Conceptual challenges with the graphical representation of the propagation of a pulse in a string

Muhammad Aswin Rangkuti[1,2,*] and Ricardo Karam[1]

[1]*Department of Science Education, University of Copenhagen, Universitetparken 5, Copenhagen 2100, Denmark*

[2]*Department of Physics, Universitas Negeri Medan, Jl Willem Iskandar Psr V, Medan 20211, Indonesia*

(Received 31 March 2022; accepted 25 July 2022; published 21 September 2022)

Student difficulties with making sense of graphs in physics have been thoroughly reported. In the study of one-dimensional waves, the issue is even trickier since the amplitude is a function of two variables (position and time). In this work, we investigate students' reasoning and difficulties with interpreting the graphical representation of the propagation of a pulse in a string. A profile $y(x)$ of the pulse was provided and students were asked to estimate the velocities of several points at the profile. This forced them to consider the time dimension, by focusing their attention on the motion of these points. This turned out to be extremely challenging to the students, who manifested several conceptual challenges which were categorized and analyzed in the first phase of the study. Based on these findings, three levels of scaffolding support were provided, where each level gradually guided the students to draw the wave profile after some time has elapsed. The scaffolding turned out to be effective, since many students managed to identify the new positions of the points successfully. The study reveals how static representations of intrinsically dynamic phenomena can be challenging for students to grasp.

DOI: 10.1103/PhysRevPhysEducRes.18.020119

I. INTRODUCTION

Graphical representations are widely used as powerful tools to represent concepts and phenomena in physics. In fact, the lack of understanding of graphical representations is often an issue of concern to physics education research (PER). The literature is vast in both the identification of misconceptions and the development of instructional strategies to circumvent them in a variety of topics such as kinematics, thermodynamics, and electrodynamics [1–4].

In wave phenomena, graphical representations are particularly challenging because the mathematical description of waves involves a function of two variables, position x and time t. In the one-dimensional case, this function is generally expressed as $y(x,t) = f(x \pm vt)$, which is not always treated in mathematics lessons and is quite difficult to grasp. Although one can choose to represent the dependence of the vertical displacement y on each of the variables x and t separately, it is crucial to understand that they are related. The function $y(t)$ describes the movement of a given particle (fixed x) when time is progressing, whereas the function $y(x)$ describes an instantaneous configuration of a wave, like a screenshot. Investigating how students try to make sense of these conceptual subtleties is the main goal of this study.

The PER literature is also comprehensive in terms of studies investigating student difficulties with wave phenomena. For example, Sadler *et al.* [5] found that students struggled to distinguish between vertical particle motion and horizontal wave propagation. For the case of transverse waves, students often concluded that matter was transported in the direction of wave propagation. Similar findings also showed that most of the students believed the particles in the air were pushed together towards the direction of motion when a sound wave is traveling [6,7]. These misconceptions occurred because students tend to treat waves as objects and use that reasoning to solve problems [6–9]. Furthermore, some students struggled to distinguish between a mathematical representation and a physical situation, e.g., most students treat the relation between velocity, wavelength, and frequency of periodic waves $v = \lambda f$ mathematically without considering how each variable is related physically [8,10].

In this paper, we explore how university physics students understand graphical representations of waves in a manner which goes beyond other studies in the literature [1–4,11,12]. More specifically, the topic of this study differs from previous ones because most of them investigated students' reasoning in the context of periodic waves [8,10,13–15]. Here, we focus on students' ability to distinguish between the horizontal movement of a pulse and the vertical motion of matter on a nonperiodical wave

*aswin.rangkuti@ind.ku.dk

profile. In particular, we provide students with the profile $[y(x)]$ of a pulse and analyze whether they can estimate the velocity of some points on the graph, therefore asking them to reason about time evolution. After assessing students' conceptions, three increasingly detailed scaffoldings were provided to see if or how they improved students' performances when solving the task.

II. PHASE I: STUDENTS' REASONING ABOUT THE GRAPHICAL REPRESENTATION OF A PULSE

A. Materials

Our investigation is based on one conceptual question, designed to explore students' understanding of the relationship between the vertical motion of the points on a string, with the (horizontal) propagation of a pulse with constant velocity in this string. Four points were located on a pulse and students were asked to (a) sort out the magnitude of their velocity, and (b) estimate whether the velocity of each point is > 0, < 0, or $= 0$.

> ***Question:*** *A pulse is moving horizontally with constant speed to the right. The profile below represents a given instant, like a picture (Fig. 1). As the pulse moves horizontally, the points move vertically (wave does not transfer matter)*
> (a) Based on the picture, sort the magnitudes of the (vertical) velocity at each point from the greatest to the smallest. Explain your reasons.
> (b) For each point, determine whether the velocity is $< 0, > 0$, or $= 0$. Explain your reasons.

From the expert perspective, one way to solve this question is to draw another profile after some time has elapsed. Figure 2 shows two wave profiles at two different instants.

In Fig. 2, the dotted profile represents the pulse after a short time interval and the red dots show the new vertical positions of the points. It can be seen that point 4 has the greatest speed because it has the greatest displacement

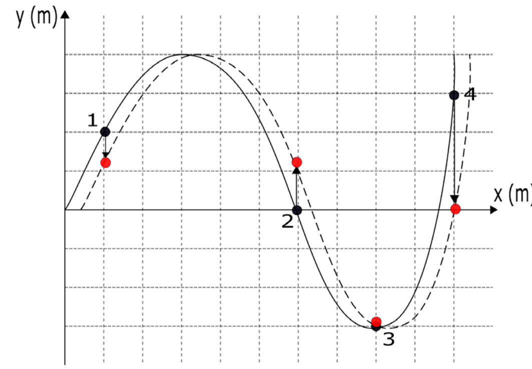

FIG. 2. A graph of two wave profiles at two instants.

compared to other points. Therefore, point 4 requires a greater speed to attain the final position. With this justification, the correct answer for question (a) is 4–2–1–3.

To answer question (b), one needs to see the motion direction of each point. The downward displacement means a negative velocity ($v < 0$) and the upward displacement means a positive velocity ($v > 0$). Using this approach, however, the challenge to answer the velocity for point 3 is inevitable because the point has already moved upward in the new profile. This is because the question asks about the velocity in the initial profile, not when the pulse move after some time has elapsed. In this case, point 3 has zero velocity. Thus, the correct answer for question (b) is that velocities of points 1 and 4 are smaller than zero, the velocity of point 2 is greater than zero, and of point 3 is equal to zero.

In fact, the expert can solve this question (a) by only drawing the slope of each point on the graph to find the speed. However, it is worth saying that the slope of $y(x)$ cannot be treated to find the velocity without knowing the relation between the slopes of $y(x)$ and $y(t)$. The slope of $y(x)$ only addresses the shape of the pulse, but indeed there is a proportionality between the slope of $y(x)$ and $y(t)$. Thus, one can infer the velocity based on the slope of each point on the graph of $y(x)$ using this relation. Figure 3 shows the slope of each point on the graph.

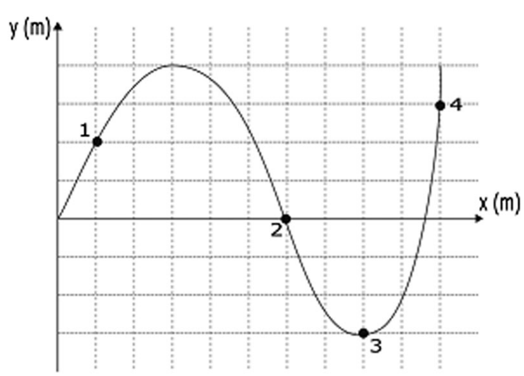

FIG. 1. Problem graph.

