Students' conceptual understanding of electric flux and magnetic circulation

Eder Hernandez,[1] Esmeralda Campos,[2] Pablo Barniol,[1] and Genaro Zavala[2,3,*]

[1]*Tecnologico de Monterrey, School of Humanities and Education, Monterrey 64700, Mexico*
[2]*Tecnologico de Monterrey, Institute for the Future of Education, Monterrey 64700, Mexico*
[3]*Universidad Andres Bello, Facultad de Ingenieria, Santiago 7500973, Chile*

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Electricity and magnetism are closely related phenomena with a well-known symmetry found in Maxwell equations. An essential part of any electricity and magnetism course includes the analysis of different field source distributions through Gauss's and Ampere's laws to compute and interpret different physical quantities, such as electric flux, electric and magnetic field, or magnetic circulation. Still, some students have difficulties with these calculations or, in some cases, identifying the differences between those quantities. We present this article to explore and compare the challenges that students experience when asked to compute the electric flux (surface integral of the electric field) or the magnetic circulation (line integral of the magnetic field) in a nonsymmetric field-source distribution with two opposite field sources inside a Gaussian spherical surface or Amperian circular trajectory. The sample consisted of 322 engineering students finishing an electricity and magnetism course. They were presented with two parallel problems. Half answered one in the electricity context and the other in the magnetism context. After a phenomenographic analysis, our results showed that the students' conceptual difficulties in both contexts can be grouped into the same categories but are not contextually parallel, as has happened when analyzing other electricity and magnetism concepts. Our results also suggest that the magnetic circulation concept is far more unfamiliar to students than the electric flux. We propose several factors that could explain this finding and suggest teaching to address the conceptual difficulties identified in our analysis.

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I. INTRODUCTION

Research has focused on students' understanding of electricity and magnetism concepts and problem-solving abilities [1]. Reference [2] reported their difficulties conceptualizing a field, its representations' complications, and the superposition principle's use. Studies in Refs. [3,4] showed that some students could not distinguish a force from a field. Other works explored the confusion between forces and fields (electric and magnetic) [1,2,5,6] or even confusion between the two contexts, known as the *interference* phenomenon [2,4,7,8]. Different explanations for each confusion can exist, such as the visual representation used [2] or the test implementation moment [4].

Literature regarding the use of Gauss's and Ampere's laws has reported that some students think of these formulas as working like a "magic box" where they can *input* some values and use the *output* as a result without analyzing its validity [2,3,9,10]. Some known conceptual difficulties in this matter are the identification of the symmetry between the field and the imaginary enclosing object [4,11] or confusing the geometrical symmetry of the enclosing object with the symmetry of the field-source distribution [4,12,13]. Specifically, regarding the electric flux, research shows that students have difficulties calculating or using it to compute other information [14] when the electric field is not a known value. There is also evidence that students confuse electric flux with electric current [15], which could be another manifestation of the interference phenomenon. In general, it has been pointed out that Gauss's law is a challenging concept for students, with the level of abstraction in the electric flux concept being one of the main reasons [11,15]. Yet, concerning magnetic circulation, research has been less extensive, with Refs. [9,16] being examples. Some studies indicate how instructors teach it compared to how textbooks present it [17]. Some offer didactical proposals to enhance the understanding of magnetic circulation by strengthening the comprehension of Ampere's law [18], as done in most studies that include this concept.

Finally, results from previous studies that used *parallel problems* (problems with similar surface features but different underlying principles) suggest that electricity and magnetism represent two very similar phenomena for some students, not because their fundamental concepts

*genaro.zavala@tec.mx

are similar, but because of their parallel surface features [2,4,7,9,19]. This generates a necessity to inquire about students' conceptual understanding in both contexts, simultaneously and with similar external characteristics [20], to have a clearer view of their conceptual difficulties with this way of comprehending electricity and magnetism.

The present study compares the difficulties students have understanding the concepts of electric flux and magnetic circulation when asked to compute them in scenarios composed of two opposite field sources inside a circlelike imaginary enclosing object. This study's relevance lies in exploring the difficulties linked to these two concepts that have yet to be widely studied and how these difficulties emerge during tests presenting parallel problems. This allows a side-by-side comparison of the challenges experienced by students in both contexts, enabling exploration of possible sources for their confusion, which would then provide instructors tools to address the students' struggles during instruction.

