


Assessment of student knowledge integration in learning work and mechanical energy

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Work and mechanical energy is a fundamental topic in introductory physics. Studies in existing literature have shown that students have difficulties in understanding work and mechanical energy, particularly the topic of work-energy theorem. To study students' knowledge integration in learning work and mechanical energy, a conceptual framework model of work and mechanical energy was developed and applied to guide the design of an assessment for measuring students' level of knowledge integration. Using the assessment, qualitative and quantitative data were collected in two high schools in an eastern Chinese city. The results reveal that the conceptual framework model can effectively represent the students' knowledge structures at different levels of knowledge integration. In addition, the assessment is shown effective in identifying unique features of knowledge integration, including context dependence and fragmentation of knowledge components, memorization-based problem-solving strategies, and lack of meaningful connections between work and change in kinetic energy. The conceptual framework of work and mechanical energy and assessment results can provide useful information to facilitate instructional designs to promote knowledge integration.

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I. INTRODUCTION

In science education, a fundamental goal of teaching and learning is to help students develop a deep understanding of essential scientific principles and concepts [1–5]. However, it has been shown that many students failed to achieve this goal after traditional instruction [6–8], which often leads students to develop memorization-based problem-solving approaches. As a result, students are often well versed in traditional textbook problems with familiar contexts where memorized examples can be applied but struggle when faced with problems in unfamiliar contexts, in which memorization and pattern matching fail to be sufficient [9,10].

In order to address this issue and promote deep learning, a number of new instructional methods have been developed. Some popular models include inquiry-based instruction, peer instruction, modeling-based instruction, and knowledge integration approaches [4,11–14]. For instance, modeling-based instruction encourages students

to understand the physical world through modeling [4]. These methods aim to help students to change their preconceptions and achieve a deep understanding of the scientific ones.

Another instructional method that has grown in popularity in recent years is the knowledge integration approach, which has been explored in several studies on different physics concepts including sinking and floating, force, and motion, light interference, momentum, Newton's third law, mechanical waves, and simple electric circuits [13,15–20]. The knowledge integration framework models students' conceptual understanding and problem-solving behaviors based on the connectedness within their knowledge structures [14,21,22]. Novice students have fragmented knowledge structures where knowledge pieces are locally connected to familiar contexts that they encountered in textbooks and lectures. During problem solving, they often directly match context features to memorized equations with little conceptual understanding [23–25]. At the intermediate level, students have developed partially integrated knowledge structures where some knowledge pieces are connected, though not to the key principles. The students can provide reasonable explanations for questions in familiar contexts but have difficulty when answering questions in unfamiliar contexts [26,27]. Students at the expertlike level have developed an integrated knowledge structure to the point that all the knowledge pieces are well connected around a few core principles with strong, global links. They are able to solve problems in novel contexts

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based on the core concepts or principles in their knowledge structures [14,28].

Conceptual framework can be considered an operational model of knowledge integration, which can explicitly represent students' knowledge structures and reasoning pathways for a particular concept [13,16–20]. In practice, the development of a conceptual framework starts with identifying the central idea of a concept, which serves as an anchor point for connecting the contextual features, intermediate processes variables, and reasoning processes. Building on the central idea, various knowledge components and contextual features are connected in a hierarchical network to represent students' knowledge structures. Once the conceptual framework of a content topic is established, it can be used to guide the design of assessment for determining levels of knowledge integration. The results of the assessment can further inform the instructional designs that help students create the needed links within their knowledge structures to improve their knowledge integration [13,16,19].

Following the previous work, this study applies the conceptual framework model to study knowledge integration in student learning of work and mechanical energy. Work and mechanical energy is an essential topic in physics, often taught in secondary schools and universities. Existing studies have reflected common difficulties in distinguishing different related concepts such as energy, work, force, and power [29–32]. During problem solving, students tend to rely on a set of nonscientific beliefs and preconceptions developed based on their experience [29,33,34]. These difficulties suggest that developing integrated knowledge structures and achieving a deep conceptual understanding of work and mechanical energy can be challenging in learning, especially through traditional instruction [35–38].

In this study, the conceptual framework of work and mechanical energy is developed and used to guide the design of an instrument for the assessment of students' knowledge integration in learning work and mechanical energy. Specifically, two areas of research are conducted:

Part 1: Develop a conceptual framework on work and mechanical energy that can be used to analyze students' learning behaviors and features from the knowledge integration perspective.

Part 2: Design an assessment instrument based on the conceptual framework and probe students' knowledge integration levels in learning work and mechanical energy.

II. PART 1: DEVELOPMENT OF THE CONCEPTUAL FRAMEWORK OF WORK AND MECHANICAL ENERGY

A. Expertlike and novice understanding of work and mechanical energy

The topic of work and mechanical energy involves a number of key concepts, such as work, kinetic energy,

gravitational potential energy, elastic potential energy, conservation of mechanical energy, and work-energy theorem [31,39]. These concepts can be grouped into three categories: work, mechanical energy and conservation, and work-energy theorem. In a typical introductory physics course, work done by a constant force is defined by $W = \vec{F} \cdot \vec{d} = Fd \cos \theta$. Mechanical energy is defined as the sum of kinetic energy and potential energy, $E = K + U$, where potential energy includes gravitational and elastic (spring) potential energy. Using Newton's second law and the definitions of work and kinetic energy, one can derive that the net work done on an object is equal to the change in its kinetic energy, $W_{\text{net}} = W_{nc} + W_C = \Delta K$, also known as the work-energy theorem [40,41]. In a conservative system, the change in the system's kinetic energy is equal to the work done by conservative forces. Since the work done by conservative forces is equal to the opposite change in potential energy, the change in the system's kinetic energy is equal to the opposite change in the system's potential energy. This result is known as the conservation of mechanical energy, which is a special case of the work-energy theorem when only conservative forces do work.

