Recognition and conversion of electric field representations: The case of electric field lines

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We conducted a study with introductory and upper-division physics students in a Mexican university to learn how students independently recognize the electric field's main characteristics in the electric field lines diagram and as a source or target representation in conversion processes. We used the theory of registers of semiotic representations and a phenomenographic approach as a framework to analyze data. The recognition and conversion abilities were explored through interpretation and construction tasks. We identified students' main difficulties in recognition and conversion in the interpretation and construction tasks. In conversion processes, we found that when the electric field lines diagram is the source representation, students do not interpret the field lines' density as the magnitude of the electric field. The difficulties of interpretation that arise in these conversion processes depend partially on the target representation. We also found that constructing electric field lines is especially difficult for students at both introductory and upper-division levels. Most students would prefer to draw vector field plots instead. We recommend that electricity and magnetism teachers and researchers be aware of the difficulties that the recognition and conversion in interpretation and construction tasks may represent for their students in understanding the electric field concept.

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I. INTRODUCTION

In the study of electromagnetism at the university level, students must understand the concept of electrical fields and their relation to physical phenomena. Identifying students' difficulties in learning the electric field concept is fundamental for developing strategies to help them overcome them efficiently. The literature in physics education research has identified the main conceptual difficulties, such as the confusion between electric field and electric force and the difficulties applying the superposition principle of the electric field. Furthermore, several studies have identified that students often misinterpret the representations of the electric field.

In a previous study [1], we explored the role that the conversions between multiple semiotic representations of the electric field may play in students' understanding of this physical quantity. We applied the theory of registers of semiotic representation [2] and a phenomenographic approach [3] to analyze the results. In the framework of the theory of registers of semiotic representations, semiotic representations are understood as symbols that represent something else (e.g., an object, a concept). The term register refers to a representation system. The term "conversion" is a synonym for a translation between two different representations or transduction in the social semiotics framework [4]. We use the terms semiotic representations, registers, and conversions consistently throughout this article in coherence with the theoretical framework.

In this study, we continue to explore this methodology with a different strategy, improved instruments, and a broader set of participants. This new approach allowed us to identify students' abilities to recognize each representation individually through interpretation and construction tasks and their conversion abilities for each representation, either as the source or the target. Conversion tasks involve interpreting the source representation and constructing the target representation. It is helpful to explore them with this different strategy to compare students' difficulties when recognizing representations individually and in conversion tasks.

We conducted a study with Mexican introductory and upper-division electromagnetism students to explore their abilities to interpret and construct representations of the electric field. The interpretation or construction tasks involve recognizing the electric field's characteristics and

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converting electric field representations. The representations for the electric field concept that we considered in this study are three widely used representations in textbooks and the classical electromagnetic theory [5,6]: electric field lines, vector field plots, and the algebraic notation of the field. This contribution is part of a group of articles focused on the interpretation and construction tasks for the three representations considered in the study. In this case, we focused on the electric field lines representation: the interpretation and construction of electric field lines and the electric field lines diagram acting as the source or the target representation.

We first present a literature review covering the main difficulties of learning the electric field concept and using the electric field line representation. We then present the theory of registers of semiotic representations as the theoretical framework and explain how the concepts of this theory are related to the aim of the investigation, followed by the methodology and data analysis strategy. We present the results in two main sections that correspond to the tasks of interpretation and construction of the electric field lines diagram. Finally, we discuss the implications of this study for teaching and learning the electric field concept at the university.

II. PREVIOUS RESEARCH

The difficulties in understanding the electric field concept have been widely studied in physics education research literature. Some students tend to confuse the concepts of electric field and electric force [7–9]. This difficulty is due to a conceptual change that should happen when switching from a Newtonian to a Maxwellian profile [7] and could be associated with an incomplete understanding of the electric field as a vector field [8]. Many students move between a Newtonian and a Maxwellian profile to explain the characteristics of the electrical field, which illustrates the difficulties of conceptualizing the electric field as a vector field [10]. Several studies have found that students have difficulties applying the superposition principle [11,12]. Some students think that only the nearest charges contribute to the net electric field or that other electrical objects can block the electric field [9,13,14].

Students' abilities to move between representations of the electric field are associated with their conceptual understanding [1]. However, each representation may elicit different difficulties due to its nature (e.g., verbal, algebraic, vectorial, graphical). Students have difficulties understanding the electric field line representation, as reported in the literature. Several studies have found evidence that some students tend to treat electric field lines as real entities or tubes that transport charge [15–17]. Some students confuse the electric field lines diagram, the electric field produced by a single charge, and the net electric field at a position [16]. When analyzing the electric field's magnitude, students may present difficulties relating the electric field line density with the magnitude of the electric field [15,18]. When analyzing the electric field's direction, some students identify it as the curve described by electric field lines instead of the tangent to the curve [14]. When applying the superposition principle, some electric field line diagram characteristics create a blocking effect because field lines cannot cross [18–20]. When analyzing students' construction of electric field lines inside different capacitors, many students would identify that field lines are drawn from positive to negative charges perpendicular to the surface. In contrast, fewer students would identify that electric field lines cannot cross [21]. In a different study by the same authors, they found that most students identified that electric field lines could not cross when analyzing the electric field of conductors [22], suggesting that different applications (i.e., capacitors versus conductors) may elicit the identification of different properties of electric field lines. It is important to recognize that these difficulties will not apply to all students; however, the most prevalent difficulties need to be identified to inform the creation of more effective instructional material.

III. DEFINITION OF THE STUDY

A. Theoretical framework

Physics education research has different theoretical frameworks and approaches for analyzing students' use of multiple representations in problem solving [23]. The theoretical framework for the current study is based on the theory of registers of semiotic representations [2]. This framework links representational use with conceptual understanding through the synergy between representation systems [24], while other well-known frameworks for semiotic representations focus on how the representation systems are used [4,25].

Figure 1 presents a diagram of the elements of the theory of registers of semiotic representations and their interrelations. Semiotic representations use symbols, rules, and associations to represent an object within this theory. Since mathematical objects can only be accessed through semiotic representations, the role of semiotics is not only to represent the object but to allow cognitive activity in the form of transformations of registers of representations [2]. The transformations can be treatments when they occur within the same register and conversions when the transformation is between two or more different registers.

Duval [2] proposes that the different types of transformation (treatments and conversions) have several sources of difficulty that affect understanding the object. One difficulty is the recognition of the characteristics of the object in the representation. This difficulty is more critical in conversions because it is necessary to recognize the characteristics in two registers that do not use the same symbols, rules, and associations to represent the object. Another source of difficulty is the direction of the conversion since students can successfully convert from one representation to the other but not vice versa. The third source of difficulty is that students associate the

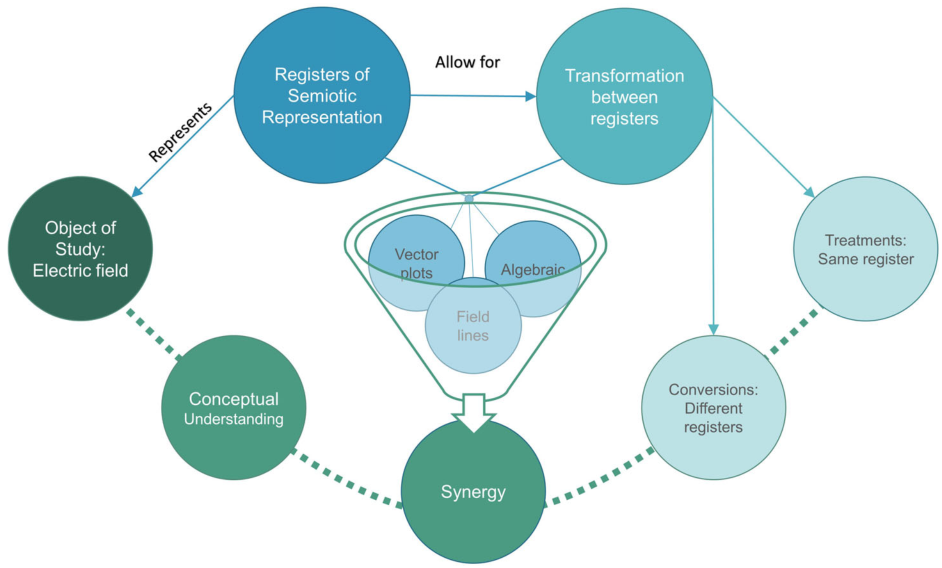

FIG. 1. Main concepts of the theory of registers of semiotic representations and their connections.

representation of an object with the object itself, so dissociation is necessary to promote the conceptual understanding of the object. For dissociation between the representation and the object to occur, it is necessary to have synergy in converting registers of representation.

The theory of registers of semiotic representations has been widely used to link the recognition and conversion difficulties with cognitive activity and understanding in mathematics education [24,26–28]. In physics education research, several studies have acknowledged the relevance of the theory [29–32], but only a few have included it as a theoretical framework [1,32]. In this study, we refer to this theory to link students' representational use of electric field lines and their understanding of the electric field concept.

To define the theory's elements in this study's context, we consider the electric field concept as the object of study. We acknowledge that the electric field is a physical quantity, but the concept is highly abstract and can only be accessed through semiotic representations. The three registers of semiotic representations we consider in this study are the electric field lines diagram, the vector field plot, and the algebraic notation. These representations are relevant in learning electromagnetism at the university level, as reported in other studies [1,33]. The study focuses on the electric field lines diagram; it explores the role of this representation in the conversion to or from the vector field plot or the algebraic notation.

