Participatory approach to introduce computational modeling at the undergraduate level, extending existing curricula and practices: Augmenting derivations

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Recent educational policies advocate a radical revision of science curricula and pedagogy, to support interdisciplinary practices, a distinguishing feature of contemporary science. Computational modeling (CM) is a core methodology of interdisciplinary science, as such models allow intertwining of data and theoretical perspectives from multiple domains, to address complex problems such as climate change and pandemics. This integrative nature of CM could support the pedagogical transition to interdisciplinary science as well. Most approaches to introduce CM in science curricula are based on learning new practices, such as VPython programming or agent-based modeling. These approaches do not integrate CM with existing content, media, and teaching practices. To facilitate this integration, we present a more gradualist design, starting from derivation models in physics. This design was implemented as a set of teacher professional development modules, and presented to a group of physics teachers interested in introducing CM to undergraduate students. The analysis of their responses indicates that even this gradual transition to CM requires teachers to significantly revise their ideas about the nature of physics and physics learning (their personal epistemologies). We discuss how the teacher professional development modules were redesigned based on this finding.

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I. INTRODUCTION AND MOTIVATION

Complex real world problems—such as climate change, pandemics, and green energy—cannot be addressed without interdisciplinary approaches, which are now central to contemporary science. Novel mathematical modeling methods based on computing, such as multiscale simulations and machine learning, are critical components of such interdisciplinary research, as these methods help integrate data from multiple domains, leading to new predictions and technologies. These integrative methods are now a key part of graduate-level learning in science, technology, engineering, and mathematics (STEM). Extending this trend, there is now a growing effort globally to introduce computational modeling (CM) at the undergraduate level, and even high school [1–4]. For instance, the Next Generation Science Standards in the United States, which is informed by cognitive science studies of science and engineering, includes computational thinking among the set of eight practices of science and engineering that education needs to support [5–7]. Similarly, India's new National Education Policy 2020 envisions a change towards educational approaches that focus on interdisciplinarity [8].

Transitioning to a whole new modeling methodology like CM, and also related science practices, is difficult for teachers and students. Detailed studies are needed to understand how they could adapt to this transition, especially focusing on instructors' experiences and views related to CM learning [9–11]. This understanding would allow the design of smoother transitions from existing pedagogical practices. To promote a gradual curricular transition towards computational modeling, and eventually interdisciplinary STEM practices, we have developed a teacher professional development (TPD) program rooted in physics derivations. In this paper, we present the initial design of this TPD program, and discuss the response of a select cohort of physics teachers to this gradualist design.

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We then discuss the redesign of the program, based on a detailed qualitative analysis of their feedback. Our findings indicate that teachers do not readily accept this transition, and this resistance is closely related to their ideas about the nature of physics and physics learning. These ideas are about the nature of knowledge, and they are usually not articulated during everyday teaching. However, such personal epistemologies come to the fore when teachers encounter curricular designs that incorporate CM, and they play a critical role in teachers' receptivity to such designs.

The study reported here took place in a developing country context. As such, the design of the TPD program was shaped by some key constraints that are central (though not unique) to such contexts. One is the dominance of the traditional lecture method as the standard format of instruction [12]. More importantly, many students cannot afford laptops, tablets, and smartphones. Further, instruction is primarily structured and guided by textbooks, as they are cheap, widely available, and often the only viable media format for most students. These constraints, and the resource-limited design context, suggested that a redesign of curricula and instruction for CM would require a gradualist approach—starting from textbook-based teaching and learning, and therefore existing curricular elements.

Such a design approach is under-explored in the CM case, even though this direction is available in principle. For instance, other pedagogical models in physics have taken a gradualist design approach, such as the *p*-prim approach advocated by diSessa [13]. In our case, it was the resource-limited context that required us to consider a gradualist approach towards CM learning. The constraints thus worked as a guiding design principle, and also provided the motivation and premise for this work. More generally, the very idea of augmenting derivations for computational modeling—which is the key conceptual and design contribution of the current work—emerged from the constraints we faced. This suggests that challenging learning environments can lead to novel design approaches, which might turn out to have useful pedagogical features in general. (An analogical case is "frugal design" for resource-limited contexts, a design principle that has led to innovations such as Paperfuge, a low-cost hand-operated centrifuge based on the whirligig toy, developed for African clinics with limited electricity access. Interestingly, the system also revealed novel rotational mechanics, which is a more general feature that can potentially support wider applications [14].) At an even wider level, our gradualist model also illustrates a systematic way to implement radical pedgogical changes promoted by new science education policies. This overall structure is important to keep in mind, as such education policy changes are necessitated by dramatic transitions in the method and scope of science. Our approach could thus provide a model to address education policy changes that will follow another radical methodological change in science that is currently ongoing—the transition to machine learning models.

The present study was guided by the following research questions: (a) What design requirements or characteristics of a TPD program would allow teachers to gradually transition to CM? (b) How can such a program be operationalized? (c) How do teachers respond to this proposed transition?

In Sec. II, we briefly review the literature on TPD studies related to introducing CM in physics, and outline the theoretical considerations employed. Section III discusses in detail the three design principles we followed in developing the TPD program, thereby addressing the first research question. The operationalization of these design principles, to develop modules for the teacher workshops, is discussed in Sec. IV. The design elements of the workshops, details of the structure and content of the modules used, and the rationale underlying their development, are described in Sec. V. Sections IV and V together thus answer our second research question. Section VI describes the pilot implementation of the program with six practicing teachers, and qualitative analysis of teachers' responses to the design and the findings of our analysis, thereby addressing the third research question. Section VII discusses how the modules were redesigned based on the study findings. We conclude with some implications of this redesign, and ongoing and future design research that extends this work.

II. OVERVIEW OF COMPUTATIONAL MODELING AND RELATED PEDAGOGICAL APPROACHES

In our view, mathematical modeling of a physical phenomenon broadly involves systematically building a quantitative framework that can (a) *enact* the behavior of the system it seeks to predict or control, and (b) support measurements that can test the model. Note that in this enactive view, physics models and their constituent representations are treated as active ("acting out") entities. This view is different from—but includes—the dominant representational conception of models, which treats models as static entities that "stand in" for the world [15]. In the enactive view, the process of building a model involves systematically "loading" dynamic aspects of the real world into a symbolic structure, which can replicate this dynamics, through changes in quantities [16]. To explore different transformations of the modeled system systematically, the enactive nature of the final symbolic structure can be "switched off," and the components of the model adapted and manipulated "offline," as if the model and its elements are static entities. Calculus-based methods were one of the first approaches to support such systematic "loading" and tracking of dynamics, as well as systematic transformations of model components. These methods have thus historically driven enactive modeling in physics.

Computational models also support such symbol-based enactment of target physical systems, and the tracking of their evolution in time and space [17]. The major difference between CM and classical mathematical modeling is that instead of relying on continuous functions and analytical procedures, CMs use discrete entities and numerical computations. The latter is required because of the way digital computers work—in a step-by-step and repetitive fashion. This difference however allows for greater flexibility and scope for modeling. In particular, it helps overcome some of the constraints set by analytical procedures, such as linearity. Also, as CMs allow connecting partial models together using logical operators, CM can be used to systematically capture very complex properties. These include time or phase-dependent modulations (say chemical reactions that are activated only at some time points, based on concentration gradients), patchy spatial features that interact with the system intermittently (say cloud patterns in a weather model), and feedback loops across levels. As an example, models of atherosclerosis requires such interlinking of parts that are themselves complex and have feedback loops. These parts include turbulent blood flow in heart valves, the subsequent mechanotransduction process in endothelial cells, the resulting biochemical responses that build up plaque in heart valves, and the further turbulence in blood flow generated by the plaque buildup.

CM approaches like agent-based modeling (ABM) allows individual physical entities (like molecules) to be modeled as autonomous agents, which can then be programmed to interact, based on simple rules. ABM is used to understand how the collective behavior of different interacting microlevel entities lead to system-level behavior. This approach allows integration and enactment of nonlinear aspects at many different levels [18], and modeling of systems with highly variable individual components. A significant trade-off in building such complex models is that they typically capture only specific cases, and generalizations across many cases is difficult. Also, predicting the state of a system (say a hurricane) at a given time point (say in 36 hours) cannot be done quickly, as the model needs to "run" (often for many hours, particularly for complex systems like the weather), so that the whole model evolves across states to reach—and enact—that time point. Such models also require high levels of optimization of computing resources.

As the above brief outline of CM shows, this approach can connect together disparate states, and feedback across many interacting levels, to enact in detail the evolution of many-layered systems—as a whole—across time. This rich capacity has led to widespread application of CM in science, particularly engineering sciences, to support complex interdisciplinary investigations. Given the wide and novel application possibilities of CM, there have been many policy efforts to integrate this modeling approach into physics education. For instance, The American Association of Physics Teachers has proposed that computational physics can be considered as a third way of doing physics, complementing the traditional theoretical and experimental ways [19,20]. To further the adoption of computational modeling as part of instruction, the Association has created new partnerships, such as the Integration of Computation into Undergraduate Physics [20].

Parallel to such policy-level efforts, many curricular-level research projects have studied ways to incorporate computational problem solving as part of introductory physics courses [11,21–24]. Chonacky and Winch have proposed a broad set of guidelines and recommendations to integrate computation into the undergraduate curriculum [25]. These include treating numerical solution approaches at par with analytic approaches in physics, introducing scientific software tools, teaching computing platforms and important algorithms as part of physics courses, and illustrating the modeling of complex and interdisciplinary systems. They also stress the importance of initiating and acculturating instructors to computational modeling and thinking, by creating supportive social networks for faculty. Another initiative to help teachers integrate computational modeling in their classrooms is "Computational Modeling in Physics First with Bootstrap" [26]. There have also been efforts at the high school level to couple computational thinking and problem solving with modeling [27].

To understand the implementation-level issues related to new educational approaches, many studies have looked at teachers' beliefs, perceptions, and attitudes towards integration of science, technology, engineering, and mathematics (widely referred to as STEM integration) in the context of TPD programs [28–30]. Focusing specifically on the integration of computing and science, some studies have investigated computation as a pedagogic tool, while others have examined computers as a technological artifact. Kong and Lai developed a framework based on TPACK (technological pedagogical and content knowledge) for designing teacher professional development programs that implement computational thinking [31]. Vasconcelos and Kim report that a program that involved coding related to scientific modeling had a positive impact on preservice teachers' epistemological understanding of scientific modeling and computer science concepts [32]. Langbeheim *et al.* found that even a short experience of computational modeling positively affected teachers' self efficacy in integrating CM into physics teaching [9].

A key finding from such studies is that teachers' positive beliefs, attitudes and comfort level play a crucial role in the integration of computers as an instructional tool in classrooms [33]. In particular, teachers' ideas about the nature of physics and physics learning play a critical role in their receptivity to curricular designs that incorporate computational modeling. Prior studies have investigated instructor perceptions, attitudes and self efficacy with regard to CM [9–11]. Vieyra and Himmelsbach explored how secondary teachers went about conceptualizing the integration of CM with physics [10]. It was found that some teachers envisaged the role of CM as facilitating an in-depth engagement with concepts in physics, while others considered CM as a means to develop skills that span multiple disciplines. This study led to a conceptual framework to assess boundary-stretching attitudes among secondary physics teachers, and how these mental states indicated the likelihood of integrating CM in their courses. A related research study investigated the factors affecting attitudes and self-efficacy of teachers in relation to the integration of CM in physics at the high school level [9]. Results showed that these mental states were positively impacted by a workshop that integrated CM with experimental measurements in physics. Similarly, Pawlak *et al.*, studied different ways in which computation was perceived by learning assistants (LAs), while teaching computational problems as part of an introductory level course in physics [11]. The authors identified four pedagogical categories, depending on whether the focus of the LAs was on (i) programming, (ii) learning physics with computation as a means, (iii) treating computation as a tool for modeling and problem solving, or (iv) considering computational problems as an opportunity for shifting student perceptions of learning. Building on these studies, we developed a set of design principles for our TPD program. These are discussed in the next section.

III. THE DESIGN PRINCIPLES

The conceptualization and design of our TPD program was based on the following three central principles.

A. Computational modeling as a vehicle for interdisciplinary science education

Recent discussions at the interface between philosophy of science and cognitive science argue that new kinds of computational modeling lead to new kinds of scientific discovery practices [7,34–38]. The specific reasons for this expansion are (i) the ability of computational modeling to integrate disparate theories and datasets, and (ii) the new thinking and imagination possibilities enabled by such integration. This argument is extended by recent discussions related to the use of agent-based modeling in science, technology, and mathematics education [39,40]. These analyses suggest that emerging interdisciplinary discovery practices and computational modeling are closely intertwined [41].

Parallel to this discussion in philosophy of science and cognitive or learning sciences, discussions in education have proposed modeling as a common theme underlying knowledge construction and sense making in science, cutting across natural sciences [42–49]. This underlying common structure (modeling) is currently not obvious to most science students and teachers, given the way textbooks package knowledge in a modular fashion, and classroom practices follow this module-based structure. Further, the close connections between the building of classical mathematical models and computational models are not well known. Developing an appreciation of modeling as common to all disciplines, and connecting this underlying structure to computational modeling, could thus help teachers and learners move to interdisciplinary thinking.

Integrating these two discussions in the literature (computational modeling leading to new interdisciplinary practices and discoveries, modeling as a common theme underlying sciences), we arrived at computational modeling as a natural vehicle for promoting interdisciplinary science practices, at the K–12 and undergraduate levels. However, CM can play this role only when it is understood as an extension of existing modeling practices, rather than as a novel and purely technical skill.

B. Gradualist approach

Our second design principle sought to augment existing curricula and teaching practices, particularly by compensating for their limitations. This approach is in contrast to models that require science teachers to learn and teach a new programming language or modeling practice such as agent-based modeling [18,22,23,39]. Both these approaches require wide student access to hardware such as laptops and tablets.

In developing country contexts, where textbooks are currently the only viable media format (as they are cheap and widely available), most students cannot afford laptops, tablets, and smartphones. This constraint suggests that any redesign of curricula for such resource limited contexts needs to start from textbook-based teaching and learning. A pedagogical design driven by this constraint is presented by Karnam *et al.*, where a simulation-based design (a touch-based interactive system to learn model-based reasoning using vectors) is connected to textbooks using QR codes, thus augmenting the textbook with interactive media [50]. This mixed media structure allowed teachers to smoothly extend their existing textbook-based teaching practices, to demonstrate (using their personal phones) the dynamics embedded in vector operations. Students could then try out these interactions, either using the teachers' phone, or borrowing a phone from someone in their community. This gradualist design approach allowed even resource-limited students to enter the simulation world without much investment, and develop a modeling based understanding of science. Such design approaches seek to create a continuum from existing media, content, and classroom practices. This design allows teachers to smoothly extend their accumulated experiences and practices, rather than learn a whole new set from scratch. Given the developing country context of our design, such a gradualist approach is a necessary requirement. Note that even in more resource-rich contexts, such a gradualist approach would allow both students and teachers to better understand and integrate new CM practices.

Applying this requirement to the objective of our workshop (enabling teachers to shift to CM), we needed to develop a method that allowed teachers to smoothly extend a large part of the content they currently teach, in a way that allow students to move easily to computational modeling.

