Analysis and comparison of students' conceptual understanding of symmetry arguments in Gauss's and Ampere's laws

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Identifying students' difficulties in understanding Gauss's and Ampere's laws is important for developing educational strategies that promote an expertlike understanding of the field concept and Maxwell's equations of electromagnetic phenomena. This study aims to analyze and compare students' understanding of symmetry when applying Gauss's and Ampere's laws to calculate the electric or magnetic field. We conducted a study to analyze how students reason regarding the symmetry conditions necessary to apply Gauss's or Ampere's laws to calculate the electric or magnetic field in three inverse problems. We applied two open-ended questionnaires with parallel surface features, one for Gauss's law and the other for Ampere's law, to 322 engineering students. The three inverse problems present different scenarios with the common characteristic that there is no sufficient symmetry to solve the electric or magnetic field from its corresponding equation. We analyzed students' answers with a phenomenographic approach, focusing on students' answers to a yes or no question and their reasoning. The main findings of the study are the descriptive categories of understanding and the comparison of the categories between contexts (outcome space). The correct reasoning is identifying the necessary symmetry to apply Gauss's or Ampere's law. The other categories refer to the surface features of each scenario to explain students' answers, applying Gauss's or Ampere's law in an oversimplified way and thinking that it would be possible but more complicated in these scenarios. The descriptive categories are related to some of the difficulties previously reported in the literature with standard problems involving Gauss's and Ampere's laws. However, inverse problems elicited variations in the types of reasoning related to the surface features of the scenarios and their parallel representations. The comparative analysis between the electricity and magnetism contexts allowed for identifying that analyzing currents can be more challenging for students than analyzing point charges. This study's findings can guide introductory and intermediate electricity and magnetism instructors to redirect their approach to Gauss's and Ampere's laws by introducing the analysis of inverse problems.

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I. INTRODUCTION

Electromagnetic phenomena are explained by Maxwell's equations, which describe the flux and circulation of electric and magnetic fields and their interrelations. Gauss's law for electricity and Ampere's law for magnetism are two of the pillars of electrostatics and magnetostatics [1]. Specifically, they identify the sources of the field through the analysis of electric flux and magnetic circulation. Understanding these laws is crucial for developing

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an expert understanding of the field concept [2] and thinking like a physicist [3].

There is also a level of parallelism present in these two equations [4]. On the one hand, Gauss's law relates the electric flux through a Gaussian surface with the net charge enclosed by the surface. On the other hand, Ampere's law relates the magnetic circulation through an Amperian trajectory with the net current enclosed by the trajectory. They both relate a closed integral to a field source, even though these electric and magnetic fields are essentially different.

There is also parallelism in how these two laws are represented and taught. In the introductory and advanced electromagnetism courses, Gauss's and Ampere's laws are usually used to calculate the electric or magnetic field in highly symmetric physics problems, where the field can be solved from the equation [2]. The highly symmetric cases where these two laws can be exploited are usually represented with parallel surface features, such as dots for point charges, vectors pointing outwards or inwards for currents, and circles for spherical surfaces and circular trajectories. We refer to parallel surface features as the similar ways in which two different physical problems can be represented. In the introductory electricity and magnetism course, the most basic physical problem that is solved with Gauss's law is the electric field of a point charge, and with Ampere's law, the magnetic field of an outward pointing current. These two problems are usually represented with parallel surface features.

The catch with Gauss's and Ampere's laws is that, even though they are fundamental concepts for explaining electrostatic and magnetostatic phenomena, they can be used to calculate the electric or magnetic field only in a few cases with a high degree of symmetry. The argument for teaching these two topics is more related to developing physical thinking than exploiting them for calculating the field [3]. The problems that can be solved using Gauss's and Ampere's laws become the "building blocks" [3] to solve situations that would be more complex otherwise, an ability that comes from developing physical thinking. Because of the high abstraction and the required mathematical ability to correctly understand Gauss's and Ampere's laws, students tend to have difficulties interpreting, understanding, and applying them. This study aims to explore students' understanding of symmetry when using Gauss's and Ampere's laws to calculate the electric and magnetic fields. We aim to compare students' difficulties when answering whether it is possible to use Gauss's or Ampere's law to calculate the electric or magnetic field in three different inverse problems in electricity and magnetism. We refer as inverse problems to those where the electric or magnetic field cannot be algebraically solved in the integral, which means that Gauss's or Ampere's laws cannot be used to calculate the field in these cases, as defined in [3,5].

The study contributes to identifying students' difficulties in analyzing the symmetry conditions for applying Gauss's and Ampere's laws to calculate the electric or magnetic field in a parallel setting. The article is organized as follows. Section II presents the literature review comparatively, identifying the difficulties previously reported in the literature in each context and both contexts. Section III presents the research gap and research questions for this study. Section IV describes the methodological approach, participants, instruments, data collection, and analysis strategy. The results are presented in Sec. V in two parts: the description of the categories and the frequency of categories. Section VI discusses the results, focusing on answering the research questions. Section VII concludes with the main takeaways of this article, implications for teaching, and future directions.

II. LITERATURE REVIEW

Several studies have explored undergraduate students' difficulties understanding and applying Gauss's and Ampere's laws effectively in the introductory and upper-division electricity and magnetism courses. While most studies have focused either on Gauss's law [5–11] or Ampere's law [3,12–14], a few studies have addressed students' difficulties with Gauss's and Ampere's laws with a complementary perspective [2,15], and with a comparative view in our preliminary study [16].

Most of these studies specify that understanding Gauss's and Ampere's laws effectively implies recognizing the physical situations where these laws are useful for calculating the electric or magnetic field. This task requires students to analyze the symmetry of the charge distribution and the definition of a Gaussian surface or an Amperian trajectory with a symmetry that fulfills three key aspects: (i) the field has the same magnitude throughout the surface or path; (ii) the direction of the field is parallel or antiparallel to the surface or trajectory vector, and (iii) having enough information to calculate the surface or path. Manogue *et al.* [3] argue that regardless of the difficulties in understanding and applying Ampere's law effectively, these topics are necessary because they require students to think like a physicist. A few studies highlight the need to analyze inverse problems where the electric or magnetic field cannot be algebraically solved from the integral, which means that it is not possible to use Gauss's or Ampere's law to calculate the field [3,5,7,16].