B. Research questions

In this study, we refer to the theory of registers of semiotic representations as a theoretical framework to approach the following research questions.

- What difficulties do students have in recognizing the main characteristics of the electric field in the electric field lines diagram?
- What difficulties do students have converting from the electric field lines representation to the vector field plot and the algebraic notation of the electric field?
- What difficulties do students have converting from the vector field plot and the algebraic notation to the electric field lines diagram?

IV. METHODOLOGY

We conducted a study with 295 engineering physics students in a private Mexican university, of which 210 took the introductory electricity and magnetism course and 85 took the upper-division electromagnetism course. The introductory electricity and magnetism course used a well-known textbook and tutorials [5,34]. It was an active-learning setting that included peer instruction and cognitive scaffolding activities and took place in a SCALE-UP environment [35,36]. The course covered the topic of electrostatics during the first six weeks of the course and electromagnetic induction in the last two weeks (which included non-Coulombic electric fields). In between, the course covered electric circuits and magnetism (8 weeks). The upper-division electromagnetism course used a standard textbook [6]. The setting was traditional, with some active-learning strategies. The course covered the topic of electrostatics for the first half of the semester (8 weeks), magnetism (5 weeks), and electromagnetic induction (last 3 weeks), covering non-Coulombic electric fields. Both courses used the three representations in lectures and learning materials, which are standard representations. However, there was no explicit instruction on how to convert between them. The students in this cohort have previously presented shared characteristics with students in the United States [37] and Spain [1]. The data collection took place in three semesters of 2018 and 2019 at the end

TABLE I. Definition of the items' objectives and type of task.

	Task	
Objective	Interpretation	Construction
Recognition of electric field lines	Item Q1.1	Item Q2.1
Conversion (source: electric field lines)	Items Q1.2 and Q2.2	
Conversion (target electric field lines)		Items Q1.3 and Q2.3

of the course, after all the course contents had been covered.

We administered two versions of a questionnaire that included three items of interpretation and three of construction, as detailed in Table I. The questionnaires were administered randomly to all students, and each student answered only one of the questionnaires. Upon administering the tests, the students were informed of the study's objective, that their participation was voluntary, and of the possibility of opting out by not answering the questionnaire. They were assured that their participation would not affect their grades. They were explicitly asked not to include identifiable information, such as names or identification numbers, in their response. Therefore, the data were treated anonymously.

A. Instrument

We present the instrument in two sections: (1) interpretation tasks: the items that require an interpretation of the electric field lines diagram; and (2) construction tasks: the items that require constructing an electric field lines diagram. All the questions were in Spanish; we present a translation of the instrument.

1. Interpretation tasks

Item Q1.1 targets the recognition of the magnitude and direction of the electric field in the electric field lines diagram presented in Fig. 2. Students interpret the magnitude in the first part of the problem (a) and the direction in the second part of the problem (b) independently. The correct answer in part (a) is to identify that the electric field is stronger at B and weaker at A. The electric field at C, D, and E is approximately the same between A and B. A complete justification should include that the density of electric field lines represents the relative magnitude of the field. In part (b), the correct answer is to describe the direction at C as an up and right diagonal and E as an up and left diagonal. The complete justification should include that the direction of the electric field at each position is tangent to the electric field lines.

The conversion items where the electric field lines diagram is the source representation are Q1.2 and Q2.2. The electric field presented in both items is the same, but in Q1.2, students convert to the vector field plot, and in Q2.2, to the algebraic notation. Both conversions are done independently. The item (shown in Fig. 3) has been validated in previous studies [1,33]. This is an interpretation task since the electric field lines diagram is the source representation. The students should recognize the characteristics of the electric field in the diagram and be able to represent the characteristics of the field in a different register of representation (i.e., the vector field plot or the algebraic notation). The correct interpretation requires the students to recognize that the magnitude of the field

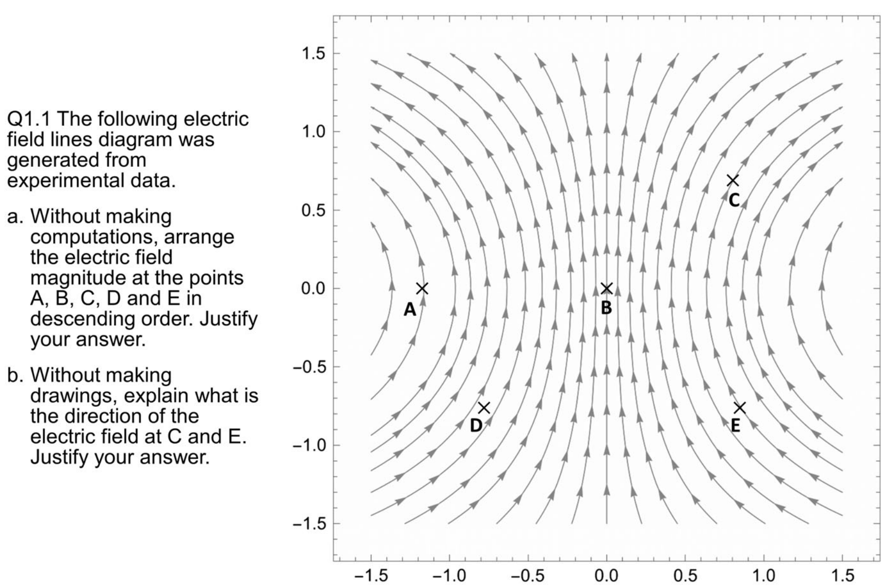

FIG. 2. Item Q1.1 targets the recognition of the magnitude and direction of the electric field in the electric field lines diagram.

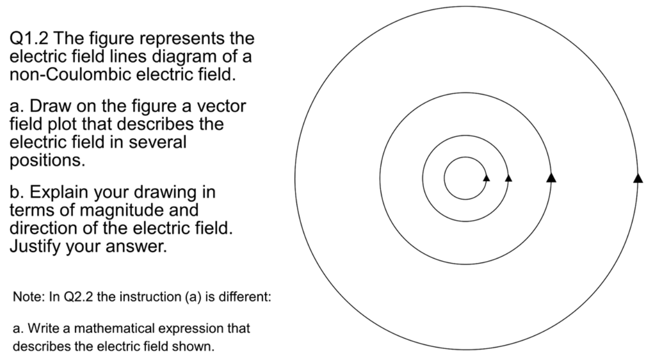

FIG. 3. Items Q1.2 and Q2.2 have the objective of conversion from the electric field lines diagram to the vector field plot or the algebraic notation, respectively.

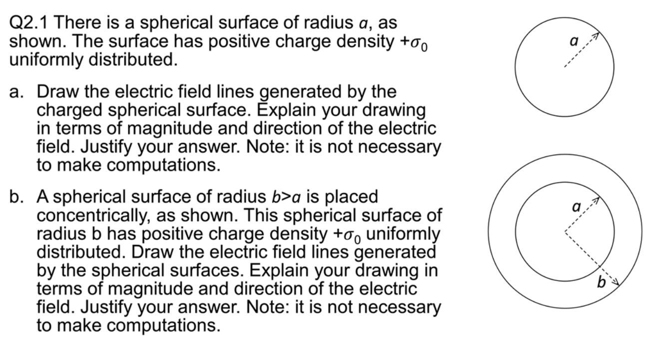

FIG. 4. Item Q2.1 targets the recognition of the electric field and the application of the superposition principle in the electric field lines diagram.

decreases with the distance from the center because the density of electric field lines decreases and that the direction is tangent to the electric field lines in a counterclockwise direction. In item Q1.2, students are expected to draw enough vectors over the same circle to visualize changes in the field direction. In addition, in positions closest to the center, students draw larger vectors that decrease as they move away from the center. In item Q2.2, students are expected to generate a mathematical expression that includes a proportionality constant, an inverse dependency of the radius, and angular direction in polar coordinates.

2. Construction tasks

Item Q2.1 targets the recognition of the electric field and the application of the superposition principle in the electric field lines diagram, as presented in Fig. 4. This is considered a construction task because the students need to create an electric field lines diagram of a situation using the principle of superposition. Students construct an electric field lines diagram of a single charged sphere in the first part of the problem (a) and two concentric spheres in the second part of the problem (b). In part (a), the correct answer is to draw electric field lines that start at the surface of the charged sphere ($r = a$) and extend radially outward, as in Fig. 5(a); inside the sphere ($r < a$), there should be no field lines. A complete justification should include that the electric field inside the sphere is zero, and outside the sphere decreases with the distance from the center. In part (b), the correct answer is to draw electric field lines that start at the surface of the first sphere ($r = a$) and extend radially outward, adding more field lines that start at the surface of the second sphere ($r = b$) and extend radially outward, as in Fig. 5(b); inside the sphere ($r < a$), there should be no field lines. The complete justification should identify the different regions of the problem and how each sphere affects each region.

The conversion items where the electric field lines diagram is the target representation are Q1.3 and Q2.3. In item Q1.3 (Fig. 6), students convert from the algebraic notation, and item Q2.3 (Fig. 7), from the vector field plot.

