Comparison of two semiotic perspectives: How do students use representations in physics?

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The study of students' use of representations is one of the main topics of physics education research and is guided by the overarching field of semiotics. In this paper we compare two semiotic frameworks, one coming from didactics of mathematics and one from physics education research; *the theory of registers of semiotic representations* and *social semiotics*, using the networking of theories methodology. A group of first year university students were audio and video recorded as they discussed concepts relating to thermal energy, a study that will be further explored in an upcoming paper. We find that analyzing the recorded data using two different semiotic perspectives provides a wider interpretation of students' representational use, a descriptive approach to how students use the representations, and an approach to the cognitive aspects of the construction of knowledge. By comparing the theoretical constructs they employ, and how they are employed in the analysis process, we identify constructs that both frameworks have in common, but also where they differ. We have found that each semiotic theory provides a different perspective regarding students' representational use. We also propose that comparing different theories may provide a space for complementing the constructs of each theory and providing a bigger picture to understand students' representational use in physics and other science, technology, engineering, and mathematics education areas.

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I. INTRODUCTION

To explain and to understand learning we must construct theories with the aim to describe the learning process. These theories are called theoretical frameworks and in this paper we will compare two theoretical frameworks that are being used in educational research—*social semiotics* (SS) [1–3] and *the theory of registers of semiotic representations* (TRSR) [4–6]—by applying each framework to the analysis of the same empirical data. We will do so by first contrasting the theoretical constructs used in both theories, and second, by using the theories to analyze the same dataset and comparing the results. This approach allows us to compare both the theoretical constructs with each other, and also how they are applied in practice. Both frameworks are used to describe meaning making or learning that occurs with the help of representations in either mathematics or physics, and, in this paper, we apply both of them in a physics education setting. The analysis builds upon, and extends, the analysis found in Ref. [7] who analyzed the same data using TRSR and the *onto-semiotic approach to mathematical cognition and instruction* [8]. We expand the analysis by also comparing the theoretical constructs of each framework to provide a deeper understanding of similarities and differences between the two frameworks.

The aim of this paper is to highlight both similarities and differences between the two frameworks in order to identify possible ways that the frameworks can be expanded and/or be used in parallel to produce a richer understanding of different learning situations. By better understanding the learning situation and how students navigate it using representations, a teacher may make better informed choices with respect to their choice of representations and possible interventions [9–12]. Thus, a better description of the learning situation may provide insight into improving the pedagogy of physics education.

The methodology used is networking of theories [13]. This methodology uses a wide range of strategies for connecting theoretical approaches: understanding different theories and making them understandable, contrasting and comparing, combining and coordinating, and synthesizing and integrating different theories. To exemplify how the analysis can be done through each theoretical lens, we gathered qualitative data. The qualitative data used for the analysis consisted of group interviews with university

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physics students in Sweden. During the interviews, which were held over Zoom, the students discussed tasks around the concept of thermal energy and were encouraged to use the annotated feature of Zoom to construct their own representations, such as text, diagrams, graphs, and equations.

In this study, we performed all the strategies proposed by the networking of theories methodology. In Sec. II, we present the first pair of strategies: understanding the theories and making them understandable. The paper begins with a short description of the field of *semiotics* on which both frameworks are built. Then follows a description of the two frameworks. In Sec. IV, we present the next pair of strategies, compare and contrast. We present a theoretical comparison of the central concepts from the two frameworks. In Sec. V, we present the next pair of strategies, coordinating and combining. This was performed by analyzing the same set of data through each theoretical framework separately, and comparing the results of each analysis by highlighting their scope and limitations. We end the paper with a discussion of the usefulness of this type of theoretical comparison, both with respect to the richness of the description during the analysis, but also with respect to the further development of the theoretical frameworks. The last pair of strategies, synthesizing and integrating locally, resulted in the redefinition of active and passive transductions in the theory of social semiotics, published elsewhere [10].

II. SEMIOTICS

The theoretical starting point of both SS and TRSR is located within semiotics. Semiotics, which can be traced back to either de Saussure (e.g., [14]) or Peirce (e.g., [15]), deals with the interpretation of various signs, how these are constructed, what they mean, and what meaning may be extracted from them. The two frameworks described in this paper deal with the meaning making that occurs when students interact, manipulate, and communicate using different representations within a subject such as physics. Representations are established signs within the physics discipline and the manipulation and construction of these representations is seen as a necessary step towards becoming a physics expert [16]. A physics concept must be either experienced, or represented in some way, for a student to have the opportunity to discern and learn it. Each representation is designed to showcase some aspects of the concept, but one representation cannot make all aspects of the concept visible to students. Thus, it is natural to apply the ideas of semiotics to explain how students use, construct, and move between different types of representation. Any study of how students use and interpret formulas, for example, becomes a study of the semiotics of formula use.

A. Representational research in PER

Representations and student's usage of representations have been investigated within the physics education research field, as evident by the two theoretical frameworks compared in this paper. The type of representation that is used when presenting physics problems affects how well students perform on the problem [17]. Thus, the student's representational competence [18,19] affects how well they can extract disciplinary knowledge and how they approach the situation. Thus, we may obtain insights into the student's understanding of the physics and their representational competency by studying how students use and construct disciplinary relevant representations. For example, Ref. [20] found that experts and novices use representations differently when solving physics-related problems. Both novices and experts use many different types of representations, but the experts solved the problem faster and moved easier between different representations compared to novices. Thus, how students use representations [21–23], move between them [10,24–26], and how they choose to construct them [27,28], all provide insights into the student's understanding of the physics content of the situation the students are engaged with. In several studies, the insight into a student's understanding is empirical. As the interest in this type of studies increases, it becomes more relevant to understand empirical data with a theoretical framework. Both social semiotics and the theory of registers of semiotic representations aim to describe how students use representations to learn, and communicate ideas within the physics discipline. Our contribution in this study is to compare and exemplify how these two theoretical frameworks can be used to analyze empirical data of representational use to provide insight into student's understanding.

B. Social semiotics

SS was initiated in 1978 by Halliday [1] as a description of language. It aimed to describe language's different parts from an interpretive and meaning-making perspective. The framework has evolved over the years and, in this paper, we will use SS as it is presented in Ref. [2] (with the additional theoretical developments of Refs. [9,29,30]). Airey and Linder [2] (p. 95) define SS as

> *"the study of the development and reproduction of specialized systems of meaning making in particular sections of society"*

and have applied SS to the study of learning physics [31–33].

Concepts within disciplines such as physics and mathematics must be represented in such a way that a learner may experience and explore them. The concept of "force" must be represented in a way that allows a learner to discern some distinct aspects of it, such as direction, magnitude, or contact point. These representations are often mathematical formulations, graphs, diagrams, or pictures. A specific representation, such as $\bar{F} = m\bar{a}$, is called a *semiotic*

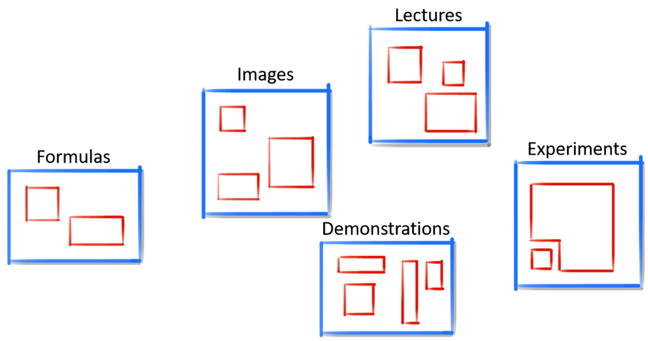

FIG. 1. Within each semiotic system, blue squares, exists many different semiotic resources, red squares.

resource and is situated within a *semiotic system*: "equations." A semiotic system is a system to construct and to represent concepts and each semiotic system is qualitatively different when compared to other semiotic systems. Equations, "graphs," "gestures," and "images" are all examples of semiotic systems, within the discipline of physics, that are used by experts to communicate in the discipline, but also to introduce concepts to novices. See Fig. 1 for a schematic picture of the relationship between semiotic system and semiotic resources. Semiotic resources are not only representations, but also activities, equipment, or anything that is used to interpret or present disciplinary information. For example; a particle accelerator is a semiotic resource because it is used to make meaning of specific aspects of subatomic physics, just as a velocity-time diagram is used to make meaning about the relation between time and velocity in a specific situation.