C. Participatory design

To develop the content of the workshop, we interviewed three expert modelers (who were also teachers), based on their modeling papers, and elicited their core modeling practices. This understanding of computational modeling informed our design of the workshop modules. As part of this process, we also held discussion sessions on philosophy and cognitive science papers examining how discoveries emerged through model building in physics. These discussions helped the modelers understand the process of model building [15,51]. To extend and apply these ideas, the modelers also participated in the design of the teaching modules, as well as the design of teaching simulations that were developed as part of the workshop. One of the expert modelers also taught some of the workshop modules.

A second participatory component focused on teachers, who also contributed to the design. Once the first design iteration of the modules stabilized, we presented the modules to a cohort of six teachers interested in including computational modeling in their teaching. Based on an analysis of the feedback from four teachers (the other two only listened), we redesigned the modules. Apart from this feedback during the initial design, we built new teaching simulations, based on suggestions by teacher-participants in the final workshops.

IV. OPERATIONALIZATION OF THE DESIGN PRINCIPLES

To operationalize these three design considerations in an integrated fashion, we first did a process analysis of modeling practices that are present in existing curricula. Modeling practices are rarely discussed or enacted in Indian classrooms, particularly in ways that reveal the many model-building considerations involved in developing a scientific model. One objective of our analysis was to identify a candidate practice that could be extended smoothly to computational modeling, as such a smooth extension was needed to operationalize our gradualist design approach.

Based on this analysis and related discussions, we identified physics derivations as a candidate modeling practice, as these could be extended towards computational modeling in ways that build on current teaching practices, content, and media. Derivations are a core component of physics instruction in India, and this modeling practice is learned by all science students [52]. This common structure thus allowed a smooth extension of existing classroom and teaching practices, as envisioned by our gradualist design approach.

However, derivation models are taught as a series of mathematical steps in Indian classrooms. To extend derivations smoothly towards computational modeling, these formal steps needed to be systematically unpacked, in two ways: one, to reveal how derivations are built up from observations and data from the real world (model building), and two, to reveal when numerical solutions are needed, how they work, and how they augment classical modeling approaches. In the workshop design, these two steps were implemented using interactive simulations. In the discussion below, we outline some theoretical justifications for these two steps, based on the following two themes:

Role of model building in derivations.—In this discussion, we present the processes of model building and model-based reasoning that led to influential derivation models, such as Maxwell's equations and the Carnot engine. These cases illustrate the necessity of model building while solving open-ended problems.

Numerical solution approaches as boundary-crossing spaces.—In this discussion, we argue that numerical solutions are natural extensions of derivation models, made possible by newer calculation technologies. As discussed in the brief review of CM in the introduction, CMs allow modeling of complex interdisciplinary problems—such as climate change, pandemics, and biomedical solutions—from a physics perspective, using modeling tools from physics, such as equations. Numerical solutions thus work as boundary-crossing spaces.

A. Role of model building in derivations

Physics derivations taught in classrooms are the final products of extensive model building by scientists [15,51,53]. However, derivations are often taught in physics classrooms as lengthy chains of mathematical operations, without any mention of the extended model-building processes that allowed scientists to arrive at these formal systems. These building processes include analogical thinking and building of spatial models, as well as complex reasoning based on both these elements (model-based reasoning). For instance, Maxwell's equations are taught in classrooms as a series of mathematical operations starting from other basic equations, even though Maxwell arrived at the equations of electromagnetism through a series of complex reasoning processes, based on analogy, model building, and model-based reasoning [51,53]. Interestingly, these reasoning processes are outlined in detail in Maxwell's notebooks, but they are not part of his final mathematical model.

Pickering discusses a similar "vanishing act" in experimental physics, where final interpretative accounts (similar to the final mathematical model) are constructed [54].

These accounts minimize the role played by specific experimental and instrumental setups, and their performances, in generating phenomena such as the Hall effect, which "can only be embodied in such devices." According to Pickering, "representational chains in science visibly terminate in machines and instruments," which means experimental results could be understood as tied to—and thus specific to—the "machinic" (i.e., experimental and instrumental) setups, and not applicable to the real world. The interpretative accounts, constructed after an experimental result is obtained, help overcome this lack of ready correspondence between the experimental system and the real world. In this view, the final account, which helps "vanish" the instrumental setup, is required in all experimental science, to support the claim that the results, while based on the specific experimental model system, correspond to aspects of the real world.

In the case of theoretical models such as Maxwell's equations, the components that vanish are the extensive analogical, model building, and reasoning operations, which the scientist developed to arrive at the model. Following Pickering's reasoning, this vanishing process is a required step—to make the claim that entities in the theoretical model correspond to actual aspects of the phenomena in the world, and not just to elements of the modeler's imagination (such as the vortices and idle wheels imagined by Maxwell while developing the model of electromagnetism). Interestingly, this view also accounts for the preference for analytical, closed form, solutions to equations, as their general nature supports the argument for a tighter correspondence relation between the model and the world. In this view, the process of "wiping" cognitive and material processes involved in model building, and constructing an overarching—and often purely mathematical—final account of theoretical models, is a core component of science practice.

As the derivations taught in classrooms only use the final "wiped" (and succinct) mathematical rendition of the model, learners are not able to see the way scientists develop this complex argument structure, to support model-world correspondence. Learners thus do not understand and appreciate this core practice, which leads to the "achievement of correspondence" [7]. Apart from this lack of understanding of model building, the exclusive use of succinct final models also leads to learners understanding derivations as just a set of mathematical procedures, whose relationship to actual phenomena in the world is not clear.

This limited view of derivation models significantly limits learners' ability to build on and extend their understanding from the study of derived models, particularly to open-ended problems. Studies show that students cannot solve open-ended problems even after solving two thousand word problems based on the derived model [55]. This is because open-ended problems, especially interdisciplinary problems, require high levels of model-building, where the student needs to build up systematic relationships between real-world structures and mathematical structures, in ways that lead to the world being "loaded" into mathematics [16]. Solving open-ended problems also requires generating new structures in the imagination, and using these imagined structures to build mathematical models. This means learners need to understand the ways in which scientists integrate imagination and reasoning processes (such as analogy and model building) with mathematical reasoning.

The above analysis suggests that to use derivations as a stepping stone to CM, the model-building "moves" in derivations, and the way these are integrated—and eventually subsumed—by mathematics, need to be made explicit. This is because the current purely mathematical understanding of derivations does not extend well to solving open-ended problems, which require analogical thinking and model-building processes. Since these cognitive processes are key to the "loading" of the real world into mathematical models, the same processes are deployed while building computational models, particularly when solving complex interdisciplinary problems.

The curricular focus on the final formal model, which presents modeling as starting and ending with mathematics, implicitly follows currently dominant science and philosophy of science discussions, which mostly do not consider the role of model-building and reasoning processes in the development of theoretical models. To address this pedagogical issue, we have developed interactive derivation systems that make explicit the process of model building. As an illustrative example, consider the system we developed for the derivation of the wave equation [56]. The derivation is presented as a process of systematically loading features of reality—particularly dynamics—into mathematics, through a series of modeling moves such as idealization, discretization, etc.

B. Numerical solution approaches as boundary-crossing spaces

A major component of the mathematical approach to learning derivation models is the development of analytical (closed-form) solutions to equations. This approach to solving equations is overemphasized in current teaching practices, even though (i) numerical solution approaches are possible in all the derivation cases discussed in the textbooks, and (ii) most contemporary modeling practices are based on numerical solutions. As discussed above, this (curricular and disciplinary) preference could be based on the generality of closed-form solutions, which better support model-world correspondence claims. Apart from this reason, it is possible that analytical (closed-form) solutions were emphasized by classical first-principles-based approaches because of a purely practical problem—the manual calculation of hundreds of numerical steps using paper and pencil, and finding the hidden patterns in the resulting numbers was very difficult and cumbersome [57].

New computational technologies opened up the possibility of performing such complex and extended mathematical operations very quickly. This, in turn, allowed building of complex numerical models that could approximate the complexities of the real world. In the classroom, the emphasis on analytical solutions to equations blocks students from accessing this rich modeling space opened up by computers. To address this issue, when teaching derivation models, numerical solution approaches to solving their equations could also be taught, along with analytical solutions when available. Such a dual structure would bring the derivation modeling practice closer to CM, and also contemporary science practices. This dual approach would also emphasize the continuity between analytical and numerical solutions.

Interestingly, numerical solution approaches also work as a key boundary-crossing space for developing interdisciplinary science practices. This is because numerical solution approaches allow different types of rules to be connected together, using logical operators. This structure is conducive to interconnecting theory and data from different levels (physical, chemical, biological, behavioral, etc.), and multiple disciplines, thus overcoming analysis constraints imposed by existing disciplinary perspectives, such as closed-form solutions. Computational modeling thus opens up new kinds of model-based reasoning, particularly suited to interdisciplinary problems [7,36]. This boundary-crossing feature makes numerical approaches a natural vehicle for the transition to interdisciplinary science education.

These two themes (role of model building in derivations, numerical approaches as boundary-crossing spaces) provided us with a good starting point to extend the practice of derivation modeling to computational modeling. Specifically, we decided to develop modules that made explicit the model-building moves embedded in derivation models, and also clearly showed the way the derivation process "loads" reality into mathematical symbols. Second, we decided to develop modules that showed how equations resulting from derivation models could be solved numerically, using software like Wolfram Alpha. Finally, we also decided to develop a series of "bridge" simulations, which allowed participants to (i) understand the continuity between derivation models and computational models, and (ii) follow the numerical solution process in a step-by-step way. Figure 1 provides an outline of the design of our TPD program.

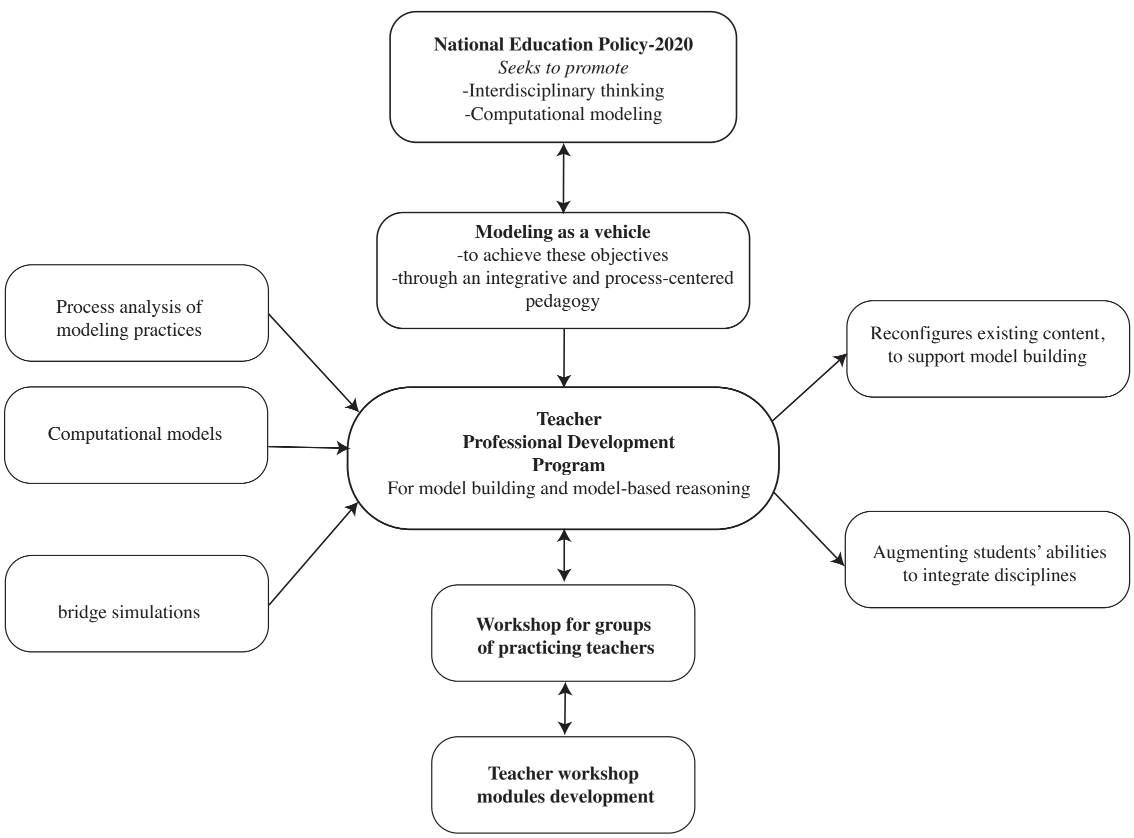

FIG. 1. A schematic of the design of our teacher professional development program and an outline of the key stages, components, and processes involved in this design.

C. Sequencing the transition

As computational modeling is a vast domain, and learning CM is a difficult process, we sequenced the workshop design in the following way. First, we decided to develop a sequence of 3 workshops: the first design, reported here, seeks to extend derivation modeling to computational modeling. This part can cater even to those who have access only to textbooks, as these modules rely only minimally on access to computers and laptops. The second workshop will focus on editing of code (to manipulate many types of existing computational models) and arrive at specific results. After this second TPD phase, which would help familiarize teachers with coding, the final workshop would involve actually building computational models.

Apart from this sequencing, we also decided to design a later session for data modeling approaches, such as machine learning, as these models are more complex, and have a different argument structure from standard simulation models. We plan to introduce this data-driven approach only after the three workshops focused on structure-based computational models. Figure 2 below outlines the design sequence. The next section outlines some of the key design elements of our teacher professional development program, centered around derivations.

V. FROM TEXTBOOK DRIVEN DERIVATIONS TO MODELING: TEACHER WORKSHOP AND THE DEVELOPMENT OF MODULES

A. Design elements of the workshop for teachers

Based on the focus on computational modeling and the gradualist approach, as well as discussions with expert modelers and teachers, we developed the following three design elements to implement the teacher workshops.

1. Reconceptualization of derivations as mathematical model building

Popular derivations at the higher secondary and undergraduate level were analyzed, to unpack and highlight the core model-building decisions, moves, and practices that went into their construction. This analysis led to a schema that illustrated how a physical phenomenon or a process gets turned into a mathematical model, through a series of interconnected steps. These steps embed, and keep "alive," the dynamic process in the world that the enactive final model seeks to replicate. This understanding could help teachers reconceptualize derivations as mathematical model building, and thereby achieve a wider perspective on modeling, which can cover most derivations.

2. Bridge simulations

A set of teaching simulation systems were designed to help teachers integrate derivation models with computational models. These publicly available systems turn textbook-based derivation models into fully manipulable interactive systems, interconnecting physical phenomena (such as oscillation), their equations, and their graphs [61]. One of the systems illustrates the way analytical solutions make way for numerical ones, as the complexity of the modeled phenomena increases from a simple harmonic oscillator to a piecewise oscillator. Textbooks that embed these simulations using QR codes would allow students to smoothly move from reading to actively manipulation (and enaction) of different types of formal systems, with many levels of complexity of oscillation. This process would allow learners to appreciate the dynamic nature of formal models, and also the continuity between equation models and CMs. These systems would also help learners integrate the multiple representations used in science learning and discovery, in a coherent way.

3. An integrative pedagogical framework

To extend derivation models smoothly towards computational models, we developed a new teaching narrative that connected: the model-building analysis of derivations, bridge simulations, and modeling tasks. This narrative allowed teachers to extend these elements to CM. The narrative was based on a standard textbook topic (oscillation), and captured the systematic evolution of mathematical model building, moving from simple derivation systems (solved analytically) to complex systems (analytical solution not possible).