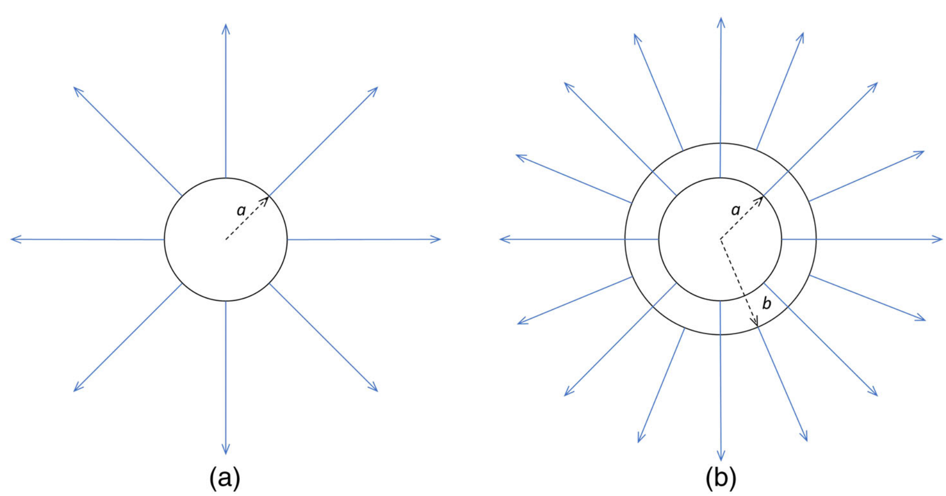

FIG. 5. Expected construction of the electric field lines (blue) in item Q2.1. (a) Corresponds to the expected answer in Q2.1 (a), and (b) is the expected answer in Q2.1 (b).

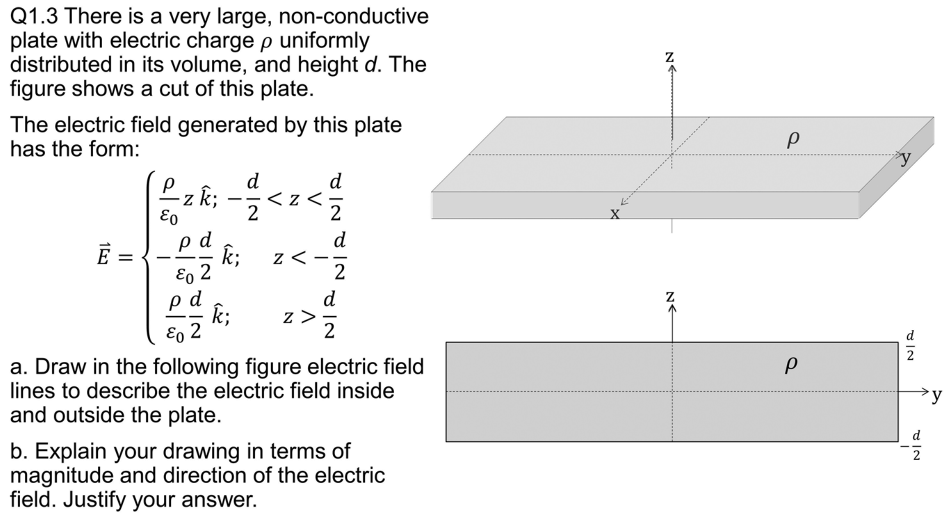

FIG. 6. Item Q1.3 aims to convert from the algebraic notation to the electric field lines diagram.

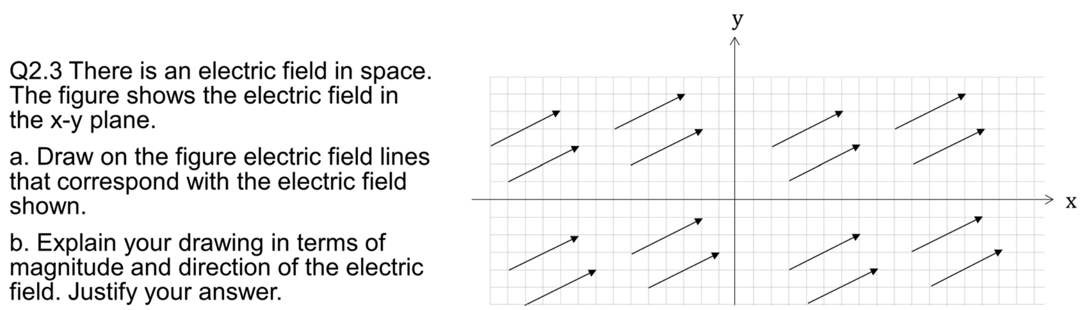

FIG. 7. Item Q2.3 aims to convert from the vector field plot to the electric field lines diagram.

Since the electric field lines diagram is the target representation of both items, they are construction tasks. To convert efficiently from the algebraic notation and the vector field plot to the electric field lines diagram, students should recognize the characteristics of the electric field in the source representation and portray these characteristics of the field in the electric field lines diagram. In item Q1.3, the correct construction [Fig. 8(a)] requires the students to recognize that the magnitude of the field increases with the distance from the x-y plane inside the plate and it remains uniform outside the plate, and to represent this variation with the density of electric field lines. Students should also recognize that the direction is upward above $z = 0$ and downward below $z = 0$. In item Q2.3, the correct construction [Fig. 8(b)] requires the students to recognize that the magnitude of the field is uniform and to represent it

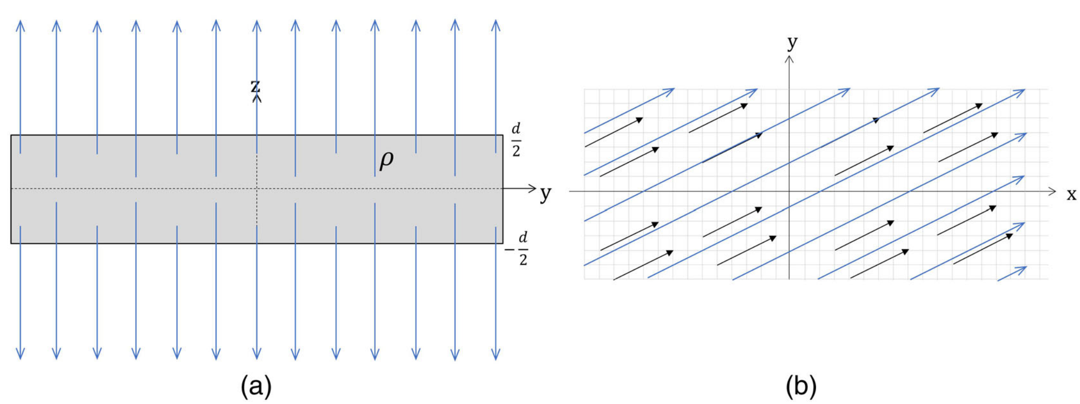

FIG. 8. Expected construction of the electric field lines (blue) in conversion items. (a) Corresponds to the expected answer in Q1.3, and (b) is the expected answer in Q2.3.

with a uniform density of electric field lines. Students should also recognize and represent that the direction is diagonal, parallel to the vectors.

B. Data analysis

The phenomenographic method was used to analyze the students' responses, as it allows for finding different ways in which people interpret significant aspects of reality through the description, analysis, and understanding of people's experiences [3]. Under this approach, conceptions and ways of understanding are not seen as individual qualities but as descriptive categories that group each person's specific cases. Descriptive categories in various situations are considered stable and can be generalized between situations, even if each individual moves from one category to another at different times. The set of descriptive categories denotes some collective understanding [3]. The phenomenographic approach to data collection and analysis referred to students' explanations for creating descriptive categories about the recognition and conversion of the electric field concept. In a previous study with 146 participants, we presented the creation and validation process of categories with an interrater reliability of 0.90 [1]. The students in this study share the same characteristics as the Mexican sample in the previous study.

The data analysis was performed in Spanish. The names of the categories were translated, as well as the examples of students' answers. As the first step for data analysis, descriptive categories were created for each question based on the answers of 20 randomly chosen students, and a group of experts reached a consensus. To create descriptive categories, we considered both the drawings and equations generated by the students and their explanations. Each question has different descriptive categories, given the nature of the questions and the type of recognition or conversion required as interpretation or construction tasks. After identifying the emerging categories, the remaining answers were analyzed and classified into them. If new categories emerged from the data, they were integrated into an iterative process. The descriptive categories have a hierarchical sequence but are not limited to defining whether a response is correct or incorrect. Instead, they describe students' most common skills and difficulties in their responses. It is important to note that several experts on the subject made and validated the categories, as experts can determine the hierarchy of descriptive categories.

The type of task differentiates the items with a recognition objective (Q1.1 and Q2.1), either interpretation or construction of the representation. The item of interpretation of the magnitude and direction in each representation (item Q1.1) was categorized independently for each characteristic. The item that involves a construction task (item Q2.1) was analyzed for the system of a single electrical field source and then for the system with two sources independently. We identified the different patterns in students' answers, their strategies of interpretation and construction of representations, and the application of the superposition principle.

The conversion-objective items (Q1.2, Q2.2, Q1.3, and Q2.3) were analyzed with a structure that reflects recognition and conversion abilities in a framework for the descriptive categories. The primary sources of difficulty that relate the use of representations to conceptual understanding in Duval's theory [2] are the recognition of the mathematical object in two representations that do not have the same characteristics and the conversion between them. All questions have the same structure of theoretical categories, even if their descriptive categories are different. This structure allows comparisons between conversion tasks. The theoretical structure of the conversion items is further explained in Sec. V C.

