in Table III. The main reason for the differences was that we wanted the students to go through the same reasoning and learning processes despite differences between the simulations.

The sims are different in the degree to which they allow for quantitative exploration. Coulomb's law sim allows us to explore numerical quantitative patterns related to how the magnitude of attractive and repulsive forces is affected by distance between the two changes and charge magnitude (see Fig. 1). Energy Forms and Changes sim, in turn, is more qualitative, but it does allow one to observe the number of energy units (energy cubes) needed to heat up a given substance by a certain number of degrees (Fig. 2). Because of these differences, we provide data for Energy Forms and Changes sim (see heat capacity activity in the Supplemental Material [30]) to ensure that students have the supports for engaging in quantitative pattern identification. For Coulomb's law activity students were not provided any data, but rather were expected to use the sim to identify quantitative patterns.

Further, students were asked to identify specific quantitative patterns in both activities before being asked to develop their final mathematical relationship (questions 5 and 4 for Coulomb's law and heat capacity activities respectively shown in the Supplemental Material [30]). No additional questions on identifying quantitative patterns using the data were asked in heat capacity activity to ensure

TABLE III. Design differences between physics and chemistry activities.

Design feature	Coulomb's law	Heat capacity
Sim design	Allows to explore specific quantitative relationships.	Quantitative exploration is supported less. However, it is possible to count the number of energy cubes it takes to raise the temperature of a substance by 1 or more degrees.
Preassessment	Develop math formula based on introductory scenario before interacting with the sim.	Develop mathematical relationship based on interaction with the simulation.
Data availability	Data were not provided because the sim allowed for quantitative exploration.	Data were provided because the sim did not fully support quantitative exploration.
Exploration of quantitative patterns and postassessment	Asked to explore the quantitative patterns with the sim prior to being asked to develop a math formula.	Asked to explore the quantitative patterns using the sim prior to being shown the data. Data were provided to students, but they were not explicitly asked to identify quantitative patterns in the data prior to being asked to develop a math formula.

TABLE IV. Scoring rubric for pre-, mid-, and postconstructed response assessment tasks.

Accurate formula provided?	Elements of student justification	Cognitive model level
No	No justification	Level 0 (no indication of MSS)
Yes	No justification	Cannot be accurately determined
Yes	Justification provided at any of the sublevels described for level 1	The corresponding sublevel 1
No	Lists only variables relevant for describing the phenomenon mathematically (Table V description sample response)	Level 1 (description)
No	Described qualitative relationships among the relevant variables (Table V pattern sample response)	Level 1 (patterns)
No	Describes qualitative causal mechanism for the phenomenon (Table V mechanism sample response)	Level 1 (mechanism)
No	Described numerical values of the relevant variables (Table VI description sample response)	Level 2 (description)
No	Described quantitative patterns among the relevant variables (Table VI pattern sample response)	Level 2 (pattern)
Yes	Justifies the mathematical relationship using specific numerical values of the variables (not identified)	Level 2 (mechanism)
Yes	Justifies the mathematical relationship by relating the mathematical operations in the equation to specific quantitative patterns observed in the simulation and/or the data (Table VII pattern sample response)	Level 3 (pattern)
Yes	Provides the causal mechanistic explanation of the equation structure (not identified)	Level 3 (mechanism)

that the activities are maximally parallel. Since the physics activity did not have any data associated with it, adding questions on quantitative pattern identification with the data in the chemistry activity would have made it significantly different from the physics one.

Finally, the preassessment format for the two activities differed. Preassessment in the Coulomb's law activity focuses on an introductory scenario that modeled the type of quantitative exploration students would eventually engage in once they started interacting with the sim. The preassessment in the heat capacity activity, on the other hand, asks students to use the sim right away to suggest a mathematical relationship. Considering the Energy Forms and Changes sim was less quantitative, it is likely that the preassessment activity for heat capacity was significantly more difficult than the one for Coulomb's law. The decision to not have an introductory scenario for preassessment in the heat capacity activity was made to ensure that student have enough time to explore the sim during the activity. The chemistry class that had ~50 min to finish the activity. Students had a 2-h lab period to finish the physics activity.