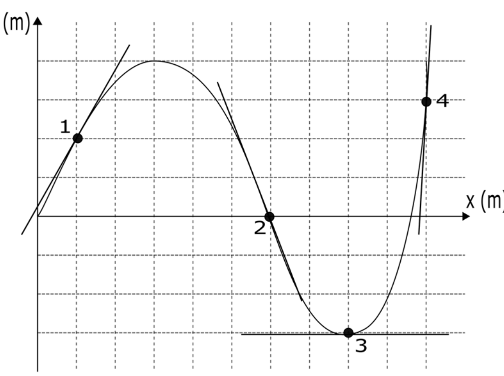

FIG. 3. Solving question (a) by drawing slope.

TABLE I. Students' conceptual challenges in phase I.

Question	Conceptual challenge	Category	Number of students
Part (a)	Difficulty in reading distance on the graph	The roller coaster erroneous reasoning	6
		Inaccurate assumption in reading vertical displacement	6
		Using horizontal position	7
		The wave profile represents wavelength	4
		Using periodical wave formulas	9
		Total	32
Part (b)	Dividing the graph into positive and negative parts		12
	Mixing between the roller coaster reasoning and dividing the graph into positive and negative parts		20
		Total	32

Based on Fig. 3, it is clear that point 4 has the greatest slope which results in the greatest speed. Therefore, using this justification, we can also find the correct answer to question (a) is 4–2–1–3.

B. Interviews

In order to explore students' reasoning as fruitfully as possible, we performed paired semistructured interviews with 32 physics students from two Indonesian universities. They were in the second or third year of their studies and all of them had already taken introductory physics courses, including basic notions of wave phenomena. Third year students, especially, had already completed an advanced course on waves.

Arksey and Knight [16] explain that paired interviews have the benefit to bridge the gap between pairs. Consequently, this condition will engage participants in elaborating more on their answers and gaining more interactions during the discussion setting. This type of interview also pushes the participants to work together to answer the questions that they might not be able to respond to individually [17]. Moreover, Houssart and Evens [18] suggest that paired interviews will be more beneficial for unseen questions, meaning that the questions are first encountered at the beginning of the interview. This should provide a collaborative working environment and encourage students to see alternative views from their answers.

We followed the interview procedures based on what has been recommended by the literature. We interviewed students in pairs, and posed the interview question for the first time at the beginning of the interview. They read the questions in a couple of minutes and eventually ask clarification questions to the interviewer. Students first responded individually to the question, and then a discussion in pairs began. In these discussions, students could defend or change their prior reasoning based on each argument from their peers. In the end, we gathered one agreed final answer from the pairs or individual answers if they could not reach an agreement.

C. Results

We found that *no* student was able to answer the questions correctly. Their reasons were diverse and their conceptual challenges are categorized based on the common difficulties encountered. Table I shows categories of students' conceptual challenges in phase 1.

In general, we found only one category of students' struggles to answer question (a), which is related to their difficulty to read the appropriate information of distance on the graph but with varieties of conceptual challenges. Meanwhile, the nature of students' reasoning to answer question (b) was based on identifying the position in Cartesian coordinates without considering other physical aspects within it.

1. Question part (a)

Difficulty in reading distance of each point on the graph of $y(x)$.—A few students translated the wave profile as a motion trajectory and used this notion to determine the displacement between two points. We call this the "roller coaster" erroneous reasoning. However, the way this error appeared differed among students. For example Diana[1] divided the profile into four parts and conceived four different motions, the origin (0, 0) moving to point 1, point 1 to point 2, point 2 to point 3, and point 3 to point 4. Point 4 will keep moving upwards. She then related her notion with the proportionality relationship between velocity and displacement. She answered that point 4 has the greatest velocity because it has the greatest trajectory from point 3 to 4. Using that notion, point 2 could also be considered having the greatest trajectory moving from point 1 to 2. When asked about this, she said that she

[1]Student names are pseudonyms.

measured the motion trajectory of point 2 from the crest of the wave profile because she assumed that "the point would start its new movement in that position." She concluded her reasoning by answering 4–2–3–1 for part (a). The following is Diana's reasoning:

> *Diana: The greater displacement of a point will result in a greater velocity. Point 4 has the greatest trajectory moving from point 3 to point 4, so point 4 has the greatest velocity. Point 2 is the second order because I calculate its trajectory from the crest of the wave profile. Next is point 3 because it is moving to point 4. Point 1 is the smallest because it has the shortest trajectory moving from point (0, 0).*

Another interviewed student, Angela, also assumed that the points on the wave are moving like a roller coaster. However, unlike Diana, she did not measure the motion trajectory of point 2 from the crest to point 3 but from the whole profile from point 2 to point 3. She then decided that point 2 has the greatest speed and sorted the magnitude of speed from the greatest to smallest as 2–4–3–1. Here we see elements of a stronger roller coaster reasoning, as she sometimes relates higher speeds with regions of lower potential energy.

Other students referred to the idea of vertical displacements to determine the velocity, but with inaccurate assumptions. One of the students observed the distance of each point to the *x-axis*. She said that the greater the distance of each point to the *x axis*, the greater its velocity and her answer for this question was 4–3–1–2. Julia, also one of our interviewed students, estimated the value of grid lines of 1 m on the *y axis* and then divided the graph into positive and negative parts.

> *Julia: If we assume each grid line on the y axis represents 1 m then point 1 has a distance of 2 m, point 2 is 0, point 3 is* -3 m*, and point 4 is 3 m. So the answer is 4–1–2–3.*

Surprisingly, we found some students estimated the horizontal position to determine the distance of each point. They said that the further away a point is from the origin (0, 0) horizontally, the greater its velocity. One of the student's reasoning is shown below:

> *Doddy: The answer is 4–3–2–1 because velocity is proportional to the distance based on the velocity formula, which is* $v = s/t$*. Velocity in point 4 is the greatest because it is located furthest compared to other points.*

We do not assume that Doddy considered points on the wave to move horizontally because he did not state any displacement of points to answer the question. Even though this reasoning is simple to understand, using horizontal displacement seems to be in contradiction to the nature of motion of particles on the string.

Assuming that a wave is always periodic.—Almost half of the students assumed that the wave profile in the question is periodic, even though this was not mentioned in the question. Although their primary goal was to find distances related to each point on the graph, these students associated the distances with wavelengths. We categorized this erroneous reasoning as the "periodicity fixation."

Ivan, for instance, assumed that the movement of each point starts from the origin (0, 0) and follows the wave profile until it reaches its respective position. This reasoning is also related to the conceptual challenge of the roller coaster. However, it was more plausible to place it into periodicity fixation because he continuously referred to the notion of wavelength in his answer. He conjectured that the distance from each point to the origin (0, 0) determines the magnitude of its wavelength. He then associated it with the proportionality between wavelength and velocity. He said that the greater the wavelength of a point [sic] the greater its velocity. With this notion, he decided that point 3 has the greatest velocity due to its greatest wavelength. This point has a $3/4$ wavelength because it consists of one hill and a half valley. Using hills and valleys to determine the wavelength is common when students learn periodic waves in these universities; one wavelength consists of one hill and one valley.