Netlogo simulations, of problems like virus transmission, were introduced at the end, to show the new modeling possibilities opened up by CM, and also to illustrate how such simulations build on equations. These systems also illustrated the real-world and societal relevance of computational modeling and thinking, and highlighted the way such models help understand complex contemporary problems in an integrated way.

The next subsection discusses in detail the way we developed the TPD modules, to meet the learning objectives of the workshop.

B. The design and development of the modules

The design of the modules was informed by insights obtained from an extensive reading (review) of the literature related to modeling. This included papers from science education research, philosophy of science, and cognitive science [15,16,42–46,48,51,53]. As discussed above, the modules were structured around derivations, to meet the dual requirement of (i) centering the discussion around modeling moves, and (ii) developing a design that extended existing teaching practices to CM.

1. Module 1: Introduction to the objectives of NEP 2020

This module introduced teachers to key goals and objectives of the National Education Policy (NEP) 2020,

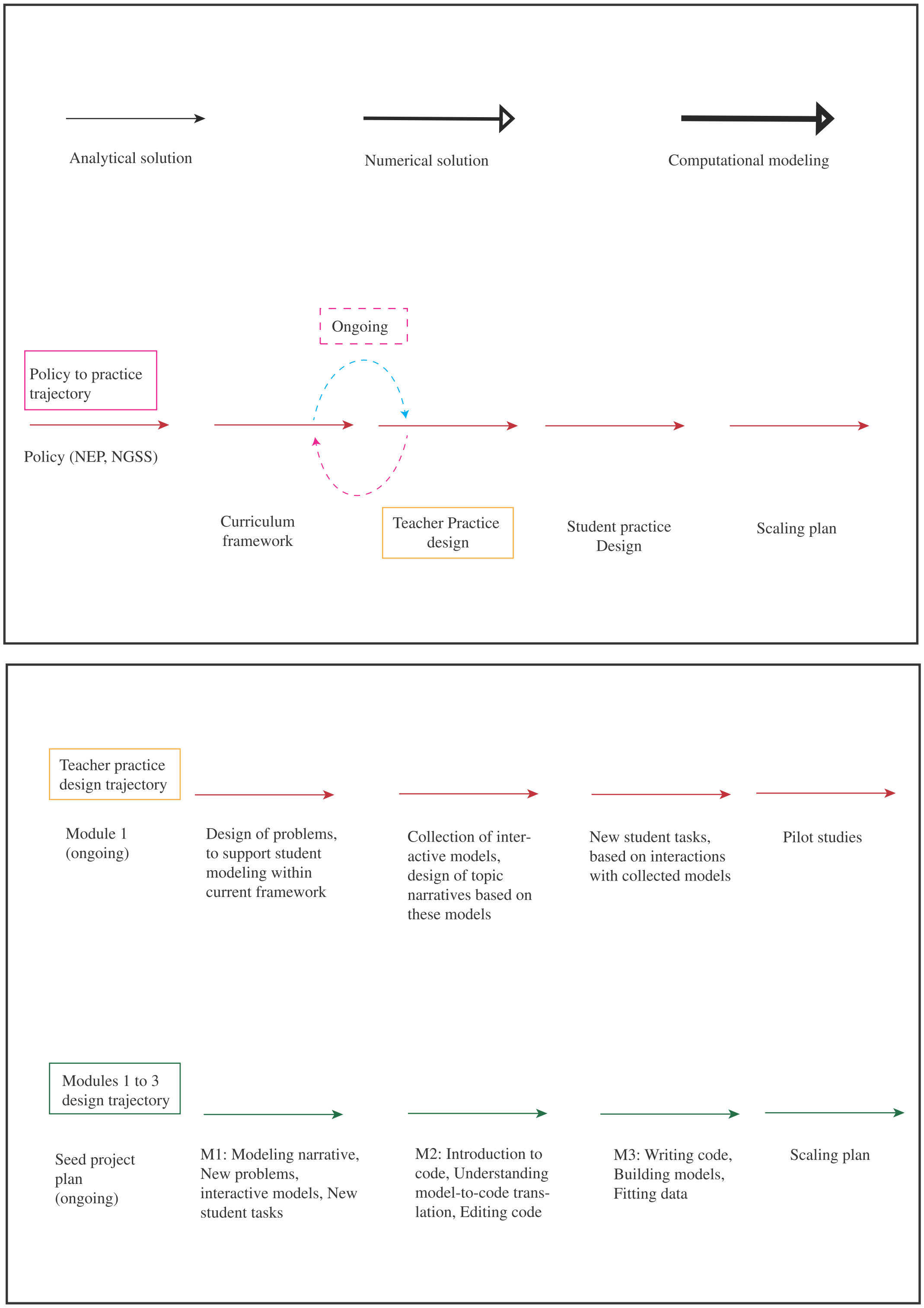

FIG. 2. Outline of the projected design sequence of the workshop for teachers.

namely transitioning to interdisciplinary thinking and computational modeling [8]. The importance of these objectives—particularly to advance research at the cutting edge of scientific frontiers, and their significance in tackling complex societal problems like climate change, pandemics etc.—were emphasized. The module highlighted one of the key aims of the workshop—to reenvision physics education with an emphasis on modeling. It then laid down a road-map, based on: recasting the pedagogy of derivations as an activity in mathematical modeling, and connecting derivations to other novel approaches to model building, like simulations and computational thinking. The close connections between computational and interdisciplinary thinking was discussed.

To capture and sustain the attention of practicing teachers, laying out the road map to a new curriculum was not enough. In addition, we needed to highlight how the workshop would benefit their day-to-day physics teaching. Taking this aspect into account, the module addressed how the new perspective on derivation (provided by the workshop) would help students organize physics knowledge in a more structured fashion. Understanding derivation as an activity in mathematical modeling would provide a unifying theme, which could help students understand most derivations as following a common model-building structure.

2. Module 2: Derivation and problem solving as mathematical modeling activities—A practitioner's perspective

The new perspective on derivations we have developed, where they are understood as an activity in mathematical modeling, emerged from discussions in philosophy of science and cognitive science. However the teacher participants in the workshop would have no background in these disciplines. To initiate the modeling discussion in a way that is aligned to teachers' day-to-day practice and discourse, we introduced the approach using the context of a familiar textbook problem—a bullet fired from a helicopter; students needed to calculate the time (t) at which the bullet will hit a target on the ground—initial velocity (u) of the bullet and vertical height (s) of the helicopter from the ground is given. The conventional approach to solving this problem is by invoking the equation $s = ut + \frac{1}{2}at^2$ and plugging in numbers that are given. A critique of this approach was the first discussion point of the module, which was led by a teacher who is also a modeler. The teacher stated a modified, and relatively open ended version of the problem, which did not specify the initial velocity of the bullet and height of the helicopter. He then asked the audience to state all the possible factors that could affect the motion of the bullet, and then listed them. The suggested factors included gravity, air resistance, weather, influence from the helicopter fan, the possibility of the bullet losing mass, among others. Multiple models and associated differential equations were constructed, by including and omitting the above mentioned factors. For example, one model considered air resistance, gravity, and variation of mass of the bullet, while another excluded all the three factors. By discussing multiple models like these, he illustrated that the problem can be tackled in different ways (depending on the precision and accuracy needed), and highlighted related trade-offs. The module sought to provide a general sense of what it means to build a model of a phenomena, and the role of judicious decisions on what to include, omit, etc., in the model. The discussion made it clear that the typical model involving the equation $s = ut + \frac{1}{2}at^2$ discussed in textbooks has no special status, and it is just one possible way to build a model, out of many.

Another key issue highlighted was the idealization tradeoff, where getting an equation that is analytically solvable becomes very unlikely as problems become more complex, especially when they required moving closer to the real world. The modeler also illustrated the wide applicability of numerical methods, and the limited application of analytical solutions. The availability of easy-to-use mathematical packages to solve equations (like WolframAlpha) was also highlighted. The emphasis on the wider applicability of numerical methods sought to dislodge the (commonly perceived) sacrosanct nature of analytical solutions, and thereby pave the way for computational approaches.

3. Module 3: Deconstruction of the derivation process from a modeling perspective

This module sought to provide a detailed account of the key steps involved in the process of building a mathematical model of a physical phenomenon. The process of derivation was deconstructed, to show the following 4 general steps: Physical phenomena → Structural diagram or schematic → Geometrical model → Algebraic model (see Fig. 3). The clear and tight connections between these steps were shown clearly, highlighting how they together form a funneling process, leading up to the succinct final equation. The derivation of the equation of motion of a simple pendulum was used as the context to illustrate this process. Towards the end of the module, the same process was shown in the case of motion of an object on an inclined plane, to emphasize that the four-step structure underlies not just the pendulum derivation, but many other derivations as well. The details involved in the 4 steps are discussed below, using the simple pendulum as an example:

Physical phenomena: There are innumerable physical situations around us involving oscillation, such as the ones illustrated in panel 1 of Fig. 3. The interesting movement or change properties underlying them, such as periodicity and isochronism, are the starting points of a characterization of the real-world phenomena, which leads up to the model. These properties are the referents for the variables in the equation, which is obtained eventually. Once the final equation is built, it can be considered as capable of acting out the dynamical (movement or change) features of the physical phenomena (which the equation is also considered to "represent," when the equation is treated as a static entity on paper).

Structural diagram or schematic: Moving from the physical phenomena to a schematic is the first major transition in our characterization. The schematic serves as a stand-in for the multitude of oscillating phenomena in the real world. This generalization develops through the methodology of idealization, leading to the schematic.

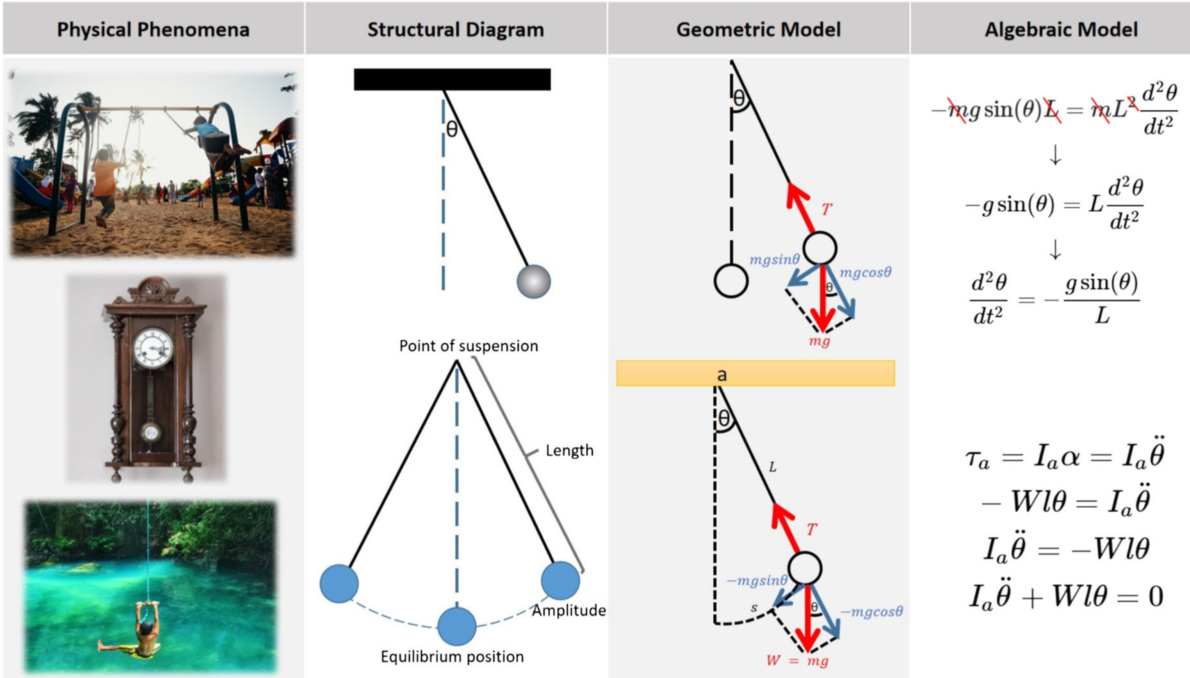

FIG. 3. Four general steps underlying many derivations in physics. The derivation of the equation of motion of a simple pendulum (analyzed in the lab frame) is deconstructed along the four steps, as an illustrative example.

The module discusses two key features of the schematic: (i) the way it dissociates and decouples the system's behavior from specific contexts as well as context-specific physical and perceptual experiences and related patterns, and (ii) the way it reveals and brings together previously obscured and scattered information about different systems, in a salient and common form. This process makes this information available for new cognitive operations, and novel manipulations.

Geometric model: In this step, the schematic is embedded in a coordinate grid. The structure and components of the schematic are chosen in such a way that it can be easily subsumed under existing mathematical concepts and variables. The causal factors affecting the motion of the pendulum bob are then identified and ordered as primary and secondary factors. Gravity, tension are considered as primary, and viscous force, buoyancy, friction among secondary factors, as the latter could be excluded in a simpler model. The coordinate grid facilitates the representation and manipulation of the causal factors in terms of vectors. Once all the details of the system are made available in the geometric model, then the transition to the fourth and final step becomes possible.

Algebraic model: Newton's second law plays a crucial role in the transition from geometrical model to algebraic model, as the details of the geometric pendulum system are fed into the differential equation that follows from the second law. The resulting equation is rearranged to form the differential equation for the motion of the simple pendulum. Since the equation is not easily solvable, the small angle approximation is invoked, to facilitate an analytic solution. The module discusses the ubiquitous role of such approximations in physics derivations, and the limitations imposed by them on the solutions.

After explaining this four-step characterization of derivations, the module compares and contrasts a pedagogy based on the four-step process with the traditional approach of teaching derivations. In the traditional approach, teaching of derivations emphasizes the mathematical procedures and manipulations involved, focusing on a specific content topic. Derivations are presented as straightforward applications of theory, rather than as building processes involving complex judgements. In such a mathematics-centered approach, what is valued is the formal knowledge elements, such as pieces that are carried over from previous derivations. These act as problem solution templates, but only when the given problem is structured similarly to previous derivations (i.e., not open-ended). In our approach, what is valued is the imagination and building processes, which are not specific to a content topic. These processes involve understanding the four-step process elements, and using them to model any given problem, including open-ended problems, across different content topics.

The mathematics-centered framing shapes current teaching approaches in India, as the emphasis is on working with and using topic-specific formal models (in mechanics, optics, etc.), which are treated as templates for solving well-structured problems. This leads to the whole pedagogical approach being focused on well-structured problems, and organized in a highly modular and additive fashion. In contrast, we advocate teaching derivations using a building approach, which can scaffold the solving of open-ended problems, across topics. We emphasize the internal logic of building a derivation, showing how this process follows a funneling structure, from the physical world to a symbol-based structure (equation). This funneling process involves key transformations, such as idealization, approximation etc., as illustrated by the four-step model. This is a more grounded approach, where the learner can start with data or questions, and when faced with the problem of predicting or explaining the behavior of an unknown system, try to solve it by engaging in the process of model building. This process may invoke existing theories if available, or it may require coming up with a model from the data, to eventually arrive at a generalization. We believe this characterization allows students to appreciate the role of imagination in modeling, the different modeling decisions that are taken, and the deep structure underlying the different reasoning practices involved in scientific modeling. The focus on building also helps learners become active participants in the derivation modeling process.