D. Implementation context

Both activities were implemented during Fall 2022 as part of freshmen undergraduate courses in a large public university in the Western part of the United States. The Coulomb's law activity was implemented in an algebra-based introductory physics course for non-physics majors. The activity was implemented in a paper-pencil formal during the laboratory session. Students worked individually, and each turned in the assignment. A total of 73 students submitted the assignment. The heat capacity activity was implemented in a one-semester introductory chemistry course for chemistry and biochemistry majors. The activity was implemented using the Qualtrics survey tool during the regular class session. Students worked in groups of 2–3 people to complete the activity. Each group was allowed to turn in one submission. There were 29 students in the class, and they submitted 16 assignments.

E. Data analysis

Student responses to all activity tasks were transferred to an excel spreadsheet format. Further, each student response to each MC task was scored directly into the corresponding sublevel of the cognitive framework. This was easy to do since each MC task was designed to probe a specific level of the cognitive model as shown in Table II. The pre-, mid-, and postassessment tasks were also directly assigned a specific cognitive model level according to the rubric shown in Table IV. The rubric was previously shown to yield high interrater reliability (IRR) [7].

The main factor in the level assignment was the degree of sophistication of the provided justification as shown in the rubric (Table IV). Briefly, level 1 justification focused on qualitative identification of cause-effect relationship describing the scientific phenomenon in question. Level 2 justification involved justifying the proposed mathematical formula using specific numerical values of the relevant variables. For example, at level 2 students might justify the correct Coulomb's law formula by stating that this formula makes sense because you multiply given values of $Q1$ and $Q2$, divide by the squatted value of the distance between the charges, and need to multiply by Coulomb's constant. At level 2 the cause-effect relationship was still at the qualitative level, similar to that of level 1. Finally, level 3 focused on justifying the proposed mathematical relationship by directly relating quantitative pattern observations to specific numerical operations in the equation and describing the causal mechanism reflected in the equation structure. For example, level 3 justification for the correctly proposed mathematical relationship for Coulomb's law would state something like

> "The magnitude of electric force is linearly related to the total amount of charge on the interacting objects and inversely related to the square of the distance between the objects. This implies that the charge should be in the numerator, and the distance squared should be in the denominator of the math equation. The equation for the force should show that the force is caused by charged objects being brought close enough to interact."

Further, at the highest sublevel students should also recognize that the math equation should be multiplied by a proportionality constant that ensures that the two equation sides are equal.

Table IV shows how combinations of no, vague, or inaccurate formulas with specific types of justifications relate to specific sublevels of the cognitive framework shown in Table I. For example, if the accurate formula was provided, but the justification was consistent with level 1, the final level assignment was at level 1. Further, if the formula provided was inaccurate, the level assignment was also based on the degree of sophistication of the justification following the same criteria as shown in Table IV. However, in this case students would not be assigned any level beyond level 1. See the results section for examples of student responses.

The final level assignment upon completion of the activity was based on the following pieces of evidence: (1) the most sophisticated justification provided at any of the three points during the activity (pre-, mid-, or post-assessment), (2) student responses to MC items probing specific sublevels as shown in Table II. The results section discusses student learning outcomes grounded in these pieces of evidence in more detail.

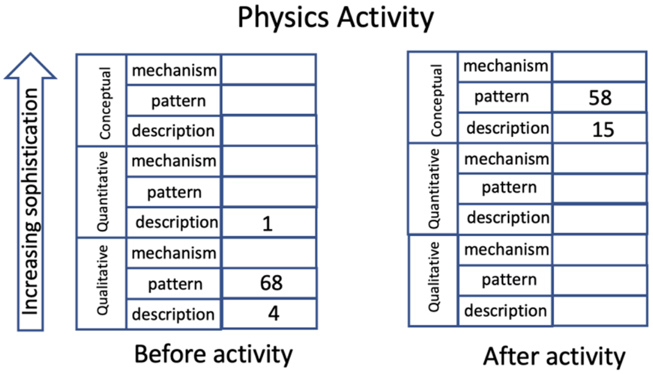

FIG. 4. Number of students and their sublevel placement before and after physics activity.

IV. RESULTS

A. Physics classroom: Overall student progress upon completion of the activity

The overall level assignment before and after physics activity is shown in Fig. 4. As shown in Fig. 4, most students started at various sublevels of level 1 (qualitative). Table V shows sample response for level 1 (qualitative). Note that even if students provide any kind of mathematical relationship before the activity (vague, inaccurate, or fully accurate), the cognitive model level was assigned based on the *degree of sophistication and accuracy of their justification*. For example, sample response for level 1 (qualitative) pattern suggests an initial inaccurate formula [$\mathbf{distance/charge = f}$]. The justification for the formula describes qualitative patterns, which is consistent with the assigned sublevel.