Ivan's reasoning became more complicated because of his notion of hill and valley. Paradoxically, he did not consider the whole wave profile to determine its wavelength, but asserted a different wavelength to each point of the profile. Moreover, he argued that point 4 is located in a new wavelength, therefore it has the smallest speed. The following is Ivan's reasoning for question part (a):

> *Ivan: The order is 3–2–1–4. I calculate the distances of each point to the origin (0, 0) to determine their wavelength. Point 3 is the greatest because it has a* $3/4$ *wavelength, point 2 has a half wavelength, and point 1 has less than a half wavelength. Point 4 is the smallest because it is located in the new wavelength.*

This type of conceptual challenge can also be seen from Johan's reasoning. He actually understood that the points on the wave move vertically, but he believed that the wave profile in the question is a sine wave. Then, he used a sine wave function $y = A \sin(kx - \omega t)$ to find a formula for velocity, as depicted in Fig. 4.

Johan finally arrived at the velocity formula $v = \omega\sqrt{A^2 - y^2}$. He then determined the magnitude of velocity using those two variables, amplitude (A) and vertical displacement (y). He said that the magnitude of velocity is maximum at $y = 0$ and minimum at $y = A$. With that analysis, he found that point 2 has the greatest velocity because $y = 0$ and point 3 has the smallest velocity because

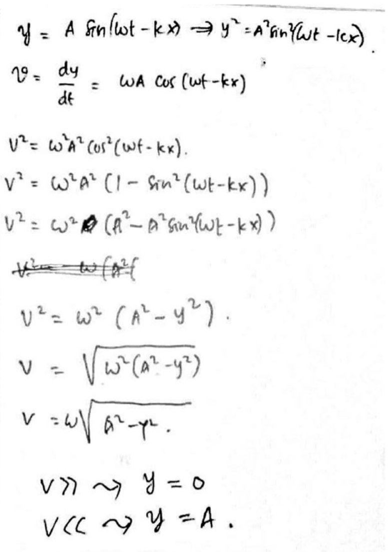

FIG. 4. Johan's derivation of a periodical wave to estimate the magnitude of the speed.

$y = A$. In the case of a periodical wave, Johan's reasoning is correct because point 2 is located at the inflection point. However, the wave profile in the question is not periodic. For point 3, in particular, his answer is correct because this point is located precisely in the crest and thus has zero velocity. Finally, he said point 1 has a greater velocity than point 4 because it has a smaller vertical displacement.

Similar to Johan, Adi also operated the concept of periodic waves to solve the problem. However, his method was based on the acceleration formula $a = -k^2 y$. He assumed that the magnitude of speed can be estimated by using the proportionality relation between acceleration and velocity. The greater the vertical displacement of each point, the greater its acceleration, resulting in greater velocity. He focused on the vertical displacement of each point and measured it based on the displacement of each point to the *x axis*. With this assumption, he then answered point 4 has the greatest speed. We asked him to clarify his answer since point 3 and point 4 have the same distance to the *x axis*. He then also considered the horizontal position of a point. Point 4 is located further horizontally than point 3, so that point 4 has the greatest speed. Adi's reasoning can be seen below:

> *Adi: I will use the formula of acceleration which is $a = -k^2 y$ because of the proportionality relation between acceleration and velocity. So, the greater y of a point will result in a greater acceleration, which also produces a greater velocity. The velocity at point 2 is zero because $y = 0$.*

With slightly different reasoning, Edy immediately noticed that the velocity at point 3 is zero by saying it is located at the position when a point will move between up and down. This reasoning was undoubtedly correct. He then noticed that point 2 is located at the inflection point and concluded that point 2 has the highest velocity. Again, this answer could be valid if the wave profile in the problem were a periodic wave. Even though Edy never stated any formula regarding a periodical wave, we infer that he also has a periodical wave fixation by his answers to the velocities of points 2 and 3.

2. *Question part (b)*

Dividing graph into positive and negative parts.—More than half of the students just simply labeled the Cartesian coordinates into negative and positive parts. The velocities of points located above the *x axis* are positive, and below the *x axis* are negative. Meanwhile, the velocity of points located exactly at the *x axis* is zero. The following is one of the student's reasoning related to this conceptual challenge:

> *Johan: The velocity of point 1 and 4 are greater than zero because they are located above of x axis, so their magnitudes must be positive. The velocity of point 2 is equal to zero because it is exactly located on the x axis. The velocity of point 3 is smaller than zero because it is located below the x axis, so its magnitude must be negative.*

Students in this group merely applied the position of each point based on Cartesian coordinates instead of considering the direction of each point when it is moving. We notice that the majority of students that hold this conceptual challenge also had a false assumption of vertical displacement to answer question (a).

Mixing between the roller coaster reasoning and dividing the graph into positive and negative parts.—Almost half of the students had a conceptual challenge by mixing two different notions to answer this question. First, they claimed that the area above the *x axis* is positive and the area below the *x axis* is negative. Then, they combined that notion with their incorrect interpretation of a moving point, the so-called roller coaster reasoning. Here is an example:

> *Indra: The velocity of point 1 is greater than zero because it is moving up to the crest of the hill. So, it requires velocity to climb the hill. I can also see that point 1 is located in the positive area of Cartesian coordinate. The velocity of point 2 is zero because it is located on the x axis. The velocity of point 3 is smaller than zero because it is located in a valley (moving down). For the same reason as point 1, the velocity of point 4 is bigger than zero.*

This group's reasoning can be associated based on how their method solves question (a). Indra, for example, reasoned that the points on the wave will move along the wave profile. Because of this conceptual challenge, his approach to answer question (b) was affected by this error. He said that points 1 and 4 are moving up because they are located at the upward profile whereas point 3 is moving down because it is located in the downward profile. Then, he compounded the error by saying that the area above the *x axis* is positive and the area below the *x axis* is negative. Indra mixed these two conceptual challenges to solve part (b). Points moving upward (point 1 and 4) will move into the positive area, so their velocities must be greater than zero. A point moving down (point 3) will move into the negative area, so its velocity must be smaller than zero. For point 2, however, he concluded it had a zero velocity because it is located at the *x axis*. Because of this, Indra's reasoning seems incoherent because he only analyzed the position of point 2 instead of applying his two conceptual challenges like he did when analyzing the other points. However, Diana, who also mixed these two aspects, indicated that velocity of point 2 is smaller than zero because it is located at the downward profile. Thus, her reasoning seems more coherent.

III. PHASE II: SCAFFOLDING SUPPORT

A. Methodology

The result from the questionnaire made it clear how challenging the posed question was to the students, which motivated us to develop instructional strategies to see if they could understand the basic conceptual issues. Three levels of scaffolding were implemented and 22 students who answered the questionnaire were selected randomly to participate in this stage, where, once again, semistructured interviews were conducted in pairs.

Methodologically, our study is similar to the one conducted by Maries *et al.* [19] who developed scaffoldings to reduce student difficulties with Gauss's law. Our scaffoldings were also designed to incorporate the experts' approach to solve the problems [20,21]. However, our supports were slightly different because we did not provide a complete explanation, but only minor hints to the students. We expected that students could build their understanding and answer the questions based on their own analysis.

The goal of these interventions was to lead the students to draw the wave profile after some time has elapsed to reveal possible changes in students' reasoning to solve the problem. Under these interventions, we expected that students could notice the displacements of each point by comparing two wave profiles at two instants to solve the problem.