Module 3 ends with a bridging simulation (see Fig. 4) that allows manipulating the schematic, equation, and a graph of a simple pendulum. Interacting with this system facilitates two learning objectives: (a) Understanding the coherence relation between the three different representations of the pendulum, and the way they are interconnected; (b) understanding the dynamic nature of the equation and the graph, and how they "act out" the behavior of the pendulum. Teachers are introduced to these learning objectives of the bridge simulation. They are then asked to manipulate the system and try out different system states and scenarios.

4. Module 4: On the limitations of analytical solutions and the generality of numerical methods

This module builds on some of the threads introduced in module 2, particularly to expand ideas related to the generality and wider applicability of numerical methods. The point that analytical solutions cannot be found for most real-world problems is emphasized [12]. Different aspects of the thinking underlying numerical methods are highlighted using examples. These are then contrasted with analytical methods for solving equations. For example, the solution to a quadratic equation obtained for the bullet problem is solved under the pretense of not knowing the quadratic formula. Different values of time are inserted in the equation, to approach the value of the solution. The idea that the process can be halted or continued, depending on the desired accuracy, is emphasized. The module then unpacks the logic underlying more complex forms of numerical solutions, such as Newton's method. The discussion focuses on the following reasoning features related to analytical solutions and numerical methods: (a) Analytical mode involves algebraic thinking and symbol manipulation, while numerical solutions involve thinking in terms of numbers and computation of numbers. (b) The output of an analytical solution is a function, while the output for numerical methods is a discrete set of numbers, obtained sequentially. Functions can be thought of as one way to organize these numbers as a general pattern, and graphs, visualization, etc., are other ways.

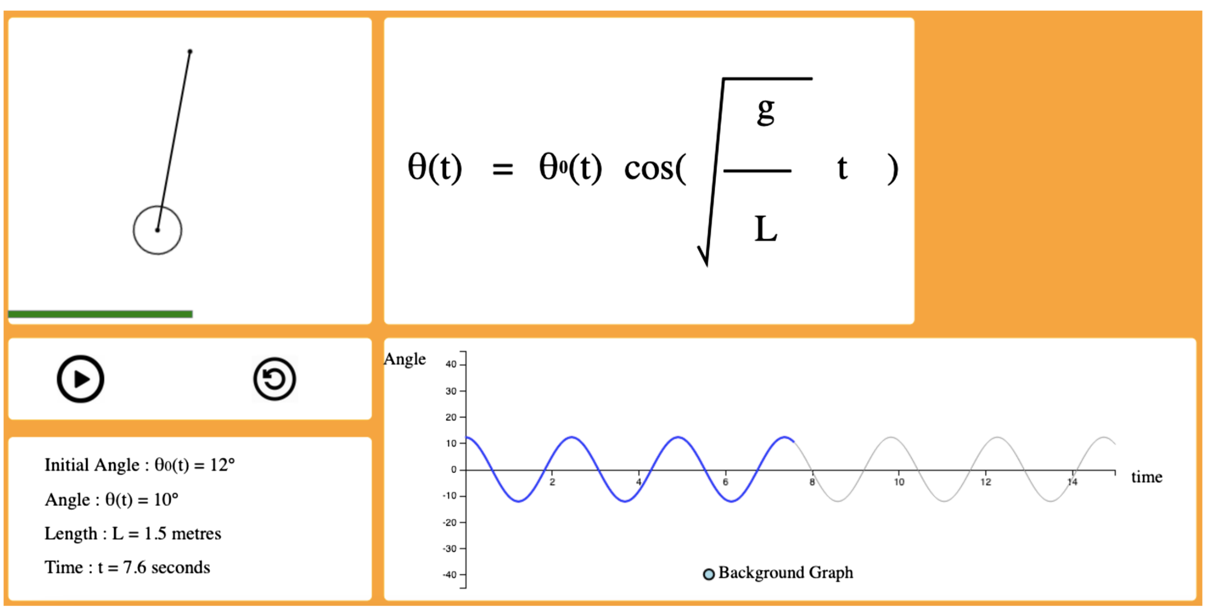

FIG. 4. A snapshot of the bridge simulation based on the simple pendulum (for access, see, Pendulum Ref. [61]). The schematic and the equation can be manipulated interactively, and the simultaneous changes in the graph can be observed.

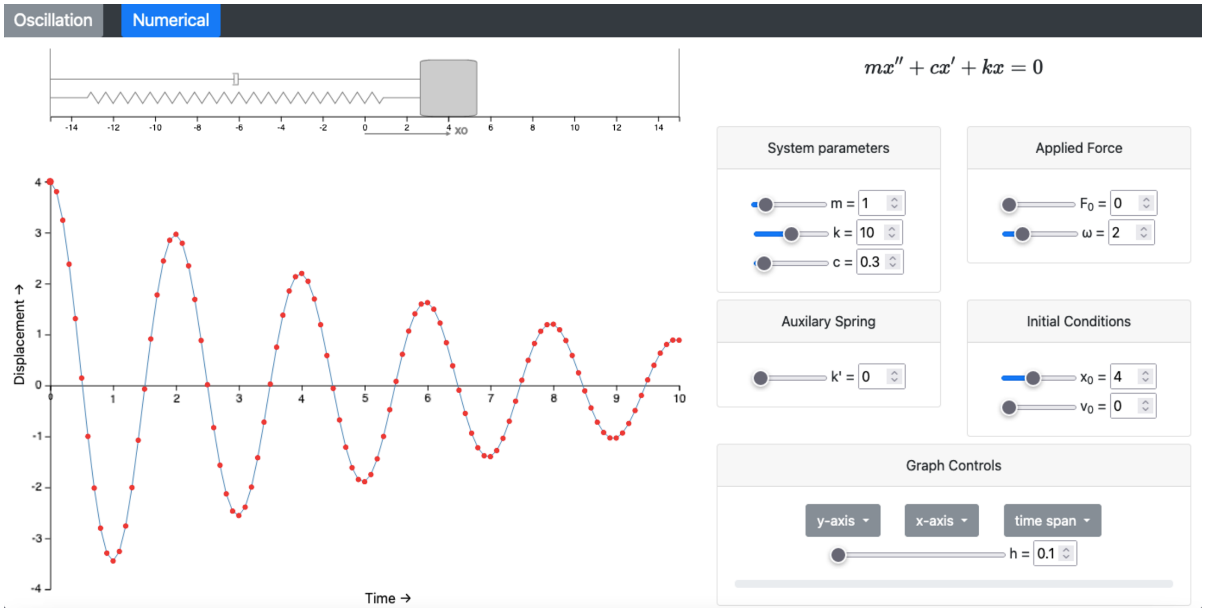

FIG. 5. A snapshot of the bridge simulation based on the spring-oscillator system (to access the system, please see piecewise oscillator [61]). The parameters can be manipulated interactively, to study the motion of damped, forced, and piecewise oscillators. The simulation has a tab that illustrates in detail the processes involved in the numerical solutions of differential equations.

This module leverages the close connection between computational modeling and numerical methods, to create a segue into computational thinking, starting from derivations. It emphasizes the point that the lack of analytical solutions does not limit our ability to solve problems, particularly given computational tools like *Mathematica*, which allow one to numerically solve complex equations quickly. This feature makes it possible to build models that tackle complex and open-ended problems close to the real world. The pedagogical emphasis is that teachers should focus on helping students to systematically formulate a problem, by engaging in the process of mathematical model building, rather than investing time disproportionately on the less crucial issue of finding analytical solutions to equations.

5. Module 5: Modeling oscillators—From simple harmonic to piecewise oscillator

This final module recaps the key arguments from the other modules. The module starts with modeling the oscillation of a spring, along the four-step characterization discussed in module 3. The discussion shows how the 4-step framework holds even when the complexity increases from the simple case to the one involving damping, and then an external driving force. An additional layer of complexity is then brought in, by turning the spring system into a piece-wise oscillator, by adding a second spring. The equation for the piecewise oscillator is not analytically solvable, and leads to discussion of numerical methods. A bridge simulation software we have developed, capturing the three-tier spring system (with damping, external force, extra spring) is then introduced (see Fig. 5). Teachers are asked to work with this system, to generate scenarios. The software can be used in three ways: (i) as an interactive tool to visualize and understand the complex set of graphs generated by different combinations of the variables of the spring system (ii) to help students appreciate the interconnected relations between the different representations of the spring system (schematic, equations, and graphs), and also the dynamic nature of equations, and (iii) to understand in detail the calculation process involved in numerical solutions. This process is presented in a different tab in the system.

The module also discusses the nature of the code underlying the software for the spring system, emphasizing the fact that it is based on a numerical solution. The connection between numerical methods and computational models is thus once again emphasized. This context is then leveraged, to briefly discuss and demonstrate a Netlogo simulation on virus transmission, showing how equations are augmented with space-time parameters, to model real-world virus transmission scenarios. The module ends by outlining the possibility of creating many such pedagogical transitions from physics derivations to interdisciplinary models, similar to the virus transmission model. The intermediary role played by the transition from analytical methods to numerical solutions, which then segues into computational modeling and thinking, is highlighted. The enabling role played by the bridge simulations, particularly to help students understand the relationship between derivation models, numerical solutions, and computational models, is also discussed.

VI. PILOT IMPLEMENTATION AND ANALYSIS OF DATA

A. Objectives and framework

The modules 1–5 discussed in Sec. V. B were presented to a group of 6 undergraduate physics teachers. Their academic interests spanned nanomaterials, polymers, and quantum physics. The objective was to seek their opinion and feedback on the modules, as practicing teachers. We invited ten teachers with known active interest in improving physics education. Out of these ten, six teachers responded, who constitute our sample. Five of the teachers had a Ph.D. in physics, and one teacher had a master's degree in physics. They had teaching experience in the range of 5–20 years. Four teachers taught in regional colleges (similar to community colleges in the U.S., primarily focusing on teaching) while the other two taught in a university, engaging in both teaching and research. Because of COVID restrictions, meetings were conducted online. We met each week for around 2 h, for 4 consecutive weeks. These meetings were recorded and later transcribed for analysis. The objective of the data analysis was to systematically capture the opinions expressed by the teachers (though $N = 6$, only 4 actively participated in the discussion), and the way these opinions changed over time, in response to the researchers' proposed design and related discussion. Specifically, we wanted to capture and characterize the dynamic interplay between the proposed design, researchers' views, and teachers' opinions, in a way that could provide actionable directions for the redesign of the workshop modules. Towards this end, we decided to adopt a qualitative approach to the analysis of the transcribed data (see Table II in the Appendix).

The first part of the analysis involved a thorough reading of the data transcripts. Since the purpose of the discussions with teachers was to seek feedback on the design of a novel pedagogical approach, we focused on the utterances that dealt with teachers' ideas about teaching and the nature of physics. These were found to be complex, due in large part to the deep entanglement between two kinds of teachers' views: about physics teaching and learning, and the nature of (physics) knowledge and knowing (excerpt 1B in Table III, in the Appendix exemplifies this). As a change in the latter was part of the workshop design (through the focus on numerical solutions and CM), the data analysis could not treat teacher feedback as fixed views about a specific topic (e.g., computational modeling) or module. We had to consider the teachers' comments as part of a *meaning-making process*. In this view, meaning is not fully predetermined, or fixed in the minds of the discussants before or during the interactions. Instead, meaning is considered as embedded within the sociocultural environment and related interactions [62,63]. Meaning is considered co-created, through agreements, contestations, and negotiations between the discussants (see Fig. 6).

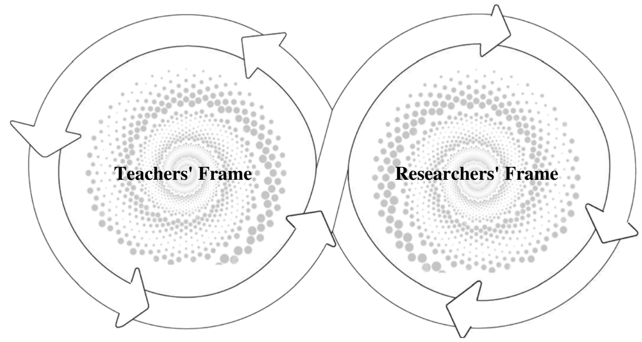

FIG. 6. An initial (pre-analysis) schematic representation of the dynamics of the meaning making process. Further analysis was conducted to reveal the specific dynamics of the meaning-making process, and to determine its implications for the redesign of the modules.

In our case, the meaning was constructed by the participants (teachers) in real-time, as they assimilated the content of the discussion into their existing knowledge structures and experiences, and also while they attempted to articulate their responses to the ongoing discussion. In parallel, researchers' interpretation of participants' responses to the pedagogic modules was based on their own intellectual and experiential repertoires. In this analysis approach, participants and researchers were equal contributors to the meaning-making process. Note that we use "participants" and "teachers" interchangeably in the context of this data analysis.

To understand the detailed nature of this dynamic process, and how it could inform the redesign, we decided to study the dynamics of the discussants' epistemological framing of the modules, and the pedagogical approach therein [67]. Consequently, we decided to base our analytical framework on a theory of personal epistemology (the views that an individual holds about the nature of knowledge and knowing). After a review of some of the accounts of this construct, we settled on the cognitive resources framework of personal epistemologies (PE) by Hammer and Elby [65,66,68,69]. This framework was chosen because it allowed us to capture and characterize the context-induced variability in participants' reasoning, about the nature and content of their disciplinary knowledge. Further, it allowed an integration of participants' views about pedagogy and personal epistemology [70]. As these features were well-aligned with the properties of the dataset and the redesign objective, the PE framework was judged the most appropriate for our analysis.

Also, from a methodological standpoint, we found the following 3 assumptions of the framework quite useful:

- Personal epistemologies inform and motivate all of knowing and reasoning.
- Personal epistemologies function as context-sensitive cognitive resources.

- Personal epistemologies combine to enact epistemic framing, which manifests as stable beliefs and/or theories about an event, object, or phenomenon.

We used these assumptions to develop 3 key analysis questions:

- What must be presumed about pedagogical practices and science cognition, to account for the teachers' stance towards teaching derivations as mathematical modeling?
- How do personal epistemologies influence the interpretation of the proposed modules?
- What are the implications of researchers' and participants' (epistemologically informed) interpretations of the proposed modules, particularly for module redesign?

Based on these questions, we devised an incremental approach, to systematically address the questions (see Fig. 7 for an overview of the method). See Table II in the Appendix for the steps of the method and their rationale.

As the design project was highly interdisciplinary, different aspects of the project (theories of modeling, numerical solutions, interactive simulations, workshop narrative, data collection, data analysis, etc.) were led by different researchers, with interests and abilities related to these components. The analysis component was led by a researcher with significant experience in qualitative analysis. As every member of the team did not have this expertise, consensus on the analysis procedures and categories were achieved through this researcher meeting with the two PIs every week, to discuss and develop agreement on the codes, the process by which they were arrived at, and the analysis logic in general. This process ensured a broad interrater agreement about the analysis. Apart from this discussion process, we also used qualitative measures to ensure coding reliability and overall rigor of the analysis. The specific approach used, and the rationale for its adoption, is provided in Part C of the Supplemental Material [71] on data analysis.