Notice that Table V also contains a sample response for the level 1 mechanism. Even though none of the students scored in this sublevel as their highest one, there were some students who provided responses consistent with this sublevel at some point during the activity but attained a higher sublevel by the end of the activity. One such example is shown in Table V.

Table VI shows sample responses for level 2 of the framework. Similarly, a level 2 mechanism sample response in Table VI was provided by a student at some point during the activity who later developed a higher sublevel.

Upon completion of the activity most students had transitioned to level 3 (conceptual) of the framework. The final level was assigned by taking into consideration

TABLE V. Sample student responses for level 1 (qualitative) of the framework for Coulomb's law.

Qualitative	Mechanism	$\mathbf{F = distance \times q}$; "the distance between the spheres and the charge affect the force"
	Pattern	Response #1: **no formula**, "as distance increases force decreases, as charge decreases, force increases" Response #2: $\mathbf{d/charge = f}$; "as distance increases, force decreases; as charge increases, force increases"
	Description	**no formula**, "magnitude of force depends on distance and charge."

TABLE VI. Sample student response for level 2 (quantitative) of the framework for Coulomb's law.

Quantitative	Mechanism	$\mathbf{F} = \mathbf{charge}\ 1 \times \mathbf{charge}\ 2/\mathbf{distance}^2$ Justification: "when separation increases force decreases"
	Pattern	Correctly answering all MC questions on quantitative pattern identification (see question 5 for this activity in the Supplemental Material [30])
	Description	$\mathbf{F} = \mathbf{charge} \times \mathbf{initial\,separation}$ Justification: "when separation increases force decreases; when charge increases (1 U to 9 U) force increases (from 0.62 N to 5.6 N)"

the following learning indicators: (1) whether students arrived at the correct formula for Coulomb's law; (2) the level of sophistication of their justification for the formula as related to the cognitive framework levels. The highest-level justification provided at any point during the activity counted for final level assignment. Importantly, all students were able to arrive at the correct formula by the end of the activity. Most of them (58) provided justification consistent with sublevel 3 pattern of the framework, which allowed us to place them on a sublevel with a high degree of confidence. These students presented the correct mathematical relationship, correctly identified all the relevant quantitative patterns in MC questions (Q5), recognized the need for the proportionality constant in the equation, and demonstrated the ability to directly relate quantitative patterns to the mathematical operations in the equation in their justification. Table VII shows sample responses for level 3 sublevels. No one attained the highest sublevel of the framework—level 3 (conceptual) mechanism. Hence, there is no sample response shown for that sublevel.

Further, there were 15 students who provided the correct mathematical relationship, but either provided written justification consistent with level 1 (qualitative) or did not provide any justification. To gauge whether these students understood the provided mathematical relationship at a higher level, we evaluated the following indicators: (1) whether the students correctly identified quantitative relationships probed in multiple choice questions (question 5 of the Coulomb's law activity shown in the Supplemental Material [30]) as the indicator of level 2 (quantitative) pattern sublevel and; (2) whether they recognized the need for the constant of proportionality as the indicator of level 3 (conceptual) description (questions 8 and 10 of the Coulomb's law activity shown in the Supplemental Material [30]). The analysis showed that all 15 students were able to correctly identify the quantitative patterns, which is consistent with level 2 (quantitative) pattern and recognize the need for the constant of proportionality. They also said why it is important and how to calculate it, which is the criteria of level 3 (conceptual) description, and hence was the level they were assigned. Overall, there is a clear progression of all 73 students from the lowest, level 1 to the highest, level 3 of the cognitive model.

B. Physics classroom: Which tasks helped students attain their highest LP level?

Figure 5 shows a diagram reflecting the path students took from preassessment to the final LP level assignment. The left portion of Fig. 5 shows where the students in physics class started in terms of providing the formula for Coulomb's law on preassessment task. There were three groups: (1) those that provided an accurate formula but no justification before the activity (29 people); (2) those that provided a vague or inaccurate formula (38 people); (3) those that did not provide any formula (6 people). The middle portion of Fig. 5 shows the types of tasks that helped students in each group attain their highest LP level. We further discuss these groups.

TABLE VII. Sample student responses for level 3 (conceptual) for Coulomb's law.