Following the conceptual framework model, the expert-like level of understanding is operationally defined based on whether students have developed a good understanding of the central idea in work and mechanical energy and can apply it to connect all the related concepts into a coherently integrated knowledge structure.

Conversely, as well documented in the literature, novice learners often hold very different views on understanding the concepts in work and mechanical energy, which are summarized and discussed next.

1. Difficulties in understanding work

Work involves both force and displacement, and as shown by research, students can have a tendency to consider only one of the two variables, i.e., either the force or the displacement, when solving work problems [42]. For instance, they often think that an object being lifted vertically upward takes more work than an object being pulled along an inclined plane to the same vertical displacement since the force is smaller in the latter case [43]. When students are asked to determine whether the work done is positive or negative, they tend to rely on memorization of similar cases they have encountered before. For example, students often believe that the friction force always does negative work since the direction of the friction force is opposite to the direction of motion in most cases taught in class [44]. In addition, students often have trouble distinguishing between conservative and nonconservative forces and believe that the work done by both types of forces is either path dependent or path independent [39,43]. For example, in the case of a block going down a frictionless ramp, some students may only focus on the length of the pathway along the ramp and consider that a

longer path will lead to a larger work, ignoring the impact of the change in the component of the gravitational force along the ramp [30].

2. Difficulties in understanding mechanical energy

Novice students often have difficulties distinguishing between kinetic energy, velocity, and mechanical energy. For example, students may believe that kinetic energy is a vector with both magnitude and direction because velocity is a vector [45]. Regarding potential energy, some students tend to believe that if the gravitational force is not zero, gravitational potential energy cannot be zero. Students may also consider the magnitude of elastic potential energy being determined by the length of the spring rather than the amount that the spring is stretched or suppressed [35]. Additionally, students have difficulties in identifying the underlying conditions for conservation of mechanical energy, especially that the potential energy needs to be considered with a system rather than a single object. For example, some students may claim that mechanical energy is only conserved when the net force on an object is zero [43,46], confusing the conservation of mechanical energy with an object having constant mechanical energy [47]. Furthermore, some students also confuse the condition for conservation of mechanical energy with the condition for conservation of momentum [31], where students believe that if the momentum of a system is conserved, its mechanical energy must also be conserved.

3. Difficulties in understanding work-energy theorem

McDermott and Lawson *et al.* found that students often inappropriately use “*compensation argument*” when comparing changes in quantities that involve two or more variables [31]. In the case of kinetic energy, defined by $K = \frac{1}{2}mv^2$, students often incorrectly predict that if object A is faster than object B, the change in kinetic energy of object A must be larger than that of object B because A being faster “matters more” than other information on its mass [31,43,48,49]. Driver and Warrington found that novice students prefer to use the kinematic approach to solve problems rather than the energy approach despite the fact that the energy approach is often more efficient with fewer steps [39,42]. In addition, Lindsey and Lawson discovered that students often fail to make the connection between work and change in kinetic energy indicating a lack of a basic understanding of the work-energy theorem [31,36]. Similarly, a number of studies conducted to examine the Chinese students’ understanding of work and mechanical energy have revealed that many students can do well on work or mechanical energy problems but often struggle in solving problems involving the work-energy theorem [46,50,51].

In summary, as shown by research, there are widespread difficulties in students’ understanding of the multitudes of

concepts involved in mechanical energy. In particular, the lack of understanding of the work-energy theorem can be a critical conceptual bottleneck that may hinder student learning toward developing a more integrated understanding of work and mechanical energy. Therefore, to promote knowledge integration, emphasis on the work-energy theorem is proposed, which is discussed next.

B. The conceptual framework of work and mechanical energy

According to previous studies using the conceptual framework model [13,16–20], the first step of developing the conceptual framework for a particular concept is to identify the central idea that serves as the anchor for an integrated knowledge structure. Through analysis of the relevant literature and textbook materials conducted by a team of two faculty and three Ph.D. students in physics education research, the work-energy theorem is identified as the central idea for the conceptual framework of work and mechanical energy, which serves as the mechanistic process that connects work and mechanical energy.

The work-energy theorem states that the work done by the net force on an object is equal to the change in the object’s kinetic energy [40,41]. In this study, the simplified case of a constant net force acting on an object is considered, which is the scope of the curriculum for Chinese high schools. For a conservative system, the work done on an object in the system by a conservative force is equal to the change in kinetic energy and the opposite change in potential energy of the object, which leads to the conservation of mechanical energy.

Building upon the physics content and the related research literature on student difficulties, the conceptual framework of work and mechanical energy is developed and shown in Fig. 1. The top layer is the central idea, the work-energy theorem. When solving problems that involve work and mechanical energy, experts often start by analyzing all the forces acting on the system and determine whether there are nonconservative forces doing work. If there is no nonconservative force doing work, the mechanical energy is conserved and can be used for problem solving. On the other hand, if there are nonconservative forces doing work, the mechanical energy is not conserved, and conservation of energy will include additional work terms for work done by nonconservative forces, which change the mechanical energy of the system.