Figure 1 summarizes a comparison of the most relevant findings in the literature on students' difficulties when understanding and applying symmetry in Gauss's and Ampere's laws. This comparison helps recognize how more difficulties have been studied in the context of Gauss's law than in Ampere's law. It also highlights the three difficulties that have been identified persistently in both contexts, which is evidence of the existing parallelism between them.

Some of the most prevalent difficulties in understanding and applying Gauss's law are confusing the symmetry of

FIG. 1. Comparison of the most relevant findings in the literature of students' difficulties when understanding and applying symmetry in Gauss's and Ampere's laws. The bubbles on the left side of the figure represent the difficulties of Gauss's law, and on the right side for Ampere's law. The bubbles in the middle are difficulties found in both contexts.

the charge distribution and the geometric symmetry [6,8,10,11], and using Gauss's law in a rote way [5,7]. Another prevalent difficulty is to think that only the enclosed charge distributions contribute to the electric field, failing to apply the superposition principle correctly [5,11], which some authors identify as a functional reduction [2,15]. Students think it is possible to apply Gauss's law only on open surfaces or confuse open and closed surfaces [6,9,10]. There is overwhelming evidence that students confuse the electric field and electric flux [6,7,9,10], which has been identified as functional fixedness [2,15]. Some studies found that students generalize the use of Gauss's law to other contexts by rote learning [8] and assume the electric field can be calculated without sufficient symmetry [11]. Specifically, in inverse problems, students may think the solution is "messy" rather than impossible [5]. Other prevalent difficulties are related to the surface features of specific problems; such as comparing a spherical and a football surface [8], assuming that if the enclosed charge or the net flux is zero, the electric field is zero [11], and failing to realize that a cylinder or a cube may not be suitable Gaussian surfaces for calculating the electric field [6].

The difficulties in understanding and applying Ampere's law are similar but have not been as comprehensively studied in the literature. Some studies have reported difficulties recognizing and using symmetry arguments as a general ability to think like a physicist [3] and not using information about the magnetic field [12]. The literature has also found the functional reduction of considering that only the sources enclosed by the Amperian path create the field [2,14,15] and the functional fixedness of confusing the magnetic field and circulation [2,14,15]. Moreover, [13] identified two conceptual aspects as sources of difficulties for students: the confusion between the sources of the field and the enclosed currents and the confusion of field and circulation; and an epistemological aspect: thinking that Ampere's law helps calculate the field in all situations.

III. PROBLEM STATEMENT

By recognizing the parallelism between several concepts of electricity and magnetism, we identified the need to analyze and compare students' understanding of symmetry when applying Gauss's and Ampere's laws to calculate the electric or magnetic field. To address this gap, we conducted a study with the objective of comparing students' difficulties when answering if it is possible to use Gauss's or Ampere's law to calculate the electric or magnetic field in three different inverse problems where the field cannot be solved from the equation.

The research questions are as follows:

1. What are students' most frequent difficulties when identifying the necessary symmetry to solve the

electric field from Gauss's law and the magnetic field from Ampere's law in inverse problems with parallel surface features?

2. How do these difficulties compare between the two contexts, electricity for Gauss's law and magnetism for Ampere's law, in the three inverse problems with parallel surface features?

It is important to acknowledge that many difficulties were reported independently in each context, as found in the literature review. The only study that analyzed Gauss's and Ampere's law in a complementary fashion was Ref. [2], and their categories were broad due to the classification into functional fixedness and functional reduction. In this study, the analysis presents the students' difficulties comparatively with a fine-grained categorization. This direct comparison between the electricity and magnetism contexts of parallel problems allows finding similarities in the frequency of categories in each context.

IV. METHODOLOGY

This study has a qualitative research design to approach the objective of comparing students' understanding and use of symmetry in Gauss's and Ampere's Laws and their relation to the electric and magnetic fields, respectively. We conducted a phenomenographic study [17,18] that uses an open-ended questionnaire to explore students' analysis of the symmetry condition for using Gauss's law to calculate the electric field or Ampere's law to compute the magnetic field. We compare the categories that emerged from students' responses in both contexts. This study is part of a broader investigation where the research team has compared students' understanding of different topics of electricity and magnetism [4,16,19,20]. This contribution focuses on students' understanding of symmetry in the context of Gauss's and Ampere's laws.

A. Participants

The participants are 322 introductory engineering students from a large private Mexican university in the north of Mexico. The students completed the calculus-based electricity and magnetism course, the last of the introductory physics courses offered by the institution to all engineering programs. The course consists of 3 h of lecture and 1.5 h of laboratory sessions per week. Students use a known textbook [21] and tutorials [22]. The course is comparable to an electricity and magnetism course in the American university curriculum regarding content, duration, and students' performance [23].

B. Instrument

The instruments are two open-ended questionnaires, one with the context of Gauss's law and the other with the context of Ampere's law. The questions exploit the parallelism between Gauss's and Ampere's laws given by Maxwell's equations and the similar surface features commonly used to represent Gaussian surfaces and Amperian trajectories. The questionnaires include three different scenarios presented in Fig. 2: "Opposite field sources," "Squarelike symmetry," and "Off-centered source." All the scenarios ask students the same context-dependent question: Using the shown surface, is it possible to use Gauss's (or Ampere's) law to calculate the electric (or magnetic) field at any point of the surface (or trajectory)? All the scenarios share the main characteristic that the student needs to verify if the given surface or trajectory satisfies the necessary symmetry to apply Gauss's or Ampere's law to solve for the electric or magnetic field, respectively.

The three scenarios are represented with parallel surface features. Electric point charges are represented with small circles, while electric currents are represented with the symbol for outward and inward vectors to represent their direction. Spherical Gaussian surfaces and circular Amperian trajectories are represented with a dotted circle. The squarelike symmetric surface and trajectory are represented with a dotted square. These representations are customary in textbooks in the electricity and magnetism classroom [22,24] and conceptual evaluations [25–27]. The electric charges are labeled as $+q$ or $-q$, and the electric currents are labeled as $+I_0$ to indicate a conventional current. Our students were familiar with these representations by the end of the introductory electricity and magnetism course.