V. RESULTS

A. Interpretation of electric field line diagrams

Table II presents examples of students correctly interpreting questions Q1.1, Q1.2, and Q2.2 and the percentages of introductory and upper-division students who answered each item correctly. In question Q1.1 (a), there are percentages more significant than 30% for the two student profiles. Observe that the recognition skills improve when moving from an introductory profile to an upper-division profile in items Q1.1 (a), Q1.2, and Q2.2. It is relevant to qualitatively identify these differences in the correct response, as they demonstrate the expertise students gain in their transit from novices to experts. In item Q1.1 (b), recognition of the direction decreases when moving from the introductory to the upper-division profile. This difference may be due to alternative interpretations where students do not resort to the electric field line diagram characteristics representing magnitude and direction (i.e., the tangent to the field line). The alternative interpretations are discussed in Sec. V A 1.

1. Alternative interpretations related to surface features of the representation

Students sometimes use surface features of the item's diagram to explain their answers. In some cases, the surface features of the problem allowed the students to reach an appropriate conclusion about the electric field, even if their explanation was incorrect. In other cases, attention to the surface features of the representation diverted students from reaching a correct conclusion regarding the electric field. In Table III, we present the categories that emerged in items Q1.1 (a), Q1.1 (b), and Q1.2, where students use the surface features of the representation to explain their answers. For further reference, we present the drawings in Fig. 9.

The category "Magnitude: Distance from the center" includes students who ordered the magnitude of the electric field correctly and explained that the magnitude is obtained

TABLE II. Examples of the correct interpretation for items Q1.1, Q1.2, and Q2.2, and the percentages of introductory (Intro) and upper-division (UD) students who answered each item correctly.

Categories	Item	Example	Intro	UD
Recognition of magnitude, the density of field lines	Q1.1 (a)	"*$B > C = D = E > A$; B the lines are closer together, D and E are at the same height approx. C equals E, and A is the smallest because the field is less dense*." Upper division, EM2019-A11	47%	70%
Recognition of direction, tangent to field lines	Q1.1 (b)	The direction in C: "*Up and left at approximately 45 degrees horizontally*." The direction in E: "*Up and right at approximately 45 degrees horizontally*." Explanation: "*Tangents to field lines give the field direction at a point*." Upper division, EM2019-A15	29%	16%
Recognition of magnitude and direction, conversion to vector field plot	Q1.2	Drawing: Several vectors that are smaller than the vectors inside denote the changes of magnitude. [See Fig. 9(a).] Explanation: "*The magnitude decreases as you move away from the center because there is less density of lines, and the direction is circular*." Upper division, EM2018-A2-11	28%	36%
Recognition of magnitude and direction, conversion to algebraic notation	Q2.2	Mathematical expression: $\vec{E} = \frac{A}{r}\hat{\phi}$ Explanation: *Direction, the field circulates, so it goes in $\hat{\phi}$; Magnitude, there is a greater magnitude to the center, so it falls to a higher radius*." Upper division, EM2018-A1-2	12%	29%

TABLE III. Difficulties related to surface features. The answers imply difficulties in recognizing the characteristics of the electric field in the electric field lines diagram in items Q1.1 and Q1.2.

Categories	Item	Example	Intro	UD
Magnitude: Distance from the center	Q1.1 (a)	"*$B > D - E - C > A$; B is greater because it is in the center, D-E-C because they are on equipotential lines. A is the smallest for being the furthest from the center*." Introductory, EM2019-I28	12%	0%
Direction: Coordinate axes	Q1.1 (b)	The direction in C: $\vec{e} = \cos 60°\hat{\imath} + \sin 60°\hat{\jmath}$ The direction in E: $\vec{e} = -\cos 60°\hat{\imath} + \sin 60°\hat{\jmath}$ Explanation: "*their x-components appear to be the same but opposite*." Introductory, EM2019-I3	11%	12%
Direction: In the direction of the line	Q1.1 (b)	The direction in C: "*In which the arrow points, following the line*." The direction in E: "*In the direction of the field line according to the arrow*." Explanation: "*Field lines show the direction that would follow a positive charge if no other forces were acting on it*." Upper division, EM2019-A11	27%	44%
Magnitude: Increases with the radius	Q1.2	Drawing: Arrows tangent to the circles to denote the direction changes, but it is observed that the arrows are more prominent in the outer circles compared to the arrows in the inner circles [see Fig. 9(b)]. Explanation: "*At each point, its direction is described by the orientation of the arrow and its magnitude by the size of the arrow*." Upper division, EM2018-A2-12	32%	25%

with the distance from the center in item Q1.1 (a). From their explanation, it cannot be determined whether the students interpreted the center as a critical position because of the density of field lines or for some other reason. Although students in this category could correctly order the magnitude of the field in the five different positions, there is no evidence that they located the density of lines as representing the magnitude of the field instead of the distance from the center.

The category "Direction: Coordinate axes" includes students who described the direction of the electric field correctly and explained it using field vectors or Cartesian coordinates in item Q1.1 (b). Students in this category did not express that the field vector is tangent to the line, so it is impossible to determine whether this representation feature was considered.

The category "Direction: In the direction of the lines" includes students who explained that the electric field has the direction given by the field lines and did not mention that it is tangent to the field line in item Q1.1 (b). These students did not recognize that the tangent of the field line at each position describes the direction of the electric field. This explanation is interesting because field lines are curves, while electric field vectors cannot have curvature. The curvature of the field lines can be considered a surface feature of the representation because it depends on the system and is not directly related to the direction of the field but by its tangent.

The category "Magnitude: Increases with the radius" includes students who interpreted that the magnitude of the field increases with the radius in item Q1.2. It is impossible to determine why students in this category interpreted the magnitude as increasing relative to the center. However, it can be inferred that they did not recognize the density as the characteristic of the field lines diagram representing the magnitude. One possibility is that students relate the size of the circles, which is a surface feature, to the electric field's magnitude.

The results in Table III show that the surface features of the electric field lines representation can direct the attention of students with less experience toward different difficulties. Specifically, in terms of recognition of the magnitude of the electric field as the density of field lines, there are difficulties in the questions where students interpret field lines. On the one hand, in item Q1.1 (a), students reach a correct conclusion, even if their explanation does not consider the density of lines. Although the conclusion is correct, this explanation is not entirely adequate, as there are other regions of the field line diagram where the density is more significant when the distance to the center is also greater. It is also noted that only introductory students follow this explanation, so there is a qualitative decrease in this difficulty as they move from introductory to upper division.

On the other hand, in item Q1.2, students come to an incorrect conclusion, as they indicate that the electric field increases with the radius. This difficulty presents a risk since when concluding that the electric field increases, students do not analyze what this implies in terms of energy conservation; in this case, an electric field that increases with the radius means that the field in infinity tends to infinity, which also implies that the energy is infinite. This analysis would be expected from an expert on the subject. From Table III, both introductory and upper-division students tend to commit this recognition difficulty, implying that it is persistent between the two profiles.

As for recognizing the electric field's direction as a tangent to the field lines, two different explanations were observed. On the one hand, some students explained it based on coordinate axes, which are not in the problem. This explanation does not represent a difficulty by itself; instead, it indicates an alternative to how students use graphical tools in their explanations. This explanation was helpful for introductory and upper-division students in virtually the same percentage. Instead, the explanation where students relate the electric field's direction to the direction of the lines does present a difficulty [14]. Students may be aware that the direction of the field is tangent to the field line but prefer to write it differently. Since we can only use the explicit information provided in students' answers, we cannot interpret that the students that answered "in the direction of the line" meant it was tangent. It is important to differentiate between a tangent direction and the direction of the line. In this case, if the direction of the electric field is the same as that of the line, it implies that the electrical field vector has curvature. Electric field vectors describe the direction of the field in a single position, so the arrow has no spatial dimensions and, therefore, cannot have curvature [38]. Students should be aware of this difference and be more cautious when interpreting the direction of the field in the electric field lines diagram. Also, such explanations can be associated with the notion that the field line represents the path a point charge would follow if placed within the electric field [39]. As for this difficulty, when moving from introductory to upper division, more students relate the field's direction to the direction of the lines. It is interesting to identify that, as students become more experienced, the difficulty of identifying the direction of the field lines as a direct indicator of the direction of the electric field is reinforced, or even of relating the field lines as trajectories through which electric charges would move.

2. Association with electric field sources

Students try to identify the electric field sources for the field line diagram they are interpreting. In a prior study, the same behavior was observed among experts, who resorted to the location of field sources in the diagram and not to the field line density to explain the magnitude of the electric field [40]. This presents different types of difficulty for the questions in this study.

TABLE IV. Answers that imply difficulties in recognizing the characteristics of the electric field in the electric field lines diagram in items Q1.1 and Q2.2. These difficulties are related to the association of the electric field with a source. We present the examples and the percentages of introductory (Intro) and upper-division (UD) students who answered each item with the corresponding difficulty.