The central idea is also closely tied to other key concepts in Newtonian mechanics, such as Newton’s second law and Hooke’s law, which may also be useful in solving problems involving work and mechanical energy. The middle layer of the conceptual framework consists of intermediate outcomes that arise from reasoning and other mathematical and logical manipulation processes as well as students’ naïve or alternative views and beliefs. The bottom layer involves contextual features directly related to the given

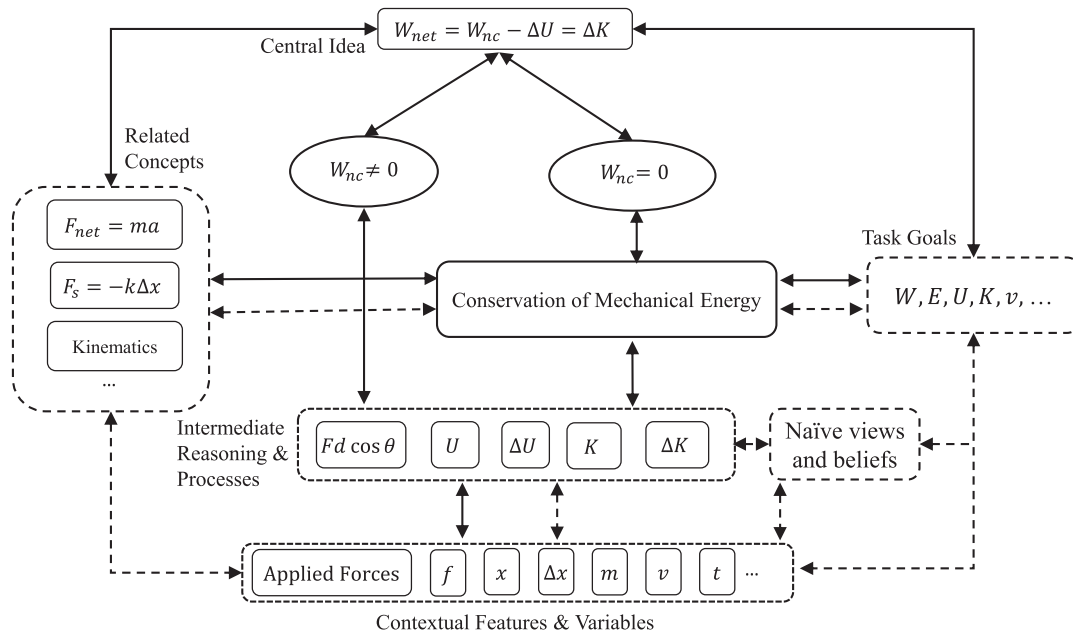


FIG. 1. The conceptual framework of work and mechanical energy. The solid arrows represent the conceptual pathways of experts, and the dashed arrows represent the conceptual pathways of novice students.

problem at the surface level. For most questions on work and mechanical energy, the task outcomes are often to calculate the work, mechanical energy, or velocity, etc., of a given system, which are shown on the right of the conceptual framework.

The arrows linking the different contextual, conceptual, and outcome components represent the possible reasoning pathways of experts and novices. The solid arrows represent experts’ reasoning pathways while the possible pathways of novices are illustrated with dashed line arrows. The central idea serves as an anchor node to integrate the connections, forming an integrated knowledge structure for experts. Therefore, when solving problems involving work and mechanical energy, experts activate the central idea in their analysis to identify relevant concepts and formulate problem-solving approaches [31]. However, novices tend to make weak, local connections between the context layer and the often memorize intermediate procedures and equations, forming fragmented knowledge structures [30]. In the case of work and mechanical energy, students are often exposed to situations where the mechanical energy of the system is conserved, which can be solved using the mechanical energy conservation equations. However, novices often generally apply the conservation law to situations where the mechanical energy is not conserved, leading to difficulties and confusions in solving work-energy theorem problems. The connections shown in Fig. 1 are represented with double-headed arrows, indicating that a connection can be initiated from either side. However, it is possible for individual students to have links in one direction among some of the connections, which are features representing partially connected networks.

C. Modeling student understanding using the conceptual framework

Using the conceptual framework, students’ misconceptions and difficulties documented in the literature can be interpreted in terms of their knowledge structures and reasoning pathways, which can be further analyzed to identify students’ levels of knowledge integration. In this study, students’ conceptual understanding and problem-solving behaviors documented in the literature can be categorized into three levels of knowledge integration, which are discussed next.

1. Novice level

Students at this level typically have fragmented knowledge structures with only local connections linking contextual features to kinematics knowledge and work or energy equations. The novices’ reasoning pathways are shown with the dashed arrows in Fig. 1. For these students, an understanding of the central idea has yet to be established, and they mostly rely on matching problem contexts with memorized equations to solve problems. For example, they can calculate the kinetic energy using $K = \frac{1}{2}mv^2$ given the mass and the velocity of an object, but when the question lacks information on either mass or velocity, it becomes nearly impossible for them to process, and most will attempt to calculate the velocity using kinematics equations [31,32,48]. The understandings of conservation of mechanical energy and the work-energy theorem are usually weak or missing among these students, and their reasoning pathways can be summarized as

“problem context → naïve views and/or memorized equations in mechanical energy or kinematics → solutions.”

2. *Intermediate level*

Students at this level have developed more connected knowledge structures and can engage in a deeper level of reasoning based on the contextual variables over the novice-level students. However, these students still tend to rely on memorized examples and equations to aid their problem solving. Students at this level are often good at solving work and typical conservation of energy problems, but they would fail on atypical conservation of mechanical energy problems involving unfamiliar contexts. For instance, intermediate-level students often fail to recognize that the location for zero potential energy can be arbitrarily set and tend to always treat the ground as having zero potential energy, which can lead to difficulties in certain atypical questions. Additionally, when faced with situations where mechanical energy is not conserved, students often use kinematic approaches, instead of energy approaches, to solve problems [39,42]. Therefore, their reasoning pathway can be summarized as “problem context → conservation of mechanical energy or kinematics knowledge → solutions.”

3. *Expertlike level*

Students at this level have developed a well-connected knowledge structure. This allows them to relate contextual variables to the work-energy theorem alongside many of the intermediate processes and related concepts, forming a comprehensive package of resources to address a wide range of problems in familiar and unfamiliar contexts. For example, when asked to calculate the work done by an unknown external force where the mechanical energy is not conserved, expertlike students can use the work-energy theorem to solve the problem [42] and may additionally use kinematic approaches to double check the outcomes [41]. Therefore, their reasoning pathway can be summarized as “problem context → work-energy theorem → solutions → kinematic check (sometimes).”