SS draws on the multimodality framework [3,34–36] to describe how semiotic resources are used and transformed. If a semiotic resource is modified, but it stays within the same semiotic system, the modification is called a *transformation* but if the modification involves the movement between two different semiotic systems it is called a *transduction*.

For example, the modification of the formula $\bar{F} = m\bar{a}$ into $GMm\bar{r}/|\bar{r}|^3 = m\bar{a}$ is a transformation because it stays within the same semiotic system of formula. Whereas the modification shown in Fig. 2 is a transduction since it involves the movement between different semiotic systems.

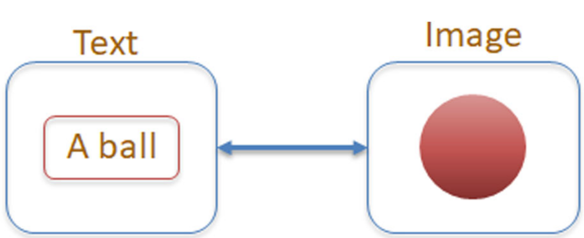

FIG. 2. A transduction is performed between the semiotic systems "text" and "image."

1. Understanding in social semiotics

Any learning situation encompasses many transformations, transductions, semiotic resources, and semiotic systems to explore and experience the problem or concept at hand. In a learning environment we wish for students to obtain a *multifaceted way of knowing* [26] which means that a student has experienced, and discerned, a concept using many different semiotic systems and semiotic resources. A student should become fluent in using the semiotic resources and the movement between semiotic systems with regards to the specific concept, or *semiotic material*, in question. Semiotic material is the content that is represented in a representation, or the ideas that the representation aims to convey. In translations and transductions, we often wish to preserve or highlight some aspects of the semiotic material. A learning situation may be described in terms of changes to the discernibility of the semiotic material through the use of different semiotic resources and translations of the semiotic resources.

Within a semiotic resource, we may also investigate how well a student may discern important or disciplinary relevant aspects. Eriksson *et al.* [37] constructed the *anatomy of disciplinary discernment* that describes a hierarchy of discernment based on disciplinary knowledge. This hierarchy aims to capture all the ways to discern disciplinary relevant aspects from a disciplinary perspective and is tied to the student's disciplinary understanding of the semiotic resource. The anatomy of disciplinary discernment is described in detail in Ref. [37] and the levels are paraphrased here; from least discernment to most discernment:

- Disciplinary identification: The student can name aspects of a representation using disciplinary specific terms.
- Disciplinary explanation: The student can explain how aspects relate to each other in the representation, in a disciplinary way.
- Disciplinary appreciation: The student appreciates the value of the representation with respect to its disciplinary content.
- Disciplinary evaluation: The student can evaluate and find flaws in the representation from a disciplinary perspective.

C. Theory of registers of semiotic representations

The theory of registers of semiotic representations was developed by Raymond Duval since the 1990s and early 2000s. Duval [4] considers that a representation is something that stands for something else, an object that can be tangible or intangible, such as ideas and concepts. A representation of an object can be physical when created by means of physical devices such as photographs, or semiotic when using symbols, rules, and associations as tools to represent the object. In this theory, registers of representation are the semiotic representational systems that allow for transformation.

In natural sciences, such as physics, chemistry, and biology, the objects of study are directly or indirectly approachable. This allows representing the objects with several semiotic and physical representations and relating the representations with the object. In contrast, mathematical objects of study are only accessible through semiotic representations [4]. Similarly, there are some highly abstract concepts in physics and other natural sciences that are only directly approachable with semiotic representations and indirectly with physical representations [6].

The theory of registers of semiotic representations suggests that the cognitive activity in mathematics resides in the use of semiotic representations that allow the development of mathematical thought [4]. The use of semiotic representations for mathematical cognitive activity creates a paradox because, on the one hand, mathematical objects are only accessible through semiotic representations while; on the other hand, the mathematical object should not be confused with its representation. The challenge in the learning of mathematics and other abstract concepts is to dissociate the object from its representations, which can only be achieved through the use of multiple semiotic representations.

Using multiple semiotic representations requires that these representational systems can be transformed. However, not all representational systems can be transformed; then, only the representational systems that allow for transformation are considered registers of semiotic representation. Identifying the registers of semiotic representation that are involved in a cognitive process is the first part for analyzing students' understanding. To identify registers effectively, it is important to understand what is a register and how registers are transformed. In Fig. 3 we present an example of the algebraic register used to describe the physical concept of energy. The speech balloon on the left side represents the spoken form of the algebraic register, a person reciting the equation, while the right side represents the written form of the equation. Even though the delivery of the information is different (spoken and written), the semiotic content is the same, the algebraic relation between energy and mass.

Transformation can be treatments, which happen within the same semiotic register, and conversions, which happen between two or more registers that denote the same characteristics of the mathematical object. For example, the modification of the formula $\bar{E} = \bar{F}/q$ into $\bar{E} = kQ/r^2\hat{r}$ is a treatment because it stays within the algebraic register. Whereas the modification shown in Fig. 4 is a conversion since it involves the movement between different semiotic systems. Figure 4 represents the conversion between the graphical register and the algebraic register. It shows the electric field lines and an algebraic representation of the same field; a student must recognize that the same mathematical object is represented in both registers. Conversions are more complex than treatments because they require the recognition of the same object in two semiotic systems that represent the same object with different characteristics. Duval identified that the recognition of the object in the characteristics of the representation is one of the main sources of difficulty, and that these difficulties depend on the direction of conversion.

By describing the learning situation using semiotic registers, representations, treatments, and conversions, we can identify when students may run into difficulties and investigate them accordingly.

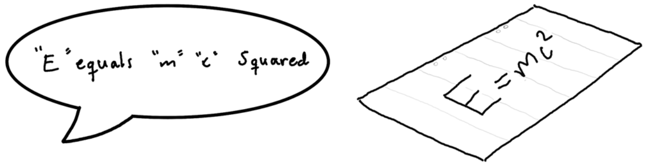

FIG. 3. Two semiotic representations of the same underlying mathematical object. Both semiotic representations are part of the same semiotic register: the algebraic register. On the left is the spoken form and on the right is the written form of the algebraic register.

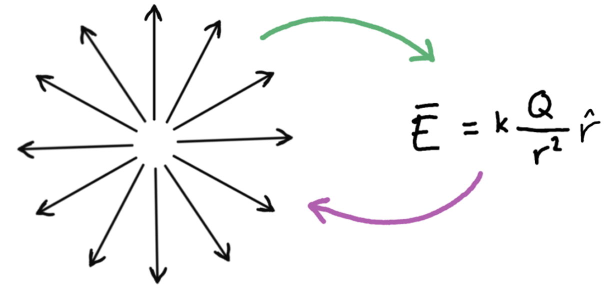

FIG. 4. A conversion between the graphical register and the algebraic register. The difficulty of performing a conversion has been shown to depend on the direction of the conversion. Here the conversion is between a field line representation of an electric field and the formulaic representation of the field. Students must recognize that both semiotic representations aim to represent the same mathematical object.