B. Findings and discussion

We first determined which aspects of the modules the researchers and teachers agreed and disagreed on. As illustrated in Fig. 8, the discussants appeared to have a shared understanding of the problem (the dismal state of undergraduate physics education in the country) and its causes (poor conceptual understanding and problem-solving skills). However they seemed to have conflicting opinions about the mechanism for pedagogic change. Specifically, while the researchers proposed to restructure the teaching of derivations to support mathematical modeling and model-building skills, the participants believed that the mathematical modeling was better linked to traditional textbook problem solving.

We then sought to determine how the discussants were differently interpreting the contents of the conversation. For this, using an open coding framework, we identified numerous repetitive key terms and concepts within the transcript. Assigning each of these terms a unique label,

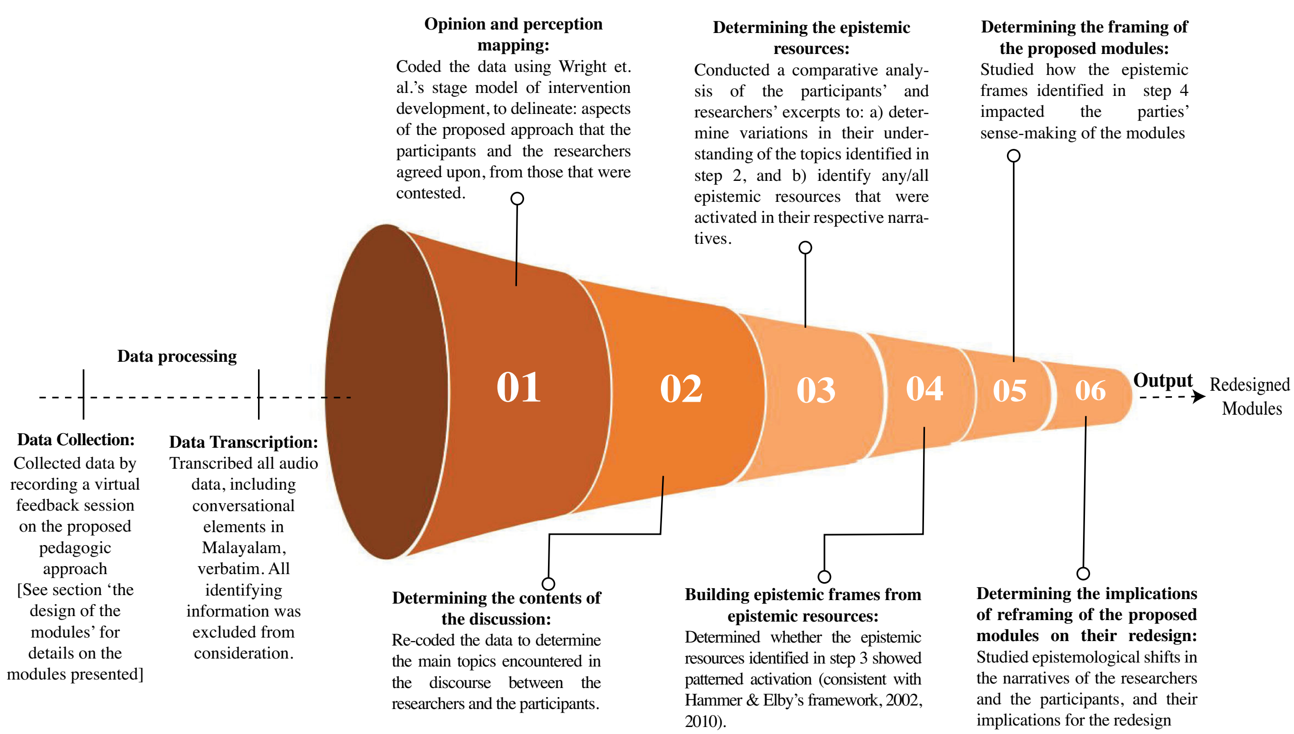

FIG. 7. Overview of the steps involved in our data analysis.

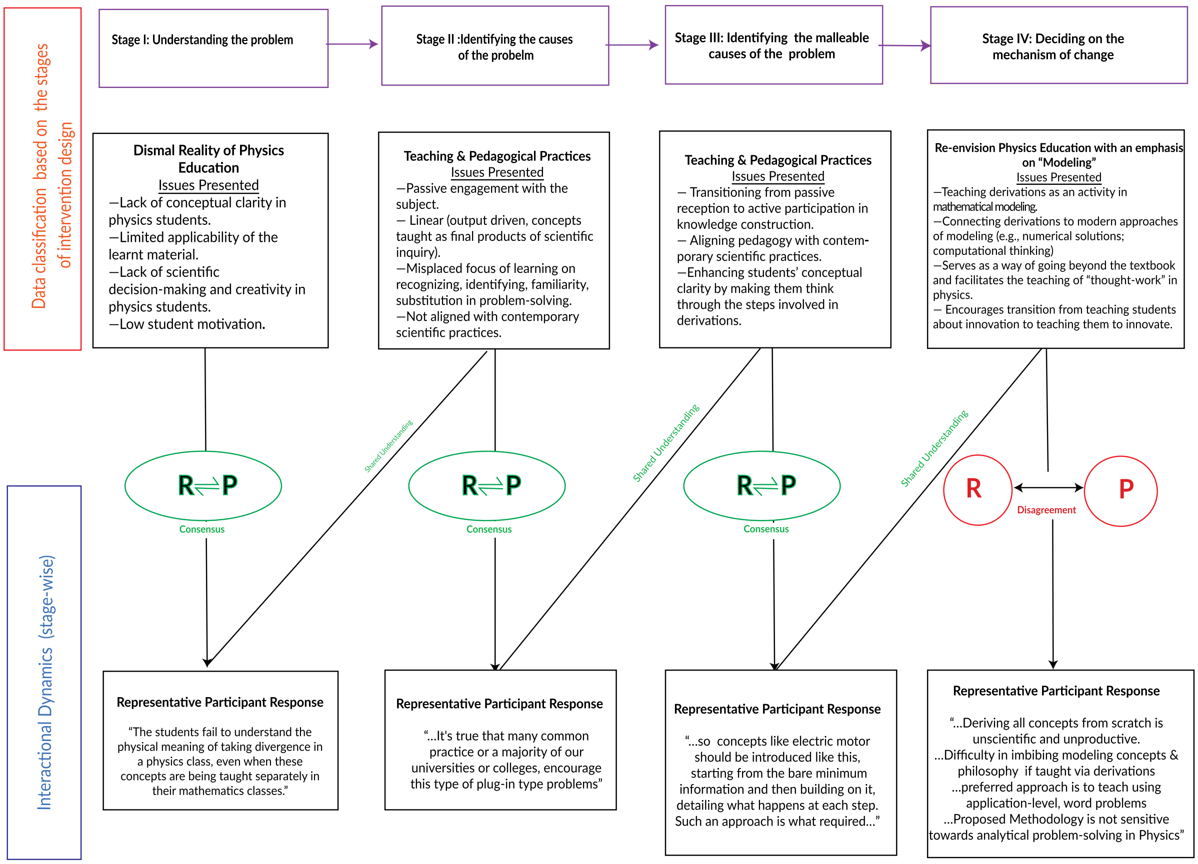

FIG. 8. An overview of teacher participants' and researchers' conflicting and accordant opinions about the modules.

we generated 11 distinct codes. These included problems, models, derivations, physics laws, problem solving, abstraction, idealization, de-idealization, perceived nature of physics knowledge, focus of preferred pedagogical approach, and assumptions about science learners. Conceptually similar codes were then clustered into 3 broader categories, namely, discourse objects, processes, and science learning, as shown in Fig. 9. A description of the clusters appears in the caption of the figure.

Next, using the procedures defined in Fig. 10, we identified the connotative and interpretive variations in the narratives of the researchers and the participants, corresponding to the codes (or topics) within a specific cluster [72,73]. The variations so identified were then used to infer the epistemic resources—the fine-grained knowledge elements that characterize participants' views about knowledge and learning—underlying the narratives. In this paper we focus on the analysis of just the following 4 codes: problems, derivations, abstraction, and assumptions about science learners (see Fig. 9). This is for two reasons. First, for identifying the patterned activation of epistemic resources, the differences in the responses of the two participant groups were more salient with these codes than others. Second, as they contributed to the final objective of the analysis, which was to provide a clear direction to the redesign of the workshop, based on participants' narratives. These codes highlighted patterns that were actionable. Analyses related to the remaining 7 codes have been uploaded as Supplemental Material [71] related to data analysis (see part A for details).

1. Connotative variations in the use of the terms "problems" and "derivations" (cluster 1)

Consider the following excerpt from researcher S1, while explaining the problem with current pedagogical practices:

A helicopter is flying at a height of 1500m and a bullet is fired from the helicopter [initial velocity not given]. When will the bullet hit the target on the ground? This is the problem. It may or may not be very precisely defined, but this is actually, you know, the kind of problem you will encounter when you close the textbook. [So] in retrospect, we know what equation to be used, what derivations to be used. But if you for a moment forget that and take these problems, just as *problems then you see that they are quite complicated. Now why, because in each of them you have to make judgments on what should be known and what should I measure so that I can get the time the bullet reaches on the ground. So you have to make judgments for*

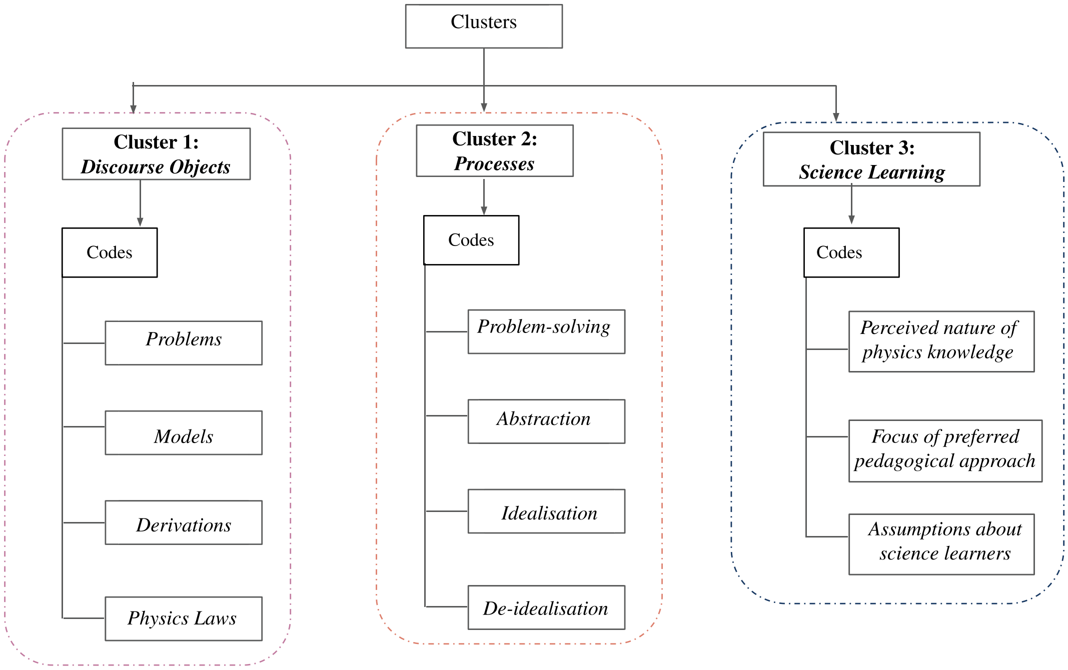

FIG. 9. A taxonomy of the codes developed using an inductive approach to data coding. As is evident from the diagram, we labeled our first cluster "discourse objects." Here, we define discourse objects as operative procedures that embed within them psychological processes such as inference, deduction, categorization, generalization, learning, and decision-making. Next, we labeled our second cluster "processes." By processes, we mean any mental or cognitive function that is involved in the acquisition, use, interpretation, manipulation, and transformation of knowledge. Finally, we labeled our third cluster "science learning." The codes contained within this cluster conveyed beliefs and assumptions about science teaching and learning.

yourself and this is a very hard part because then you are like a practitioner.

S1 employed the adverb "just as" to establish a sense of equivalency between physics (word) problems and derivations. On the other hand see how participant or teacher H used the connector "even though" in the following excerpt to maintain a distinction between the two:

Instead of transferring this derivation of an equation into modeling, are we actually doing, are we actually aiming to do this process of problem solving into modeling? Because in the presentation by, I think researcher S1, even though *he started with the equation* $s = ut + \frac{1}{2}at^2$, *it's not the derivation of that equation that he has modeled here. I think so. What he has modeled is how to approach a problem. How to dissect a problem into different parts, how to do modeling on that problem.*

This example indicates that the discussants differed in how they configured the relationship between problems and derivations. Perceived relationships between disciplinary constructs (e.g., problems and derivations) are considered to influence perceptions about the level of coherence between disciplinary concepts and ideas [74]. In this view, since participant H distinguished problems from derivations, there is a low perceived level of coherence between these disciplinary constructs. Alternatively, when the relationship between the constructs of a discipline is perceived as being integrated and osmotic, as was the case with researcher S1, a high level of coherence between disciplinary constructs and ideas is perceived (see level I themes in Table III in the Appendix). Consistent with Hammer and Elby's framework, we have attributed these difference to "differences in personal epistemologies pertaining to the content of physics knowledge" [65,66]. Further details of this analysis, including the full excerpts, can be found in Table III in the Appendix.