Conceptual	Pattern	$\mathbf{F} = (\mathbf{Q1} \times \mathbf{Q2}/\mathbf{d}^2) \times \mathbf{C}$ Justification: as distance increases, force decreases exponentially by a factor of x^2; as charge increases, force increases by a factor of X
	Description	Sample response #1: $\mathbf{F} = (\mathbf{Q1} \times \mathbf{Q2}/\mathbf{d}^2) \times \mathbf{C}$ Justification: as distance increases, force decreases, as charge increases, distance increases Identifying quantitative patterns identification (Q 5): all correct Recognizing the need for a constant and provide sample calculations (Q 8 &10): yes Sample response #2: $\mathbf{F} = (\mathbf{Q1} \times \mathbf{Q2}/\mathbf{d}^2) \times \mathbf{C}$ Justification: none Identifying quantitative patterns identification (Q 5): all correct Recognizing the need for a constant and provide sample calculations (Q 8 &10): yes

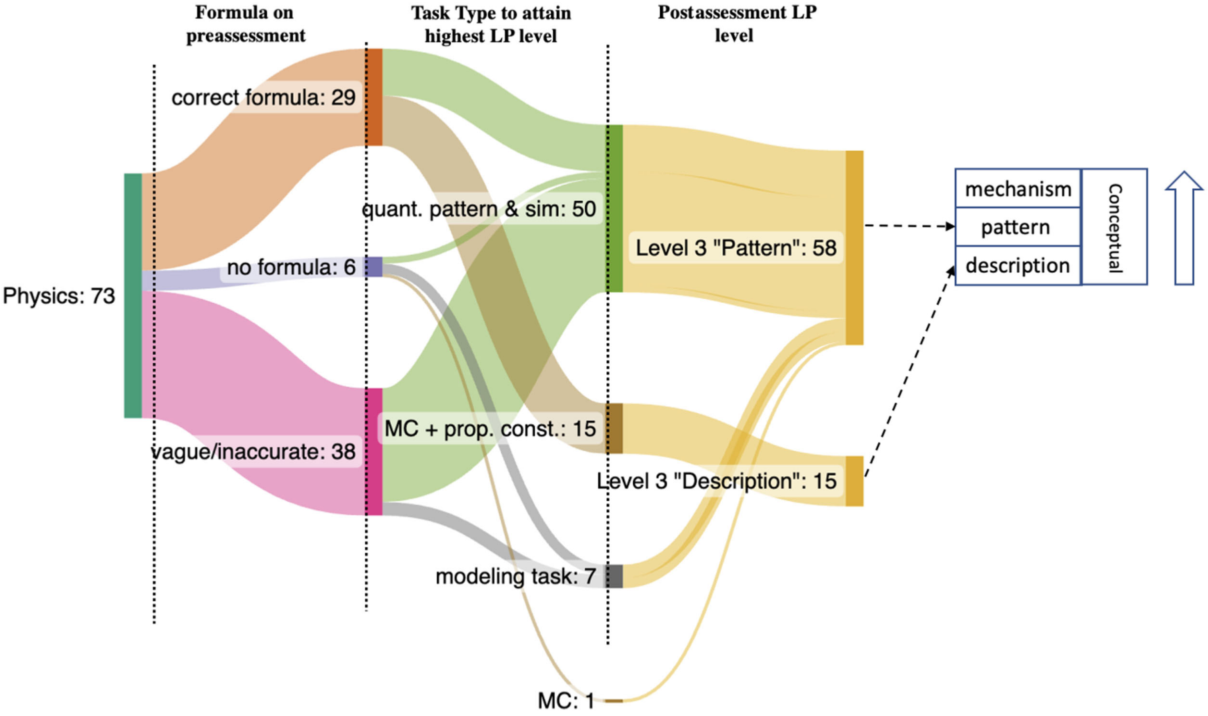

FIG. 5. Student learning path in physics activity.

1. Students who did not provide a formula before the activity

There were six students who did not provide any formula before the activity (Fig. 5). All six students eventually provided justifications consistent with level 3 (conceptual) pattern. This finding suggests that the activity was helpful in supporting student progress to the highest, level 3, of the cognitive model. Moreover, the majority (5 out of 6 students) did not need MC list of possible formulas to figure out the mathematical relationship. Rather, they were able to figure out the correct formula upon completing the tasks focused on using the sim to identify the quantitative patterns, which is reflective of engaging in MSS at level 2 (quantitative) pattern level. Therefore, engaging in MSS at level 2 (quantitative) pattern seems to be critical in helping students transition to level 3.