III. RESEARCH QUESTIONS

We found the need to compare and exemplify how the two theoretical frameworks: Social semiotics and the theory of registers of semiotic representations can provide insight into the analysis of a student's understanding through the use of representations. Our main research question is:

> *What further insight about students' understanding through the use of representations in physics learning can be gained by contrasting and comparing two theoretical frameworks?*

To be able to answer these questions, four separate questions were identified that would together provide a comprehensive theoretical overview of both frameworks.

- How do the theoretical frameworks describe a represented object?
- How do the theoretical frameworks describe representations?
- How do the theoretical frameworks describe changes in representations?
- How do the theoretical frameworks describe the understanding of physics?

The bullet list with questions will be addressed in the following section and the main research question will be answered in Sec. V G.

IV. THEORETICAL COMPARISON

Representation systems are central parts of both theories. However, given that the TRSR is focused on the cognitive activity in mathematics, the main tenet is that we can only access mathematical objects through semiotic representations. While in SS, the semiotic material can also be discerned with physical devices, such as photographs or measurement data. Therefore, using the TRSR to analyze students' representational use while learning physics is limited to highly abstract physics concepts (such as the electric field [6]), while SS allows for a broad range of physics concepts. Also due to the different nature of each theory, the TRSR allows linking students' ability using several representation registers with their understanding of a concept, while SS focuses on disciplinary discernment. As we can see, both theories have their strengths and limitations, which will be analyzed with more detail throughout this section.

We have summarized the comparison in Fig. 5.

A. Similarities

Both SS and the TRSR describe representations used by students to learn. Both frameworks also identify changes to the representations as important aspects in the learning process. In Table I we connect concepts in the two theories with each other. However redundant it may be, the first comparable aspects are the representamen and the semiotic object in each theory: in SS the semiotic object is the semiotic material and the representamen is the semiotic resources; in the TRSR, the semiotic objects are mathematical objects, and the representamen are registers of semiotic representation. In Table I we also include the physical representations, but Duval explicitly states that the case of mathematical objects does not allow for physical representations [4].

Another aspect where we can find similarities is the structure with which both theories describe the changes of representation: whether they happen within one representation system, or if they involve more than one. The TRSR considers that cognitive activity in mathematics happens through the transformation of registers of representation. In this theory, transformations can be treatments, when the transformation occurs in the same register, and conversions, when two or more registers are transformed. Similarly, in the SS theory, translations can be transformations when they happen in one semiotic system, and transduction, when two or more semiotic systems are involved. Within the transductions in SS, we identify active and passive transductions; active transductions can be compared to conversions with recognition in the TRSR [10], while passive transductions are comparable to conversions without recognition in the TRSR. This implies that recognition is a cognitive aspect in the TRSR.

Finally, the cognitive aspects have some similarities in both theories. We attempt to compare disciplinary discernment in SS with the cognitive activity in the TRSR, specifically with recognition and dissociation. These two cognitive aspects in the TRSR are identified as sources of difficulty in the learning of mathematics. Pertaining to

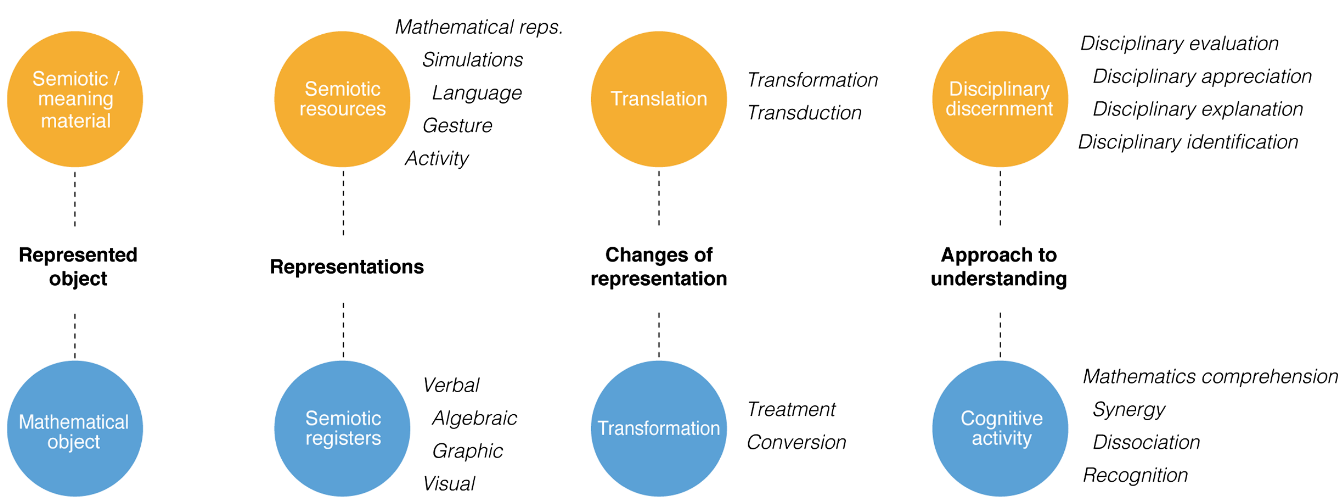

FIG. 5. Different areas that could be analyzed. With the social semiotics approach on the yellow side and the theory of representations of semiotic registers on the blue side. Image created by Dr. Elias Euler for this paper.

TABLE I. Similarities between social semiotics and the theory of registers of semiotic representations.

Aspect	Social semiotics	Theory of registers of semiotic representations
Semiotic object	Semiotic material	Mathematical object
Representamen	Semiotic resources	Semiotic and physical representations
Representation change	Translation	Transformation
Changes within one system	Transformation	Treatments
Involving one or more systems	Active transduction	Conversion with recognition
	Passive transduction	Conversion without recognition
	Transductive link	Transitional auxiliary representations
Cognitive aspects	Disciplinary discernment	Recognition + Dissociation
Understanding	Multifaceted way of knowing	Mathematical comprehension

understanding, SS describes students' multifaceted way of knowing when they can refer to several semiotic resources, while the TRSR describes mathematical comprehension when there is synergy between representations: students recognize the mathematical object in several registers, convert between them and dissociate the object from the representation.

As a big picture (see Fig. 5), this comparison proves that the two theories are sufficiently similar in their structure. However, while analyzing several examples, we found that the theories have subtle differences. This finding is important because the two theories are similar enough to allow for integration of knowledge, but different enough to provide contrasting lenses to tackle the research objectives with different angles, which may lead to enriched insight. We describe some of the differences in the subsequent section.

B. Differences

The most relevant difference between the two theoretical approaches is that the TRSR is limited to the cognitive activity around mathematical objects, while SS has a broader application to physics and science. This is a big difference because from there, all other differences emerge. The TRSR is focused on how transformations of representation define the cognitive activity in mathematics because there is no other way around. But in physics, we have other ways (like obtaining data) so the SS refers to how the representations are used, and it does not focus solely on cognitive activity. Since TRSR is limited to mathematical objects and other highly abstract concepts, the link between representational use and understanding is inevitable, while SS allows for more scientific and physical concepts even if it is not too focused on understanding. This is where the two theories can interact and learn from each other.