Scaffolding is associated with providing suitable support to a learner to overcome something that is difficult to achieve [22]. Originally, the term scaffolding was used to describe a series of steps for a learner to achieve a better performance [23]. Nowadays, scaffolding is used as an intervention to help not only an individual person, but also pairs and teams in many fields, including physics [24,25].

1. Scaffolding level 1

The purpose of scaffolding level 1 is to provide an illustration to the students of the characteristics of the wave profile after a short time interval. The interviewer demonstrated physically with his hands how to create a single pulse on a string that is moving to the right with constant velocity. Students were then asked to draw the next wave profile after some time has elapsed on the graph in the question. Students were also asked to locate the displacement of points in the new wave profile. Students who failed to draw the correct wave profiles in this stage were given intervention level 2.

2. Scaffolding level 2

In this level, the PhET simulation called "wave on a string" [26] was introduced to the students. This simulation presents the real condition of a vibrating string, and it has a variety of features that are suitable to our wave profile. This simulation can be modified into different situations, for example, showing how a string oscillates with or without reflections. The vibration source can be created manually with the possibility of adjusting damping and tension. Moreover, if the users want to see the movement on the string in detail, a slow-motion feature can be applied to the system.

Students were asked to use the simulation to reproduce the pulse that was given in the question, and they were left to explore the simulation without any help. Students who were able to generate the pulse in the simulation were asked to draw the new wave profile again after a short period of time. Then they were asked once again to answer the question. Even though some students did not create the same wave profile, we asked them to answer the same question because we wanted them to realize that the shape of the wave remains the same when it is progressing. Students who failed to use the appropriate features in the simulation were given intervention stage 3.

3. Scaffolding level 3

In this final support, we showed to the students how to create a pulse moving to the right with a constant velocity. They were instructed to use a manual vibration source, set the damping to zero, choose no-end string, and use the slow-motion feature to see the vibration in detail. After successfully creating the correct simulation, they were asked once again to draw the wave profile after some time has elapsed and then answer the questions once more.

B. Results

Students' performance in the scaffolding environment was diverse at each level with noticeably scaffolding level 1

TABLE II. Students' results in the scaffolding (SCL) environment for question (a) ("1" indicates that students answer the question correctly, "0" indicates that students answer the question incorrectly).

Pair	Student	SCL I	SCL II	SCL III	Drawing
1	1	0	0	0	Failed
	2	0	0	0	Failed
2	3	0	0	1	Succeeded
	4	0	0	1	Succeeded
3	5	0	0	0	Succeeded
	6	0	0	0	Succeeded
4	7	0	0	0	Failed
	8	0	0	0	Failed
5	9	0	0	0	Succeeded
	10	0	0	0	Succeeded
6	11	0	0	0	Succeeded
	12	0	0	0	Succeeded
7	13	0	0	1	Succeeded
	14	0	0	1	Succeeded
8	15	0	1	0	Succeeded
	16	0	1	0	Succeeded
9	17	0	1	0	Succeeded
	18	0	1	0	Succeeded
10	19	0	1	0	Succeeded
	20	0	1	0	Succeeded
11	21	0	0	0	Failed
	22	0	0	0	Failed

TABLE III. Students' results in the scaffolding environment for question (b) (1 indicates that students answer the question correctly, 0 indicates that students answer the question incorrectly).

Pair	Student	SCL I	SCL II	SCL III	Drawing
1	1	0	0	0	Failed
	2	0	0	0	Failed
2	3	0	0	1	Succeeded
	4	0	0	1	Succeeded
3	5	0	0	0	Succeeded
	6	0	0	0	Succeeded
4	7	0	0	0	Failed
	8	0	0	0	Failed
5	9	0	0	1	Succeeded
	10	0	0	1	Succeeded
6	11	0	0	0	Succeeded
	12	0	0	0	Succeeded
7	13	0	0	1	Succeeded
	14	0	0	1	Succeeded
8	15	0	1	0	Succeeded
	16	0	1	0	Succeeded
9	17	0	1	0	Succeeded
	18	0	1	0	Succeeded
10	19	0	1	0	Succeeded
	20	0	1	0	Succeeded
11	21	0	0	0	Failed
	22	0	0	0	Failed

being very challenging for the students. Tables II and III show students' results in the scaffolding environment.

These results show that students' performance has improved by looking at their success in answering the question with correct reasoning after scaffolding level II. Although some students still hold robust erroneous views, the complex understanding of graphs of $y(x)$ and $y(t)$ appeared to be solved by some of our students.

1. Students' results for scaffolding level 1

We found that all students had difficulties imagining the nature of a pulse moving to the right with a constant velocity. Most of the students drew the new wave profile smaller because they said that the wave will lose its energy after moving a bit and its amplitude will diminish slowly. Figure 5 shows Citra's drawing exemplifying this difficulty.

Based on her drawing, Citra's difficulties were not only due to the notion of losing energy, but that she also struggled with locating points in the new profile. She started drawing the new profile from the origin and assumed that the points on the wave would only oscillate in the fixed *x-axis* position (except point 1). The way she located the new positions of the points also seems inconsistent. When asked for the reason for that choice, she simply said that she located the points randomly in the new wave profile.

Hendri, another of our interviewed students, also thought that the wave will lose its energy. However, unlike Citra, he started drawing the new wave profile after a short time interval to the right from the initial profile. Figure 6 shows Hendri's drawing in scaffolding level 1.

His choice to locate the new positions of the points was based on his conceptual challenge to answer question (a). Based on Fig. 6, he implemented his notion of roller coaster reasoning to locate the displacement of points on the new wave profile. He thought that points on the wave are moving along the wave profile and the numbers with prime symbols indicated this conceptual challenge.

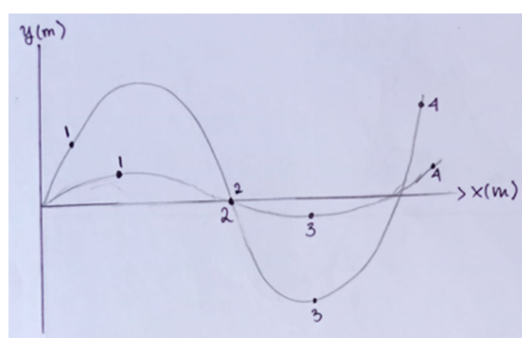

FIG. 5. Student's drawing with the next profile become smaller.

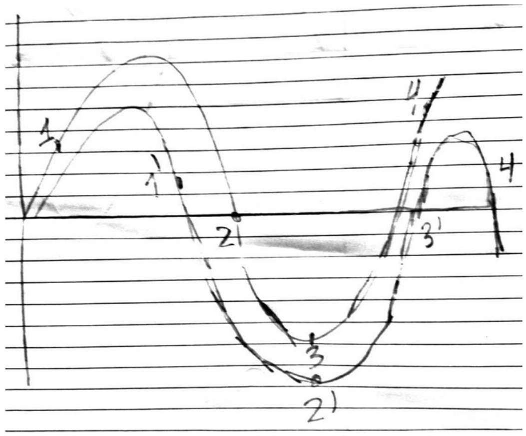

FIG. 6. Hendri's drawing in scaffolding level 1.

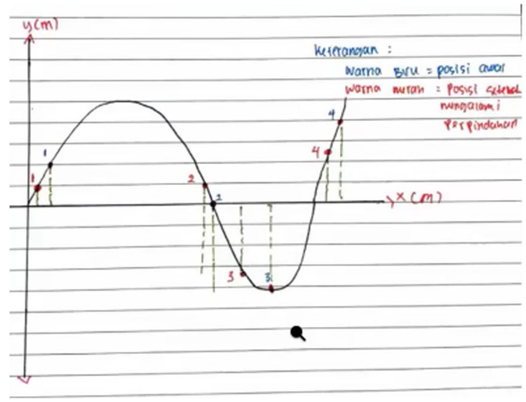

FIG. 8. Yuda's difficulty drawing two wave profiles in one graph.