2. Students who provided vague or inaccurate formula before the activity

Out of 38 students who provided vague or inaccurate formula (Fig. 5), all eventually figured out the formula before the MC list of possible formulas (before question 7) and provided justifications consistent with level 3 (conceptual) pattern. This finding suggests that the activity was helpful is supporting progress to the highest, level 3, of the cognitive model. Moreover, all 38 students needed only the tasks focused on engaging them in the level 2 (quantitative) pattern type of blended MSS (questions 5 and 7) to figure out the correct formula. This is consistent with the trend for the group described above suggesting that engagement in level 2 pattern MSS is critical for helping students transition to level 3.

3. Students who provided the formula before the activity

Out of 29 students who provided the formula before activity (Fig. 5), no student provided justification beyond level 1 for the proposed formula. Fourteen students out of these 29 students later provided justification consistent with level 3 (conceptual) pattern after interacting with the simulation and answering MC questions focusing on the quantitative pattern identification (question 5). Similar to the two groups described above, engagement in level 2 (quantitative) pattern MSS is critical for helping students transition to level 3. Overall, most students (57 out of 73) provided an accurate formula and justification after engaging in level 2 pattern type of MSS suggesting that these tasks are critical for helping students attain level 3.

The remaining 15 students correctly answered the MC questions focused on identifying quantitative patterns (question 5) and recognized the need for the proportionality constant in the equation, which is reflective of level 3 description but did not provide any justification at any point during the activity. There is no evidence that they can relate the identified quantitative patterns to the equation structure indicating they are at level 3 description. They need instructional supports to relate the quantitative patterns to the equation structure, level 3 pattern.

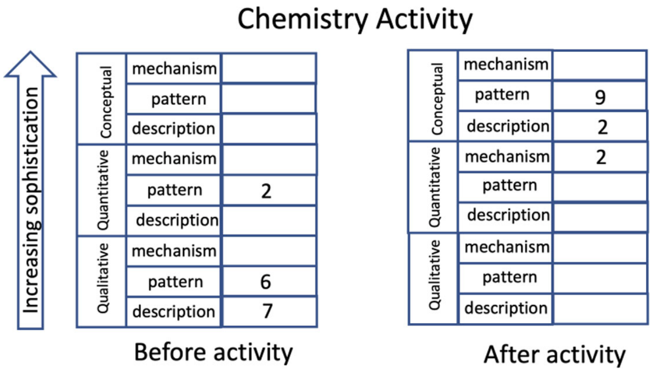

FIG. 6. Number of student groups and their sublevel placement before and after activity. Note that two groups did not provide justification for the formula or answers to other assessment questions which makes it impossible to determine their post-activity level.

C. Chemistry classroom: Overall student progress

We followed the same process as described for physics activity to assign LP levels to student responses on the questions probing their ability to engage in MSS at various levels for the heat capacity activity. Figure 6 shows the overall student group progress by the end of the activity. Note that students in the chemistry classroom worked in groups and submitted the activity responses in groups as well. As a result, Fig. 6 shows progress of the groups of students rather than individual students.

Sample student responses for each level are shown in Tables VIII–X. No one attained the highest sublevel of level 3 (conceptual), so there are no sample responses for that sublevel.

Overall, there is a clear progression of all the groups to the higher levels of the cognitive framework, except for the two groups that did not provide all responses. Next, we will discuss how this progress occurred during the activity focusing on the tasks that promoted the growth.

D. Chemistry classroom: Which tasks helped students attain their highest LP level?

Figure 7 shows a diagram reflecting the path students took from preassessment to the final LP level assignment in chemistry activity. Only 1 group started with providing the correct formula, but they did not provide any justification for it. This group eventually provided justification at level 2 (quantitative) mechanism level based on the numerical values of the data provided to them (see Table IX mechanism sample response).

The remaining 14 groups started with no formula initially. Out of 14 groups, 5 groups developed the correct mathematical relationship and justification consistent with level 3 pattern upon completing the tasks on identifying the quantitative patterns (Question 4) and exploring the data provided to them. These five groups therefore did not need to see the list of possible formulas to develop the correct mathematical relationship. The remaining nine groups

TABLE VIII. Sample student responses for level 1 (qualitative) of the framework for heat capacity.

Qualitative	Mechanism	An increase in given amount of a substance (g) with an increase in the change in final temperature requires a higher amount of energy to be put into the system.
	Pattern	As you add more mass, it will become more difficult to increase the temperature of the substance.
	Description	The change in temperature, mass, and volume

TABLE IX. Sample student response for level 2 (quantitative) of the framework for heat capacity.