2. Connotative variations in the use of the term "abstraction" (cluster 2)

Analyses similar to the one just discussed were done to characterize connotative variations in the use of the term abstraction. For instance, participant H emphasizes abstraction in the context of end-of-textbook problem solving. He subscribes to a lower level of engagement with the abstraction process, and views physics knowledge structure as a collection of distinct facts or pieces of information (see Table IV in the Appendix for details). Alternatively, researcher M emphasizes practicing abstraction in the context of mathematically modeling derivations. He subscribes to a higher level of engagement with the abstraction process, and conceives physics knowledge as having a mechanism-like structure (see level I themes in Table V in the Appendix). Consistent with the cognitive resources framework of personal epistemologies, we attribute these

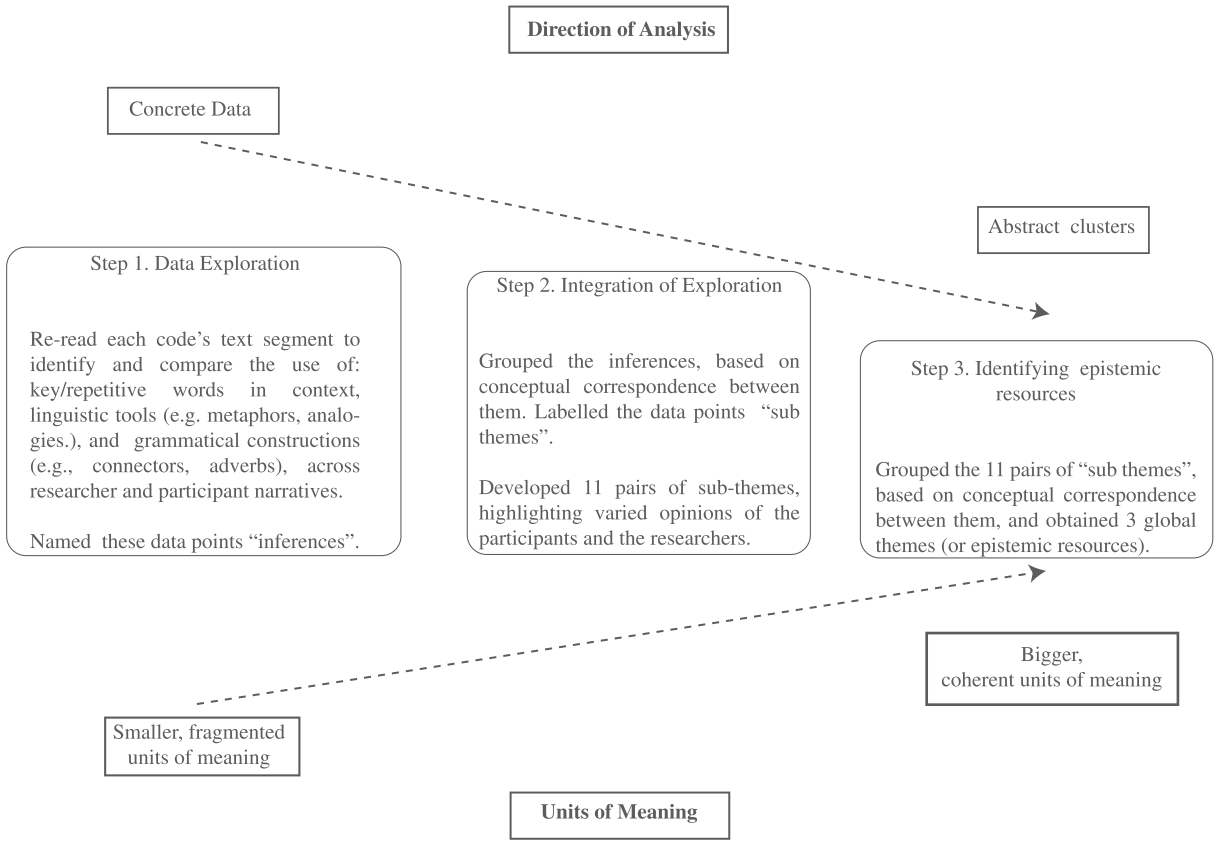

FIG. 10. Overview of coding procedures used to study connotative and interpretive variations in discussants' use of technical terms or concepts.

differences to "differences in personal epistemologies pertaining to the structure of physics knowledge" [65,66]. Further details of this analysis, including full excerpts, can be found in Table IV in the Appendix.

3. Interpretive variations in assumptions made by researchers and participants about science learners (cluster 3)

We then comparatively analyzed participants' and researchers' excerpts to understand differences in their assumptions about science learning. In Table V in the Appendix, assumptions about science learners appear to mark one such point of distinction. Specifically, while teacher or participant A considers physics knowledge structure to be independent of the learner's self, researcher M considers it to be intertwined with the learner's self. Similarly, other differences were observed in the subjects' assumptions about the nature of physics knowledge (formalist vs intuitive), as well as in the focus of their proposed pedagogic interventions (promoting mastery over learnt content vs teaching disciplinary methodologies). See Table V in the Appendix for further details of the analysis, including full excerpts.

The next step in the analysis examined whether there was a pattern to the activation of epistemic resources (ER) identified within the discussants' narratives. Consistent with the cognitive resources framework, we assessed whether the ERs were locally coherent and mutually reinforcing within the discussants' narratives [65,66]. Theoretically, coherent epistemic resources are defined as those that have a higher degree of correspondence between them, and continuous resources as those that mutually reinforce each other's activation. To determine whether the epistemic resources demonstrated any of these characteristics, we recoded and reorganized participants' and researchers' excerpts in accordance with the procedures described in step 4, Table II in the Appendix. Table I presents the major findings from these analyses.

The analysis presented in Table I revealed that researchers and teacher participants differed in how they framed mathematical modeling as a pedagogical activity. These differences were identified from the variations observed in the activated epistemic resources (ER) across the parties' narratives, as well as the influence these resources exerted on one another. For instance, consider the participants' ER that "the content of physics knowledge constitutes distinct pieces of facts and information contained within derivations and problems." This ER functioned as a precursor to their other ER, that "physics knowledge is cumulative and universal." Similarly, in the case of researchers, their ER,

TABLE I. Mapping participants' and researchers' framing of the proposed pedagogic approach.

Stages of framing	Selected excerpts: Researchers	Selected excerpts: Participants	Domain of framing: Task attributes	Observations about the content of framing: Based on comparative analysis of excerpts
Pre-Task stage	Researcher S1: *In retrospect, we know what equation to be used, what derivations to be used. But if you for a moment forget that and take these problems just as problems, then you see that they are quite complicated. Now why, because in each of them you have to make judgments on what should be known and what should I measure so that I can get the time the bullet reaches on the ground.*	Participant H: *Instead of transferring this derivation of an equation into modeling, are we actually doing, are we actually aiming to do this process of problem solving into modeling? Because in the presentation by researcher S1, even though he started with the equation* $s = ut + \frac{1}{2}at^2$*, it's not the derivation of that equation that he has modeled here. I think so. What he has modeled is how to approach a problem. How to dissect a problem into different parts, how to do modeling on that problem.*	Task's purpose or goal	Here, the participant frames the task's intended purpose as teaching students skills of problem solving whereas researcher S1 frames it as teaching them skills in model building. Further, notice that the frames problem solving as modeling and model building as modeling include a marinade of views about knowledge (its content: fragmented vs conceptual mechanism; and structure: cumulative vs holistic) and science learning (thinking like a scientist vs conventional problem solving).
In-Task stage	Researcher M: *In our modeling approach what we advocate is that we should emphasise imagination...we have a physical system, we have a physical reality and we have to model this. So, how do we move from the physical reality to a schematic? What are the idealizations that are required? What is the imagination, what is the mental simulation of the phenomenon that we require? So we consider the learner, the student, as an imagining and thinking agent and we walk through the key practices involved like how we move from physical phenomenon to a schematic, what is the role that idealization is doing, then okay after idealization we do what and the general picture of the modeling is given to students so that they are able to see derivation as a building process, or they're able to think through all the steps.*	Participant H: *My suggestion is that we introduce a problem before the students and see how we can dissect the problem, as researcher S1 presented. May be every problem can be approached in that manner. How it can be dissected, how can we take it to the bare minimum like theoretical minimum and all because the philosophy of physics is that, going to the bare minimum? So, how can we take it to the bare minimum? What all we require to solve it to the bare minimum? What all data we do not have, in the problem of firing the gun from the helicopter, what all data we require, when it is said that firing is done from the helicopter, we need the height, initial velocity. Such details may be extracted from students through discussion, already the physics they know. Like, students who come after going through higher secondary education will be aware of these equations,* $s = ut + \frac{1}{2}at^2$*, will know Newton's laws of motion. To cite an example in mechanics, in continuation to what researcher S1 has mentioned, giving them a problem and splitting students into groups, to reach the bare minimum, how we can solve it, what all data is required. Now, what all complications can be added to it like air velocity, air friction can be added. When such factors are added, what modifications are to be made in the equations? This can be evolved through different group discussions. I feel this is better, and that too problem based.*	Task or activity	Here, the frames problem solving as modeling and model building as modeling can be seen implicating the pedagogical design of the task or activity. Specifically, while researcher M configures the activity to facilitate examination and comprehension of empirical observations in service of knowledge re-construction, participant H frames it as comprising stringing together sets of formulae and equations to analytically solve complex word problems. Yet again, notice that the frames are constituted by a marinade of views about science learning (thinking like a scientist vs conventional problem solving) and knowledge (its content: fragmented vs conceptual mechanism; and structure: cumulative vs holistic). Also evident in the excerpts is the influence that science learning and knowledge views exert on the parties' expectations about learning skills. Specifically, in the researcher's approach, these include abstraction and formalization of physics concepts; and integration of content-specific knowledge. The participant's approach, on the other hand, emphasizes learning skills such as compartmentalization of problems and their specifications as compartments or flows or as routines and procedures.

(Table continued)

TABLE I. *(Continued)*

Stages of framing	Selected excerpts: Researchers	Selected excerpts: Participants	Domain of framing: Task attributes	Observations about the content of framing: Based on comparative analysis of excerpts
Post-Task stage	Researcher M: *In the modeling approach, what we are advocating is that if we teach students derivations in a way that the general features of the modeling process are made clear to them, they will be in a better position to attempt novel problems and they will be able to develop competency in modeling...it will be easier for students to learn other modeling approaches like computational modeling, simulation etc., because there will be features such as idealization that will be similar to both derivations and computational models.*	Participant H: *It is better to impart the modeling approach through problems rather than as part of derivations. If we present it like a methodology or philosophy behind the derivations as steps one [physical phenomenon to a schematic diagram], two [schematic diagram to a geometric model], and three [geometric model to algebraic model], students will not be able to imbibe or even transfer it to another application. What is our aim here as teachers, is not whether or not the students are able to define such a schema, but that it enters the students' minds, it is transferred when they come across another problem. So, I feel rather than presenting the schema in an abstract form, it is better to present it as a hands-on problem-solving activity.*	Task's outcome	Here, the frames problem solving as modeling and model building as modeling can be seen implicating learning outcomes of the task or activity. Learning outcomes in turn, are seen as influencing the level of depth in students' engagement with the discipline. Here also, views about knowledge (its content: fragmented vs conceptual mechanism; and structure: cumulative vs holistic) and science learning (thinking like a scientist vs conventional problem solving; relationship between the learner and that which is learnt) seem to interact to attribute task-specificity to learning outcomes.

that the "content of physics knowledge constitutes conceptual and mathematical knowledge elements," served as a cue for activating their other ER, that "physics knowledge is contextual and holistic in nature." The stances the participants and the researchers adopted towards physics knowledge also appeared to co-activate the learning opportunities they envisioned for their students. The pedagogical approach described in the following excerpt by teacher participant H provides an example:

My suggestion is that we introduce a problem before the students and see how we can dissect the problem, as researcher S1 presented. Maybe every problem can be approached in that manner. How it can be dissected, how can we take it to the bare minimum like theoretical minimum and all because the philosophy of physics is that, going to the bare minimum? So, how can we take it to the bare minimum? What all we require to solve it to the bare minimum? What all data we do not have, in the problem of firing the gun from the helicopter, what all data we require, when it is said that firing is done from the helicopter, we need the height, initial velocity. Such details may be extracted from students through discussion, already, the physics they know.

In this view, teachers are required to play an active role in reducing or decomposing physics word problems to smaller problems or elements. The students, on the other hand, are required to engage in higher-order thinking skills needed to decompose such complex, but concept or topic-centric, problems. Thus, the proposed pedagogical approach seems to be framed such that it retains its focus on developing students' mastery over textbook content. In contrast, in the researchers' proposal, students are required to engage with the different stages of modeling, in order to arrive at an understanding of a physical phenomenon. The following excerpt from researcher M provides an illustrative example:

In our modeling approach therefore, what we advocate is that we should emphasize imagination...we have a physical system, we have a physical reality and we have to model this. So, how do we move from the physical reality to a schemata? What are the idealizations that are required? What is the imagination, what is the mental simulation of the phenomenon that we require? So we consider the learner, the student, as an imagining and thinking agent and we walk through the key practices involved like how we move from physical phenomenon to a schematic, what is the role that idealization is doing, then okay after idealization we do what and the general picture of the modeling is given to students so that they are able to see derivation as a building process, or they are able to think through all the steps.

In this view as well, the pedagogical approach is concerned with teaching physics content as given in textbooks, but it seeks to do this by teaching the characteristics underlying the modeling of a phenomenon in physics. Taken together, these patterns indicate that the parties' epistemic resources did not function in isolation. Instead, they reinforced each other, within the researchers' and the participants' respective narratives.

As a result of the extended interactions, the discussants' initial frames got modified dynamically, and a reframing was achieved. Figure 11 schematically shows how the frames of researchers and teachers interacted and informed each other as the discussion unfolded. Tables V and VI in the Supplemental Material [71] exemplify the framing-reframing process.

VII. REDESIGNING THE MODULES—DESIGN 2

As discussed above, the detailed qualitative analysis of teachers' responses to the modules revealed the activation of different epistemic resources, and personal epistemologies that were different from the ones the researchers used to develop the workshop design. Broadly, this analysis revealed that while the teachers agreed with the need for a redesign, their responses to the design were very different from what was expected by the designers. It is worth noting here that this broad finding does not *directly* lead to design guidelines that help redesign the modules.

Arriving at actionable design guidelines from these identified differences required another analysis step, where the differences needed to be integrated, from a design perspective. This step required an analysis where broader structures—such as the overall design, and also the feasible design pathways—were kept in mind. One overall objective of this design-centered analysis was to develop guidelines for an integrated redesign approach, rather than many particular design changes based on different aspects of teacher comments. In particular, it was felt that if the differences between researchers' and teachers' personal epistemologies could be integrated as a specific "stance" taken by teachers during the discussion, a redesign could broadly seek to address this position.

This type of integration, driven by specific design objectives, is common in design analyses. Importantly, this type of integration is not based entirely on clustering methods used in standard qualitative approaches. Instead, they include a creative element, and follow clustering methods that embed design considerations, such as design objectives, feasibility, affective patterns, and user agency. An illustrative example is the method used by graphic design groups to develop candidate logos, say for an organization, based on the firm's type of work. The designers would first generate, as a collective, many labels, terms and drawings, through free association with the firm and its work, in a large external "mood board." This serves as a reference point that converges the group's initial spontaneous impressions, and also potential design directions. These graphic and word elements are then clustered through a more deliberative process, using "affinity mapping." This collective process reveals points of design intervention, which are developed into candidate logos.