A few students could not draw two wave profiles separated by a short time interval. Ivan, for example, could only visualize the next wave profile after it moves a complete wavelength. He drew the next wave profile from the end of the original profile, which can be seen in the red profile from Fig. 7.

Based on Ivan's drawing, it is impossible for him to imagine a wave profile after a short time interval. The way he located points on the red profile also indicated he had a roller coaster erroneous view. For question (a), we categorized Ivan's reasoning in the periodical wave fixation because he analyzed the problem by considering the wavelength to solve it. However, in this stage, we also found that Ivan holds a roller coaster reasoning.

One of our interviewed students, Yuda, recognized that the wave profile will always be identical when it is progressing. Only the points will be moving up and down vertically. However, he did not have a picture how to draw the two wave profiles in one graph. Yuda's drawing can be seen in Fig. 8.

Figure 8 shows that Yuda did not manage to draw the new profile after a short time interval. He only drew one wave profile and located the points at two different conditions. The blue dots represent points at the original profile and the red dots represent the displacement of points after a short time interval. In the beginning, we thought that he had a roller coaster reasoning because the points appeared moving along the wave profile, but this was not his reasoning. He said that point 1 will be moving down, point 2 will be moving up, point 3 will be moving up, and point 4 will be moving down after some time has elapsed. Points on the wave only move vertically and his reasoning regarding this was correct. When we asked him to draw once again the two wave profiles in one Cartesian coordinate, he was still puzzled about how to do that.

Some students understood that the shape of the wave profile will be identical when it is progressing. The difficulty arrived when they located the displacement of points on the new wave profile. Figure 9 shows that Edy could draw two wave profiles at two instant times and the new wave profile is represented with the dotted line. The displacement of points was still inaccurate except point 3 which was located correctly, showing that point 3 is moving upward vertically. However, his reasoning regarding the motion direction of the points was correct. We also noticed that the dotted profile he drew looked like a repeating continuous pattern which is the characteristic of periodical waves.

On the other hand, Indra drew his new wave profile as if it was traveling as shown in Fig. 10. This result came

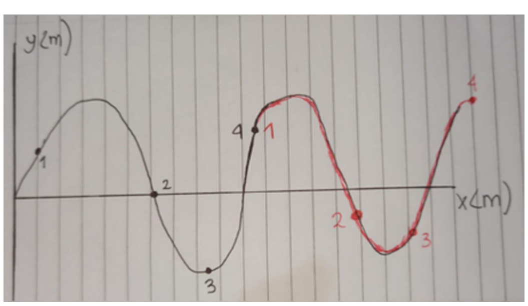

FIG. 7. Ivan drew the next wave profile after a complete wavelength.

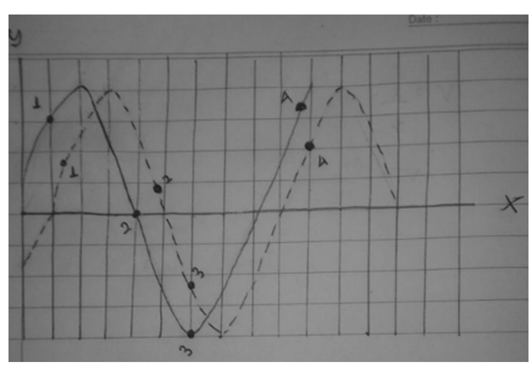

FIG. 9. Student's difficulty to place the displacement of points on the new wave profile.

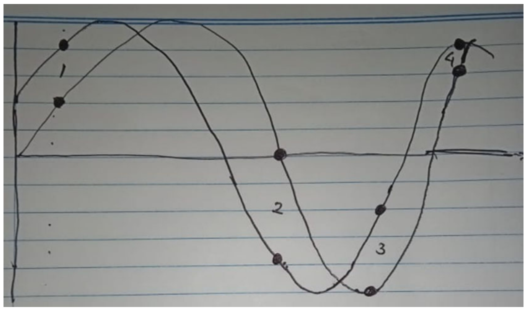

FIG. 10. Indra's drawing as if the wave moves to the left.

because of his notion about the motion direction of the points. He said that point 1 will move upward after a short time interval. His reasoning would be correct if the wave were moving to the left. From the positions of the points on the new wave profile, it shows that Indra understood that each point moves vertically.

2. Students' results for scaffolding level 2

Seven pairs of students were not able to use the simulation correctly to imitate the wave profile. All pairs eventually tried to use different settings in the simulation several times. However, students tended to use periodic wave vibration in the simulation instead of using manual vibration. We also noticed most of them hold a periodical wave fixation in phase I. One pair of students used the manual setting, but the damping was not set to zero. The simulation output was the wave that loses its energy when it is progressing, and its amplitude diminishes after some time has elapsed. We noted that this pair had difficulty drawing the new wave profile smaller in intervention level 1.

Regarding the reflection, two pairs applied the fixed end and no end setting, respectively, and the rest of the pairs applied the loose end setting. They paused the simulation and showed a periodical wave profile at an instant time to the interviewer. However, most of them realized that the way they used the simulation was incorrect because it was not a single pulse moving to the right with a constant velocity.

Four pairs of students could set the features properly and imitate the similar wave profile in the simulation. However, only three pairs applied the precise settings so that they could create the same profile as in the question. Meanwhile, one pair of students failed to move the manual vibration precisely, so the result of their wave profile was not similar to the question. However, at this point, they noticed that the wave profile will always be identical, and the points are moving vertically in a straight line when it is progressing.

Students who managed to notice this were asked once again to draw the wave profile after some time has elapsed and locate the positions of the points. Figure 11 shows one of student's drawings after scaffolding level 2.

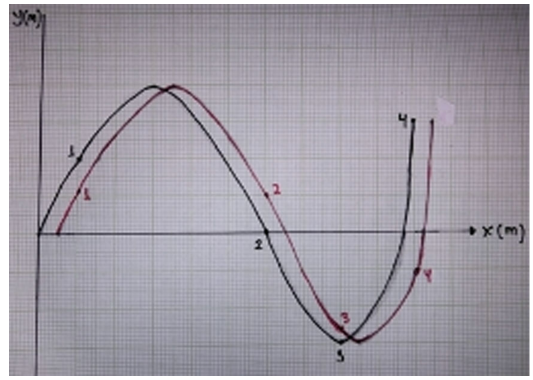

FIG. 11. Student's drawing after scaffolding level 2.

With their drawings, most of them immediately noticed that point 4 has the greatest displacement compared to the other points. They then relate this notion to distinguish the difference of displacement of each point to solve question part (a). One interviewed student, Edy, who was categorized as having a periodical wave fixation in phase I, changed his reasoning to answer this question. He now focused on the displacement of each point and said that point 4 has the greatest velocity because of its displacement.

> *Edy: I think the greater the displacement of a point, the greater its velocity. We know that the velocity is proportional to the displacement. So, the velocity in point 4 is the greatest because it has the greatest displacement. Also, we can see from the simulation that point 4 has the greatest speed moving downward compared to other points.*

In contrast, one pair of students who also managed to grasp the conceptual understanding of a traveling pulse using the simulation did not use this support to change their prior reasoning to solve the problem. We note that this pair failed to create identical wave profiles in the simulation consistent with the question, but they understood the concept behind it. Figure 12 shows one of their drawings after scaffolding level 2.