Quantitative	Mechanism	**E required to change the T of a given amount of substance = $C \times \Delta T \times m$** Justification: As seen the temp is dependent on the mass and not simply addition of division but multiplication of numbers.
	Pattern	There is a direct, linear relationship with how the temperature of a substance can be changed and with the amount of energy added to the substance
	Description	Adding 8 energy cubes to water increases the temperature by 1 degree

TABLE X. Sample student responses for level 3 (conceptual) of the framework for heat capacity.

Conceptual	Pattern	**E required to change the T of a given amount of substance = $C \times \Delta T \times m$** Justification: Temperature change by 10 changed the joules in the system by a factor of 10. If you double the mass of the substance, it doubles the joules in the system.
	Description	provided correct calculations of the specific heat capacity constants for brick, water and olive oil using the data provided

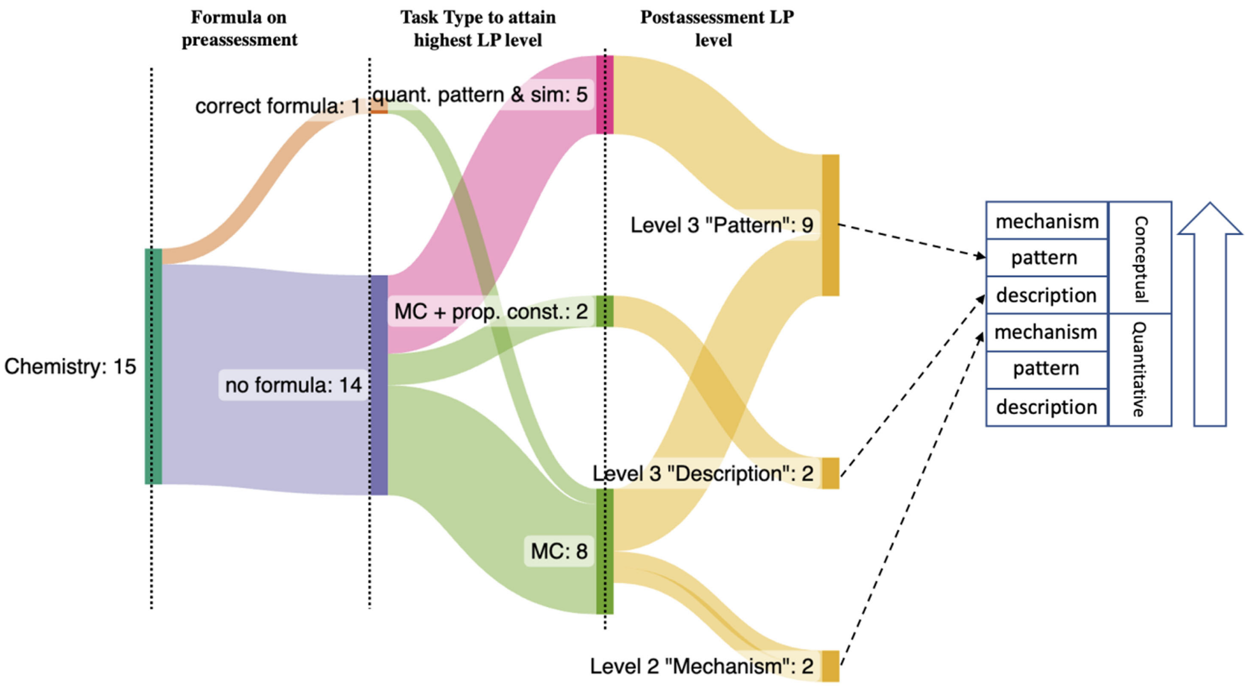

FIG. 7. Student learning path in chemistry activity.

identified the correct mathematical relationship from the MC list of possible formulas provided to them (Question 6). *This is the main difference between the chemistry and physics classes: more students in the Chemistry class needed to see the list of possible formulas to identify the correct one compared to physics class. In the Physics class most students identified the mathematical formula by exploring the simulation prior to seeing the list of possible formulas. We discuss the possible reasons and implications of this finding in the next section.*

Out of these nine groups, 4 groups progressed to level 3 pattern as reflected in their justification of the correct formula. Two groups provided justification at level 3 description but not relating quantitative patterns to the equation structure in their justifications. One group progressed to level 2 mechanism as indicated by them correctly identifying the formula and justifying their formula using the numerical values from the data provided to them. The remaining 2 groups correctly identified the formula by the end of the activity but did not provide justification at any point nor answer questions 4, 7, and 8, so their postactivity level cannot be accurately determined.