In summary, students in different levels of knowledge integration show unique types of reasoning pathways that can be mapped in the work and mechanical energy conceptual framework. Assigning students to these levels would usually require targeted assessments and interviews to determine reasoning patterns and then matching characteristics of these patterns to the conceptual framework. In the next part of research, the established conceptual framework is applied to develop an assessment for the evaluation of students' knowledge integration. Quantitative data are collected to analyze features of students' knowledge structures and determine their levels of knowledge integration. In addition, follow-up interviews are conducted to further examine students' reasoning pathways, which provide confirmative validation of the quantitative outcomes.

III. PART 2: ASSESSMENT OF KNOWLEDGE INTEGRATION IN STUDENT LEARNING OF WORK AND MECHANICAL ENERGY

A. Design of the work and mechanical energy test

According to previous studies on student difficulties in work and mechanical energy, most students are fairly comfortable with work, less comfortable with conservation of mechanical energy, and have trouble with the work-energy theorem [31,39,46,50,51]. This may be because, in problems of work done by a constant force, only a few simple variables are involved, which can be successfully solved with “plug and chug” method using memorized equations. On the other hand, problems on conservation of mechanical energy often involve more variables with complex settings, including kinetic energy, potential energy of different types, and conditions for conservation of mechanical energy, etc., which are difficult to be easily solved with memorized equations or solutions, leading to difficulties among novice students. Finally, to successfully solve problems on the work-energy theorem, students need to deal with all the context settings and conditions related to work and conservation of energy. Furthermore, students also need to have a good understanding of the work-energy theorem (the central idea) that connects the concept of work to the concept of conservation of mechanical energy. Therefore, problems on the work-energy theorem are often very challenging for novice-level and even intermediate-level students [31,39], which can be interpreted with the conceptual framework model as the result of lacking a good understanding of the central idea. Thus, success in solving these problems can be an indicator of students achieving an integrated knowledge structure with a good understanding of the central idea of work and mechanical energy.

Based on the analysis of students' learning behaviors using the conceptual framework of work and mechanical energy, an assessment instrument, referred to as the work and mechanical energy test, was developed to measure student level of knowledge integration in work and mechanical energy. The work and mechanical energy test consist of 20 multiple-choice questions designed in three concept groups including work (seven questions), conservation of mechanical energy (CME) (eight questions), and work-energy theorem (WET) (five questions). The complete test is included in the Supplemental Material [52].

In addition, the assessment design uses a mixture of both typical and atypical questions, which is a strategy shown effective in designing conceptual-framework-based assessment [18–20,53]. Typical questions include those that are commonly used in instruction as examples or homework problems. On the other hand, atypical questions are designed to engage the central idea, which often make use of unfamiliar contexts that students rarely encounter within the traditional curriculum. The design features of the test questions are shown in Table I.

TABLE I. Design features of the work and mechanical energy test. Based on the distribution of the context and concept features, it can be suggested that the traditional instruction seems to have more examples on work and conservation of mechanical energy, which show more typical questions. On the other hand, the work-energy theorem is somewhat less popular, and the related questions are often unfamiliar to students.

Concept type	Context type	Questions
Work	Typical	Q1, Q2, Q3, Q4, Q5, Q7, Q12
CME	Typical	Q9, Q11, Q14, Q15, Q18
	Atypical	Q6, Q13, Q20
WET	Atypical	Q8, Q10, Q16, Q17, Q19

The questions in the work and mechanical energy test are mostly adapted or modified from existing instruments including the Energy Concept Inventory [49], the Energy Concept Assessment [48], the Energy and Momentum Concept Survey [43,54], and several Chinese physics college entrance examination questions. A benefit of using existing questions is that the test questions can establish a basic level of validity and reliability. Details of the modification, classification, and an analysis of the reliability and validity of the test are given in the Supplemental Material [52]. Although most questions were designed in prior studies for assessment of various features of student conceptual understanding of work and mechanical energy, the selective use and modification of these questions in this study are guided based on the conceptual framework for assessment of knowledge integration, which aims for a research goal different from those of the previous studies. Therefore, the assessment design in this study is not a repetition of the previous work but can provide new insights into students' learning from the perspective of knowledge integration.

B. Data collection

The research subjects of this study include a total of 329 grade-10 students from two Chinese high schools in seven classes. All the students had previously learned the physics content on work, mechanical energy, and conservation of mechanical energy at a conceptual level in their middle school physics courses when they were in 8th grade. At the 10th grade, these topics were repeated and the work-energy theorem was first introduced. The instruction of these topics made emphasis on extensive problem-solving practices, which were taught in 20 lessons (40 min each) during a 4-week period. The work and mechanical energy test was given as a post-test after the students completed learning the relevant content knowledge. Students were allowed 40 min to complete the test, and most of them finished the test in the allotted time.

One week after the posttest, a total of 30 students were selected for think-out-loud interviews from the same pool of students with 10 in low performance level (0%–40%), 10

in medium performance level (40%–80%), and 10 in high-performance level (80%–100%). Ten questions from the work and mechanical energy test were selected as interview questions. Each interview lasted about 30 min.

The main purpose of this study is to evaluate students' knowledge integration in learning work and mechanical energy. The data analysis focuses on differences in students' performances on questions with different conceptual and contextual designs, which are used to make inference on students' levels of knowledge integration. Statistical significances in comparing results of different question sets are determined using a one-way ANOVA and are further explored using *t* test and Cohen's *d* effect size. Interview data are used to further explore details of student reasoning and validate the conclusions drawn from statistical analysis.