The three scenarios are examples of inverse problems where it is impossible to calculate the field with the given surface or trajectory. The correct answer for all these questions is to identify that the field cannot be calculated using the given surface or trajectory because it does not comply with the necessary symmetry between the field and the surface or trajectory (e.g., the field is not constant at all points of the surface or trajectory, or the angle between the field and the surface or trajectory does not simplify the dot product). The opposite field sources scenario presents an electric dipole inside a spherical Gaussian surface for the electricity context and a set of two conventional currents, one pointing outward and the other inwards, inside a circular Amperian trajectory in the magnetism context. The squarelike symmetry scenario presents a positive charge inside a cubic Gaussian surface for the electricity context and an outward electric current inside a square Amperian trajectory for the magnetism context. The off-centered source scenario consists of a positive charge inside a spherical Gaussian surface, not located in the center of the surface for the electricity context. Parallelly, the outward electric current is not in the center of the circular Amperian trajectory in the magnetism context. It is important to note that the squarelike symmetry scenario has been previously analyzed elsewhere [16]. In this study, the results are different from the previous work because we provide a fine-grained analysis of one of the categories ("Identifies

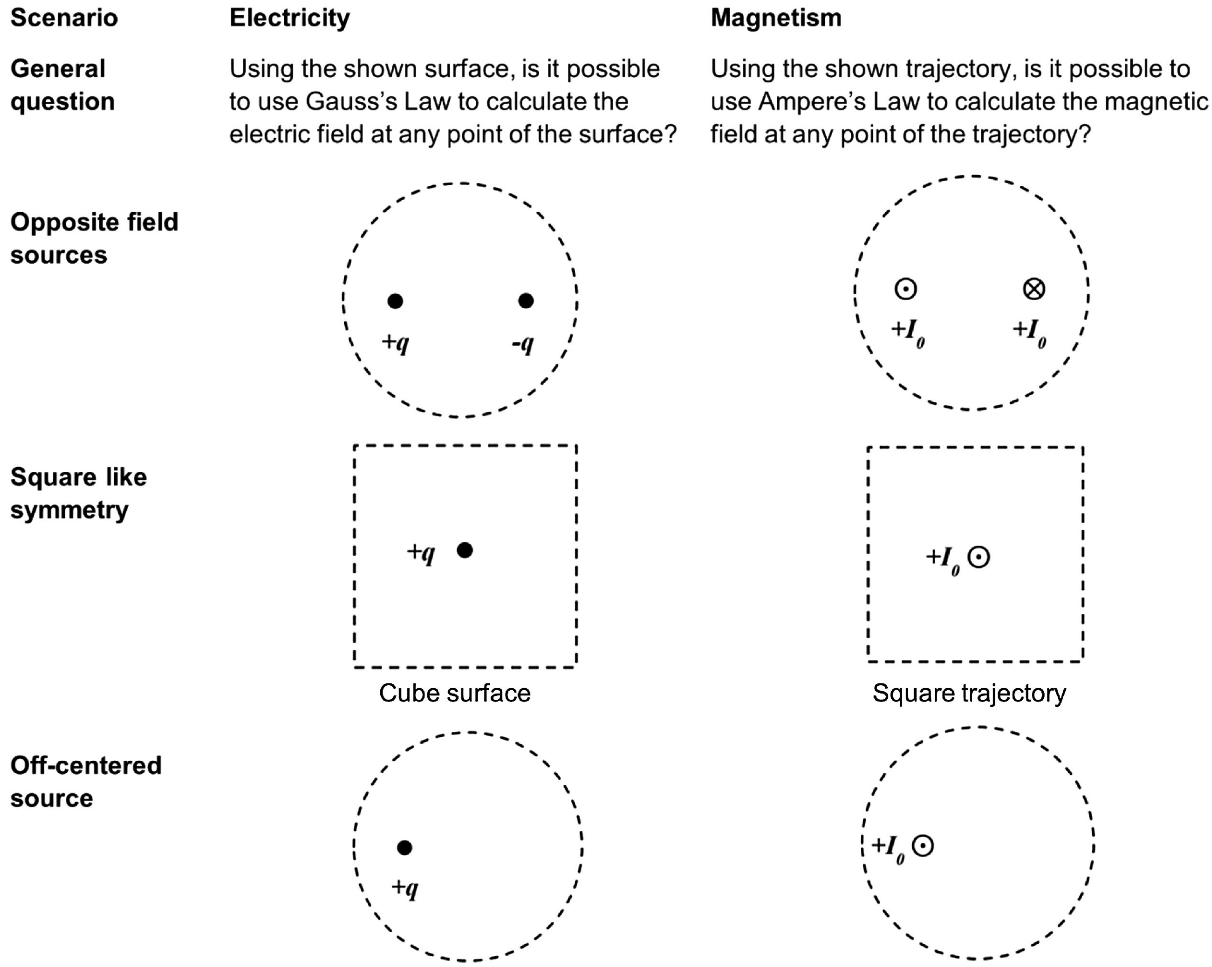

FIG. 2. Three different inverse problems where the field cannot be solved from Gauss's or Ampere's laws. The scenarios use parallel representations for the two contexts.

only an enclosed field source" in [16]) and a comparison with the other inverted problems.

C. Data collection and analysis

We randomly administered the questionnaires to all participants during the last electricity and magnetism course laboratory session. A total of 162 students received the electricity version and 160 the magnetism version. The tests were administered, responded to, and analyzed in Spanish. The analysis followed a phenomenographic approach, which focuses on analyzing the different ways people experience and understand phenomena. This approach recognizes that each person's experience and understanding are unique; however, it acknowledges that commonly shared experiences and conceptions can be grouped to describe a collective intellect [17]. The categories that describe the collective intellect are the main result of phenomenographic research. Phenomenographic research is mainly interested in the variations between the categories that emerge from qualitative data, which results from open-ended instruments [18]. The method has been used in physics education research to analyze open-ended written questions and to derive students' understanding and difficulties when learning physical concepts [4,28–32]. We use open-ended written questions because they provide the advantage of finding the collective intellect of a large number of participants on different topics of electricity and magnetism [4,29].

Two of us performed the data analysis. We identified emerging categories from the answers of a 20-student random sample, reaching a consensus between the experts. We individually analyzed the remaining participants' answers and classified them into emerging categories. We compared the classifications done by the two of us. When more categories emerged, we included them in the analysis in an iterative process. We used Cohen's kappa to measure the interrater reliability of our analysis, attaining an average of 0.95 for the electricity test and 0.97 for the magnetism test. This statistic considers that agreement can occur by chance. A value greater than 0.75 represents excellent agreement beyond chance [33]. Table I presents the details of the interrater reliability for each scenario and context.