Characteristics	Item	Example	Intro	UD
Magnitude: Distance from a source	Q1.1 (a)	"*A, C = E, D, B; A is closer to the charge, C and E are at the same distance, but both are closer than D, B is two times the distance of D.*" Introductory, EM2019-I6	21%	5%
Association with an electrical source	Q2.2	Mathematical expression: $\vec{E} = -\frac{\sigma}{\varepsilon_0}\frac{1}{r^2}\hat{\phi}$ Explanation: "*The electric field decreases with distance and maintains an azimuth direction.*" Upper division, EM2018-A1-7	23%	10%
Association with magnetic source	Q2.2	Mathematical expression: $\vec{E} = \frac{\mu_0 I}{2r}\hat{\phi}$ Explanation: "*As it moves away from the center, the intensity decreases, which is seen in factor $1/r$. The field 'circulates,' so it goes in the direction of $\hat{\phi}$ in the proposed expression.*" Upper division, EM2018-A1-3	13%	10%

The category "Distance from a source" includes students who explained that the electric field magnitude at each point is obtained by the distance between the point and the electrical field source in item Q1.1 (a). The field line diagram provided does not display any electric field sources, so the students imagine the location of one or several sources. It is impossible to determine if students imagine or place their sources in the center of the diagram, at the left, right, bottom, or upper edges, if they think of positive or negative charges, or imagine one or more sources in different locations. As the placement of field sources, in this case, depends on each student, the students accommodated the field magnitude in different orders.

The category "Association with an electrical source" includes students who associated the electric field line diagram with an electric field source in item Q2.2. This includes students who used the law of Coulomb, σ, ε_0, or some dependence on the square of the inverse of the distance ($1/r^2$) in their mathematical expression. In some cases, students related the direction to the angular coordinate, and in other cases, to the radial direction. This presents difficulty in recognizing the magnitude of the field, as students do not resort to field line density as an indicator of magnitude but use other explanatory elements, such as the location of an electric field source. In some cases, this interpretation was also linked to difficulty recognizing the field's direction as the tangent to the electric field lines.

The category "Association with a magnetic source" includes students who related the field line diagram to magnetic type sources in item Q2.2. When answering the questionnaire, students were already familiar with nonconservative electric fields and knew the variation of the magnetic field as a source for the electric field. Some students associated the field line diagram with a magnetic field source because they included some current indicators and the permeability constant in empty space in their mathematical expressions. However, their mathematical expression meets the conversion requirements: it includes a proportionality constant (in this case, an electric current, or permeability in empty space, a reverse dependence on r and angular direction). Other students wrote an expression related to Faraday's law and explained that a change in magnetic flux must generate a non-Coulombic electric field. By making this expression, we identified that students associated the electric field lines diagram with a source. Still, they did not relate the density of field lines to the magnitude of the field nor the tangent to the lines with the direction.

Based on the results presented in Table IV, it is generally observed that the difficulties of associating with some type of source decrease when moving from an introductory profile to an upper-division profile. The association with a field source is not necessarily incorrect, and it has been observed that some experts prefer such explanations [40]. In these cases, in item Q1.1, this association resulted in incorrect conclusions since, by the nature of the diagram shown, it is impossible to accurately determine the location of the electrical field sources. In item Q2.2, some students satisfactorily converted to the algebraic notation. It should be remembered that the objective is not to evaluate whether students come to a correct or incorrect conclusion but to identify the different paths and, more importantly, how these different paths can lead to difficulties in recognizing the characteristics of the electric field in the field line diagram and the conceptual understanding of the electric field.

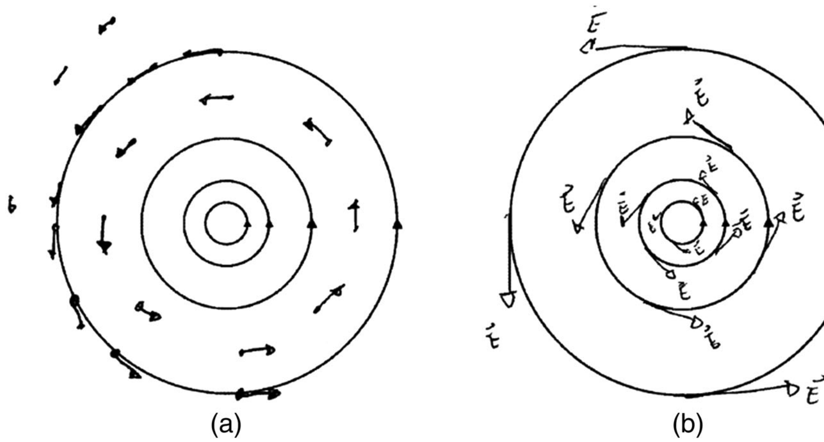

FIG. 9. Examples of (a) recognition of magnitude and direction of the electric field lines diagram and (b) difficulty of recognition “Magnitude: increases with radius”.

B. Construction of electric field line diagrams

Studying the construction skills of electric field line diagrams has allowed us to identify how students use the characteristics of the electric field line diagram to represent the magnitude and direction of the electric field. This is one of the steps required to perform the conversion to the electric field line diagram and demonstrate the recognition of the characteristics of the diagram that represent the electric field. The items where students are asked to draw electric field line diagrams are Q2.1, Q1.3, and Q2.3.

TABLE V. We present the correct construction for items Q2.1, Q1.3, and Q2.3, along with an example and the percentages of introductory (Intro) and upper-division (UD) students who answered each item correctly.

Characteristics	Item	Example	Intro	UD
Interpretation and drawing	Q2.1 (a)	Drawing: electric field lines that start at the edge of the sphere and point radially outwards [see Fig. 10(a)]. Explanation: “*The field points out in a radial direction due to positive charges. In terms of magnitude, the field is larger when there is a higher concentration of lines. There is no field in* $r < a$.” Upper division, EM2019-A11	0%	5%
Principle of superposition	Q2.1 (b)	Drawing: Electric field lines in a radial direction out of the sphere of radius a, interspersed with the lines out of the sphere of radius b [see Fig. 10(b)]. Explanation: “$r < a$, *the field is 0*”, “$a < r < b$ *the field is radial and only depends on the outside in* $r > b$ *is radial, but with more magnitude because now a and b contribute*.” Upper division, EM2019-A2	18%	41%
Conversion from algebraic notation to electric field lines	Q1.3	Drawing: Electric field lines that increase in density in the region within the plate and whose density remains constant outside the board [see Fig. 10(c)]. Explanation: “*Outside the plate, the field lines are equally spaced and with the direction of E to show that it is a uniform field. Inside, the direction is equal, but closer to* $z = 0$, *there is less density of field lines, and these increase in density per unit area as z increases, as well as the magnitude of E*.” Upper division, EM2019-A13	2%	5%
Conversion from vector field plot to electric field lines	Q2.3	Drawing: Electric field lines throughout the region, evenly spaced and in the correct direction [see Fig. 10(d)]. Explanation: “*Arrows on lines and curvature, in this case, null, show direction, and the spacing between lines shows magnitude.*” Upper division, EM2018-A1-5	15%	19%

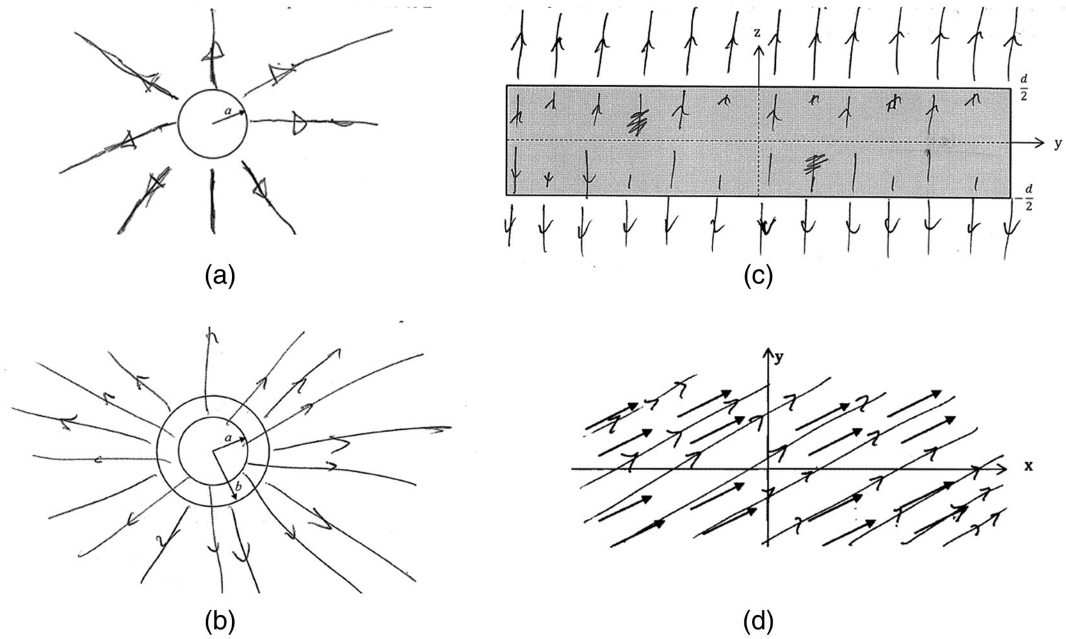

FIG. 10. Example of the complete construction of an electric field lines diagram for items (a) Q2.1 (a), (b) Q2.1(b), (c) Q1.3, and (d) Q2.3.

In item Q2.1 (a), students interpret a physical situation and make an electric field lines diagram to represent it. In item Q2.1 (b), the situation is modified, and students make a new diagram applying the principle of superposition. In items Q1.3 and Q2.3, students convert from an initial representation (algebraic notation and vector field plot, respectively) to the electric field lines diagram. We present the correct construction of electric field line diagrams in these items in Table V. For further reference, we present the drawings in Fig. 10.

It is relevant to note that the analysis of students' answers to item Q2.1 (a) focuses on the interpretation of the physical situation and the construction of the electric field lines diagram, while the analysis of students' answers to item Q2.1 (b) focuses only on the ability to apply the principle of superposition. The percentages shown in Table V regarding the principle of superposition also consider the students who did not draw an electric field lines diagram but applied the superposition principle accurately. Only three students could apply the superposition principle while constructing an appropriate electric field lines diagram.