C. Assessment outcomes

Students' scores on questions designed with different concept and context types are given in Table II. The results show that students scored the highest on the work problems, medium on the CME problems, and lowest on the WET problems, which are consistent with the expectations discussed previously. A one-way ANOVA shows significant differences between scores on the three concept types [$F(2, 984) = 153.592, p < 0.001$], which are more clearly demonstrated with pairwise *t* tests [$t_{(W,CME)}(328) = 13.093, p < 0.001, d = 0.858$; $t_{(W,WET)}(328) = 7.746, p < 0.001, d = 0.463$; $t_{(CME,WET)}(328) = 22.190, p < 0.001, d = 1.447$]. Students' scores on the typical and atypical questions are also consistent with expectation, which show that students' performances on atypical questions are significantly lower than the typical questions [$t_{(T,A)}(328) = 20.419, p < 0.001, d = 1.113$]. Overall, the analysis outcomes suggest that the question design using concept and context types is effective in distinguishing the different aspects of students' conceptual understanding of work and mechanical energy.

To examine how students at different overall performance levels may respond to the different question designs, students' score distributions on different concept types are plotted across their levels of total scores in Fig. 2. As shown in Fig. 2, students with low total scores (score $\leq 40.00\%$) had low mean scores across all concept types. These students were only able to correctly solve a few work

TABLE II. Students' scores on three concept types and two context types. The scores shown are on a percentage scale.

Concept type	Context type	Score of concept		Score of context	
		Mean ($N = 329$)	SE	Mean ($N = 329$)	SE
Work	Typical	77.77	1.00	70.82	0.95
CME	Typical	58.92	1.37		
	Atypical			49.28	1.17
WET	Atypical	47.87	1.26		

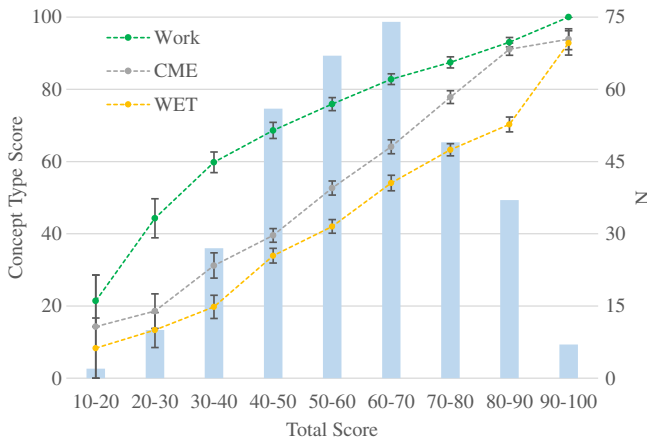


FIG. 2. Score distributions on questions of different concept types across total scores in percentage scale. The error bars represent standard errors. The frequency of total score distribution is shown as a bar chart in the background, with the absolute count of students falling in each range of total score.

questions but would fail on most CME and WET questions, which indicates that these students lack even the basic understanding of the central idea and are at the novice level of knowledge integration. In problem solving, these students primarily rely on matching surface features of context with memorized equations and examples, which usually fails on questions with novel contexts.

As the total score increases to the medium range (40.00% < score ≤ 80.00%), performance gaps between the different question types are more pronounced, indicating that students in this score range have begun to perform well on simple and more complex typical questions using memorization but without establishing a deep understanding of the central idea. As the total score further improves, the performance on work questions quickly reaches the ceiling, and students’ performance on CME questions starts to show significant improvement. Meanwhile, students’ performance on WET questions also begins to catch up. The results indicate that the students have developed partially integrated knowledge structures with a basic understanding of the central idea that allows them to apply their knowledge in some atypical contexts.

Finally, students with high total scores (score > 80.00%) show a minor difference between their scores on different question sets, suggesting that they have achieved a deep understanding with an integrated knowledge structure. These patterns of scores on different questions reveal a general progression of student knowledge integration that matches well with novice, intermediate, and expertlike levels discussed in part 1 and summarized in Table I.

Based on the conceptual framework developed in part 1, students at the three levels of knowledge integration are expected to have different performances on questions with different designs of concept types and context saliency. As indicated by the performance gaps among the questions with different designs shown in Table II and Fig. 2, the total score of the test appears to be a useful indicator for the different knowledge integration levels. Here a score division of 0–40, 40–80, 80–100 (in percentage) for indicating the novice-intermediate-expertlike levels of knowledge integration is proposed and summarized in Table III. However, the knowledge integration levels are based on the assessment outcomes that are population dependent. This specific score division scheme reflects only a reasonable approximation of the data in this study and should not be generally extended to other contexts and populations. Nevertheless, this work demonstrates the possibility of identifying a quantitative categorization scheme to model knowledge integration as well as its utility in teaching and learning.

To sum up, the quantitative analysis suggests that question designs using the three concept types and the typical and atypical questions can provide effective quantitative measures to categorize students into different knowledge integration levels. Based on analysis of the differences among student performances on questions designed with different concept types and typical and atypical contexts, students can be categorized into three levels of knowledge integration according to their total scores on the test. To further examine the reasoning pathways of students in each level of knowledge integration, a qualitative study is conducted with think-out-loud interviews, which is discussed next. The interview results can provide confirmative evidence to further validate the

TABLE III. Summary of total score and question set scores for each knowledge integration level. Standard errors are given in brackets. The *p* values reflect the significance of the one-way ANOVA analysis for the differences among mean scores of the novice, intermediate, and expertlike students in each question design.

Level	Score range	N	Concept type			Context type	
			Work	CME	WET	Typical	Atypical
Novice	0.00–40.00	39	53.85 (2.90)	27.11 (2.91)	17.52 (2.59)	43.80 (1.93)	18.27 (2.05)
Intermediate	40.01–80.00	246	78.63 (0.98)	58.13 (1.28)	48.04 (1.32)	71.10 (0.81)	49.03 (1.03)
Expertlike	80.01–100.00	44	94.16 (1.17)	91.56 (1.49)	73.86 (2.19)	93.18 (0.99)	78.13 (1.58)
<i>p</i> value			<0.001	<0.001	<0.001	<0.001	<0.001

score division (0–40, 40–80, 80–100) for determining the students' knowledge integration levels.