TABLE I. Interrater reliability of the phenomenographic analysis for each scenario and context.

Scenario	Electricity	Magnetism
Opposite field sources	0.94	0.97
Squarelike symmetry	0.94	0.96
Off-centered source	0.98	0.98
Average	0.95	0.97

V. RESULTS

We analyzed students' answers for each scenario independently, focusing on two aspects: (i) whether the student answered that it was possible to use the surface or trajectory to calculate the field (Yes or No), and (ii) their reasoning. We classified the blank answers and answers without explanation into "unanswered." Once all answers were classified, we selected the categories considering a 5% threshold. The categories that did not fulfill the 5% threshold were included in the category "other." However, we kept only one category below the 5% threshold for comparison between scenarios. We present the results for students' understanding of the use of symmetry for calculating the electric or magnetic field using Gauss's or Ampere's law. We describe the categories presenting examples for each relevant scenario in Tables II–VII. All the examples presented in the results section were translated from Spanish. We present the frequency comparison between all the scenarios and contexts in Table VIII.

A. Description of categories

"*Lack of symmetry*": The students answered that it is impossible to calculate the field with the given Gaussian surface or Amperian trajectory and supported their answers with arguments related to the symmetry between the field vector and the surface or trajectory. This category emerged in all the scenarios and contexts. We consider the reasoning in this category the closest to the scientifically accepted explanation that emerged from this cohort of students. We provide examples for each scenario and context in Table II.

"*Surface features restrict*": The students answered that it is impossible to calculate the field with the given Gaussian surface or Amperian trajectory and supported their answers with arguments related to the surface features of the surface or trajectory or how the charge is distributed within. For the students in this category, the reason why it is impossible to calculate the field with the given surface or trajectory is related to surface features of the scenario, instead of the analysis of the symmetry conditions. The surface features are expected to differ for each scenario because they are specific characteristics of the physical situation that can distract students from analyzing the symmetry conditions necessary to exploit Gauss's or Ampere's laws to calculate the field. Since the surface features that emerged depend on the scenario, we provide the description and an example for each scenario and context in Table III.

In the opposite field sources scenario, the surface feature is that the net enclosed field source is zero, which could lead students to analyze correctly that the electric flux or magnetic circulation is zero or to the incorrect conclusion that the electric or magnetic field is zero. These are surface features of the scenario because students correctly answer that it is impossible to calculate the field by referring to arguments that only apply to this specific scenario where the enclosed field source is zero. Some students would propose that it is impossible to calculate the field because you would need only one charge at the center of the sphere or one current at the center of the trajectory. This answer was also treated as the surface features of the scenario because students do not necessarily identify the necessary symmetry but may memorize a scenario where they can use Gauss's or Ampere's law to calculate the field.

In squarelike symmetry, the surface feature is that the Gaussian surface is cubic or not spherical and that the Amperian trajectory is square or not circular. These are surface features of the scenario because it is an explanation that applies only to similar scenarios that enclose a field source with any other surface or trajectory different from the expected (a sphere or a circle). Students that use the

TABLE II. Examples of students' reasoning in the category lack of symmetry for each scenario.

Scenario	Electricity	Magnetism
Opposite field sources	"*No. The distance and position of each charge affects the field in different parts of the surface*." (Student E14)	"*No. There will be points where the wire is closer* [to the trajectory] *than others*." (Student M105)
Squarelike symmetry	"*No. This is not a surface where the points are at the same distance from the charge. To apply the equation, the electric field must be constant throughout the surface. Since it is at a different distance at some points, that is not the case*." (Student E26)	"*No. Because the magnetic field is not constant throughout the trajectory*." (Student M11)
Off-centered source	"*No. Because we don't have the same distance from the charge to the sphere at all points*." (Student E42)	"*No. The angle between* $\mathbf{B}$ *and* $d\mathbf{l}$ *won't be constant*." (Student M30)

TABLE III. Description and example of student's reasoning in the category surface features restrict for each scenario.

Scenario	Electricity	Magnetism
Opposite field sources	Description: No, because there is no enclosed charge. Example: "*No, it can't be calculated because there is no enclosed charge*." (Student E40)	Description: No, because the enclosed current is zero. Example: "*No, since* $I_{\mathrm{enclosed}} = 0$." (Student M01)
Squarelike symmetry	Description: No, because the surface is cubic or not spherical. Example: "*No, because it's a square surface*." (Student E16) "*No, the surface must be circular*." (Student E77)	Description: No, because the trajectory is a square or is not circular. Example: "*No, it is not possible using a square trajectory*." (Student M18)
Off-centered source	Description: The charge is not at the center. Example: "*No, because the charge is not at the center of the spherical surface*." (Student E28)	Description: The current is not at the center. Example: "*No, because the current-carrying wire is not at the center of the trajectory*." (Student M79)

surface features of this scenario do not necessarily analyze and identify the necessary symmetry conditions between the field vectors and the surface or trajectory. Rather, the surface features argument may come from using Gauss's and Ampere's laws in a rote way [16].

In the off-centered source scenario, the surface feature is that the field source is not at the center of the sphere or the trajectory. This scenario is important because the field source is not symmetrically oriented with the surface or trajectory, but the surface or trajectory has geometric symmetry. This surface feature makes evident that explicitly stating that the distance from the field source to all the other points on the surface or trajectory varies is different from stating that the field source is not at the center. The former implies acknowledging that the field is different at all the points of the surface or trajectory, while the latter can result from rote learning of familiar situations without necessarily understanding the symmetry conditions.

"*Surface features permit*": The students answered that it is possible to calculate the field with the given Gaussian surface or Amperian trajectory and supported their answers with arguments related to the surface features of the surface or trajectory or how the charge is distributed. For the students in this category, the reason why it is possible to calculate the field with the given surface or trajectory is related to surface features of the scenario. The surface features can be different for each scenario because they are specific characteristics of the physical situation that can distract students from analyzing the symmetry conditions necessary to exploit Gauss's or Ampere's laws to calculate the field. Since the surface features that emerged depend on the scenario, we provide the description and an example for each scenario and context in Table IV.

In the opposite field sources scenario, there are two emerging surface features that lead students to the incorrect conclusion that it is possible to calculate the field. One is related to the characteristics of the Gaussian surface or Amperian trajectory, whether it is spherical or circular, closed, or symmetric. The other surface feature is related to the field source distribution, specifically that the field is zero because the net field source is zero. These are surface features of the scenario because students answer that it is possible to calculate the field by referring to arguments that only apply to this specific scenario where the enclosed field source is zero. Moreover, they may rely on surface features of the representation that lead them to their answer, such as having a circle with two symmetrically located opposite field sources.