A big difference between the two frameworks relates to the underlying division of representations. In SS, representations are divided into semiotic systems which groups representations in terms of how concepts are represented. For example, a written formula and a spoken formula are divided into *formula* (written) and *speech* (spoken). In the TRSR, the registers are not defined in the way they represent a concept, but by the content of the representation. Thus, a written formula and a spoken formula would be part of the same register: the algebraic register. However, the algebraic register takes on the written or oral form.

Derived from the previous differences, the relation between representational use and student understanding becomes critical. As emphasized before, SS refers to how students use representations in physics and other scientific contexts. When describing the how, SS classifies the representational abilities of students in the anatomy of disciplinary discernment. This structure provides insight into whether students are able to identify, explain, appreciate, and evaluate the disciplinary conditions of the representations that they are using. In the case of the TRSR, the relation between representational use and understanding is given through the synergy between representations. In this theory, the registers and their transformations are defined with the tenet of cognitive activity. The terms that describe the cognitive activity are the recognition of characteristics of the concept and the representation, and the dissociation between the concept and its representations. So, recognition and dissociation play an essential role in the cognitive activity, and they are part of the treatment and conversion of registers. In TRSR, representational use and understanding are directly linked, while SS creates this link indirectly, through the description of disciplinary discernment. The two theories together provide a broader picture where we can analyze the disciplinary discernment of students based on their representational use, as well as their understanding of the physical phenomena.

V. EMPIRICAL COMPARISON

The methodology used is networking of theories [13]. In this section, we present the strategies of combining and coordinating the theories, by analyzing the same set of qualitative empirical data with both theoretical approaches. In our comparison between the two frameworks we aim to describe both their underlying theoretical constructs but also showcase how they are applied in an analytical situation. In Sec. V G we provide a list of theoretical constructs for both frameworks and how they line up with each other. Later in Sec. V G we provide some results from the analysis of applying both frameworks on the same set of data.

A. Data collection

To be able to perform a comparison on how the two theoretical frameworks are applied to analyze a learning situation, we gathered data where students used representations to discuss and explore the concept of thermal energy as an example. The aim of the data collection was not to investigate the students' understanding of thermal energy, but to capture data that has a wide range of usages of representations by the students. We present this comparison of frameworks in a way that either or both frameworks can be applied to different topics of physics education where semiotic representations are used to derive student's understanding of physical concepts (not exclusively about thermal energy). It is important to note at this point that the connection between representational use and understanding is made through the analysis of such data within each of the theoretical frameworks.

1. Participants

The participants that took part were first year university physics students and first year physics teacher students at a well-known university in Sweden. The participation was voluntary and the participants were recruited through a physics course during their first year. All participants signed a consent that complied with The General Data Protection Regulation [GDPR, Regulation (EU) 2016/679].

2. Digital group interviews

The interviews were done with two or three students at a time over Zoom. During the interview, the students were encouraged to discuss the task at hand, but also to explore tangents where they produced or used different representations. Using the Zoom annotate function, the students could draw, point and write directly on the PowerPoint where the tasks were presented, see Fig. 6 for an example of this.

The PowerPoint presentation, the students faces and discussions, and the dynamic annotations were all recorded using the Zoom record function. Excerpts of the data are presented in the transcripts (Tables II, III, IV, and V), together with a figure to provide context to the discussion if necessary.

B. Data selection

Not all the data, nor all results of the analysis, will be presented in this paper because this paper's focus is on the comparison between the two theoretical frameworks. A small subset of the data will be presented and analyzed by both frameworks so that a comparison can be made. Parts of a single interview with physics teacher students are presented here because of the richness of representational manipulations and constructions. It should not be seen as representation of the student body as a whole, but a carefully selected sample to exemplify representational manipulations that may be described using the frameworks of SS and TRSR. The students are Fredrik, Gustaf, and Hela,[1] with Kim being the interviewer. A full analysis of the collected data will be presented in another paper that focuses on the construction of representations using social semiotics as a lens.

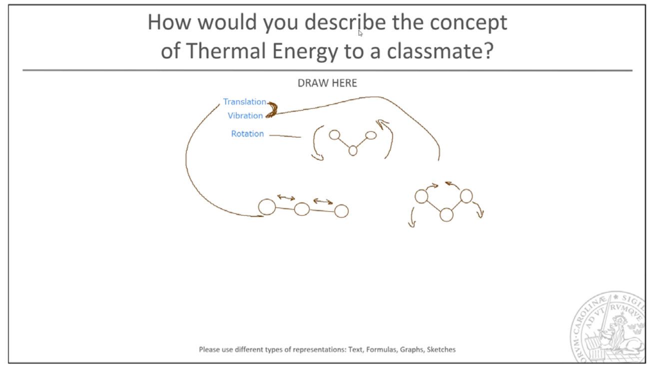

FIG. 6. Students discuss and solve the task together using the Zoom annotate function where they can draw and write using different shapes and colors. In the above example, two students discussed "How would you describe the concept of thermal energy to a classmate?" and drew representations of molecules in motion.

The selected data are chosen to showcase how the frameworks describe different representational manipulations and how these are related to the meaning-making process.

C. Quality assurance

In this section we wish to address the steps we have taken to ensure that the research presented here is of high quality. We draw upon the categories described in Guba and Lincoln [38] as a first check.

Credibility.—The category aims to ensure that the findings are credible from the participants' point of view. We have achieved this by, during the interviews, allowing the participants to speak freely about the study and the questions and if there were aspects they felt that we missed. They were also encouraged, both in written and spoken form, to contact us if they wished to add anything to the data. The data were also processed as a whole, although, we do not present all the data in this paper, all the data were transcribed and analyzed.

Dependability.—The category aims to ensure that the study is repeatable if done with the same cohort, context, and researchers. This was achieved by keeping track of the

[1]The names are fictional to preserve the students' anonymity.

TABLE II. Transcript of a transduction from speech to formula of the semiotic material of thermal energy.

1	Fredrik	We have the formula for heat. [Gustaf nods]. The "Q" equals to, what is it, "$mc\Delta T$" ?
2	Gustaf	Yeah.
3	Fredrik	Should I write it down... I can write it down
4	Kim	Yes, please do. [Fredrik draws a "Q"]
5	Fredrik	We have "m" "c" "ΔT"
6		[Fredrik writes "mass" and draws an arrow from the word mass to the "m" in the formula.]
7	Fredrik	"c" is the.... [Draws and arrow pointing at "c"] what is this called?
8	Gustaf	Heat Capacity...
9	Fredrik	It's called Heat Capacity... Specific Heat Capacity, yeah.
10		[Fredrik writes "Heat Cap" at the arrow pointing at "c"]
11	Fredrik	And ΔT" is the, well, change in temperature.
12		[Fredrik draws an arrow pointing at "ΔT"]

TABLE III. Transcript of a transformation of a formula. Continuation of Table V.

25		[Fredrik is looking up the formula on a formula sheet]
26		[Fredrik begins to write down the formula $\Delta U = nC_v\Delta T$]
27	Fredrik	I am just copying the formula.
28		[Fredrik adds: $= f/2nR\Delta T$]
29	Gustaf	Yeah, sure
30		[Fredrik adds: $= f/2Nk\Delta T$]
31	Kim	And what does that formula say?

study methods and by carefully designing the PowerPoint itself. The intent of the questions and what each slide in the PowerPoint is aimed to capture is documented and constructed with outside expertise. The analysis process employs established theoretical constructs from both frameworks with predefined definitions, which reduces the mislabeling of aspects, ensuring that the same events would receive the same treatment if the study was repeated.