Based on the results in Table V, we observed that students have a small percentage of skills in building electric field line diagrams efficiently. These skills are analyzed in items Q2.1 (a), Q1.3, and Q2.3, where correct answer percentages were less than 20% for both introductory and upper-division students. In question Q2.1 (b), the application of the superposition principle was considered regardless of whether students drew field lines or vector field plots. Therefore, this question is not an indicator of students' abilities to draw electric field lines but to apply the principle of superposition in the physical situation presented. When passing from an introductory to an upper-division profile, we qualitatively observed that their skills for drawing field line diagrams improved roughly. However, when applying the principle of superposition to answer item Q2.1 (b), we observed that their skills improved, indicating that students improve this ability regardless of representation.

1. Difficulties in drawing field line diagrams

In analyzing the questions requesting students to draw diagrams of electric field lines, some categories emerged that reflected their difficulties when making these diagrams. The predominant ones were that, in some situations, students draw field lines only in a region of the system, they fail to represent the magnitude of the electric field by employing the density of field lines, or they draw the field lines by joining vectors on a continuous line, neglecting that the direction is tangent to the line. These difficulties are presented in Table VI. For further reference, we present the drawings in Fig. 11.

In the category "Identified only outer region," we classified students who only identified the outer regions in items Q2.1 (a) and Q2.1 (b). Students with this difficulty in item Q2.1 (a) described that the electric field outside the sphere is radially outward and decreases with distance but did not explicitly identify that the field is zero inside the sphere. It is important to note that even if they did not explicitly mention it, students might have recognized that the field is zero inside the sphere, as they did not draw field lines in that region. In item Q2.1 (b), students who presented this difficulty only distinguished the region outside the *b*-radius sphere. They explained that in this region, the electric field is greater or

TABLE VI. Answers that imply difficulties in constructing the electric field lines diagram in items Q2.1, Q2.3, and Q1.3. We present the examples and the percentages of introductory (Intro) and upper-division (UD) students who answered each item with the corresponding difficulty.

Characteristics	Item	Example	Intro	UD
Identified only the outer region	Q2.1 (a)	Drawing: Electric field lines on the sphere's surface, with radial direction outwards. Inside the sphere, they did not draw lines [see Fig. 11(a)]. Explanation: "*The electric field is radial from $r > a$ and decreases with the distance, so the field lines are separated as they move away from the source*." Upper division, EM2019-A14	11%	21%
	Q2.1 (b)	Drawing: Electric field lines that start on the radius surface of $r = b$ and have a radial direction outward [see Fig. 11(b)]. Explanation: "*The magnitude of the field is greater (x2) because more charges are present. The direction is the same as in the previous item. Charges move to the surface, but it is as if there is a point charge in the center of the spheres at twice the magnitude*." Introductory, EM2019-I19	23%	17%
Difficulty with field lines density	Q1.3	Drawing: Evenly spaced field lines inside and outside the board [see Fig. 11(e)]. The lines point up where $z > 0$ and down where $z < 0$. Explanation: "*The equation of E tells you, mostly with the sign, the direction of the field and how they are all in the direction k, so all the lines are vertical. Given the function, the field within the surface decreases as you approach $z = 0$*." Upper division, EM2019-A20	11%	19%
	Q2.3	Drawing: lines that are more concentrated in the center of the diagram and less concentrated at the drawing's upper and lower left ends, as shown in Fig. 11(c). Explanation: "*The direction is towards the diagonal x-y, as the arrows show, and in the center, there is a higher density of lines because there is greater magnitude*." Upper division, EM2018-A1-2	2%	12%
Join vectors on a continuous line	Q2.3	Drawing: Field lines that connect the vectors [see Fig. 11(d)]. Explanation: "*It seems to me that it makes sense because field lines are a representation of the electric field, so yes, they should be constant to each other. If we compare them, field lines take a continuity criterion a little further*." Upper division, EM2018-A1-8	22%	31%

twice the magnitude of the previous case, but they did not explain the field inside the spheres. Students in this category applied the principle of superposition only on the outside of the two spheres. Some students mentioned that the charges were concentrated on the outer surface, which could mean they did not understand that the region between the two spherical shells was hollow.

The category "Difficulty with field lines density" includes students who did not represent the magnitude of the electric field using the density of field lines when converting from the algebraic notation in item Q1.3 or the vector field plot in item Q2.3. In item Q1.3, some students interpreted the function correctly, describing how the electric field depended on z inside and was uniform outside the plate. However, their drawings showed field lines that did not match their interpretation. In item Q2.3, students related the number of vectors to the magnitude of the field and reflected it through the separation between lines. They were not spaced evenly when plotting the field lines, so they did not represent a uniform electric field.

The category "Join vectors in a continuous line" groups the students that joined vectors to convert from the vector field plot to electric field lines in item Q2.3. In some cases, they mentioned that there is continuity. When constructing electric field line diagrams, it is not always sufficient to ensure continuity by joining the vectors, as the field vectors need to be tangent to the field lines, and the density of field lines must represent the relative magnitude of the electric field.

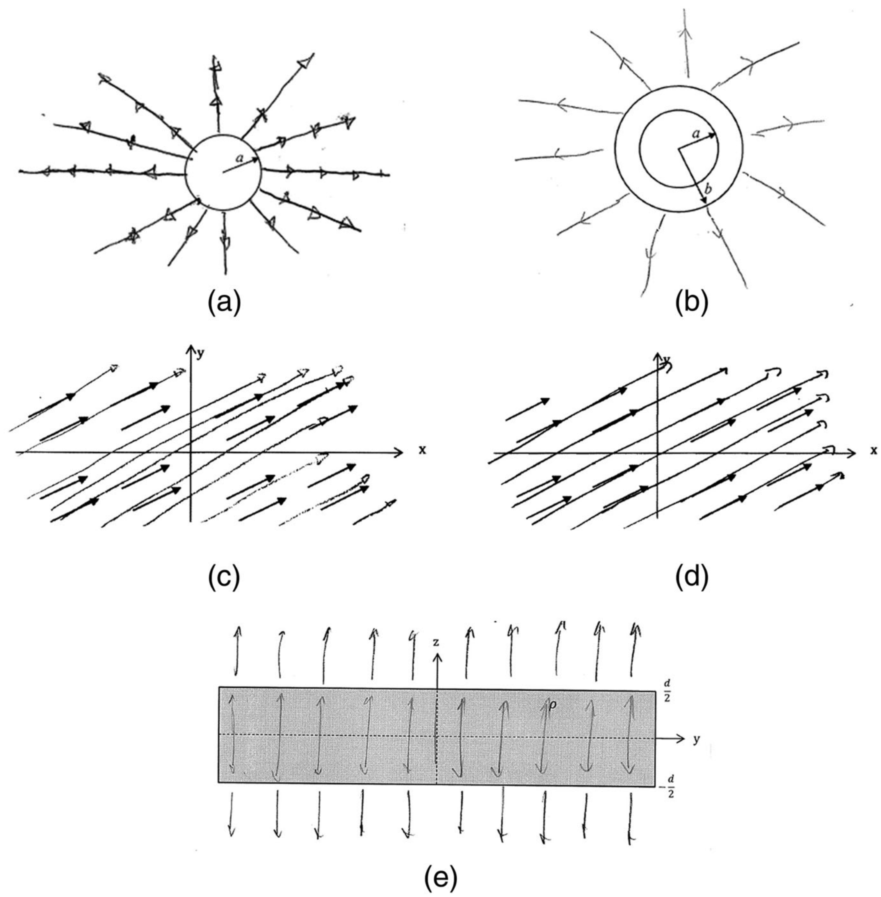

FIG. 11. Drawing (a) shows an example of the difficulty "Identified only outer region" in item Q2.1 (a), while drawing (b) shows an example of the same difficulty in Q2.1 (b). Drawing (c) shows an example of the "Difficulty with field lines density" in item Q2.3. Drawing (d) shows an example of the difficulty "Joined vectors in a continuous line" in item Q2.3. Drawing (e) shows an example of the "Difficulty with field lines density" in item Q1.3.

Based on the results presented in Table VI, the main difficulties students have when charting field lines are that (1) they draw and describe the electric field in some regions of the system and avoid drawing and describing the electric field in other regions, (2) they have difficulty representing the magnitude of the field through line density, and (3) they join the arrows into continuous lines when converting from the vector field plot, which leads to neglecting the correct representation of the direction. There are higher percentages of students presenting these difficulties when moving from an introductory profile to upper division; however, it may mean that more advanced students are trying to draw electric field lines, thus showing difficulties.

2. Replacing field line diagram with vector field plot

Many students did not draw electric field lines but drew a vector field plot. We grouped these students in the category "Drew vector field plot," which emerged in all the construction tasks. It is not possible to determine whether students confuse one representation with another or if they avoid drawing field lines. The results in Table VII and Fig. 12 provide evidence that most students prefer drawing vector field diagrams over electric field line diagrams. When converting from the vector field plot to electric field lines, upper-division students better distinguish between these two representations and attempt to draw field line diagrams more frequently than introductory students.