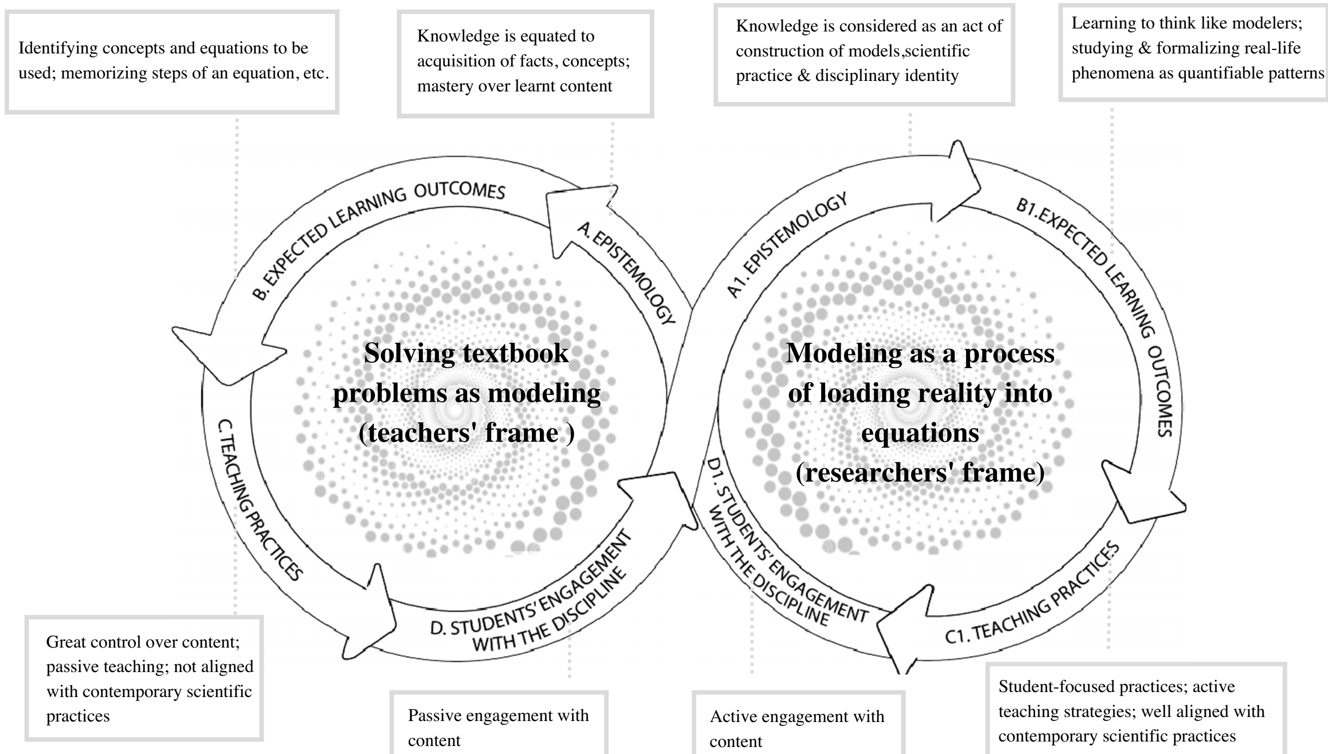

FIG. 11. The two circular structures capture the teachers' and researchers' epistemological frames in an integrated way—textbook problem solving as modeling, and modeling as a process of loading reality into equations, respectively. The spirals signify stability and local coherence in the activation of their respective epistemic resources. Note that this integration is driven by design considerations, and does not follow directly from the qualitative analysis results.

Our design analysis followed a somewhat similar approach, where the key nature of the teachers' feedback (the differences in personal epistemologies), and design parameters (such as the overall nature of our design, and feasible redesign directions) were brought together. For this, we started by identifying the differences between teachers' and researchers' positions, as indicated by excerpts in cluster 1 (Connotative variations in the use of the terms problems and derivations), cluster 2 (connotative variations in the use of the term abstraction), and cluster 3 (interpretive variations in assumptions made by researchers and participants about science learners). We then sought to cluster these differences under a common rubric, keeping in mind the nature of the design and feasible design pathways, to identify points of intervention. This discussion indicated an integrated "teacher stance" (on the ideal way to develop physics expertise), as a candidate design guideline (described below).

(i) *Teacher stance 1.—A textbook-based epistemology of physics*: Teachers' conception of physics appears to privilege problem solving as the central way to develop expertise. Derivations are considered as providing template results and knowledge elements for solving textbook problems. We term this stance "text-book based" because it follows the structure of standard textbooks, where the procedural approach that is characteristic of analytical solutions is valued. Textbooks present these solutions in a privileged manner, emphasizing the generality of the solutions obtained.

Apart from this stance, which was developed by the design-driven analysis of teachers' utterances reported above, a second stance ("within-classroom stance") was inferred from a set of teacher comments on classroom logistics and time management. As these comments were focused on operational issues, and not conceptual aspects of the design, we are not reporting these in detail in this paper.

(ii) *Teacher stance 2.—Within-classroom stance of the teachers*: Teachers' primary commitments and concerns were focused around the way the proposed modules required them to redesign their classroom teaching, and the utility and implementability of the proposed modules to develop such a redesign. Larger goals emphasized by the researchers, such as educational transformation and curricular revision, were not treated as central concerns.

Based on this design-focused characterization, our effort to redesign the modules focused on reducing the tension between our design approach and these stances of teachers. Broadly, the redesign sought to emphasize convergence points, and then build on them, to address the above stances. In this direction, the following three major features were developed and incorporated into the modules during the redesign.

A. Bridging the gap between derivation and problem solving

We designed the modules by considering derivations and problem solving as related activities, both involving mathematical modeling at its core. However, the analysis of teacher discussion around the modules revealed that this similarity between the two was not reflected in the teachers discourse. This indicates that they still perceived these activities as distinct, and considered our emphasis to be centered around derivations.

To highlight the point that both derivations and problem solving are two manifestations of mathematical modeling, we modified the narrative. Particularly, this idea was made more explicit in module 2, which discussed mathematical modeling using a problem involving context (calculation of time taken by a bullet to hit the ground, when fired from a helicopter). A new activity was introduced, wherein teachers would model the oscillation of a pendulum in a viscous medium. This task was carefully worded and framed, in such a way that the traditional distinction between derivation and problem solving was blurred. Specifically, the task could be equally perceived as deriving the equation of motion of a pendulum oscillating in a viscous medium, or as solving a problem that involved tweaking the simple pendulum derivation.

In addition, we developed two more tasks based on a similar framing and narrative (blurring the distinction between derivation and problem solving), where the underlying modeling aspect was emphasized. These two tasks required teachers to engage in modeling: (i) the oscillation of a string fixed at both ends (ii) variation of temperature (in space and time) in an isolated hot rod.

B. Bridging analytical and numerical solutions

Given the textbook emphasis on analytical methods, our narrative on numerical solutions was perceived by teachers as (i) highlighting the deficits of analytical methods, and (ii) as a call to adopt new computational tools.

In the redesign, we sought to highlight the role of numerical methods in enriching physics, rather than the role of computation in physics. The importance of numerical methods in solving physics problems closer to the real world—such as pendulum oscillating in a viscous medium, variation of the temperature on a hot rod, etc.—was highlighted. We also added a segment where teachers could walk through the way equations are solved numerically, using Matlab. A related refinement is a new segment in the oscillating spring simulation, which shows the difference equations generated to solve the differential equation, and the way the values of the parameters are computed step by step, starting from the initial condition. A graphical representation of the computed values is displayed simultaneously alongside.

Apart from these changes, the Netlogo simulation module (on virus transmission) was also redesigned, to emphasize the role of equations in modeling this problem, and the way numerical solutions help augment the problem space that can be modeled. This section explains how the equations are extended to incorporate computational methods, thus segueing into simulations.

C. Facilitate the within-classroom stance of teachers

To help teachers embed the perspective gained from the workshop modules within their classroom teaching, we included more readily implementable resources, including new simulations. In addition to the bridge simulations (simple pendulum, spring oscillator) described earlier, we developed a system that turned the 4-step characterization of modeling (described in module 3) into an interactive learnware. The system is based on the derivation of the equation of motion of a string fixed at both ends, starting from real-world phenomena such as a guitar and a bridge. The system allows teachers and students to move through the key modeling steps and moves in the derivation, turning the real-world phenomenon into a mathematical equation.

To further enable a more classroom-centric stance, we also redesigned the presentation structure of the workshop, from the “preaching model” we started off with (which predominantly involved module presentations based on slides) to a more interactive structure, where we did a set of modeling activities with teachers. Also, a discussion session was added at the end of each module, highlighting how the modeling points illustrated by the module could be embedded within existing teaching patterns. These discussions also helped strengthen the interleaving of the key take-home points, which were spread across the modules in the first design.

VIII. CONCLUDING REMARKS AND FUTURE DIRECTIONS

The redesigned modules were used to conduct a series of 6 workshops for undergraduate physics teachers in one of the states (Kerala) in India. There were 18–20 participants in each workshop (total around 110 teachers, nearly 10% of UG physics teachers in the state). Insights from each workshop were used to structure the subsequent workshop. The goal was to help teachers approach derivations and problem solving from a modeling perspective, an integration that can subsume a large section of the content they currently teach. This perspective can then be extended to CM.

As shown in Fig. 12, uncovering the modeling elements within each existing content module (in textbooks) and then building on them is central to our gradualist design. This approach smoothly extends the current modular organization of content, in a way that the logical complexity (rather than content complexity) goes up systematically. Once students are sufficiently familiar with elements of CM through this gradual structure, a natural progression

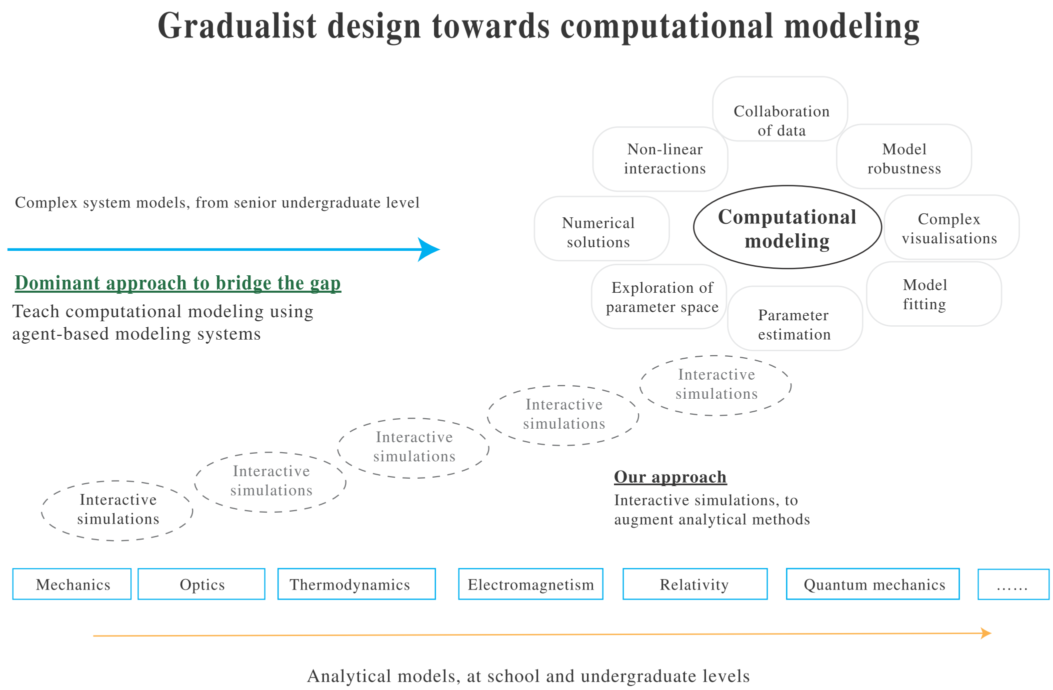

FIG. 12. Students are taught topics like mechanics, optics, thermodynamics, etc., as sequential modules, before moving to advanced topics like quantum mechanics (trajectory shown at the bottom of the figure). Introducing computational modeling (top right) in the senior undergraduate years introduces a whole set of new and unfamiliar practice elements, which makes physics difficult for most students. A dominant approach at this level is to teach CM using-agent based modeling (in green), which is disconnected from the core physics courses, and thus lacks pedagogical continuity and coherence. This problem is mitigated in our approach (the diagonal ovals), based on bridge simulations and numerical solutions as boundary crossing spaces. This structure creates a smoother transition.

towards computational modeling and interdisciplinary thinking could be developed.

In the next step, we plan to include modules on editing existing computational models, to a subset of teachers who have attended the current workshops. The existing work and available resources based on VPython programming will be helpful in this stage [22–24]. Interested teachers from this subset will then be given training to build models from scratch. Our eventual goal is to provide a full-length pilot implementation of a proof-of-concept seed design, illustrating a scalable TPD program that facilitates the transition to CM. While the conceptual and theoretical framework of the current design is developed in the context of physics, it can be extended to other sciences, by leveraging the practice of building models that underlie these disciplines. This structure would extend the emerging consensus in philosophy of science, cognitive science, and science education, that modeling is a quintessential practice that cuts across sciences.

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APPENDIX: DETAILS OF DATA ANALYSIS

Table II gives the rationale and details of the key steps involved in the qualitative analysis of our data. The analysis generated codes which were then clustered into broader categories. Tables III, IV and V provide full excerpts and details of the connotative and interpretive variations in the narratives of the researchers and the participants. These correspond to the four codes discussed in the paper.

TABLE II. Details of the steps involved in the analytic approach. Broadly, steps 1–3 correspond to the 1st assumption in Hammer and Elby's cognitive resources framework of personal epistemologies (assumption *a* in analytical framework), and steps 3–4 and steps 4–6 with their 2nd (or assumption *b* in analytical framework) and 3rd (or assumption *c* in the analytical framework) assumptions, respectively [65,66]. Also note that the steps serially extend into one another, meaning that the output generated by one step becomes the input for the subsequent step. This is why they have been described as being incremental.

Steps	Rationale	Procedure
1. Opinion and perception mapping	This step facilitated a systematic approach to data analysis by allowing us to structure the discourse based on our research objectives. Further, it enabled us to de-contextualize the numerous agreements and contestations between the researchers and the participants from their immediate context and to subsequently understand their meanings and implications from a design perspective.	We used Wright, Wimbush, Jepson, and Doi's model of intervention development to reorganize our data [64]. Specifically, we used the first four stages of the model viz. Defining and understanding the causes (of the problem), clarifying which causes or contextual factors are malleable, identifying how to bring about change, and identifying how to deliver the change mechanism, to dissect, classify and organize the researchers' excerpts. This was done to establish correspondence between these design-related excerpts and the stages of the model. We then classified and organized the participants' responses, relative to the opinions expressed in the excerpts of the researchers. This step culminated into an understanding of the aspects of the modules' design that the researchers and the participants agreed upon and those that were contested.
2. Determining the contents of the discussion	Identifying the main topics of discussion enabled us to monitor our understanding of the discourse, and particularly, the arguments made by the parties in support of their stances towards the modules. It also helped us in the selection of appropriate analytic techniques for further analysis.	We re-read the transcript and identified words, phrases, and topics that were recurrent within and across the narratives of the researchers as well as the participants. During our search, we paid special attention to terms that were previously identified as reflecting contesting interpretations or opinions between the parties in step 1. We assigned each identified word or phrase a distinct descriptive code to facilitate further analysis.
3. Determining the epistemic resources	Ascertaining epistemic resources in the participants' and researchers' narratives allowed us to examine how they were understood by the researchers and the participants and to determine if there were any connotative and interpretive similarities and/or variations associated with their expression or usage.	We clustered the codes developed in step 2 into more abstract categories. The decision to group a set of codes was based on the level of conceptual correspondence (or similarity) between them. After the grouping process, we selected representative participant and researcher excerpts for a specific cluster of codes and conducted a comparative analysis. As part of this analysis, we studied the linguistic elements of the excerpts (e.g., repetitive terms or keywords, metaphors, analogies, use of connectors, adverbs, and other grammatical elements) to determine interpretive and connotative similarities and variations in how the participants and the researchers made sense of the codes (or topics) under scrutiny. The analysis culminated with the identification of a specific epistemic resource in the participants' and researchers' narratives. This process was then repeated for the remaining clusters of codes to enable the identification of other epistemic resources in the discourse.