Confirmability.—The category aims to ensure that the study can be corroborated, or confirmed, by outside researchers. As the aim of this particular paper is to compare two theoretical frameworks, and not the particulars of the data from the data collection, we do not expect the data collection to be replicable with the information presented here. However, we expect that other researchers will come to the same conclusions if they apply both theoretical frameworks to their own data with the intent to compare them.

Transferability.—The category aims to ensure that the study can be generalized or transferred to other contexts. As the study's focus is on the comparison between theoretical frameworks, we expect that the methodology can be transferred to any dataset where students are using representations to learn. However, this study was explicitly designed to capture students' usage of representations and other datasets may have been captured with other focuses. The application of the frameworks used in this paper may not be suited for a dataset captured to study, for example, student attitudes or motivations.

D. Analysis

Below follows some example analyses when social semiotics and the theory of registers of semiotic representations are applied to describe a learning scenario where students solve and discuss physics concepts. In the transcripts, the formulas the students say out loud have been transcribed into formulas to be easier to read. We study how the use of multiple representations is related to understanding, by analyzing the data of using multiple representations with different theoretical frameworks. Each framework has its own way of describing understanding. In SS, we relate representational use with disciplinary discernment, while in TRSR, we relate it to recognition and dissociation.

TABLE IV. Example of disciplinary discernment.

1	Fredrik	So it would try to reach equilibrium which would mean that the gold would lose heat.
2	Kim	So what would happen to the representation in that case?
3	Fredrik	The volume would decrease.
4	Hela	In that case...
5	Fredrik	Well
6	Hela	Where you say that... it's warmer than the room you put it in, the change in temperature is relative... to the environment... and.. wait I just lost my train of thought for a second.... Right, yeah, if you define the zero point as the environment then it would be the opposite of what we said earlier.

E. Analysis using social semiotics

Social semiotics looks at the actions the students perform with respect to the representations. Such as, how they choose to construct them, what they deem necessary and relevant, and how they manipulate the representations to highlight important aspects.

1. Identifying transductions

In the following transcript, Fredrik, with the help of Gustaf, performs a transduction between the semiotic systems speech and formula. We can see that Fredrik unpacks [31] the semiotic resource when he adds arrows and words to what the formula represents. The transduction results in the semiotic resource we see in Fig. 7, which is an unpacked version of the formula Fredrik and Gustaf spoke about.

In Fig. 7, we have used the concept of transduction from social semiotics to describe the specific aspect of the meaning-making process. As part of the transduction process, we see that Fredrik unpacks the new semiotic resource to highlight different aspects. The aspects the Fredrik highlights are aspects that he has deemed relevant for the situation, either to understand it himself, or to communicate the meaning to the others. Unpacking, filtering, and highlighting has been identified as important parts of the transduction process [25,39,40]. In Ref. [10] Svensson *et al.* introduces the distinction of *passive* and *active* transductions and defines them as follows:

Active transduction: The student shows engagement with the semiotic material during the transduction.

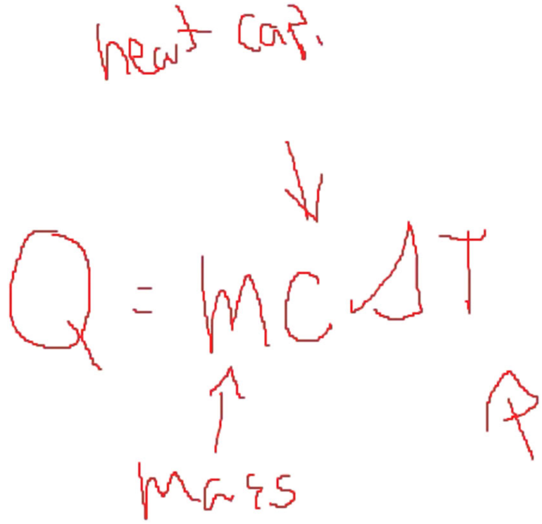

FIG. 7. Fredrik writes down the formula for thermal energy, $Q = mC_v\Delta T$, but also modifies it by adding arrows and words to explain it. Fredrik unpacks the representation and highlights different aspects, so that it will be easier to discern the meaning of the formula.

Passive transduction: The student does not show engagement with the semiotic material during the transduction.

In Table II we see that Fredrik engages with the semiotic material and we further identify the transduction as an active one.

2. Identifying transformation

In Table III, Fredrik performs a transformation of the formula as it is rewritten into different configurations. Transformations stay within the same semiotic system, in this case; the semiotic system formula, and describes manipulations and rewrites of semiotic resources. In the transcript, Fredrik performs a translation when he rewrites $\Delta U = nC_v\Delta T$ as $f/2nR\Delta T$ and once more when it is written as $f/2Nk\Delta T$. However, we cannot say that Fredrik or Gustaf engages with the semiotic material of the formula.

In Fig. 8 we see the sequence of expressions that Fredrik wrote down. The act of writing it down sets the stage for new types of manipulations and discussions. In line 31 of the transcript in Table III we see the interviewer ask for an explanation of the formula. If the formula had not been written down, the interviewer would not be able to direct the conversation in new directions. Thus, even if the actual transformation was passive, it set the stage for interventions and new translations to be performed by the students.

3. Semiotic resources and DRAs

In transcript II, we see that the students use specific words (spoken and written) and formulas to describe "heat" in this case. These are the semiotic resources they employ to communicate their knowledge to their peers. The semiotic resources are chosen because they provide access to the disciplinary relevant aspects (DRA) [41,42] of the situation. The DRAs are aspects that the discipline has deemed relevant for the situation, such as "mass," "specific heat capacity," "temperature" and the relationship between them. The semiotic resources are established within the physics discipline and provides a common language for the students to use when exploring the concept. In Fig. 7 we see that Fredrik has identified some of DRAs of the situation and is providing opportunity for Gustaf and Hela to discern them as well by unpacking the formula.

4. Disciplinary discernment

In Table II we see that the students engage with the semiotic material of the representation and Fredrik

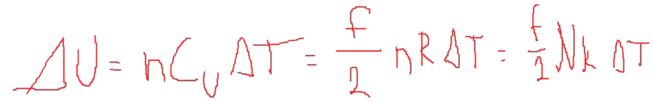

FIG. 8. Fredrik copies the formula, but without engaging with the semiotic material.

highlights different aspects of the concept by writing words and drawing arrows (lines 6, 7, 10, and 12). From this, we can say that Fredrik has, at least, reached the "*disciplinary explanation*" level of the disciplinary discernment hierarchy.

In Table IV, we see that Hela comes to the realization that the interpretation of the representation is not unique and that it produces valid, correct results if interpreted in another way. We interpret this as Hela reaching the *disciplinary appreciation* level of the disciplinary discernment hierarchy because she acknowledges that the representation can be engaged with in different ways and that those ways of engaging with the representation are also valid.

F. Analysis using registers of semiotic representation

In this section, we analyze the same example from the theory of registers of semiotic representations perspective.

1. Identification of registers of semiotic representations

We start this analysis by defining the registers that the participants Gustaf and Fredrik used: the algebraic and verbal registers. The algebraic register includes letters, numbers and symbols to represent a mathematical relation between physical quantities, such as heat, mass and the change of temperature. The verbal register uses words and sentences to represent the definitions of the physical quantities.

The episode in Table II presents how both registers appeared in the written and oral form in this example. In line 1, Fredrik recites the formula for heat, saying: "Cue equals to, what is it, em, cee, delta tee." This is an example of the algebraic register in the oral form, notice how Fredrik says the names of the consonants ($\mathrm{Q} = \mathrm{cue}$, $\mathrm{m} = \mathrm{em}$, $\mathrm{c} = \mathrm{cee}$, $\mathrm{delta} = \Delta$, and $\mathrm{T} = \mathrm{tee}$) to represent the physical quantities involved, and the name of the symbol $=$, equals to, to represent the relation between them. In line 5, Fredrik writes down the formula as he speaks, which is an example of the algebraic register in the written form. In lines 8 and 9, we see an example of the verbal register in oral form, when Gustaf and Fredrik assign the definition of heat capacity to the letter c in the equation. In line 10, Fredrik writes heat capacity, which is an example of the verbal register in its written form.