In the squarelike symmetry scenario, the surface features are only related to the characteristics of the Gaussian surface or Amperian trajectory, whether it is a cube or a square, closed, or symmetric. These are considered surface features of the scenario because it is an explanation that applies only to similar scenarios that enclose a field source with any other surface or trajectory that have a squarelike symmetry.

In the off-centered source scenario, the surface feature is that the surface or trajectory is spherical, circular, or closed. In this scenario, the surface feature of symmetry was not relevant, given that the field source is not symmetrically oriented with the surface or trajectory. However, having a spherical surface or a circular trajectory was a convincing argument for some students to state that it is possible to use Gauss's or Ampere's law to calculate the field.

Moreover, in the three scenarios and the two contexts, the surface feature of having a closed surface or trajectory was relevant. This implies that students memorize that Gauss's or Ampere's laws describe the flux or circulation through closed surfaces or trajectories without considering all the symmetry conditions that need to be met to calculate the field. In some cases, the geometrical symmetry may reinforce this idea.

"*Enclosed field source:*" The students answered that it is possible to calculate the field with the given Gaussian surface or Amperian trajectory and mainly supported their answers with arguments related to the fact that the charge or current is enclosed. This category emerged in all the scenarios and contexts. Mathematically, Gauss's and Ampere's laws pose that the flux or circulation is proportional to the enclosed charge or current. Students may fix on the "imaginary enclosing object" surface feature and misinterpret that if

TABLE IV. Description and examples of student's reasoning in the category surface features permit for each scenario.

Scenario	Electricity	Magnetism
Opposite field sources	Description: Characteristics of the surface (i.e., closed, spherical, symmetric) or the charge distribution (the net charge is zero). Examples: "*Yes, because it is a closed surface*." (Student E04) "*Because it's round*." (Student E02) "*Yes, but when calculating, it will be zero because the internal charges cancel out*." (Student E88)	Description: Characteristics of the trajectory (Circular, round, closed, etc.) or the current distribution (the net current is zero). Examples: "*Yes. Because it is a closed trajectory*." (Student M27) "*Yes. It can be calculated with spheres and circles*." (Student M18) "*It would simply be zero because there is no enclosed charge*." (Student M7)
Squarelike symmetry	Description: Characteristics of the surface (i.e., closed, cube, symmetric). Examples: "*Yes, it is a closed surface, so we can calculate it*." (Student E17) "*Yes, because it is a cubic surface, and not circular*." (Student E89)	Description: Characteristics of the trajectory (i.e., square, closed, symmetric). Examples: "*Yes. It can be applied to any closed trajectory*." (Student M74) "*Yes, it doesn't matter that it's a square trajectory*." (Student M10)
Off-centered source	Description: Characteristics of the surface (i.e., closed, spherical). Example: "*Yes. Gaussian surfaces are spheres*." (Student E99)	Description: Characteristics of the trajectory (i.e., circular, closed). Example: "*Yes. Ampere's law can be used when there is symmetry. In this case, the circular trajectory is symmetric with the wire and the current*." (Student M96)

there is a field source enclosed by a Gaussian surface or Amperian trajectory, it will be possible to calculate the field using said surface or trajectory, regardless of the necessary symmetry conditions to compute the field. We provide examples for each scenario and context in Table V.

"*Authority fallacy*" The students answered that it is possible to calculate the field with the given Gaussian surface or Amperian trajectory and supported their answers with the arguments related to the authority fallacy that because Gauss's and Ampere's Laws are physical laws, they can always be used to calculate the field, regardless of whether the symmetry conditions are met. In some instances, students would only write the equation for Gauss's or Ampere's law and/or solve for the field without considering the integral and the dot product. This category emerged in all the scenarios and contexts. We consider that some students functionally reduce these laws to formulas that apply to every scenario and any physical quantity involved [14,16]. We provide examples for each scenario and context in Table VI.

"*Difficult solution:*" The students answered that it is possible to calculate the field with the given Gaussian surface or Amperian trajectory but that it would be a very complicated solution. In some cases, they write the left side of Gauss's or Ampere's laws, referring to the definition of electric flux or magnetic circulation. This category emerged in all contexts. We provide examples for each scenario and context in Table VII.

Table VIII summarizes the frequency for each descriptive category by scenario and context. It is important to note that the first two categories, lack of symmetry and surface features restrict, group students that answered correctly that it is impossible to calculate the field with the given surface or trajectory. The rest of the categories, surface features permit, enclosed field source, authority fallacy, and difficult solution group students that answered that it is possible to

TABLE V. Examples of students' reasoning in the category enclosed field source for each scenario.

Scenario	Electricity	Magnetism
Opposite field sources	"*Yes, because the circle represents the Gaussian sphere, and the 'enclosed' charges are inside the circle*." (Student E32)	"*Yes, there are two enclosed currents*." (Student M99)
Squarelike symmetry	"*Yes, there is a known surface and an enclosed charge*." (Student E38)	"*Yes, because it depends on the enclosed current*." (Student M91)
Off-centered source	"*Yes, the formula uses* q_{enclosed}; *it does not matter where the charge is, as long as it is inside*." (Student E60)	"*Yes, to calculate the field, we require an area and an enclosed current*." (Student M72)

TABLE VI. Examples of students' reasoning in the category authority fallacy for each scenario.

Scenario	Electricity	Magnetism
Opposite field sources	"*Yes, because Gauss's law can be used in any way.*" (Student E67)	"*Yes, it is possible because Ampere's law allows to calculate the magnetic field in a trajectory.*" (Student M15)
Squarelike symmetry	"*Yes, Gauss's law allows us to know the electric field in any position of the surface.*" (Student E51)	"*Yes, because if we use it* [Ampere's law], *it would be easier to find the magnetic field.*" (Student M43)
Off-centered source	"*Yes, Gauss's law does not discriminate against charges that are not in the center.*" (Student E13)	"*Yes, Ampere's law works for these cases.*" (Student M47)

TABLE VII. Examples of students' reasoning in the category difficult solution for each scenario.