II. METHODS

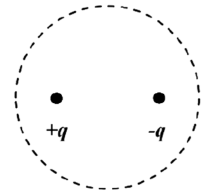
This research was conducted in a large private Mexican university with 322 participants enrolled in several introductory, calculus-based electricity and magnetism course classes previously described [2–4,7], which employed a known textbook [21] and tutorials [22]. Each class (group) comprised 30–40 students.

We designed two open-ended parallel problems, each showing two opposite field sources: (a) enclosed by a spherical Gaussian surface in the electricity context and (b) a circular Amperian trajectory in the magnetism context, as shown in Fig. 1. The electricity problem asked the students to compute the electric flux through the Gaussian surface shown in the figure. The magnetism problem asked the students to compute the magnetic circulation through the Amperian trajectory shown in the figure.

Each problem was randomly administered to the participants at the end of the electricity and magnetism course. In total, 160 students answered the electricity problem and 162 answered the magnetism problem. The course, these problems, and the data analysis were conducted in Spanish. This article presents English translations of the most relevant results.

We used a phenomenographic approach to analyze the data from open-ended questions [23], as previously used in other studies. For example, Refs. [2,4,6,7] created categories based on the similarities between students' answers and kept only those categories sustained by 5% or more of the sample in both contexts, electricity and magnetism. The solutions that did not fulfill this requirement were listed under the category labeled "*other*." We then used Cohen's kappa to validate our analysis, attaining an average of 0.95 for the electricity version and 0.94 for the magnetism counterpart, thus surpassing the minimum recommended value of 0.75 [6].

Electricity question. The following figure shows two point charges +q and -q. An imaginary spherical surface encloses both charges. Compute the electric flux (surface integral of the electric field) through the shown imaginary spherical surface. Explain your reasoning.



Magnetism question. The following figure shows a wire carrying a $+I_0$ current in direction out of the page (⊙) and a wire carrying a $+I_0$ current in direction into the page (⊗). An imaginary circular trajectory encloses both currents. Compute the magnetic circulation (line integral of the magnetic field) along the shown imaginary circular trajectory. Explain your reasoning.

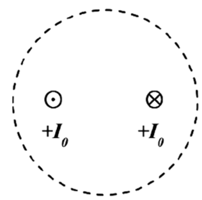


FIG. 1. The parallel problems used for this research. The electricity version presented two fixed point opposite charges, enclosed by a Gaussian spherical surface. The magnetism version gave two conventional currents flowing in opposite directions, enclosed by an Amperian circular trajectory.

III. RESULTS AND DISCUSSION

Table I lists the six major categories, an example, and their frequency per context. First, we present the "physics principles" category, which groups the answers closest to the expected response. This category had more students in the electricity than the magnetism sample, which differs from previous studies, like Ref. [7], where the magnetism version usually had more students under the equivalent *best answer* category than its electricity counterpart. The *moment of implementation* is considered a critical factor for this phenomenon, as proposed in Ref. [8]. Since the test was implemented shortly after the magnetism concepts were taught, these ideas were more readily available to generate an answer that involves field sources, regardless of their nature, and usually produces interference as a side effect. This result hints that Ampere's law is not the students' first option when asked to *compute* the magnetic circulation. Reference [11] presented a similar effect: more students correctly *analyzed* a similar scenario using Gauss's law than its magnetism counterpart, which suggests students lack familiarity with the *magnetic circulation* concept.

The "field sources cancel out" category was the most common in the electricity and the second most common in the magnetism sample. In this category, students needed to provide explicit evidence of conceptual understanding of

TABLE I. Results of the six main categories derived from the data analysis. Each category includes an example.