2. Identification of conversions between registers of representation

Having identified the registers that are used, we look into the transformations that take place in this example. In Table II, we see that from lines 1 to 5, Fredrik and Gustaf are using the algebraic register in the written and oral forms. We might confuse the algebraic register in its oral form with the verbal register, but it is important to remember that the oral form of the algebraic register follows the symbols and associations of algebra, instead of assigning meaning through verbal representation. The participants start to assign meaning to the algebraic representation with the verbal register as followed. In line 6, Fredrik writes the word mass and uses an arrow to relate the word mass with the letter m in the equation. This action indicates a change of register, from the algebraic register in its written form, to the verbal register in its written form. In this situation, the arrow acts as a transitional auxiliary representation to denote the conversion between registers (as seen in Fig. 7). From lines 6 to 12, we witness a series of conversions from the algebraic to the verbal register, both in written and oral forms. The students start assigning meaning to the algebraic register using the verbal register, this is the cognitive activity that underlies these conversions.

The example presented in Table III cannot be analyzed with the theory of registers of semiotic representations by itself, because precisely in this moment the students do not present cognitive activity by using the representation (Fredrik explicitly states "I am just copying the formula"). However, it is interesting to analyze what happened right before and immediately after this example.

Right before the transcript in Table III, the participants had the conversation presented in Table V. In line 13, Kim prompts the students to think of other representations for thermal energy, to which Gustaf responds with a verbal description and connects the concept with kinetic energy. In line 15, Kim asks the participants to draw this concept in some way, prompting for a conversion between the verbal description and a pictorial representation, a sketch. In lines 16 to 22, Hela makes this conversion between the verbal and pictorial register, since she explains how her sketch connects to the concept as she draws.

In line 23, Kim asks the participants to relate this sketch with a formula, to which Hela identifies there is a formula that relates temperature and kinetic energy. Then, the transcript in Table III takes place. Within this context, we can see that, even though the participants copy the formula from the textbook, they identified that there is a relation between this formula and Hela's sketch and verbal description. In this scenario, there is a conversion between three registers of representation: verbal, pictorial and algebraic registers. According to Duval, the cognitive activity requires the conversion between at least two or more registers [4].

Now let us see what happened immediately after Kim prompted the participants to think of a definition in line 31 of Table III. An excerpt of the conversation that followed is in Table VI. As we can see, Fredrik and Gustaf started converting from the algebraic to the verbal register by assigning a physical meaning to the elements of the equation, identifying whether they were variable or constant and what they meant. During this conversation, they not only assigned a meaning to the letter "f" in the algebraic register, they went on to "n" and "N" in Fig. 8. The rest of this conversation is omitted for its length.

TABLE V. Transcript of a conversion between verbal register and a sketch. Continuation of Table II.

13	Kim	Do you have any other way of representing thermal energy, except for this equation or formula?
14	Gustaf	I think that vibration of the inner molecules is a good way of explaining it as well. Kinetic, as we were saying first, the kinetic energy.
15	Kim	Can you draw it in some way?
16	Hela	I can try.
17		[Hela begins drawing a blue rectangle to represent a container]
18	Hela	I mean, if we have some container... That isn't a very straight line... and some molecules...
19		[Hela Draws some smaller shapes in the container, representing molecules. Hela adds more molecules to the container].
20	Hela	Then I guess I can try to show that they move back and forth.
21		[Hela adds "action lines" to some molecules]. See Fig. 9.
22	Hela	At least if it is... more solid
23	Kim	Another question I have:... What is the relationship between the box with the molecules and the formula?
24	Hela	I think, if you wanted to connect them, you'd have to take an extra step and use... I think we have a formula as well, for temperature in terms of kinetic energy.

It is really interesting to see how the social interaction between peers allowed them to create conversions together. In the TRSR, the analysis of treatments and conversions is usually done in the individual level, because it refers directly to cognitive activity. However, using it in a social interaction has allowed us to encounter an example where conversions can happen socially. This is an area of opportunity where social semiotics and the TRSR can complement each other to provide a bigger picture.

It is relevant to acknowledge that, in these examples, the cognitive activity was somehow prompted by the interviewer, which led to the conversions exemplified. This shows how with the right guidance and prompts, students can start engaging with the material and having cognitive activity through conversions. Moreover, the guidance and prompts can take place in a social environment, allowing instructors to include conversion in their active and collaborative learning design.

G. Results

Here we present the results from the theoretical comparison of the two frameworks and the application of both theories.

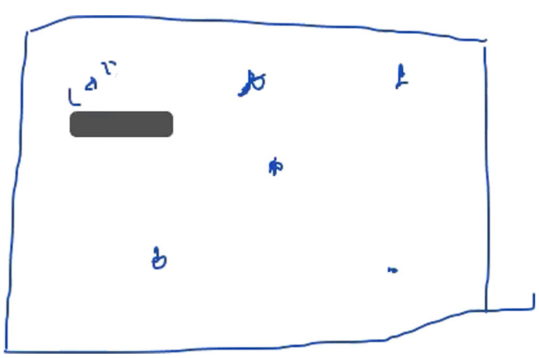

FIG. 9. Hela's sketch as constructed in Table V. Hela has just added action lines to the molecule on the top left corner; line 21 in the transcript.

We also provide a result that emerged from the comparison of the framework; the development and construction of new theoretical constructs within the frameworks.

From Sec. V D, we can see that both theories aim to identify how the students manipulate different representations of thermal energy and how they move between the different representations. In Table II, in combination with Fig. 7, we see a movement from speech to formula and Fredrik is engaging with the physical concepts of the exercise. In SS, we identified this movement as an active transduction and in TRSR we identify it as a conversion with recognition.

1. Results from the social semiotics analysis

SS provided us with a language to describe the students' manipulations of the different representations they interacted with, such as the transduction we see in Table II together with Fig. 7. From previous work (e.g., [9,25]) we know that transductions may help with the discernment of aspects. In Table IV we have applied the ideas of disciplinary discernment to describe Hela's discernment of the representation and its uses from a disciplinary perspective.

By describing the learning situation in terms of transformations, transductions, and disciplinary discernment levels, we gain a rich understanding of the learning situation. From the descriptions, we can identify what

TABLE VI. Transcript of a conversion between peers. Continuation of Table III.

31	Kim	And what does that formula say?
32	Fredrik	Well, it is the formula for internal energy... for an ideal gas
33	Gustaf	Well, it's change in internal energy right
34	Fredrik	Yeah, and we have a couple of constants...
35	Gustaf	Is the f also a constant? or what is the f?
36	Fredrik	That would be the frequency of the vibration
37	Gustaf	Right, yeah

translations prompted students to discern something in a new way and use this information to better understand the social construction of meaning and the potential problems that comes with learning physics. Using the knowledge of *what* translations afford, such as unpacking and highlighting aspects, interventions can be deployed that force students to grapple with specific aspects that they were unaware of or may find difficult. The intervention on line 31 in Table III is an example of applying this knowledge to make students grapple with the content of the formula.