D. Interview results

Think-out-loud interviews were conducted with a subgroup of students after they took the work and mechanical energy test. A total of 30 students participated in the interviews with 10 students from each level of knowledge integration determined by their scores using the rubric shown in Table III. The students were given a subset of the questions from the work and mechanical energy test and were asked to provide their answers and explain their reasoning in detail. Specifically, three work questions (Q2, Q3, Q7), four CME questions (Q6, Q14, Q18, Q20), and three WET questions (Q10, Q16, Q19) were used in the interviews. Each interview lasted about 30 min and was recorded and transcribed for further analysis.

1. Novice level

For the work questions, novice-level students were only able to solve problems that are isomorphic to those that they had encountered before by matching memorized equations,

such as $W = \vec{F} \cdot \vec{d} = Fd \cos \theta$, to the context features of the questions. However, the students at this level often lacked the understanding of the dot product and ignored the angle between the force and displacement, where they would memorize the simplified one-dimensional version of the work equation $W = Fd$. This simplification often leads to difficulties in determining the sign of the work as shown by the following interview excerpts:

Student A: (Answered C to both Q2 and Q3) "According to the work formula $W = Fd$, the displacements in these two questions are both zero, so the work done by the friction and the work done by the gravity are both zero".

Student B: (Answered C to Q2) "In the process of rising, the friction of the ball is downward, so the friction in this process does negative work; in the process of falling, the friction is upward, so it does positive work. And because the distances of the two processes are equal, the same positive and negative work cancel each other out, so the work done by air resistance in the whole process is zero."

The two students, like most novices, recalled the simplified nonvector formula for work calculation, which leads to difficulties in determining the sign of work in cases with changing directions of forces and/or paths. Specifically, student A seemed to have ignored the paths and literally plugged in the displacement variable, which may have come from memorization of class examples that involve mostly conservative forces. Meanwhile, student B apparently lacked the understanding of the role of the angle between force and displacement in work calculation and

used the direction of the force to determine the sign of the work.

Regarding CME and WET problems, which require the application of conservation of mechanical energy or work-energy theorem and contain multiple variables in complex relations, novice-level students often lack a basic understanding of these relations and can only match context features with equations. In problem solving, these equations were used as isolated fragments for direct plug-in of the given variables without understanding of any further connections among the different variables, which can be exemplified with the following interview excerpts:

Student C: (Answered E to Q14) "According to the kinetic energy formula $K = \frac{1}{2}mv^2$, if the mass is bigger, it can get more energy. According to Hooke's law, if the compression of the spring increases, the force of the spring on it also increases, then it may do more work on the puck, and it will gain more energy."

Student D: (Answered D to Q10) "Because the distance that the ball falls is $H + h$, only gravity does work, and mechanical energy is conserved. So according to $mg\Delta h$, the answer is D." (The correct answer is C.)

Apparently, student C was able to recall the formula for kinetic energy but lacked the understanding on the physics processes that lead to the change in kinetic energy such as through doing work or change in potential energy. The student's reasoning is based solely on the variables involved in the formula, i.e., the mass, without connections to other physics processes. Although the student also knew the idea of Hooke's law but had an incorrect understanding on how work is done by the spring force and lacked a connection to the kinetic energy that was mentioned in the previous sentence. The student's reasoning reveals obvious fragmentation in knowledge components, which is expected among novice-level students.

Student D only considered the distance that gravity does work in Q10 but failed to recognize that the level of zero potential energy is artificially defined. From the student's explanation, it can be hinted that student D was automatically assuming that the ground level has zero potential energy. This is likely caused by students' experience in instruction, which often set the ground level with zero potential energy. Failing to recognize the relative nature of potential energy demonstrates that the student had only achieved surface-level manipulation of the memorized equations without understanding the connections to more complex relations underlying the equations.

In addition, several students at the novice level seemed to be more comfortable using kinematics knowledge in solving energy problems. In problems involving kinematic variables, the context often activates students into using kinematics-based, instead of energy-based, understanding and equations in their reasoning to determine speed, distance, and/or force. As revealed by the interview excerpt

from student E, when determining the final speeds of the three balls launched from the same height with the same initial speed in different directions, the student only reasoned with kinematics knowledge, and the related concepts in mechanical energy were not activated. This kind of behavior further demonstrates that the students had not yet developed a basic understanding of mechanical energy and had very limited connections between contextual features and concepts in mechanical energy.

Student E: (Answered D to Q6) “The velocity will increase downward in the vertical direction, which is in the same direction as the initial velocity of B. Therefore, B will have the largest speed when it hits the ground. The initial direction of C’s velocity is horizontal, and the combined angle with the increased velocity in the vertical direction is relatively small, so I think the speed of C is the second largest. A has the largest angle between its initial velocity and the vertical change of velocity, so the speed of A is the smallest.”

In summary, novice-level students were able to use memorized equations to solve some work-related problems in familiar contexts but often had difficulties in questions designed with changing directions in forces and pathways. In addition, students at this level demonstrated persistent difficulties in working with CME and WET problems. As revealed by students’ explanations, the problem contexts were more likely to activate kinematics knowledge than concepts in CME and WET. Students’ explanations to CME and WET questions further reveal that the students lack a basic understanding of the work-energy theorem, which is the central idea of all energy-related concepts. Without a solid understanding of the central idea, students were not able to make meaningful connections among the many variables, equations, and concepts of CME and WET, which likely lead them to revert to using the more familiar kinematic methods in solving problems. The interview results are consistent with the quantitative assessment outcomes shown in Table III, which show that students at this level can only correctly solve a small number of work problems and often fail on CME and WET problems. The interview results further confirm that students in the total score range of 0%–40% behaved as novice-level students in knowledge integration.