Category	Description	E	M
Physics principles	Students explicitly stated that since the total enclosed field source is zero, the electric flux (or magnetic circulation) is also zero, following Gauss's or Ampere's laws. E52: "*There is a negative charge and a positive one of the same magnitude. The net charge is zero, so there is no net flux.* $\Phi = \frac{q_{in}}{\epsilon_0} = \frac{0}{\epsilon_0} = 0$"	20%	0%
Field sources cancel out	Students stated that the electric flux (or magnetic circulation) is zero because sources "*cancel out*" as if they eliminated each other but without mentioning fields nor Gauss's or Ampere's laws. E11: "*0. The charges in the surface cancel each other.*" M12: "*It is zero.* $\odot$ *cancels out with* $\otimes$."	30%	22%
Flux-circulation-field confusion	Students explicitly stated an answer consistent with an electric or magnetic field analysis rather than electric flux or magnetic circulation. E76: "*The flux goes from + to −. The flux is to the right because of the charges.*" M69: "*It goes clockwise. Right-hand rule.*"	5%	26%
Fields cancel out	Students stated that the electric flux (or magnetic circulation) is zero because its corresponding field is zero. E72: "*It is 0. They are at the same distance and have the same electric field magnitude, but since they are opposite,* $\lvert\vec{E}\rvert = 0 \therefore \Phi = 0$." M15: "*Does not exist. The magnetic fields of the wires superpose.*"	5%	12%
Electricity and magnetism combination	Students combined elements of the two contexts or tried to answer a problem of one given context with concepts of its counterpart context. E29: "$q \oint \vec{B} \cdot dA$. *Subtract the charges to obtain the flux inside the imaginary sphere.*" M57: "$\oint \vec{B} \cdot A$"	12%	2%
Mathematical definition	Students wrote Gauss's law $\Phi = \int \vec{E} \cdot d\vec{A}$ or Ampere's law $\int \vec{B} \cdot d\vec{l}$ integral form. E19: "$\Phi_{\text{electric flux}} = \int_S \vec{E}\, d\vec{A}$ *because it is electricity going through an area.*" M43: "*The magnetic circulation is* $\int B \cdot dl$. *I used the formula to know this.*"	5%	5%
Other	Unclassifiable answers or groups of answers with frequencies lower than 5% in both contexts.	6%	19%
Unanswered	Unanswered	17%	14%
	Total	100%	100%

either Gauss's or Ampere's laws. For instance, student 11 in the electricity sample stated, "*The charges in the surface cancel out.*" There is no way to know if this student was thinking about these charges algebraically adding up to a net enclosed charge that then is used in Gauss's law or if they only memorized that. Since they are two opposite charges of the same magnitude inside a Gaussian surface, the flux is zero. This is similar to Ref. [7], who exposed some rote answers needed to prove conceptual understanding of the analyzed concept.

The third category is "flux-circulation-field confusion," the first category we identify as a conceptual difficulty. In this category, all the students confused electric flux and electric field or magnetic circulation and magnetic field. The significant difference in frequencies between contexts in this category could indicate the difficulties students have understanding how magnetic circulation differs from a magnetic field. Others have pointed out that students have trouble discerning the difference between force and field, such as Refs. [1,6]. Other data suggest that students cannot identify and distinguish an electric field from an electric flux [13]. However, more needs to be written about the confusion between a magnetic field and magnetic circulation, such as Refs. [9,16], hinting why this is the most common difficulty for the magnetism context in our research. We propose students' lack of familiarity with the *verbal form* of the "magnetic circulation" concept as a plausible explanation for our result. We could make a direct comparison with Gauss's law, which is usually taught by first defining the "electric flux" as the surface integral of an electric field, before talking about the net enclosed charge and its effects [21]. However, it is not uncommon that Ampere's law is taught without emphasizing the name of the magnetic circulation on the left-hand side of the equation, nor analyzing it apart from the law, which could be observed in Refs. [18,19] and in the course textbook [21], where it is never referred to as magnetic circulation, but as the *line integral of the magnetic field* (which is why the students were given this clarification in the test). This also unveils another possible conceptual difficulty: students may struggle with vectorial calculus concepts such as line and surface integrals. Combining these ideas and the fact that the magnetic field is usually represented with circular field lines around the source could lead to the reasonable conjecture that students are confusing magnetic field and magnetic circulation.

The fourth category was "the fields cancel out," which also encompasses answers representing conceptual difficulties. We propose two possible explanations, which could relate to our results in other categories. The first one is that some students focus only on the fact that there are two opposite charges inside the enclosing object, incorrectly deducing the field is zero inside of it and not correctly using the superposition principle (because, if they properly did, they would notice that the field is *not* zero). A similar finding was reported in previous research, like Refs. [11,13], where it was also highlighted that the "symmetry" of the field source distribution could be misleading. This idea could be like the one that defines the category the field sources cancel out; if the sum of the field sources is zero, then the fields should also add up to zero.

The second possible explanation for this category could be the confusion between the concepts of electric field and electric flux or between the magnetic field and magnetic circulation [5,6,9]. There is a chance that students are thinking about the electric flux or the magnetic circulation as proportional to the net-enclosed field source but incorrectly referring to them as electric field or magnetic field, respectively, which would be only a semantics problem. However, the students may think of electric flux and field as the same concept or the magnetic circulation and field, similar to the data derived and classified in the flux-circulation-field confusion category. Since there is no evidence of either, the data were collected separately.