2. Results from the semiotic registers analysis

The analysis using the TRSR yielded some interesting results. For instance, we found an example where we had both the verbal and algebraic register in their written and oral forms. This allowed us to identify clearly the characteristics of the algebraic and the written registers and how they are used by students in their cognitive activity. We identified that, when converting between the algebraic and the verbal registers, students assign meaning to the algebraic register by using the verbal register. This is evidence of cognitive activity, and of the relevance of the interaction of the verbal register with other representations [43,44].

We later found that one student presented a conversion between three registers of representation, showing synergy between the representations. To have synergy between the registers implies that students can dissociate the concept from the representation, and recognize the characteristics of the concept in several representations. This is evidence of understanding in the light of the TRSR. A new finding within this theory was to see conversions happening as social interactions between peers, and also when being prompted by an instructor.

H. Developing the frameworks

An unforeseen, but welcome, effect of this study was the publication of Ref. [10]. In which SS are expanded to better describe transductions. In TRSR, conversions are separated into conversions with recognition and; conversions without recognition. This was not the case in social semiotics, and the authors realized that it was an important distinction and introduced the notion of *active* and *passive* transductions.

VI. DISCUSSION

From the outset, we observed that the two theoretical frameworks could be closely related due to the fact that both frameworks appeared to have constructed similar theoretical constructs to explain similar phenomena in the learning space. For example, both frameworks identified the movement from formula to graph to be of importance for the learning process. The ideas of transduction, from SS, and conversion, from TRSR, are very similar and are an indication that this specific aspect of the learning situation is important.

Based on the analysis shown in this paper, we can conclude that the main difference between the frameworks comes from the lens they apply to describing the different theoretical constructs in each theory. SS applies a lens that aims to understand how communication and meaning making is made in specialized groups, using specially constructed semiotic resources. Thus, SS describes how these semiotic resources are used and what they afford in the physics discipline, and avoids describing what happens inside a person's head.

TRSR uses a lens of cognition in its analysis and ties the manipulation of representations to the understanding of different concepts. This is one of the differences of the two frameworks. However, when the frameworks are used to analyze data, the identification of different specific uses or manipulations of different representations looks almost the same, they use different words and slightly different division of concepts.

Another difference is the notion of single student versus groups of students. SS is, as its name suggests, a framework to describe the meaning making and communication in specialized groups. In SS, the notion of communication and the interaction between students becomes a central part of the analysis. However, TRSR is focused on a single student's manipulation of representations.

The third big difference is TRSR's focus on mathematical objects with no physical representation such as imaginary numbers. TRSR makes a distinction between representations of physical objects and ideas that can only be manifested through representations. With SS we can study the concept of friction using different types of semiotic resources such as push a block on a bench; free-body diagrams; formulas, but TRSR is only designed to study mathematical concepts with no physical representation.

A. Using both frameworks

Based on our analysis in this paper, we find that both theories can be used in parallel to describe learning situations where representations are used, such as in physics or mathematics. We may incorporate the social aspect of SS to study how TRSR describes the manipulation of representations in groups, such as the conversion that was prompted by the interviewer in Table VI. We may also apply the directionality of conversions from TRSR to the idea of transductions in SS to get a better understanding of the process itself. In an analytical sense, we do not see any problem to extracting useful concepts from one theory and applying them to the other. We suggest that researchers do this in order to obtain a richer description of the learning situation.

SS is often used together with variation theory of learning (VTL) [30,45–47] to analyze the learning situation. VTL provides a framework to deal with the recognizing and separating a concept from its representation and is related to the ADD. VTL provides a mechanistic description of the process of discernment and ADD provides a way to describe types of discernment.The findings of this study can be put into practice by using SS and TRSR combined with didactic strategies in regular physics classes, by placing the use of semiotic representations as the core objective of the didactic strategy. This would derive in different opportunities for research into practice and calls for studies of varied scopes.

VII. IMPLICATIONS

Both theories have very similar theoretical constructs that describe the same aspect of the learning situation. Based on this, it should be possible to make valid comparisons between studies performed using SS and TRSR, as long as you take care to account for the different lenses the theories use.

Based on the work done in this project, a paper outlining the ideas of Active and Passive transductions was written. The authors found an aspect in TRSR, namely the conversions with and without recognition, that could not be described in SS with the tools at the time. A direct consequence of this comparison was the further development of one of the frameworks. See Ref. [10] for the paper that was a direct result from this study. The development of more theoretical constructs to better describe the learning situation is mirrored in Ref. [13]. On page 170 they describe the construction of *general epistemic need* as a result of using different theoretical frameworks to describe the learning situation.

We thus suggest that these types of comparisons between similar theoretical frameworks are good in several aspects; they provide an overview of both theories and aim to pinpoint the difference between them; they also help researchers discover aspects in each framework that may need to be improved or added.

Based on our findings, we are also confident in that both frameworks can be used in tandem to provide a deeper description of the learning situation that captures the two different perspectives.

Another very important implication of this comparison is the realization that we can, by cross-examining different concepts of different theoretical frameworks, begin to discover fundamental pieces of what learning entails, or what makes learning possible. If the same ideas, or concepts, are present in both frameworks, this indicates that the concept itself may represent a fundamental piece of whatever the frameworks aim to describe. However, it may just be a necessary result based on some common assumption of the frameworks as well.

We also argue that, by combining both frameworks when describing the students' usage of representations in a physics education setting, pedagogical practices may stand on firmer grounds. Interventions and scaffolding [48] should be informed by an understanding of how students use representations and what the interaction between student and representation affords for learning.

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[1] M. A. K. Halliday, Language as social semiotic: The social interpretation of language and meaning, in *Language in Society*, edited by W. L. Ballard (Cambridge University Press, London, 1978), Vol. 9, pp. 84–89.

[2] J. Airey and C. Linder, Social Semiotics in university physics education, in *Multiple Representations in Physics Education*, edited by D. F. Treagust, R. Duit, and H. E. Fischer (Springer, Cham, 2017), Vol. 10, Chap. 5, pp. 95–122.

[3] T. van Leeuwen, *Introducing Social Semiotics* (Routledge, London, 2004), p. 301.

[4] R. Duval, A cognitive analysis of problems of comprehension in a learning of mathematics, Educ. Studies Math. **61**, 103 (2006).

[5] R. Duval and A. Sáenz-Ludlow, Un análisis cognitivo de problemas de comprensión en el aprendizaje de las matemáticas, Comprensión y aprendizaje en matemáticas: perspectivas semióticas seleccionadas **1**, 61 (2016).

[6] E. Campos, G. Zavala, K. Zuza, and J. Guisasola, Students' understanding of the concept of the electric field through conversions of multiple representations, Phys. Rev. Phys. Educ. Res. **16**, 010135 (2020).

[7] L. R. Pino-Fan, I. Guzmán, R. Duval, and V. Font, The theory of registers of Semiotic representation and the onto-semiotic approach to mathematical cognition and instruction: Linking looks for the study of mathematical understanding, in *Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics Education*, edited by K. Beswick, T. Muir, and J. Fielding-Wells (Hobart, Australia, 2015), pp. 33–40.

[8] J. D. Godino, C. Batanero, and V. Font, The onto-semiotic approach to research in mathematics education, ZDM Math. Educ. **39**, 127 (2007).

[9] K. Svensson and U. Eriksson, Concept of a transductive link, Phys. Rev. Phys. Educ. Res. **16**, 026101 (2020).