2. Intermediate level

For the work problems, intermediate-level students were able to correctly determine the sign of work, which is evident from the interview response shown below:

Student F (Response to Q7) “I choose D. The slider moves to the right, and both the elastic force and the friction force do work to this system. Since the small block is stationary at points A and B, the work done by

the elastic force and the friction force must sum to zero. In the process from point A to point O, the elastic force does positive work, and then in the process from point O to B, because the direction of the force is opposite to the direction of motion, the elastic force does negative work. The friction always does negative work from A to B since it is opposite to the direction of motion.”

As revealed from the interview, student F seemed to have developed a good understanding of the relations and conditions among force and displacement in determining the sign of the work, which is a significant improvement over the novice-level students who lacked such understanding and were only able to memorize the equation. Furthermore, student F also demonstrated a basic understanding of the connection between work and kinetic energy (the central idea), which allowed the student to conclude that the sum of the work done by the elastic force and the friction force must be zero as a result of the change in kinetic energy being zero.

However, although the intermediate-level students were able to solve the CME problems in familiar contexts, they still had difficulties in understanding the conditions for mechanical energy conservation in unfamiliar contexts, which is revealed from the following interview excerpts:

Student G: (Response to Q18) “The correct answer is G. Here we have two identical blocks at the same height. If the gravity does work, the work should only be related to mass and height, and so the work done by the gravity is the same. Since it is a smooth inclined surface, there is no frictional force, so the mechanical energy is conserved. If the two wooden blocks are stationary at first, the final kinetic energy will also be the same, and so are their speeds.” (Answered C to Q20) “The two blocks in Figures A and B can be regarded as a system. Since there is no friction on a smooth horizontal surface, their mechanical energy is conserved.”

Student H: (Answered C to Q20) “Our teacher often emphasizes the conservation of mechanical energy, so I think that mechanical energy is conserved in all systems.”

Q18 is a typical question on CME, and student G was able to solve this question correctly by directly applying the conservation law. However, based on the two students’ responses to Q20, which is an atypical question, it can be seen that both Student G and H had incorrect answers, considering the mechanical energy being conserved in the situation of an inelastic collision. Most students at the intermediate level believed that as long as there are no friction and other external forces, the mechanical energy should be conserved. A small number of students in the intermediate level even believed that mechanical energy is always conserved in all systems, similar to student H, which is an obvious indiscriminate between mechanical

energy and energy in general. These students' beliefs were also likely influenced by instructional materials where most examples were practices of CME and the only nonconservation cases were often the ones with friction. The mechanism of nonconservation of mechanical energy in an inelastic collision was rarely discussed.

In solving WET problems, most intermediate-level students seemed to understand that when friction or other external forces are present, the mechanical energy of the system is not conserved. However, as seen from the interview excerpts below, many students at this level appeared to have a limited understanding of the work-energy theorem, which could lead them to use kinematics to solve these problems instead of applying the central idea. Although some of the WET problems can be solved with kinematics, using the work-energy theorem is a more efficient and straightforward method. Therefore, the students' tendency in using kinematics to solve WET problems is an indication of their weak understanding of the central idea.

Student I: (Answered C to Q10) "I first calculate the ball's speed when it hits the ground by the equation $v_f^2 - v_i^2 = 2ax$. In this case, $a = g$, $x = H + h$, it is easy to figure out $v_f^2 = 2g(H + h)$, which multiplies $(1/2)m$ to give the ball's kinetic energy as $mg(H + h)$ when it hits the ground. Since the tabletop is the zero potential energy surface, the gravitational potential energy when the ball hits the ground is $-mgh$. Therefore, the mechanical energy which is equal to the sum of the kinetic energy and potential energy is mgH ."

Here, student I recognized that the mechanical energy is equal to the sum of the kinetic energy and potential energy, but the student was not able to make connections to the concepts in WET and CME. Instead, the student used kinematics to find the final speed and then the final energy surface, the gravitational kinetic energy. Although student I obtained the right answer, the reasoning used in solving the problem indicates a weak understanding of the central idea.

A number of students at this level also revealed similar reasoning pathways. They often tend to use kinematics equations first to determine speed, distance, and/or force and then plug these variables into the energy formula to calculate the energy terms. This kind of behavior further demonstrates that students were only able to manipulate the energy-related equations at the variable level as a computational tool without understanding the underlying concepts in mechanical energy.

As revealed from the interviews, the intermediate-level students appeared to have formed partially connected knowledge structures that span across kinematics, work, and mechanical energy, allowing them to successfully solve most typical questions. However, their knowledge structures lack the central idea that can serve as the anchor point to integrate all the different components. When encountering novel contexts, such as when mechanical energy is not

conserved, the students were not able to apply the central idea (work-energy theorem) and often revert to using kinematics knowledge to solve problems.

In summary, students at the intermediate level have formed a relatively complete understanding of work. They were able to solve problems for the conservation of mechanical energy in familiar contexts, but their understanding of the conditions for the conservation of mechanical energy was incomplete. On the other hand, the central idea (work-energy theorem) was rarely mentioned and applied in problem solving, and instead, kinematics knowledge is often used to solve problems that are most applicable to using the central idea. The interview results are consistent with the quantitative assessment outcomes shown in Table III, which show that students at this level can correctly solve most of the work questions, about 60% of the CME questions, and only a few questions on WET. The results further confirm that students in the total score range of 40%–80% behaved as intermediate-level students in knowledge integration.

3. Expertlike level

The expertlike level students were able to correctly solve almost all work, WET, and CME problems with the desired reasoning pathways. In particular, students at this level appeared to have clearly understood the conditions of conservation of mechanical energy, which can be revealed from the interview excerpts below:

Student J: (Response to Q6) "I choose G, because the three balls fall at the same height and have the same initial speed, and only gravity does work and the work done is the same. According to conservation of mechanical energy, the final speed is also the same." (Response to Q20 with choice A) "Because there is a sticky puck in the first process, and the puck can be deformed after being squeezed. It is inelastic deformation, and mechanical energy is not conserved. Then the second process in the spring, which I think is elastic deformation, and the mechanical energy is conserved."