(Table continued)

TABLE II. *(Continued)*

Steps	Rationale	Procedure
4. Building epistemic frames from epistemic resources	It allowed us to determine the distinctiveness in the participants' and the researchers frames of reference for the proposed pedagogic modules.	We identified and selected participant and researcher excerpts that reflected their ideas for ameliorating the existing curriculum. We then classified these excerpts into 3 domains viz. (proposed) "task's purpose", "task or activity", and "task's outcomes". Then, by using the epistemic resources identified in step 3 as data codes, we determined the parties' moment-to-moment epistemological framing of their proposed pedagogical ideas. We then compared the epistemic framing reflected through their respective pedagogical ideas to determine if their epistemic resources demonstrated patterned activation. Additionally, we also assessed if the patterned activation in these epistemic resources differed across the participants and the researcher excerpts (more on this in the findings section).
5. Determining the framing of the proposed modules	It allowed us to examine how the participants' and researchers' framing impacted their sense making of the proposed modules.	We determined the researchers' and the participants' sense making of the proposed modules by addressing the following question: What is the activation of epistemic resources doing? What specific element(s) of sense making is being altered? Note that this was more of a reflective step than a procedural one. Here, considering the context within which participants' and researchers' epistemic resources were activated was crucial.
6. Determining the implications of epistemic reframing on the modules' redesign	It allowed us to understand whether and how the negotiations between the researchers and the participants impacted their sense making of the proposed modules. This, in turn, enabled us to ascertain how the reframing process implicated the modules' redesign.	We selected excerpts from the transcript that represented attempts made by the researchers and the participants to reconcile or resolve contestation between them. We then used these excerpts to determine the dynamics of epistemological reframing (e.g., propositions being reframed, propositions being negotiated, and reframing technique being used). As a final step in the analysis, we determined the parties' updated sense making of the proposed modules. Once again, we were directed by the question: how do variations in the activated epistemic resources as well as their context of activation impact the parties' sense making of the proposed pedagogic approach? The implications drawn from this analysis were further used to inform the redesign of the proposed modules.

TABLE III. Connotative variations in the use of the terms "problems" and "derivations" in the narratives of the researchers and the participants.

Excerpt and its context (Researchers)	Excerpt and its context (Participants)	Domain of comparison (Codes)	Inferences drawn based on the comparison of opinions expressed	Preliminary themes (Level I themes)	Global themes (Level II themes)
Excerpt 1A (Researcher S1 explaining the problems with current pedagogical practices in physics higher education): *A helicopter is flying at a height of 1500m and a bullet is fired from the helicopter. When will the bullet hit the target on the ground? This is the problem. It may or may not be very precisely defined, but this is actually, you know, the kind of problem you will encounter when you close the textbook. [So] in retrospect, we know what equation to be used, what derivations to be used. But if you for a moment forget that and take these problems, just as problems then you see that they are quite complicated. Now why, because in each of them you have to make judgments on what should be known and what should I measure so that I can get the time the bullet reaches on the ground. So you have to make judgments for yourself and this is a very hard part because then you are like a practitioner.*	Excerpt 1B (Participant H responding to researcher S1's concerns about current pedagogical practices in physics higher education): *Instead of transferring this derivation of an equation into modeling, are we actually doing, are we actually aiming to turn this process of problem solving into modeling? Because in the presentation by, I think researcher S1, even though he started with the equation* $s = ut + \frac{1}{2}at^2$*, it's not the derivation of that equation that he has modeled here. I think so. What he has modeled is how to approach a problem. How to dissect a problem into different parts, how to do modeling on that problem. So my doubt is, are we focussing on converting this problem solving into model building? Further, there seems to be an emphasis on the idea that the student has to learn from scratch, even Newton's equation, everything with theory. I don't know whether putting forward such an idea is scientific. Because science, we already have so much knowledge accumulated there. Ok and we can build from there, right. So, aiming at deriving all the things that are already discovered will be counterproductive.*	1. Problems	1. About problems Consensus in narratives: 1A. A physics problem is composite in nature (i.e., it is solved by the resolution of a series of interrelated problems). 1B. Ill-defined problems share the following characteristics: they represent a real- world scenario; their resolution requires students to make interpretive decisions; they can have multiple paths to resolution; the information needed to solve them is absent or not concretely defined; the action required to solve them is not explicitly stated.	Level I themes based on 1.A.1, 2.A.1 and 2.B.1 (i) The participant adopts a content- oriented view of physics problems and derivations (here, content is used in the traditional sense of describing the content of coursework in physics). As such, conceptual associations (or qualitative, mechanism-based associations) and mathematical formalism appear to be asynchronous, consistent with a formal view of physics knowledge.	Level II theme based on sub-themes i-ii (i) Participants and researchers differ in terms of their epistemic beliefs about the content of physics knowledge
		2. Derivations	Apparent differences in narratives 1A.1. Definition of a complex problem: Participant H appears to focus more on approaching a complex quantitative problem (as is evident when he suggests introducing noise elements into word problems to make them more complex). Conversely, for researcher S1, the nature of a problem that gets ascribed as a "complex" problem in physics, is not always quantitative. Rather, framing the problem quantitatively is part of what makes the problem complex. 2. About derivations Consensus in narratives: 2.A. Current teaching of derivations is an act of observational learning Apparent differences in narratives 2.A.1 Meaning of derivation: Participant H tends to appeal to a mathematical definition of derivations (evident from the consistent use of the phrase "derivation of an equation"). Researcher S1, on the other hand, seems to appeal to a conceptual or semantic definition of derivations (given that he considers derivations to be ill-defined, real-world problems). 2. B.1 Perceived relationship between problems and derivations: For participant H, derivations are equivalent to physics problems in so far as the learner is required to identify relationships between mathematical and physics concepts built into the representation and its subsequent interpretation and application. In other words, both derivations and problems are considered to share a semantic structure that is tied to the relationship between mathematical logic and physics concepts. Researcher S1, on the other hand, re-casts derivations as problems that are ill- defined. Thus, the commonality between derivations and problems is not derived from their semantic structure but from their content and form.	(ii) The researcher adopts a conceptual view (or a qualitative, mechanism-based view) of physics problems and derivations. Consequently, conceptual associations and mathematical formalism appear to be synchronous, consistent with a formal and conceptual view of physics knowledge.	

TABLE IV. Connotative variations in the use of the term “abstractions” in the narratives of the researchers and the participants.

Excerpt and its context (Researchers)	Excerpt and its context (Participants)	Domain of comparison (Codes)	Inferences drawn based on the comparison of opinions expressed	Preliminary themes (Level I themes)	Global themes (Level II themes)
Excerpt 2A (Researcher M explaining the purpose of teaching derivations via modeling): *Moving on...okay...so we go from the physical phenomenon to the schematic, this is which what teachers often draw on the board, and they then define the simple pendulum. Now, let us unpack the thinking behind the construction of this schematic. It is an act of scientific imagination, made possible through a method or through the methodology of idealization. So what we have done is that a lot of details like colour, material particular temperature etc., we have ignored these properties as we have judged that they are irrelevant to the problem at hand. In addition to that, what we have done is that we have created constructs like point particle, massless string, etc. So, in the schematic, usually, we usually use a massless string, a perfect sphere or a point particle particularly in the simplest case we use a point particle. So we have created concepts which actually dont exist in reality.*	Excerpt 2B (Participant H responding to researcher M’s ideas about teaching physics derivations via modeling): *So my suggestion is that, what we can do is, introduce an ill-defined problem before the students, maybe by splitting into different groups, and see how we can dissect the problem...how can we take it to the bare minimum like theoretical minimum and all because the philosophy of physics is that—going to the bare minimum... so, how can we take it to the bare minimum? What all we require to solve it to the bare minimum? What all data we do not have? Such details may be extracted from students through discussion of the physics [concepts] they know. In continuation to what researcher S1 has mentioned, [we could] give students a problem and split them into groups, and ask them to reach the bare minimum. After this, ask them: what all complications can be added to it like air velocity etc. When such factors are added, what modifications are to be made in the equations? This [approach] can be evolved through different group discussions.*	3. Abstraction	1. About abstraction Consensus in narratives: 3A. Abstraction functions as a decomposition strategy wherein a complex phenomenon or problem is considered independently of its attributes or properties. 3B. Both parties appear to emphasise movement across levels or degrees of abstraction in their proposed pedagogies. Apparent differences in narratives 3.A.1. Object of Abstraction: This point of difference is derived from the question: “what does the process of abstraction target?” or “upon what does the process of abstraction operate?” From participant H’s narrative, it is evident that numerical problems pertaining to the concept of “motion” will be targeted for abstraction. Specifically, the participant claims that abstraction could be achieved by formulating problems such that they remain devoid of their contextual details (or in his words: “by reducing the problem to its theoretical minimum”). On the other hand, researcher M considers an empirical phenomenon (i.e., the motion dynamics of a “simple” pendulum) that is represented by the ill-defined problem as the object of abstraction.	3. Abstraction Level I Themes (based one 3.A.1, 3.B.1, and 3.C.1) (iii) For the participant, abstraction is a decomposition strategy that transitions from re- contextualisation of an ill-defined physics word problem and into performing de-contextualised mathematical operations needed to solve the problem. As a consequence, the relationship between empirical phenomenon and mathematical formalism gets framed as being one of interconnectedness. Physics knowledge, in turn, is considered to be cumulative and universal in its structure.	Level II theme based on sub- themes iii and iv (ii) Participants and researchers differ in terms of their epistemic beliefs about the structure of physics knowledge

(Table continued)

TABLE IV. *(Continued)*

Excerpt and its context (Researchers)	Excerpt and its context (Participants)	Domain of comparison (Codes)	Inferences drawn based on the comparison of opinions expressed	Preliminary themes (Level I themes)	Global themes (Level II themes)
			3.B.1. Scope of Abstraction: This point of distinction is derived from the question: which elements of a problem can be subjected to abstraction? In his narrative, participant H explicitly entertains the possibility of reducing a numerical problem to its theoretical minimum. Here, the participant's intention is to devoid the problem of its context to make it difficult to determine the steps involved in its resolution. Thus, for the participant, problem definition is the element that needs to be subjected to abstraction. In other words, in the approach described by the participant, abstraction gets framed in terms of the question: "what needs to be done to resolve this problem?" While the modelling approach that researcher M proposes also targets problem definition for abstraction, other elements of the problem such as its meaning in the broader context and subsequently, the rationale for the choice of methods deployed for its resolution, also play a role in the process. In other words, in the researcher's approach, abstraction gets framed in terms of questions as: "why is method x needed to solve the problem?" and "what is the meaning of the problem p in the broader, empirical context?" Note that these questions are considered in addition to the question that was framed in the approach described by the participant. Consequently, the researcher seems to propose problems with a broader scope of abstraction.		
			3.C.1. Movement among levels of abstraction: The final point of distinction is derived from the question: "what degree of scope (broad vs narrow) and level of detail (less vs more) does a problem entail?" The pedagogical approach that participant H proposes starts with an ill-defined world problem that the students are expected to interpret in order to fathom the qualities of the solution and the form it might take. In subsequent steps, the students are required to select and execute appropriate mathematical methods and procedures that are needed to obtain the desired solution. Since ill-defined word problems are considered to operate at a higher level of abstraction and mathematical procedures at a lower level of abstraction, the movement among levels of abstraction is expected to flow in the following direction: high level → low level. The pedagogical approach that researcher M describes also begins with an ill-defined problem which is followed by the selection and implementation of appropriate mathematical procedures. However, as an additional step, the students are also required to interpret the output obtained in light of the complex system being studied. The last step described in the researcher's approach is considered to operate at a higher-level of abstraction. Consequently, the movement among the levels of abstraction is expected to flow in the following direction: high level → low level → high level.	(iv) For the researcher, abstraction is a abstraction strategy wherein assumptions about the empirical phenomenon are expected to provide scaffolding for further intellectual re-structuring of the phenomenon. As a consequence, the relationship between empirical phenomenon and mathematical formalism is considered as being one of coherence (i.e., the two would interact to form a unified whole). Physics knowledge, in turn, is considered to be contextual and holistic in its structure.	

TABLE V. Interpretive variations in the assumptions about science learners.

Excerpt and its context (Researchers)	Excerpt and its context (Participants)	Domain of comparison (Codes)	Inferences drawn based on the comparison of opinions expressed	Preliminary themes (Level I themes)	Global themes (Level II themes)
Excerpt 4A (Researcher M explaining the assumptions that the modeling approach makes about science learners and how it differs from traditional approaches): *So in the traditional approach as I told the learner sits, listen to whatever the teacher is telling, and accept it as it passively... okay take down the notes, everything is being told, like everything is handed over in a platter, there is no thinking opportunity for the student, every answer or every key step is explained to the student, whereas in a modeling approach what we advocate is that we consider the learner, as an imagining and thinking agent and walk through the key practices involved like how we move from physical phenomenon to a schematic, what is the role that idealization is doing, then okay after idealization we do what and the general picture of the modeling is given to students so that they are able to see derivation as a building process, or they are able to think through all the steps.*	Excerpt 4B (Participant A sharing his concerns about implementing the modeling approach in class): *...a classroom will always have a mixture of students with diverse capabilities and learning potential. How will we be able to cater the needs of this diverse class of students through the modeling approach or how will we be able to visualize the modeling approach to a class of diverse students?*	4. Assumptions about science learners	About assumptions about science learners (based on underlined text in Excerpt 4A and Excerpt 4B) Consensus in Narratives: 4A. Science learners are expected to develop content clarity and mastery in the subject area. 4B. Science learners are expected to practice and master higher-order problem solving and logical thinking skills. Apparent Differences in Narratives	Assumptions about science learners Level I themes (based on 4.A.1) (v) For the participant scientific knowledge is considered to be independent of the learner?s self.	Level II theme based on sub-themes v-vi
			4.A.1 Nature of relationship between the object of learning and the learner: The concern that participant A expresses frames modeling as a concept that the students would need to acquire, just like other concepts that are taught in a physics class. He further discusses student capabilities as a potential hindrance in achieving this goal. Consequently, the nature of the relationship between the object of learning and the learner appears to be one of engagement. Researcher M, on the other hand, considers learners to be active, imagining and thinking agents. In the pedagogical approach he describes, students are expected to construct their own understandings of physical mechanisms based on what they experience, their prior knowledge of its disciplinary contents, and the novel ideas or knowledge that they come in contact with. Consequently, the nature of the relationship between the object of learning and the learner appears to be one of construction.	(vi) For the researcher, scientific knowledge is considered to be experiential and constructive. In other words, the learner and the learning content are interactively linked.	(iii) Participants and researchers differ in their assumptions about physics pedagogy.

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availability of large datasets and powerful computational machinery have made possible the building of highly complex models that address very difficult real-world problems, such as drug discovery. The base neural network architectures used in these models—such as feed-forward connections, the RELU activation function, backpropagation of error, and recurrent nodes—were developed to model complex cognition problems (primarily language and vision), and have been available for more than 50 years. However, these architectures were not used to develop models to address complex science problems, such as protein folding or the glass transition. Again, this suggests the converse—that model building and theorizing in science during the last 50 years was focused on building structure-based simulations, rather than models built using data, particularly large datasets that tune complex neural network architectures. Our proposal about computing driving the transition to numerical modeling approaches and CM is based on a consolidated understanding of these and other learning and discovery cases, which together indicate that the invention of novel representational and computational tools lead to significant cognitive changes. These, in turn, enable new theoretical and methodological advancements in science. Our proposal about the emphasis on analytic approaches—that they emerged from practical difficulties related to numerical solutions—follows from this general idea. It is a converse argument (similar to the other converse cases above), where the nonavailability of efficient numerical computing and visualization tools led to an emphasis on theorizing and modeling approaches that drew on—and also optimized—the capabilities of the available representational and computing tools.

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