The fifth main category is "electricity and magnetism combination." This category grouped the answers where students combined elements from both contexts to generate an answer. Students use aspects of Ampere's law when referring to Gauss's law, probably because it was recently taught, consistent with results in studies like Ref. [4]. This also explains why it was a far more frequent incident in the electricity context than in the magnetism context and why it is the most common difficulty that emerged from our study's electricity problem. Although this is not the typical form of interference found in the literature, we concur that these elements and concepts occur because students were taught both electricity and magnetism. If this had not happened, students would not have learned the required elements to produce these combinations. This is why we propose that these answers also represent a form of interference.

However, we suggest a second explanation for this one-sided result: preconceptions. Research shows that students' previous ideas about new concepts taught are critical to consider in achieving conceptual understanding [24]. In some cases, up to 1 of 3 students have preconceptions where they relate electricity concepts to magnetism ideas formed by their day-to-day experience [25]. Since electric flux and magnetic circulation are both new concepts for the students, their preconceptions could lead them to think about a magnetic field while trying to solve an electricity problem.

The "mathematical definition" category groups the answers of students who tried to use Gauss's and Ampere's laws as "magic boxes," as found previously in studies using parallel problems such as this one with Refs. [2,4,7] and others [9,16]. This result may be related to some of the answers grouped under the category the fields cancel out because some students clarify that they use the left-hand side of the equations but are missing information to compute the required value or they lack the mathematical tools to do so, which would be similar to the findings reported by Refs. [2,4,26]. The difference between students in these two categories would be that some deduce that the electric flux (or magnetic circulation) should be zero, while others leave the equation unsolved. It has been reported in other studies also [12] that some students consider this type of problem possible but messy, which is related to needing more mathematical knowledge to solve the problem.

Examining the physics principles and field sources cancel out categories together (namely, the ones that we do not consider to be conceptual difficulties), we found that they comprised about 50% of the sample in the electricity test but only 22% in the magnetism test. We interpret this as an indication of the difficulty of the electric flux and magnetic circulation concepts, along with Gauss's and Ampere's laws, an idea reported several times [6,9,12,13,16] previously. These data are also complemented by two of the three categories considered conceptual difficulties. In previous research, the students' difficulties with the two contexts were not only the same but were also parallel. However, this study shows a disparity in the number of students experiencing these difficulties when computing a value. Confusion between fields and other quantities presented in previous studies, like Refs. [2,7], were half the difference between contexts compared to the data obtained in this study, as observed in the flux-circulation-field confusion category. This repeats if we look at the category fields cancel out, with twice as much difference between contexts as in our previous study [2]. In both cases, magnetism is the more challenging context for students. The differences between the number of unclassifiable answers shown in the other category, with around triple the frequency in the magnetism context, reinforce this idea: "computing" the magnetic circulation is more troublesome than in its electricity counterpart, at least to understand.

IV. CONCLUSIONS

In general, this article explored and compared the difficulties related to the electric flux and magnetic circulation concepts when there is a nonsymmetric field source distribution, exploiting the existing parallelism between electricity and magnetism. Among our main results, we found that more than half of the students could compute the electric flux, but only about a fifth could compute the magnetic circulation. The known confusion between electric flux and field and between magnetic circulation and field emerged more frequently in the magnetism context. Our evidence suggests that the magnetic circulation concept is more challenging for students than electric flux. The simultaneous analysis of the two contexts revealed that the difficulties related to electric flux and magnetic circulation were parallel, as found in previous studies [2,4,7]; however, in this case, the frequencies of the categories were not parallel. This result could be due to two factors: (i) the students' different perceived complexity of the concepts and (ii) the question structure since, in previous works, the students were asked to analyze, while in our study they were asked to compute a value. This last idea could be a good starting point for future research.

For instruction, we recommend explicitly defining the concepts of electric flux and magnetic circulation and having students work with these concepts before approaching Gauss's and Ampere's laws. This work can be used as the basis for collaborative tutorial-type activities, not only to introduce students to the concepts but also to work the quantities mathematically. By doing this, students could analyze the properties of each physical quantity individually, which also helps to make sure they understand those concepts with their corresponding fields. We invite instructors and researchers to address the conceptual difficulties associated with magnetic circulation, a concept that, up to now, has been understudied.

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