In addition, as revealed from the interview excerpts below, these students demonstrated a good understanding of the central idea (work-energy theorem) and were able to successfully apply it to solve problems.

Student K: (Response to Q16 with choice A) "For the kinetic energy at the endpoint, use the work-energy theorem: $W_{\text{net}} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, because the initial kinetic energy is not the same, but the net work is the same, so the final kinetic energy is not the same." (Response to Q19 with choice C) "The net force is not 0, because the person moves on an arc, there must be a centripetal force. The magnitude of the friction force will change, because the angle of the arc with the horizontal changes with the person's position, then its supporting force will change too. And if his kinetic

friction coefficient remains the same, the friction force will change. If the speed is constant, the kinetic energy is constant, and the work done by the net force is 0. Mechanical energy is not conserved because although the speed and kinetic energy remain the same, the gravitational potential energy will change.”

Student L: (Response to Q10 with choice C) “With the tabletop being the zero potential energy surface, the initial gravitational potential energy is mgH . Because only gravity does work, then mechanical energy is conserved, so the mechanical energy before landing is also mgH . Therefore, C is the right answer.”

Student M: (Response to Q10 with choice C) “In the process of falling, only gravity does work. According to the work-energy theorem, $W_{\text{net}} = \Delta K$, it can be calculated that the kinetic energy of the ball at the moment before it hits the ground is $mg(H + h)$. Since the tabletop is the zero potential energy surface, the gravitational potential energy at the moment before the ball hits the ground is $-mgh$. Therefore, at the moment before the ball hits the ground, the mechanical energy is mgH .”

As revealed from student K’s responses to Q16 and Q19, when faced with a system where the net force is not zero, this student was able to solve the problem correctly with the work-energy theorem. In addition, students at this level appeared to be able to use both conservation of mechanical energy and the work-energy theorem. For example, in solving Q10, Student L used conservation of mechanical energy while student M used the work-energy theorem. Both students also demonstrated a good understanding of the relative definition of zero potential energy.

In summary, students at the expertlike level had developed a good understanding of the central idea and a well-integrated knowledge structure connecting kinematics, work, and mechanical energy in an organized network. The interview results are consistent with the quantitative assessment outcomes in Table III, which show that students at this level can correctly solve all the work and CME problems and most problems on WET. Results of the interviews further confirm that students in the total score range of 80%–100% behaved as expertlike level students in knowledge integration.

IV. CONCLUSION

In this study, the conceptual framework of work and mechanical energy is developed and applied in the assessment of students’ knowledge integration in learning. Based on the assessment outcomes, student understanding of work and mechanical energy is categorized into three progressive levels of knowledge integration: novice, intermediate, and expertlike.

Novice level: The students appeared to lack the understanding of the central idea and their knowledge structures

were fragmented without connections to the central idea. In problem solving, they primarily focused on context features, such as speed, force, mass, and distance, which were used to match memorized equations. Students at this level were able to solve some work problems with memorized equations but often fail on CME and WET problems. In addition, they were not able to provide meaningful mechanistic explanations to support their answers.

Intermediate level: The students at the intermediate level appeared to have developed a basic understanding of the central idea and could apply it in limited familiar contexts. The knowledge structures of these students were partially integrated with some connections to the central idea. They were able to solve most work and CME problems in familiar contexts and provide reasonable explanations. However, they had difficulties in solving CME problems in unfamiliar contexts and often failed on WET problems. Intermediate students also relied on memorized examples to help them solve problems, though less so than the novice students.

Expertlike level: The expertlike students demonstrated a good understanding of the central idea and developed an integrated knowledge structure anchored by the central idea. They were able to solve all the work and CME problems and most problems on WET, while providing sound explanations for their answers. Instead of relying on memorized formulas or examples, the expertlike students demonstrated a deep understanding of work and mechanical energy, applying the central idea consistently in both familiar and novel contexts.

The results of this study reveal a general progression of knowledge integration from fragmented structures to integrated ones. In this process, establishing a good understanding of the central idea, the work-energy theorem, plays an essential role in integrating the different knowledge components. However, assessment outcomes show that after traditional lecture-based instruction, most students are at the intermediate level of knowledge integration with weak understanding of the central idea and partially connected knowledge structures. The conceptual-framework-based assessment of work and mechanical energy can help identify unique features of knowledge integration as well as specific connections that students need to develop to improve knowledge integration. For example, based on the research outcomes, it can be suggested that instruction design should make more emphasis on teaching the central idea (the work-energy theorem) and connecting it to different contexts, problems, and other concepts related to mechanical energy. A number of specific connections and understandings were also revealed to be weak among students including the relations between work by net force and change in kinetic energy, the relative definition of gravitational potential energy, the conditions for conservation of mechanical energy, etc. These results can provide concrete useful information to guide future instructional

design to promote knowledge integration and deep learning among students.

Although consistent results have been observed from assessments and interviews, there are a few limitations. First, since the schools selected in this study are the high-ranked ones in the region, the number of novice-level students is low, which limits the scope of outcomes regarding the reasoning pathways of the novice students. In addition, only half of the test items are selected to serve as interview questions, which also limits the depth and completeness of the analysis. To obtain a more fine-grained understanding and to further improve the validity of the research outcomes, future research is warranted to extend the population with more diverse backgrounds.

In conclusion, based on the previous research on student understanding of topics in mechanical energy, this study develops a conceptual framework for work and mechanical energy, which is used to guide the development of an assessment tool for probing knowledge integration in student learning of work and mechanical energy. The

assessment results categorize students into three levels of knowledge integration which are further examined with interviews. The results of this study provide encouraging evidence that the conceptual framework model can be used to analyze the extent of students' knowledge integration, which can further inform instruction design that aims to promote deep understanding.

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