Scenario	Electricity	Magnetism
Opposite field sources	"*Yes. The integral sums the electric field in each differential of area.*" (Student E107)	"*Yes, it is possible to integrate.*" (Student M42)
Squarelike symmetry	"*Yes. The only inconvenience is that we must include cosines to the formulation because the r is different in each angle.*" (Student E76)	"*Yes. It would be the sum over each side, taking it as a long wire and using the formula.*" (Student M107)
Off-centered source	"*Yes. We use the surface and enclosed flux with a differential of area.*" (Student R34)	"*Yes. Using the dl of the trajectory, we can calculate the magnetic field at any position of the trajectory.*" (Student M26)

TABLE VIII. Percentages for each descriptive category by scenario and context.

Scenario	Opposite field sources		Squarelike symmetry		Off-centered source	
Category	Electricity	Magnetism	Electricity	Magnetism	Electricity	Magnetism
Lack of symmetry	6%	4%	11%	8%	12%	15%
Surface features restrict	12%	8%	17%	17%	11%	10%
Surface features permit	26%	19%	17%	17%	9%	11%
Enclosed field source	15%	15%	13%	14%	28%	21%
Authority fallacy	14%	22%	14%	12%	12%	9%
Difficult solution	1%	4%	7%	8%	6%	6%
Other	13%	12%	16%	16%	15%	16%
Unanswered	13%	16%	5%	8%	7%	12%
Total	100%	100%	100%	100%	100%	100%

calculate the field with the given surface or trajectory. The name of the category mainly describes the reasoning that supports students' answers.

VI. DISCUSSION

We analyze the results by comparing students' difficulties when answering if it is possible to use Gauss's or Ampere's law to calculate the electric or magnetic field in three different inverse problems where the field cannot be solved from the equation. We found that the students' most frequent difficulties in the two contexts are very similar. The difficulties found in the two contexts can be related to the difficulties previously reported in the literature. Table IX in the Appendix summarizes how the main categories found in our analysis can be related to the difficulties reported in the literature. The categories are not identical mainly because of the nature of the inverse problems that we presented.

A. Symmetry arguments

The first category in Table VIII, lack of symmetry, is the symmetry argument where students identify that the scenario lacks sufficient symmetry to solve the electric or magnetic field using Gauss's or Ampere's law. The evidence points out that around 10% of students across the three scenarios and the two contexts could identify the necessary symmetry conditions. However, in the opposite field sources scenario, we found fewer students identifying that the symmetry conditions are not met, while in the off-centered source scenario, we found the most students.

This is expected when analyzing the visual aspects of the two scenarios: in the opposite field sources scenario, the two field sources are symmetrically located, while in the off-centered source scenario, it is easy to see that the charge is not symmetrically located with reference to the surface or trajectory. Several studies have pinpointed that recognizing and using symmetry arguments in inverse problems, such as the ones posed by Gauss's and Ampere's laws, is necessary for thinking like a physicist [3]. Students use Gauss's law in a rote way instead of considering symmetry arguments [5,7], and students do not use information about the magnetic field when using Ampere's law [12].

B. Surface features

The category surface features restrict was more frequent than the symmetry argument in the opposite field sources and the squarelike symmetry scenarios, but not in the off-centered source scenario. This makes sense because, in the off-centered source scenario, the lack of symmetry is more visually evident than in the other two. Therefore, even if students do not analyze the symmetry between the field source distribution and the surface or trajectory, they can use other arguments related to surface features to reach the correct conclusion that it is not possible to use Gauss's or Ampere's laws to calculate the field in these three inverse problems. Some interesting finding was the variations between the surface features that students found useful in the different scenarios. In the opposite field sources scenario, they referred to the net charge or current enclosed. In the squarelike symmetry, they referred to the surface or trajectory as being a cube or square. In the off-centered source scenario, they observed that the source was not in the center of the sphere or circle. These variations are similar to those found in the surface features permit in the opposite field sources and the squarelike symmetry scenarios but different in the off-centered source scenario. It is noticeable that similar surface features may lead students to a correct conclusion in this category or to an incorrect conclusion in the surface features permit category. It is also important to notice that the characteristic of closed surfaces or trajectories was not relevant in this category. In the off-centered source scenario, the surface feature that led students to the correct conclusion was related to the position of the field source, while the surface feature that led students to the incorrect conclusion was related to a closed surface or trajectory.

The category surface features permit included the students who focused on a particular characteristic of the scenario. This category was found across scenarios and contexts, accounting for around 15% of students. Other studies have reported similar behavior, such as Refs. [2,15], who acknowledged that students prepare *ad hoc* explanations for different situations. This category is related to the confusion between the symmetry of the charge distribution and the geometric symmetry [6,8,10,11] because it includes the arguments where students identified that the spherical surface, circular trajectory, cube surface, and square trajectory are symmetric, regardless of how the field source is distributed within the surface or trajectory [16]. Some of the students in this category were confusing the geometric symmetry and the symmetry of the field source distribution.

It is interesting to analyze the behavior of this category for the different scenarios. This category is the most relevant for similar reasons in the opposite field sources and the squarelike symmetry scenarios. On the one hand, in the opposite-field sources scenario, there are two possible surface features, those related to the characteristics of the surface and those related to the field source distribution. In this case, there is visual symmetry due to how the field sources are located within the circle representing the surface or trajectory. There is also the characteristic of having zero net enclosed charge or current. Combining these two aspects attracts students to analyze the scenario according to its surface features instead of applying physical principles. This finding is related to [6], when presenting students with a problem where the enclosed charge was zero, students assumed that the electric field was zero. On the other hand, in the squarelike symmetry scenario, the surface features focus on the characteristics of the cube surface or square trajectory and implicitly on the field source distribution since it is located right at the center of the cube or square, which makes it visually symmetric. When comparing two problems with charge distributions with different geometries (a sphere and a football), students presented difficulties interpreting the Gaussian surfaces as mathematical tools [8]. Students can also present difficulties in realizing that a cylinder or a cube may not be suitable for calculating the electric field [6]. Finally, in the off-centered source scenario, the category was less frequent, with most students focusing on the fact that the surface was closed. This difficulty is more related to that reported by Singh in several studies [6,9,10] that students confuse open and closed surfaces, or they aim to apply Gauss's law to open surfaces. In this case, these students acknowledge that Gauss's or Ampere's law can only be used with closed surfaces or trajectories, but they oversimplify it by assuming that having a closed surface is the only criteria that need to be met for calculating the electric or magnetic field.