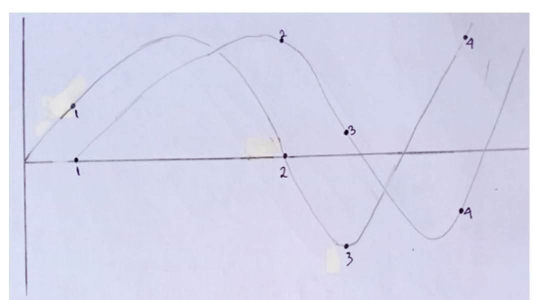

FIG. 12. A correct drawing from a student, but it did not help him to change his prior reasoning.

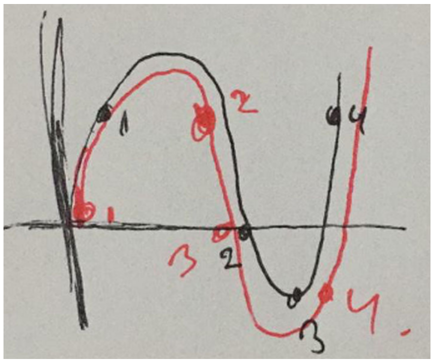

FIG. 13. A student still could not imagine how to draw two identical wave profiles in scaffolding level 3.

Their drawing was still inaccurate because of their mistake drawing the initial wave profile, affecting their drawing on the new profile. We intended to ask them to draw the wave again with the correct initial profile, but we decided to ask them first whether they wanted to change their answer. They said that they still hold to their prior reasoning, and they did not know how to answer the question based on their drawing. Even though the difference of displacement of each point it is clearly seen in their drawing, they still assumed that the greatest speed was point 4 because it is located the furthest horizontally.

For question (b), the group of students who conceived the concept of traveling pulse in this level managed to recognize the motion direction of each point correctly. Three pairs changed their answers by observing the motion direction of each point in the simulation. They said that points moving downwards will have a negative velocity, and points moving upwards will have a positive velocity. They said that points 1 and 4 are moving downward, points 2 and 3 are moving upward.

It is worth noting that our scaffolding is a bit tricky for point 3. When we emphasized that the question asked the velocity at the initial profile, then at that point, they realized that velocity in point 3 is equal to zero. The following is one student conversation in scaffolding level 2 to solve question part (b):

Irvin: Points 1 and 4 are moving downward so their velocities are negative. Point 2 is moving upwards so its velocity is positive. Velocity in point 3 is equal to zero.

Ruth: Yes, I agree. Point 3 is located in the greatest vertical displacement. It is the position where a point in the wave could move farthest. In that condition, the velocity of a point is equal to zero.

3. Students' results for scaffolding level 3

At this stage, the remaining pairs were able to visualize the wave profile with the simulation and they had a better understanding of the problem. However, there were still three pairs who were unsuccessful in drawing the wave

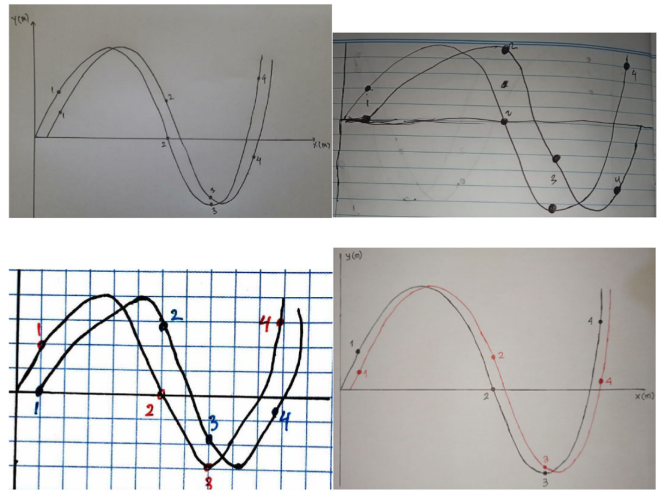

FIG. 14. Students' drawings after scaffolding level 3.

profiles. From their pictures, we noticed that they could not imagine how to draw two wave profiles at two instant times in one Cartesian coordinate. Figure 13 shows students' difficulties drawing the new profile after some time has elapsed.

Diana managed to improve her conceptual understanding from the simulation but failed to draw the correct wave profile. She could explain the motion direction of points correctly, but when we asked her to draw the new wave profile, she was unable to do that. She tried several times to draw several wave profiles, but none were correct. From Fig. 13, it seems like Diana could not imagine how to draw two identical wave profiles crossing each other in one coordinate Cartesian.

Nevertheless, four pairs of students were finally able to draw the wave profile and locate the points correctly in scaffolding level three. Two pairs could answer questions (a) and (b), one pair only could answer question (b), and one pair failed to answer questions (a) and (b).

Figure 14 shows that all the remaining pairs were able to draw two wave profiles correctly but only two pairs could notice the different displacement of each point to answer question (a). Meanwhile, we still found one pair holding a strong conceptual challenge and they did not change their prior reasoning even though their drawing was correct. They still insisted on dividing the graph into positive and negative parts and determining the magnitude of the speed based on the incorrect assumption of vertical displacement. However, for question part (b), three pairs could analyze the motion direction of points and provide the correct reasoning to answer the question.

IV. DISCUSSION

A. Students' difficulties before scaffolding support

Before the scaffolding support, students held strong conceptual challenges regarding these wave phenomena.

None of the students' reasoning is scientifically acceptable and some are quite difficult to understand. The dominant difficulty for question part (a) is students' incorrect interpretation of distance on the graph of $y(x)$ and how they relate that with the proportionality relation between velocity and distance ($v = s/t$). Noticeably, most of them did not mention time explicitly in their reasoning and a few of them assumed that time is fixed. Overall, most students tried to find distances randomly to make assertions about the velocities of each point.

The first conceptual challenge arose when a few students misinterpreted the wave profile as the motion trajectory, which we called roller coaster erroneous reasoning. Some studies show that students assumed that the wave pushes the particle in front of it forward when it is traveling and they often treat waves like objects [7,27]. Wittmann [7] found that students related points in the wave to the movement of an object in kinematic and failed to distinguish between propagating object and propagating wave. He described this conceptual challenge like a surfer riding on an ocean wave because it moves everything in front of it. In our study, students also mentioned that the points move along the wave profile because of the disturbance on the string, which can be assumed that they treat the disturbance like a kick to a ball. However, this reasoning seems to be contradictory because students were aware that the wave only transfers energy which was already emphasized in our question. Therefore, the strong assumption that a wave should be treated as an object prevented them from solving the problem correctly.

Another reason why students failed to distinguish the concept between pulse and particle motion on the wave is the confusion between the graphs of $y(x)$ and $y(t)$. This can be even more complicated when the students combine it with the assumption that a wave is periodic. In our study, almost half of the students assumed that a wave is always periodic, which is in agreement with several findings [6,7,28]. The confusion between $y(x)$ and $y(t)$ is identified by one study showing that students failed to sketch different graphical functions using the graph of $y(x)$. For instance, students misunderstood that $y(x)$ can be directly transformed to sketch the graph $v(x)$ by looking at the slope of the graph [28]. Another difficulty came when students were asked to interpret the wave properties within the graphs of $y(x)$ and $y(t)$. The result shows that most of the students were confident that two wave profiles represent the waveform [6]. In periodic waves, one can use $y(x)$ to determine the wavelength and $y(t)$ to determine the wave's period. Our wave profile was meant to represent just a pulse, thus it should not be used to determine wavelength.