In item Q2.1, students drew electric field vectors in different positions smaller as they moved away from the sphere, denoting that the magnitude decreases with the sphere's radius on the outside. They described that the electric field outside the sphere is radially outward and decreases with distance. Their explanation is consistent with the vector field plot they constructed. In item Q1.3, students correctly interpreted the magnitude and direction of the electric field. They drew vectors in several positions inside and outside the plate to describe the electric field through the length and direction of the arrow. In item Q2.3, students correctly described the electric field's characteristics and drew some vectors in the empty spaces of the drawing. In some cases, they separated each vector's vertical and horizontal components. Surprisingly, this category emerged in this question, as the initial representation was the vector

TABLE VII. Answers imply difficulties constructing the electric field lines diagram in items Q2.1, Q1.3, and Q2.3. We present the examples and the percentages of introductory (Intro) and upper-division (UD) students who answered each item with the corresponding difficulty.

Characteristics	Item	Example	Intro	UD
Drew vector field plot	Q2.1 (a)	Drawing: vectors are smaller in the regions farther from the sphere. There are no arrows inside it, and the student drew a zero. The arrows have a radial direction outwards. [See Fig. 12(a).] Explanation: "*As the distance from the sphere* (r) *increases, the field decreases. The field on the sphere is zero*." Introductory, EM2019-I15	59%	67%
Drew vector field plot	Q1.3	Drawing: Vectors in the appropriate directions, pointing upwards and growing between $0 < z < d/2$ and remaining constant in $z > d/2$, and its counterpart in regions where $z < 0$. [See Fig. 12(c).] Explanation: "*Outside the plate: the field points out of each cap with constant magnitude. Within the plate: the magnitude of the field is proportional to* z, *points in* k *for* $z > 0$ *and in* $-k$ *for* $z < 0$". Upper division, EM2018-A2-6	70%	69%
Drew vector field plot	Q2.3	Drawing: drew the vertical and horizontal components of each vector in the original drawing [see Fig. 12(b)]. Explanation: "*The electric field will have x and y components, and their magnitude and direction would be given by* [the angle]." Upper division, EM2018-A1-9	28%	17%

field plot, implying that some students do not distinguish between vector field plots and electric field lines. Another possible explanation is that drawing field line diagrams is more complicated, and students opt for a more straightforward representation.

C. Conversion between representations of the electric field

We present the results of the conversions having the electric field lines diagram as a source or target representation in Table VIII. The descriptive categories of the

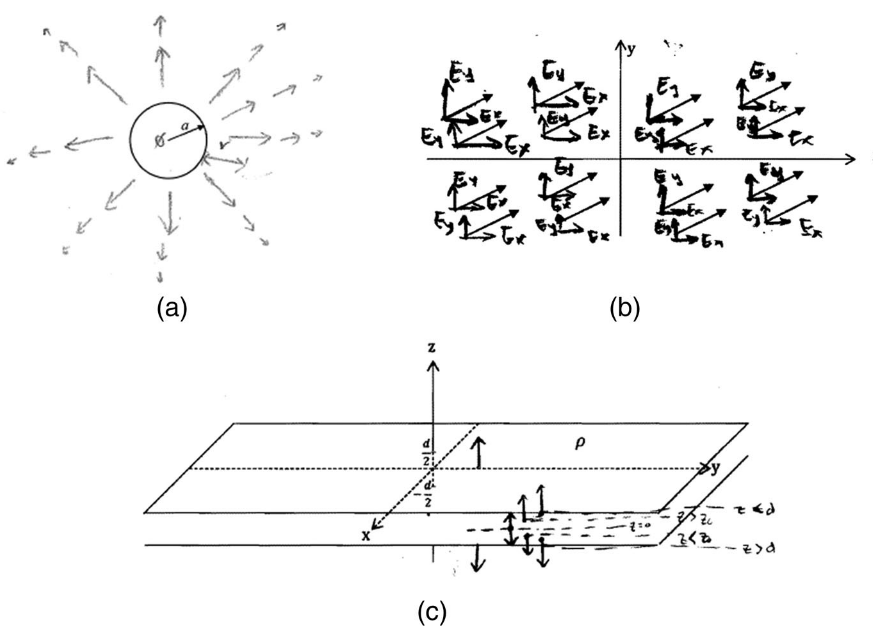

FIG. 12. Examples of the difficulty of "Drew vector field plot" in the different items. The drawing in (a) corresponds to item Q2.1 (a), in (b) with item Q2.3, and in (c) with item Q1.3.

TABLE VIII. Results of the recognition and conversion, having the electric field lines diagram as a source or as a target representation. We present the results for each conversion process according to the theoretical structure of categories that determine the success in recognition (Rec) or conversion (Con) for introductory (Intro) and upper-division (UD) students.

Source representation			Field lines		Field lines		Vector plot		Algebraic	
Target representation			Vector plot		Algebraic		Field lines		Field lines	
Cat	Rec	Con	Intro	UD	Intro	UD	Intro	UD	Intro	UD
A	High	High	19%	24%	12%	29%	15%	19%	2%	5%
B	High	Med	9%	12%	0%	0%	22%	31%	35%	47%
B’	Med	High	0%	0%	4%	10%	0%	0%	0%	0.0
C	Med	Med	43%	35%	45%	35%	30%	29%	46%	41%
D	Low	Low	29%	29%	39%	26%	33%	21%	17%	7%
		Total	100%	100%	100%	100%	100%	100%	100%	100%

conversion items were classified into a theoretical structure based on the success of recognition and conversion in each process. The same theoretical structure was introduced in a preliminary study [1]. The categories follow a hierarchical order. Category A means that students present a high level of recognition and conversion. In categories B and B’, either the recognition or the conversion is identified as medium level because they presented difficulties. In category C, both the recognition and conversion are identified as medium level because they presented difficulties. Category D groups the students with a low level of recognition and conversion, meaning that their difficulties could not be identified in the categories above, or they did not answer the question. The results are presented for all the conversion processes for both introductory and upper-division profiles.

When the electric field lines diagram is the source representation (items Q1.2 and Q2.2), the recognition difficulties represent about 70% of students in all cases (categories B’, C, and D). This implies that around 30% or less of students can interpret the electric field lines diagram effectively in the conversion processes, consistent with the results presented in Table II. Some of the difficulties of recognition in conversion processes depend on the target representation. The target representation (either vector field plot or algebraic notation) would elicit students to interpret the source representation differently. For example, recalling Sec. V A, students drew vectors with a radially increasing magnitude when converting from the electric field lines diagram to the vector field plot. In contrast, they tried identifying the sources when converting from the electric field lines diagram to the algebraic notation. It is interesting to learn how the different registers of representation may interact in conversion processes to consider a possible source of difficulty in the conversion process itself.

When the electric field lines diagram is the target representation, over 80% of students had difficulties with conversion (categories B, C, and D), which are intrinsically related to constructing electric field lines. This means that less than 20% of students could construct electric field lines that efficiently represented the magnitude and direction of the electric field. This behavior was observed when converting from the vector field plot and the algebraic notation and when the electric field lines diagram was the only representation, as shown in Table V. There were slightly fewer difficulties when converting from the vector field plot. This might be because the vector field plot and the electric field lines diagram are visual representations and can serve as intermediate steps [33].

VI. DISCUSSION

We discuss the findings of the recognition and conversion of the electric field line diagrams. We attempt to answer the research questions as follows. We first focus on the recognition of the characteristics of the electric field in the electric field line diagrams, focusing on the interpretation and construction abilities. We then discuss the conversion between electric field line diagrams, vector field plots, and algebraic notation, focusing on the electric field line diagram as the source or the target representation. We conclude with some remarks on the relevance of these findings and their implications for teaching the electric field concept.

A. Recognition of electric field lines diagrams

Regarding the interpretation of electric field line diagrams, less than 30% of students recognized the electric field’s characteristics in the diagrams provided. Only in recognition of the magnitude in item Q1.1 (a) are percentages greater than 30%. Students’ main difficulties in interpreting these diagrams were related to the surface features of the physical systems represented. These surface features diverted the students’ attention (around 10% and 45% of students in items Q1.1 and Q1.2 in Table III), creating difficulties in recognizing the density of field lines as the magnitude of the electric field and the tangent of the field lines as the direction of the electric field. Another recurring difficulty in interpreting the field line diagram is that students tried to identify which electrical field sources generated this diagram (around 5% and 25% of students in items Q1.1 and Q2.2 in Table IV).

Regarding the construction of field line diagrams, we observed that less than 20% of students drew electric field line diagrams considering both tangential direction and line density to represent the electric field effectively. The main difficulties in drawing field line diagrams are identifying regions where there is an electric field (up to 25% of students in item Q2.1 in Table VI), representing the magnitude of the field through line density (up to 20% of students in Q1.3 and Q2.3 in Table VI), and joining vectors in continuous lines (up to 30% of students in item Q2.3 in Table VI). The most recurrent difficulty was that students drew vector field plots instead of electric field line diagrams (up to 70% of students in items Q2.1, Q1.3, and Q2.3 in Table VI). With the existing evidence, it is impossible to determine whether this difficulty happened because students did not distinguish between the two representations or were not sufficiently familiar with the characteristics of the field line diagram that represents the electric field.

B. Electric field line diagram as the source representation

The results presented in Table VIII suggest that students who recognize the characteristics of the electric field in the field lines diagram can effectively convert to the vector field plot and the algebraic notation. Category B is relatively low in the two conversions where the electric field line diagram is the initial representation, implying that conversion difficulties are few once a good recognition is done. However, less than 30% of students efficiently recognize the electric field line diagram (considering categories A and B). Most students (about 70%) struggle to recognize the electric field's characteristics in the field line diagram (considering categories B', C, and D).