C. Oversimplification of Gauss's and Ampere's laws

The category enclosed field source is related to the known difficulty of considering that only the enclosed sources create the field [2,5,11,13–15]. However, the difficulty is not fundamentally the same. Due to how the questions were framed in this study as compared to others, the difficulty found is along the line of reasoning that as long as there is a charge enclosed by a surface or trajectory, it will be possible to calculate the electric or magnetic field using Gauss's or Ampere's law. In other studies, the difficulty of enclosed charges stems from an issue of misapplication of the superposition principle in the context of Gauss's or Ampere's law. This difficulty may stem from the functional reduction of oversimplification of Gauss's and Ampere's law or from the functional fixedness of confusing the field with flux or circulation [2]. The evidence shows that this difficulty is the most relevant in the off-centered source scenario. This is consistent with the interpretation of functional fixedness: Since the field source is not at the center, the scenario is not symmetric; if students have the preconceived notion that it is possible due to confusion between field and flux or circulation, they can look for the conditions to use Gauss's or Ampere's laws that the scenario does meet, which is that there is an enclosed charge or current regardless of its position.

The category authority fallacy is when students explain that Gauss's and Ampere's laws apply to every situation (which is true, they are the physical laws that explain electromagnetic phenomena, after all). However, the questions were not if Gauss's or Ampere's laws apply to the three scenarios but whether they could be exploited to calculate the electric or magnetic field. Thinking that Gauss's and Ampere's laws can be used to calculate the field in every situation might stem from the widely known confusion between electric field and electric flux in the context of Gauss's law [2,6,7,9,10,15], and the confusion between magnetic field and circulation in the context of Ampere's law [2,13–15], a relation posited in the preliminary study [16]. The relation with this known difficulty is along reasoning that since Gauss's law applies to every case of electric flux, it should be the same to every case of the electric field; and a likewise reasoning for the magnetic field, magnetic circulation, and Ampere's law. Another possible explanation found in the literature for this difficulty was a generalization of the use of Gauss's law in other contexts by rote learning [8] and assuming that the electric field can be calculated without sufficient symmetry [11]. This category was prevalent across scenarios and contexts, accounting for around 15% of students. In this category, it is interesting to note that the highest percentage found was in the opposite field sources in the magnetism context (22%), and the lowest percentage was also in the magnetism context but in the off-centered source scenario (9%). This behavior evidences that these two inverse problems may seem very different to students, and they can elicit different solving strategies.

D. Difficult solution

The category difficult solution is related to the finding by [5] that with problems that cannot be solved, students tend to believe that the solution is difficult instead of impossible. The evidence found in this study shows that this difficulty was persistently found in the three different scenarios, accounting for less than 5% of students in the opposite field sources scenario and between 5% and 10% in the squarelike symmetry and the off-centered source scenarios. Interestingly, the frequency was higher in the squarelike symmetry scenario because students may think the solution would involve the cosines and the Pythagoras theorem for changing between polar and Cartesian coordinates.

E. Parallelism between the electricity and magnetism contexts

The frequency of each category in the three scenarios is consistent across the electricity and magnetism contexts, except for a few cases outlined next. This analysis considers differences above 5% between contexts for the same scenario. The first difference that emerges is in the opposite field sources scenario for students who answer that it is impossible to calculate the field, regardless of their reasoning category ($E = 18\%, M = 12\%$). This difference implies that this scenario in the magnetism context is more difficult for students. The evidence suggests that analyzing two currents in opposite directions is more challenging for students than analyzing two charges with different signs.

Another two differences emerged in the opposite field sources scenario when comparing the two contexts: Authority fallacy ($E = 14\%$, $M = 22\%$) and surface features permit ($E = 26\%$, $M = 19\%$). Students tend to reason that Ampere's law is useful for this type of problem without considering symmetry conditions more than Gauss's law. However, in the electricity context, they have a higher tendency to analyze with surface features, such as having a spherical surface or having a net enclosed charge of zero. This pattern could be related to the difficulty of analyzing the system with two currents. Students would prefer to oversimplify the use of Ampere's law rather than analyze the case. With Gauss's law, students may be more familiar with the representation and analyze the scenario with a special focus on its surface features. This occurrence is similar to the off-centered source scenario, where the difference between electricity and magnetism was evident in the enclosed field source category ($E = 28\%$, $M = 21\%$). In this case, there is a lower tendency to apply Ampere's law by describing that the current is enclosed. This pattern, again, could be explained by the difficulty of analyzing currents instead of charges.

Another difference between the two contexts emerged in the number of unanswered or unexplained answers. The data show a tendency to leave blank questions or unexplained answers more often in the magnetism context than in the electricity context, as has been observed in a previous study [20]. Students can find the topics of magnetism more challenging than the topics of electricity due to several factors, such as the mathematical formalism required to perform vector products and the need to work in three dimensions [28,34]. When studying electricity and magnetism in introductory physics, students had learned about the magnetic field with a less formal approach through the interactions of magnets in high school. Moreover, one can assume that students have dealt with conservational fields (i.e., gravitational and electric in static conditions) before learning magnetism. Learning magnetism in the introductory physics course is, perhaps, the first time that students encounter a nonconservational field in its mathematically formal representation, including Ampere's law.

VII. CONCLUSION

This contribution presents a study of students' understanding and application of symmetry arguments in Gauss's and Ampere's laws to calculate the electric and magnetic fields. The study was conducted with a phenomenographic approach to identify students' most frequent answers and reasoning to inverse problems that use parallel surface features in the context of electricity for Gauss's law and magnetism for Ampere's law. The instrument consisted of three different scenarios for inverse problems: opposite field sources, squarelike symmetry, and off-centered source. All scenarios were presented in the electricity and magnetism contexts independently and randomly to 322 students finishing the electricity and magnetism introductory course. The analysis resulted in a set of descriptive categories that emerged in the three different scenarios and the two contexts, allowing for the direct comparison between scenarios and contexts. The main findings of the study revealed that the descriptive categories are related to some of the difficulties previously reported in the literature with standard problems involving Gauss's and Ampere's laws. However, the use of inverse problems elicited variations in the types of reasoning related to the surface features of the scenarios and their parallel representations. This was most evident when comparing between contexts.