V. DISCUSSION

We have presented an instructional model for teaching MSS in STEM courses. The instructional model (Fig. 1) is grounded in a previously validated cognitive framework for MSS (Table I) and leverages PhET simulations to support student engagement in MSS at all sublevels of the framework. This study demonstrated the use of the instructional model to design learning activities for supporting development of MSS skills in undergraduate freshmen physics and chemistry courses and presented the implementation results of these activities. The results indicate that the activities are effective in helping students in both disciplines develop higher MSS proficiency. We will further discuss how our findings answer the study RQs and provide recommendations for instructors on using this approach for supporting development of MSS.

We have demonstrated that the learning sequences developed following this instructional model for Coulomb's law in Physics and Heat Capacity in chemistry help most undergraduate students transition from the lowest to the highest levels of the cognitive framework. Moreover, we discovered that students in both classes started mostly at the lowest level of the cognitive framework, level 1. This indicates that students had very little understanding of the mathematical relationship underlying the phenomenon under study and how the relationship described the scientific observations but made great progress in completing the learning sequence. Student response evidence indicates that most students were able to successfully engage in MSS at the highest level (conceptual) pattern type of sensemaking. These results indicate that the instructional sequences developed following the instructional model presented here (Fig. 1) are effective in helping students develop higher MSS proficiency, which addresses RQ1.

Further, engaging students in level 2 (quantitative) pattern type of MSS seems to be critical to helping them transition to level 3. This finding indicates that learning tasks focused on engaging students in identifying specific quantitative patterns are effective in helping them reach level 3 and build connection between the quantitative patterns and the mathematical structure of the underlying relationship. This answers RQ2 of the study *RQ2: What instructional elements are helpful in supporting students' progress along the levels of the cognitive model?*

The instructional model for MSS presented here provides an effective roadmap that can be used by instructors in any STEM discipline for designing activities aimed at supporting MSS development. Specifically, aligning the learning tasks with specific sublevels of the cognitive framework ensures that instructors can identify specific ideas that students are struggling with [26,29]. The current study demonstrated that one of the most common student struggles in both classes was relating the quantitative patterns identified using the simulation to the equation structure, which is reflective of level 3 (conceptual) pattern type of MSS. While most students in both classes were able to attain this sublevel by the end of the activity, there were still several students who didn't provide justifications for their formula consistent with this sublevel. However, those students still demonstrated engagement in MSS at all the lower sublevels of the framework, indicating considerable progress. This finding suggests that the instructional approach grounded in the cognitive framework is effective for both organizing and guiding instruction: it offers actionable information on student MSS proficiency to guide how to support students to overcome their difficulties. For example, an instructor might choose to spend additional time on helping students transition to level 3 (conceptual) pattern by providing additional learning opportunities focused on helping student make connections between the quantitative patterns consistent with level 2 (quantitative) pattern and the equation structure for various phenomena. An example of such learning opportunity would be an activity focused on supporting students in identifying quantitative patterns in a provided data (either table or graph format), relating those patterns to mathematical relationships (for example, directly or inversely proportional relationships), and translating these observations to specific mathematical operations among the relevant variables (for example, division for inversely proportional relationships).

Another important aspect of this instructional approach is grounding the MSS in student exploration of the interactive computer sims, such as PhET sims. Evidence from the current study suggests that PhET sims are effective in supporting engagement in MSS at all the sublevels of the cognitive framework shown in Table I. This suggests that interactive computer sims such as PhET sims help students authentically engage in MSS. Specifically, PhET sims offer an authentic way to explore the phenomenon in question, identify the relevant variables and their associated numerical values (level 1 and 2 description, respectively), gauge the scientific and mathematical meaning of the unobserved variables (level 3 description), explore qualitative and quantitative relationships among the relevant variables (level 1 and 2 pattern, respectively), investigate the cause and effect relationships among the relevant variables to develop qualitative or quantitative understanding of the causal mechanism (level 1 and 2 mechanism, respectively), build a deeper understanding of the relationship between the quantitative patterns and the mathematical operations in the formula (level 3 pattern), and the causal structure of the mathematical relationship (level 3 mechanism). These unique features of PhET sims remove cognitive complexities of static learning environments, such as the need to verbally describe the phenomenon, introduce data tables reflecting relevant quantitative patterns among the variables, or encounter measurement errors associated with hands-on experiments, all of which can significantly complicate engagement in MSS.