When converting from the electric field line diagram to the vector field plot, the main difficulty is that students do not associate the density of field lines with the magnitude of the field (around 30% of students in item Q1.2 in Table III). Instead, students associate other characteristics of representation (in this case, concentric circles increase in size) with the magnitude of the electric field [Fig. 9(b)]. This difficulty coincides with what Bollen *et al.* [33] found, who observed similar behavior in their students without the context of the electric field. When converting from the field line diagram to algebraic notation, the main difficulty is that students try to associate the diagram with a field source, whether electrical or magnetic, rather than interpreting the characteristics of the field with the information provided by the representation (around 30% of students in item Q2.2 in Table IV). This difficulty is like the behavior observed by Campos and Zavala [40] when different physics teachers interpreted electric field line diagrams. Both difficulties have in common that students do not interpret the density of electric field lines as an indicator of the field's magnitude when converting to another electrical field representation.

When converting from the electric field line diagram to algebraic notation, students identified an inverse dependence between the magnitude of the electric field and the distance from the center (regardless of the observed tendency to associate the electric field with a field source). However, when converting from the electric field line diagram to the vector field diagram, the difficulty of direct dependence between the magnitude of the electric field and the distance from the center (increasing magnitude) emerged. This behavior differs from the results found by Bollen *et al.* [33] because, in their study, students first responded to the conversion of the electric field line diagram to the vector field diagram, followed by conversion to algebraic notation. The vector field diagram was an intermediate step between the representation of field lines and the algebraic notation. This may have caused students to drag the difficulty from the first conversion to the second. When asking students for each conversion independently, this difficulty did not arise when converting from the field line diagram to the algebraic notation.

C. Electric field line diagram as the target representation

The results in Table VIII suggest that most students have difficulty converting to the electric field line diagram, regardless of the initial representation. Categories A and B, which denote efficient recognition, contain 35% of introductory and 50% of upper-division students. However, approximately 80% of students have conversion difficulties (categories B, C, and D). The same behavior is observed in both cases: converting from vector field diagram to field lines and from the algebraic notation to field lines. This implies that there is good recognition in the initial representations, but difficulties arise in representing the characteristics of the electric field through the electric field lines diagram. The main difficulty encountered in both conversions was that students drew vector field plots rather than electric field lines (up to 70% of students in items Q1.3 and Q2.3 in Table VI). The difficulties encountered suggest that drawing field line diagrams that adequately represent the magnitude and direction of the electric field is difficult for students, who can benefit from other visual representations in these cases of the vector field plot.

D. Overview of students' recognition and conversion of electric field lines

We present an overview to describe students' understanding of the representation (recognition), their conversion abilities to and from other relevant representations, and their understanding of the electric field concept. In general, we observed that many recognition difficulties were related to the electric field's magnitude rather than its direction. In the case of electric field lines, a significant part of students attempted to associate the representation with an electrical field source. This behavior has been previously observed [40] and did not associate the density of field lines with their magnitude, which is consistent with other studies [15,18].

One of the most relevant findings in this study is that when students interpret the electric field line diagram by itself, they relate field lines density to the electrical field magnitude (over 45% in item Q1.1). However, when they perform a conversion task from the field line diagram to either of the other two representations, they do not associate the density of lines with the magnitude of the electrical field (less than 30% in items Q1.2 and Q2.2). Still, they use other elements that depend, in part, on the target representation of the conversion. This behavior is similar to that observed by McGee and Martinez-Planell [24], who found that students understood each representation register but were not synergistic when converting between them. Their study demonstrated that this skill could be acquired through explicit instruction. In this case, in item Q1.1, where the only tools students have to answer about the electric field come from the electric field line diagram, students effectively turn to these tools. However, in items Q1.2 and Q2.2, where the electric field line diagram interacts with another representation, whether the vector field diagram or the algebraic notation, students find different tools to explain the electric field, which can help or hinder the conversion process.

Duval [2] explains that the main difficulty of the conversion process is recognizing the mathematical object (in this case, the electric field) in two representations with no characteristics in common. In this study, we observed that the coordination of the recognition in two representations, as an interpretation or a construction task, raises difficulties. This coordination of recognition in two representations with no common characteristics is called synergy between representations [1]. The synergy between representations is high when the two representations interact effectively to represent the characteristics of the electric field and promote their understanding. This study found that the conversions that include the electric field line diagram (items Q1.2, Q1.3, Q2.2, and Q2.3) present low synergy. The results of the study indicate that, for students, electric field lines are difficult to interpret in conversion processes, but they also present a challenge to draw. This could explain the low synergy in conversions where this representation is included. The synergy between the representations is related to the dissociation between the representations and the object. This finding could add to the discussion that students think electric field lines are real entities [15,16]. However, more in-depth research would be needed to support this hypothesis.

VII. CONCLUSION

This study used the theory of registers of semiotic representation and a phenomenographic analysis to link the difficulties of recognition and conversion between representations with students' conceptual understanding of the electric field. The study's methodology allows for identifying the difficulties of recognition and conversion that students have in the electric field lines' representation through phenomenographic analysis [3,41] and links the difficulties encountered with their conceptual understanding of the electric field through the theory of registers of semiotic representations [2]. As part of the phenomenographic analysis, the categories describe the main difficulties that arise from the students' explanations.

The most relevant finding in this study is that it is different for students to recognize the characteristics of the electric field in the electric field lines representation by itself and in conversion tasks (both when interpreting and constructing field lines diagrams). The evidence collected in this study implies that students can recognize the characteristics of the electric field in the electric field lines diagram more successfully in the interpretation task alone than when they need to convert to the vector field plot or the algebraic notation. When constructing an electric field lines diagram by itself, most students resorted to making a vector field plot instead. However, the proportion of students drawing vectors decreased in conversion tasks from the vector field plot to the electric field lines diagram.

The evidence presented in this article implies that students understand the representation differently by itself and in conversion tasks, and furthermore, that interpretation and construction tasks elicit different recognition and conversion abilities. During the conversion processes, difficulties arose related to the source or the target representation, which relates to the synergy between representation registers. The synergy between the electric field lines diagram and the other two representations was low, indicating students' difficulties understanding the electric field concept. In terms of the theoretical framework, the relation between synergy and conceptual understanding is given through the dissociation between the representations and the object. In this case, the more students distinguish between three or more representations, recognize the characteristics of the field in each of them, and convert between them, they will achieve a higher conceptual understanding because they will understand that the representation is not the real object. In the case of the electric field concept, researching students' recognition and conversion of the field's magnitude and direction and the application of the superposition principle in the electric field lines diagram is the first approach to understanding the synergy between this representation and the vector field plot and algebraic notation. As part of our work, other studies will focus on the other two representations by themselves. Further studies can investigate the synergy regarding other characteristics of the field and its related concepts, such as its sources, energy, and interactions with matter.

We acknowledge some of the study's methodological limitations. The study has limitations related to the sample of students, the country where the study occurred, the topics and representations used, and the instruments. We do not aim to generalize the results of this study to students of all contexts worldwide. However, the characteristics of the students analyzed in this study have shared characteristics with students from the United States and Spain in previous studies. The results of this study could resonate with instructors and researchers in other contexts and with what they may observe in their students. It would be interesting to expand the scope of this research to other countries and students from different cohorts by employing quantitative methods and retaining the perspective of the theoretical framework of semiotic representations. Another limitation of the study is the choice of representations for the electric field concept (electric field lines, vector field plot, and algebraic notation). Even though these are some of the most common representations of the electric field, there is language (which would require an analysis of semantics and semiotics together) and other graphical representations that could provide a deeper insight into students' understanding. Moreover, the choice of instrument for conducting this research limited the depth of analysis that could be reached. In particular, in questions Q1.2 and Q2.2, the source representation was a field line diagram from a non-Coulombic electric field that could be unfamiliar to some students if the instruction did not cover that topic. However, we used a question that was used previously [1,33] and standard textbooks for both students' levels [5,6] in which the electric field lines for a non-Coulombic electric field are covered. Conducting interviews could have provided insight for interpreting the causes of some of the findings.

In science teaching, it is crucial to know students' difficulties in understanding to identify possible tools that help them overcome them. This study found that the recognition and conversion difficulties differed in interpretation and construction tasks. Instructors should not assume that students understand the representations by themselves and in conversion tasks, nor that they can interpret and construct them effectively without explicit instruction. It is essential to explain the conversion process between different representation registers to contribute to teaching and learning in science. For example, instructors could plan for introducing electric field lines without showing the charge distribution and analyzing the field's magnitude based only on the density of field lines. Instructors should introduce activities to make explicit conversions to and from the electric field lines diagram and other representations, where the characteristic of each representation is discussed, and the difficulties that arise in each conversion are addressed. This research suggests that a good synergy between representations is necessary for a conceptual understanding of abstract properties. Dedicating time and practice to coordinating various representations of the electric field concept is crucial to promote a conceptual understanding of this physical quantity. Based on the results of this research, professors and researchers in the area can consider the difficulties posed by some conversion tasks, contributing to the discussion about the teaching-learning processes of the electric field concept at the university level.

This research focuses on students' understanding of the electric field concept through representations. Semiotic representations are natural for any topic of physics learning, so there is the opportunity to conduct further research about students' recognition and conversion abilities of multiple representations in any topic of physics education, focusing on students' understanding of the representations and concepts.

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