It is clear that the periodical wave fixation is robust among the students. Many referred to periodical properties and formulas like $y = A \sin(kx - \omega t)$ to solve the problem. The profile provided in the question even had a lack of symmetry, which did not prevent students from thinking in terms of sines or cosines. This periodical wave fixation was detected in one study that found more than two-thirds of the students had a very strong belief that the motion of a particle will form a sine wave pattern when they were presented with three different $y(t)$ graphs [6]. Another study also found that students drew the sine wave curve when they were asked to transform the graph of $y(x)$ which is represented in sawtooth shape into different graph functions [28]. During the interview, we asked students about their decision to use periodical waves to solve the problem and the majority of them said that they are only familiar with sine or cosine waves, which was also found in similar studies [6,7,28,29].

In a curved graph of $y(t)$, one can see the slopes at the points to find whether the velocity is positive, negative, or equal to zero but not with the graph of $y(x)$.[2] Again, mixing the reasoning between the graph of $y(x)$ and $y(t)$ and treating a wave like an object greatly complicated the students attempts to answer question (b). This phenomenon can be found when students were presented with a position-time graph located above of *x axis*. Many of them could not imagine that the points have negative velocity due to the position of the graph in the Cartesian coordinate [30]. Although the function of the graph is different since we plot the graph of $y(x)$, we can relate that finding by how students respond to answer question (b). Most of our students just simply observed where the points are located in Cartesian coordinates, whether a point is in the $+y$ *axis*, $-y$ *axis*, or exactly in the *x axis* without considering the motion direction of the points on the wave. In the curved graph of $y(t)$, students simply observed the position of points on the graph instead of analyzing the slope of each point to find the velocity [11]. Moreover, many students in our study mentioned that a point located in the *x axis* treated having a zero or lowest velocity due to its zero position, which was also one of the highlighted findings in Eshach [30] and Mcdermott [11].

B. Students' performances after scaffolding support

In our scaffolding, we tried to address the conceptual challenges that were found in phase I by creating two approaches. The complex relationship between the motion of points and the pulse on the wave was addressed with a simulation. Here, students who thought that the points on the wave move horizontally would finally see that the points only move up and down. This also tackles the conceptual challenge of using the position of points in Cartesian coordinates to define the sign of velocity since students can notice the motion direction of points. Particularly, for point 2, students realized that the velocity

[2]In this situation (pulse moving with constant horizontal speed) one can indeed use the slope at $y(x)$ to infer velocity (dy/dt). This is related to the fact that the pulse also satisfies the transport equation, which states that dy/dx is proportional to dy/dt.

in that position is not zero but that this is the case of the points located in the crest or trough of the graph. The problem of determining distance to obtain the velocity on the graph of $y(x)$ was addressed by asking students to draw the second wave profile after it moves a bit. This created a cognitive conflict among students which used inappropriate ways to define the distance or vertical displacement of each point on the graph.

Scaffolding level 1 was less effective because we found that all the students failed to draw the new wave profile. They believed that the wave would lose its energy and the amplitude would slowly shrink. Our result is in line with a study from Wittmann [9], who presented a pulse at $x = 0$ and $t = 0$ on the graph of $y(x)$ which propagates in the x direction to the students. He then asked them to draw the condition of the pulse after moving at $x = x_0$. Most of the students sketched the amplitude lower after moving at $x = x_0$ due to energy losses. This reasoning could be accurate if dissipative effects were taken into account. However, in the idealized situation, students should recognize that the shape of the pulse will remain the same when it is moving.

Scaffolding level 2 and 3 were more helpful to the students. Some of them have improved their performances when the PhET simulation was used, and it helped students distinguish the (horizontal) wave motion from the (vertical) particle motion. One study reported significant improvement in performance of the students using this simulation, showing that 71% of them correctly recognized the nature of velocity of the points of the string of a violin compared to only 21% of students in traditional teaching [31]. The simulation brings a dynamic component to our static wave profile because the wave is traveling. The students can now focus on the motion of some specific points when the wave is progressing.

In essence, to create the wave profile precisely like in the problem, students need to use the manual feature to move the string, set the damping to zero, and use no end string. How to move the source of vibration is also essential. There are four important steps to create the wave profile exactly like in the question: (i) the string should be placed from above first; (ii) pull the string downward at a specific position; (iii) pull the string back upward at the initial position; (iv) pull the string downward again and place it in the middle position between the whole movement upward and downward. To make a more precise wave profile, students need to pull the string a little bit faster downward than the upward movement. To observe the motion in detail, students need to apply the slow-motion feature.

Four pairs of students were able to create the correct profile without any help on the simulation and changed their prior reasoning. We note that students had different performances when they precisely imitated the wave profile to match the question compared to the students who could only create a similar pulse. Three pairs who managed to simulate the identical pulse immediately spotted that the shape of the profile will remain the same and realized that there was a significant difference between point 4 and the other three points. The motion in point 4, especially, is more noticeable to observe because it moves downward faster. Thus, it was easy for them to determine that point 4 has the greatest velocity even though they had not yet considered the concept of displacement in their reasoning.

One pair who only managed to create a similar pulse did not change their prior reasoning despite successfully drawing the correct wave profile. Although there is a mistake in failing to draw the wave profile consistent with the question, they understood that the shape of the wave will always be identical when it is progressing. One of their drawings can be seen in Fig. 12, showing two identical wave profiles in a graph. The difference of displacement and direction can be clearly seen from those two wave profiles, but they kept insisting on using their prior reasoning. As a side note, we did not provide any further hints after students finished their drawings.

In scaffolding level 3, all the remaining pairs were able to use the simulation to improve their performances. However, three pairs still failed to draw the correct profile and solve the question. Lin and Singh [32] suggested that the scaffolding's effectiveness depends on students' initial knowledge and skills. Based on our results in scaffolding level 2 and 3, it is not guaranteed that our designed scaffolding was effective for all the students even though we have set a clear goal in our interventions which is strongly suggested when designing the scaffolding environment [32]. The careful design of our intervention was not enough to help students change their prior reasoning despite their successful drawing of the correct wave profile.

Students' performances were also diverse in every level of scaffolding. We found that a few students were able to grasp the consistent shape of the wave when it progressed, and our intended goal of this support was accomplished only by implementing scaffolding level 2. In the end, all the students were able to draw the correct wave profile and the purpose of our scaffolding supports were fulfilled. However, a few students still hold their robust alternative conception and did not use that support to change their prior reasoning. Our findings are similar to some studies that implemented scaffolding support as a tool to help students overcome their conceptual challenges. They found outcomes varied when the students were engaged with various scaffolding support levels. These studies suggested that the level of competence is probably the reason why student performances are diverse in each scaffolding environment [24,33].

V. CONCLUSION

Our study highlights some of the main difficulties regarding the propagation of a pulse in a string. All our students initially tried to extract inappropriate information to find the distance on the graph of $y(x)$. This becomes even more complex because many students are mostly exposed to periodic waves and always think that the nature of waves is periodic. In consequence, they hold a robust erroneous reasoning if they are presented with an unusual wave profile. Because of their simplicity, it is understandable to use regular and periodic waves in teaching. However, this practice may convey to capture a thorough understanding of wave phenomena.