Moreover, the data suggest that the sims that allow for direct and obvious exploration of the quantitative patterns among the variables might be more effective in helping students transition to level 3 than those that do not. This is reflected in the difference between the learning paths among the physics and chemistry students: most physics students developed the math relationship right after tasks focused on identifying quantitative patterns among the relevant variables (Fig. 4), while most of the chemistry students needed to see the MC list of possible formulas to finalize the mathematical relationship in addition to completing tasks on quantitative pattern identification (Fig. 6). Even though the chemistry simulation was supplemented with additional data tables showing the amount of energy it takes to heat up different amounts of the four substances by various number of degrees most student groups (10 out of 15) did not develop the formula and justification from exploring the data and the simulation. This finding suggests that a sim that does not support straightforward exploration of quantitative patterns (level 2 (quantitative) pattern type of MSS), even with supplementary data, is not as effective in helping student develop the mathematical relationship and progress to level 3.

An alternative explanation for the observed differences is that students in the chemistry classroom were not able to engage in effective quantitative pattern exploration because they were not explicitly asked to identify quantitative patterns among the relevant variables in the provided data but were only asked to do that in the simulation (question 4), which does not allow for a straightforward identification of the patterns. Therefore, future research should focus on introducing slight modifications to the chemistry activity focused on adding MC questions asking students to identify specific quantitative patterns in the data, parallel to question 5 in the physics activity. Introducing this instructional support could potentially be sufficient for students to develop the mathematical relationship upon exploring the data and without the need to see the MC list of possible formulas.

Finally, the data show that no student attained the highest sublevel of the framework-level 3 (conceptual) mechanism. This finding suggests that simply asking students to justify their proposed relationship and explain the causal mechanism described by the relationship is not sufficient for eliciting this level of MSS. A way to address this shortcoming might be to scaffold student exploration of the sim and data (when applicable) to help students identify specific causal relationships and relate them back to the equation structure.

There are several limitations of this work that are important to mention. First, the instructional model presented in this study is demonstrated using a small number of topics and students. In the future it would be beneficial to expand the design of the learning sequences to include more relevant topics across various STEM disciplines, including chemistry, physics and biology among others and pilot the sequences with a larger number of students. Additionally, future research needs to address ways of helping students explicitly engage in level 3 (conceptual) type of MSS as they pursue the instructional sequences. More research is needed on designing and evaluating the effectiveness of instructional supports for engaging students in MSS at the highest sublevel of the framework across STEM topics and disciplines. Finally, the learning approach needs to be extended to support engagement in MSS with other important mathematical tools, such as vectors and vector operations.

VI. CONCLUSION AND RECOMMENDATIONS FOR INSTRUCTORS

This work introduces an instructional model for developing MSS skills across STEM disciplines. The model is grounded in a previously validated cognitive framework and as demonstrated in this study is a straightforward and effective way of helping students develop higher MSS proficiency. The mode is flexible and can be easily adopted to the learning goals of specific STEM courses by identifying computer sims that support engagement in MSS in the context and with the math formula of interest and designing tasks aligned to specific sublevels of the framework as demonstrated here. Notably, the forma of all tasks can be converted to MC if needed to ensure quick scoring. However, we highly recommend using several constructed response tasks specifically focused on evaluating the justifications provided by students. This is the most straightforward way to evaluate their MSS level and identify the types of supports needed to help them build MSS proficiency. Often students do not see the need to write justifications, which should be explicitly addressed at the classroom level. For example, a number of students either provided no justification or provided justification consistent with level 1 (qualitative). These students need learning supports that help them build connections between the quantitative patterns and the equation structure, including all the relevant variables and the mathematical operations among the variables, and talk about these ideas in the justification. One way to achieve this goal could be presenting students with contrasting cases showing justifications for a given formula consistent with different levels of the framework, and discussing why some justifications are more sophisticated and precise than others.

ACKNOWLEDGMENTS

We would like to thank the University of Colorado Boulder physics and chemistry instructors, Dr. Eleanor Hodby and Dr. Robert Parson for providing feedback on the design of the instructional sequences and piloting the sequences in their classrooms. We would also like to thank Dr. Georg Rieger from the University of British Columbia for providing feedback on the design of the learning sequences and sharing his pedagogical and subject-matter expertise at all stages of this project. This work would not be possible without their support, input, and encouragement. This work was supported by the Yidan Foundation.

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