The students' answers were divided into six relevant categories, of which identifying the necessary symmetry to apply Gauss's or Ampere's law is the correct reasoning. The other categories refer to the use of surface features of each scenario to explain their answer, applying Gauss's or Ampere's law in an oversimplified way and thinking that it would be possible but more complicated in these scenarios. Overall, the frequencies in all scenarios were similar and consistent across contexts. Some highlights include that in the opposite field sources and the squarelike symmetry scenarios, the most frequent difficulty was related to surface features. In the off-centered source scenario, the most frequent difficulty was related to the enclosed charge or current. This consistency across scenarios and contexts was achieved with inverse problems that pose the same question to six different situations. The identification of variations was possible due to the phenomenographic approach and the consistent emerging categories.

The comparative analysis between the electricity and magnetism contexts allowed the identification of relevant difficulties that can be context dependent. One of the findings pinpoints that analyzing currents can be more challenging for students than analyzing point charges. This difficulty could explain why, in the context of Ampere's law, there was a higher tendency to recur to the authority fallacy category (an oversimplification of Ampere's law) in the opposite field sources scenario and a lower tendency to recur to the enclosed field source category in the off-centered source scenario. Another result of the comparative analysis was that the categories related to surface features were more prevalent in the electricity context in the opposite field sources scenario, which presents visual symmetry. It was also observed that more students left their answer blank or unexplained in the magnetism context, possibly due to the time of instruction and the degree of maturity achieved on the topic.

The findings of this study can help introductory and intermediate electricity and magnetism instructors to redirect their approach to Gauss's and Ampere's laws by introducing the analysis of inverse problems. The difficulties found in this study can guide the design of tutorials or teaching learning sequences that help students analyze inverse problems and develop physical thinking. We recommend instructors include active learning activities where the students have the opportunity to reflect on the application of Gauss's and Ampere's laws to calculate the electric or magnetic field, respectively. These reflections should emphasize the difference between electric field and flux and magnetic field and circulation. We recommend using the cases where these laws can be exploited to calculate the field by critically analyzing the necessary symmetry conditions, letting students reflect on the magnitude of the field at every position of the surface or trajectory and its direction relative to the surface or trajectory, such that the dot product can be simplified and the field magnitude can be extracted from the integral. This suggestion implies that Gauss's and Ampere's laws should always be treated formally; when starting to work with them, the laws should be written in their full integral or differential form instead of starting with a simplified version. It is also important to present students with inverse problems where the field cannot be extracted from the integral, or the dot product cannot be simplified. The problems presented in this study can be a starting point, along with other symmetries and charge distributions. This way, students have the opportunity to reflect that, even though Gauss's and Ampere's laws are physical laws that explain electromagnetic phenomena, they cannot always be exploited to calculate the electric or magnetic field. To benefit from the parallelism between Gauss's and Ampere's laws, we suggest that instructors focus on tackling these difficulties first during Gauss's law and reinforce the tactic during Ampere's law.

There is still the need to analyze and compare the symmetry arguments applied to Gauss's and Ampere's laws. This study followed a phenomenographic approach with a large number of participants and a rigorous iterative analysis procedure. However, there was noise present in the answers that did not fit any category. This methodologic characteristic does not allow for generalization of the results. It would be valuable to investigate students' understanding of the symmetry arguments in Gauss's and Ampere's laws through quantitative instruments that use inverse problems. The analysis would also benefit from the use of semistructured interviews for getting a deeper insight into students' reasoning and understanding. It would also be valuable to approach the understanding of Gauss's and Ampere's laws through other lenses, such as the understanding of the concepts of flux and circulation and the principle of superposition applied in Gauss's and Ampere's laws.

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APPENDIX: DIFFICULTIES COMPARISON

In Table IX we present a comparison of the difficulties found in our study and reported in the literature in the context of Gauss's and Ampere's law.

TABLE IX. Difficulties reported in the literature related to (but not necessarily identical to) the difficulties found in our study. The difficulties reported in the literature were identified in introductory physics courses and/or in upper-division courses.

Difficulties found in our study	Difficulties reported in the literature: Gauss's law	Difficulties reported in the literature: Ampere's law
Symmetry arguments: Arguments related to the symmetry necessary between the electric or magnetic field vector and the Gaussian surface or Amperian trajectory	Confusing the symmetry of the charge distribution and the geometric symmetry [6,10].	Recognizing and using symmetry arguments [3].
	Considering the shape of and charge distribution [8,11].	Not using information about the magnetic field [12].
	Misapplying superposition arguments instead of geometrical symmetry arguments; using Gauss's law in a rote way instead of considering symmetry [5,7].	
Enclosed field source: Arguments related to the fact that the Gaussian surface or Amperian trajectory encloses a charge or current.	Only the sources enclosed by the Gaussian surface create the field: Functional reduction [2,15].	Only the sources enclosed by the Amperian path create the field: Functional reduction [2,14,15].
	Only the enclosed charge distributions contribute to the electric field [5,11].	Conceptual aspect: Confusion between the sources of field and the enclosed currents [13].
	Confusion between flux and field [6,7,9,10].	Conceptual aspect: Confusion of field and circulation [13].
Authority fallacy: Arguments related to the authority fallacy that because Gauss's and Ampere's Laws are physical laws, they can always be used to calculate the field in any situation (regardless of whether the symmetry necessary for this end is met).	Confusion of field and flux: Functional fixedness [2,15].	Confusion of field and circulation: Functional fixedness [2,14,15].
	Confusion between flux and field [6,7,9,10].	Conceptual aspect: Confusion of field and circulation [13].
	Generalizing the use of Gauss's law in other contexts by rote-learning (Football symmetry) [8].	Epistemological aspect: Thinking that Ampere's law is useful for calculating the field in all situations [13].
	Assuming the electric field can be used without sufficient symmetry [11].	
Surface features: Arguments related to surface features of the Gaussian surface or Amperian trajectory.	*Ad hoc* explanation [15].	*Ad hoc* explanation [15].
	Difficulties interpreting Gaussian surfaces as a mathematical tool (Sphere v. Football) [8].	
	Assuming that if the enclosed charge or the net flux is zero, the electric field is zero [11]	
	Difficulties in realizing that a cylinder or a cube may not be suitable Gaussian surfaces for calculating the electric field [6]	
	Applying Gauss's law only on closed surfaces [6,9,10].	
Difficult solution: Students point out that it is possible but that it would be a very complicated solution.	Believing that the solution is "messy" rather than impossible [5].	

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