[10] K. Svensson, J. Lundqvist, E. Campos, and U. Eriksson, Active and passive transductions—definitions and implications for learning, Eur. J. Phys. **43**, 025705 (2022).
[11] R. Tytler and V. Prain, Supporting student transduction of meanings across modes in primary school astronomy, Frontiers Commun. **7**, 119 (2022).
[12] B. Waldrip, V. Prain, and J. Carolan, Using multi-modal representations to improve learning in junior secondary science, Res. Sci. Educ. **40**, 65 (2010).
[13] A. Bikner-Ahsbahs, T. Dreyfus, I. Kidron, F. Arzarello, L. Radford, M. Artigue, and C. Sabena, Networking of theories in mathematics education, in *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education*, edited by M. M. F. Pinto and T. F. Kawasaki (Organising Committees, PME 34, Belo Horizonte, Brazil, 2010).
[14] J. E. Joseph, *Saussure* (Oxford University Press, Oxford, 2012).
[15] C. S. Peirce, *The Essential Peirce: Selected Philosophical Writings*, edited by the Peirce Edition Project (Indiana University Press, Bloomington, IN, 1998), Vol. 2.
[16] M. T. Chi, P. J. Feltovich, and R. Glaser, Categorization and representation of physics problems by experts and novices*, Cogn. Sci. **5**, 121 (1981).
[17] P. B. Kohl and N. D. Finkelstein, Student representational competence and self-assessment when solving physics problems, Phys. Rev. ST Phys. Educ. Res. **1**, 010104 (2005).
[18] T. S. Volkwyn, J. Airey, B. Gregorcic, and C. Linder, Developing representational competence: Linking real-world motion to physics concepts through graphs, Learning: Res. Pract. **6**, 88 (2020).
[19] S. Saleh Mozaffari Chanijani, S. Saqib Bukhari, P. Klein, J. Kuhn, M. Al-Naser, and A. Dengel, A study on representational competence in physics using mobile eye tracking systems, in *Proceedings of the 18th International Conference on Human-Computer Interaction with Mobile Devices and Services Adjunct* (Association for Computing Machinery, New York, 2016), https://dl.acm.org/doi/proceedings/10.1145/2957265.
[20] P. B. Kohl and N. D. Finkelstein, Patterns of multiple representation use by experts and novices during physics problem solving, Phys. Rev. ST Phys. Educ. Res. **4**, 010111 (2008).
[21] M. D. Cock, Representation use and strategy choice in physics problem solving, Phys. Rev. ST Phys. Educ. Res. **8**, 020117 (2012).
[22] E. T. Torigoe and G. E. Gladding, Connecting symbolic difficulties with failure in physics, Am. J. Phys. **79**, 133 (2011).
[23] K. Svensson, U. Eriksson, and A.-M. Pendrill, Programming and its affordances for physics education: A social semiotic and variation theory approach to learning physics, Phys. Rev. Phys. Educ. Res. **16**, 010127 (2020).
[24] K. Svensson and U. Eriksson, Concept of a transductive link, Phys. Rev. Phys. Educ. Res. **16**, 026101 (2020).
[25] T. S. Volkwyn, J. Airey, B. Gregorcic, and F. Heijkenskjöld, Transduction and science learning: Multimodality in the physics laboratory, Designs for Learning **11**, 16 (2019).
[26] J. Airey and C. Linder, A disciplinary discourse perspective on university science learning: Achieving fluency in a critical constellation of modes, J. Res. Sci. Teach. **46**, 27 (2009).
[27] V. Prain and R. Tytler, Learning through constructing representations in science: A framework of representational construction affordances, Int. J. Sci. Educ. **34**, 2751 (2012).
[28] R. Tytler, V. Prain, P. Hubber, and B. Waldrip, *Constructing Representations to Learn in Science*, edited by T. Russell, P. Vaughan, H. Peter, and W. Bruce (Sense Publishers, Rotterdam, 2013), pp. 1–209.
[29] U. Eriksson, Disciplinary discernment: Reading the sky in astronomy education, Phys. Rev. Phys. Educ. Res. **15**, 010133 (2019).
[30] M. Eriksson, U. Eriksson, and C. Linder, Using social semiotics and variation theory to analyse learning challenges in physics: A methodological case study, Eur. J. Phys. **41**, 065705 (2020).
[31] J. Airey and U. Eriksson, Unpacking the Hertzsprung-Russell diagram: A social semiotic analysis of the disciplinary and pedagogical affordances of a central resource in astronomy, Designs for Learning **11**, 99 (2019).
[32] J. Airey, U. Eriksson, T. Fredlund, and C. Linder, On the Disciplinary Affordances of Semiotic Resources, in Book of Abstracts: The First Conference of the International Association for Cognitive Semiotics (IACS) (Lund University, 2014), pp. 54–55.
[33] C. Linder, Disciplinary discourse, representation, and appresentation in the teaching and learning of science, Eur. J. Sci. Math. Educ. **1**, 43 (2013).
[34] G. Kress, *Multimodality: A Social Semiotic Approach to Contemporary Communication* (Routledge, London, 2009).
[35] G. Kress, C. Jewitt, J. Ogborn, and T. Charalampos, *Multimodal Teaching and Learning: The Rhetorics of the Science Classroom*, Bloomsbury Classics in Linguistics (Bloomsbury Publishing, New York, 2014).
[36] J. Bezemer and G. Kress, Writing in Multimodal Texts, Written Commun. **25**, 166 (2008).
[37] U. Eriksson, C. Linder, J. Airey, and A. Redfors, Introducing the anatomy of disciplinary discernment: An example from astronomy, Eur. J. Sci. Math. Educ. **2**, 167 (2014).
[38] E. G. Guba and Y. S. Lincoln, Epistemological and methodological bases of naturalistic inquiry, Educ. Commun. Technol. **30**, 233 (1982).
[39] T. S. Volkwyn, J. Airey, B. Gregorcic, and F. Heijkenskjöld, Multimodal transduction in upper-secondary school physics, ISEC 2018, NIE, Singapore, in 8th International Conference on Multimodality, Cape Town, South Africa, 2016 (2018), https://www.diva-portal.org/smash/get/diva2:1081896/FULLTEXT01.pdf.
[40] E. Patron, C. Linder, and S. Wikman, Qualitatively different ways of unpacking visual representations when teaching intermolecular forces in upper secondary school, Sci. Educ. **105**, 1173 (2021).
[41] T. Fredlund, C. Linder, and J. Airey, A social semiotic approach to identifying critical aspects, Int. J. Lesson Learning Studies **4**, 302 (2015).
[42] T. Fredlund, J. Airey, and C. Linder, Enhancing the possibilities for learning: variation of disciplinary-relevant aspects in physics representations, Eur. J. Phys. **36**, 055001 (2015).

[43] M. G. Caligaris, M. E. Schivo, and M. R. Romiti, Communication competence in mathematics: Analysis of the evolution of calculus student skills throughout their freshmen year, Int. J. Learning, Teaching Educ. Res. **18**, 49 (2019).

[44] K. Ramatlapana and M. Berger, Prospective mathematics teachers' perceptual and discursive apprehensions when making geometric connections, African J. Res. Math. Sci. Technol. Educ. **22**, 162 (2018).

[45] F. Marton, Phenomenography: A research approach to investigating different understandings of reality, Qual. Res. Educ.: Focus and Methods **21**, 141 (1986).

[46] M. Ling Lo, Variation theory and the improvement of teaching and learning, Ph.D. thesis, Gothenburg studies in educational sciences, Gothenburg University, Gothenburg, 2012.

[47] F. Marton, *Necessary Conditions of Learning* (Routledge, New York, 2015).

[48] I. Verenikina, Understanding scaffolding and the ZPD in educational research, in *Proceedings of the International Education Research Conference (AARE - NZARE), Auckland, New Zealand* (2003), https://ro.uow.edu.au/edupapers/381/.