The second issue is the difficulty of extracting information from physics graphs. We found many students quickly fixated on unsuitable features on the curved graph that led them into a broader area of erroneous reasoning. Many phenomena on waves are presented in the graph, and this condition requires a comprehensive understanding regarding how to relate correct information on the graph into physical concepts.

In our study, designing scaffolding support becomes a challenge since students' response to these interventions is diverse and not all of them in the end successfully answered the question correctly. Our purpose of scaffolding support to lead students to draw the new wave profile after some time has elapsed was literally achieved. However, not all students used that help to change their prior reasoning. This happened because of the prior knowledge and procedural competences of the students. Further studies are needed to answer why these phenomena happened among university physics students.

ACKNOWLEDGMENTS

We thank all the subjects, especially our students, who took part in this study. Many thanks to Christian Joas and Magdalena Kersting for their feedback on this paper. Muhammad Aswin Rangkuti acknowledges a Ph.D. fellowship from Ministry of Education, Culture, Research and Technology of Indonesia.

---

[1] R. J. Beichner, Testing student interpretation of kinematics graphs, Am. J. Phys. **62**, 750 (1994).

[2] L. Bollen, P. Van Kampen, and M. De Cock, Students' difficulties with vector calculus in electrodynamics, Phys. Rev. ST Phys. Educ. Res., **11**, 020129 (2015).

[3] M. Planinic, Z. Milin-Sipus, H. Katic, A. Susac, and L. Ivanjek, Comparison of student understanding of line graph slope in physics and mathematics, Int. J. Sci. Math. Educ. **10**, 1393 (2012).

[4] E. B. Pollock, J. R. Thompson, and D. B. Mountcastle, Student understanding of the physics and mathematics of process variables in P-V diagrams, AIP Conf. Proc. **951**, 168 (2007).

[5] P. M. Sadler, C. A. Whitney, L. Shore, and F. Deutsch, Visualization and representation of physical systems: Wavemaker as an aid to conceptualizing wave phenomena, J. Sci. Educ. Technol. **8**, 197 (1999).

[6] I. Caleon and R. Subramaniam, Development and application of a three-tier diagnostic test to assess secondary students' understanding of waves, Int. J. Sci. Educ. **32**, 939 (2010).

[7] M. Wittmann, R. N. Steinberg, and E. F. Redish, Understanding and affecting student reasoning about sound waves, Int. J. Sci. Educ. **25**, 991 (2003).

[8] E. M. Kennedy and J. R. de Bruyn, Understanding of mechanical waves among second-year physics majors, Can. J. Phys. **89**, 1155 (2011).

[9] M. C. Wittmann, The object coordination class applied to wave pulses: Analysing student reasoning in wave physics, Int. J. Sci. Educ. **24**, 97 (2002).

[10] M. Kryjevskaia, M. R. Stetzer, and P. R. Heron, Student understanding of wave behavior at a boundary: The relationships among wavelength, propagation speed, and frequency, Am. J. Phys. **80**, 339 (2012).

[11] L. C. McDermott, M. L. Rosenquist, and E. H. Van Zee, Student difficulties in connecting graphs and physics: Examples from kinematics, Am. J. Phys. **55**, 503 (1987).

[12] M. Planinic, Z. Milin-Sipus, H. Katic, A. Susac, and L. Ivanjek, Comparison of student understanding of line graph slope in physics and mathematics, Int. J. Sci. Math. Educ. **10**, 1393 (2012).

[13] I. Caleon and R. Subramaniam, Addressing students' alternative conceptions on the propagation of periodic waves using a refutational text, Phys. Educ. **48**, 657 (2013).

[14] M. Kryjevskaia, M. R. Stetzer, and P. R. Heron, Student understanding of wave behavior at a boundary: The limiting case of reflection at fixed and free ends, Am. J. Phys. **79**, 508 (2011).

[15] M. Kryjevskaia, M. R. Stetzer, and P. R. Heron, Student difficulties measuring distances in terms of wavelength: Lack of basic skills or failure to transfer?, Phys. Rev. ST Phys. Educ. Res. **9**, 010106 (2013).

[16] H. Arksey and P. T. Knight, *Interviewing for Social Scientists: An Introductory Resource with Examples* (Sage, Newbury Park, CA, 1999).

[17] J. C. Greene, V. J. Caracelli, and W. F. Graham, Toward a conceptual framework for mixed-method evaluation designs, Educ. Eval. Policy Anal. **11**, 255 (1989).

[18] J. Houssart and H. Evens, Conducting task-based interviews with pairs of children: consensus, conflict, knowledge construction and turn taking, Int. J. Res. Method Educ. **34**, 63 (2011).

[19] A. Maries, S. Y. Lin, and C. Singh, Challenges in designing appropriate scaffolding to improve students' representational consistency: The case of a Gauss's law problem, Phys. Rev. Phys. Educ. Res. **13**, 020103 (2017).

[20] S. F. Chipman, J. M. Schraagen, and V. L. Shalin, Introduction to cognitive task analysis, in Cognitive Task Analysis (Psychology Press, 2000), pp. 17–38, https://www.taylorfrancis.com/chapters/edit/10.4324/9781410605795-9/introduction-cognitive-task-analysis-susan-chipman-jan-maarten-schraagen-valerie-shalin.

[21] R. E. Clark and F. Estes, Cognitive task analysis for training, Int. J. Educ. Res. **25**, 403 (1996).

[22] E. A. Davis and N. Miyake, Explorations of scaffolding in complex classroom systems, J. Learn. Sci. **13**, 265 (2004).

[23] D. Wood, J. S. Bruner, and G. Ross, The role of tutoring in problem solving, Child Psychol. Psych. Allied Disc. **17**, 89 (1976).

[24] C. Lindstrøm and M. D. Sharma, Teaching physics novices at university: A case for stronger scaffolding, Phys. Rev. ST Phys. Educ. Res. **7**, 010109 (2011).

[25] R. Rahmani, M. Abbas, and G. Alahyarizadeh, The effects of peer scaffolding in problem-based gaming on the frequency of double-loop learning and performance in integrated science process skills, Proc. Soc. Behav. Sci. **93**, 1994 (2013).

[26] https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html.

[27] H. Eshach and J. L. Schwartz, Sound stuff? Naïve materialism in middle-school students' conceptions of sound, Int. J. Sci. Educ. **28**, 733 (2006).

[28] D. J. Grayson and D. Donnelly, Using education research to develop waves courseware, Comput. Phys. **10**, 30 (1996).

[29] Ö. Özcan, Investigating students' mental models about the nature of light in different contexts, Eur. J. Phys. **36**, 065042 (2015).

[30] H. Eshach, The use of intuitive rules in interpreting students' difficulties in reading and creating kinematic graphs, Can. J. Phys. **92**, 1 (2014).

[31] C. E. Wieman, K. K. Perkins, and W. K. Adams, Oersted Medal Lecture 2007: Interactive simulations for teaching physics: What works, what doesn't, and why, Am. J. Phys. **76**, 393 (2008).

[32] S. Y. Lin and C. Singh, Effect of scaffolding on helping introductory physics students solve quantitative problems involving strong alternative conceptions, Phys. Rev. ST Phys. Educ. Res. **11**, 020105 (2015).

[33] J. J. Van Merrienboer and J. Sweller, Cognitive load theory and complex learning: Recent developments and future directions, Educ. Psychol. Rev. **17**, 147 (2005).