# Phenomenographic analysis of students' conceptual understanding of electric and magnetic interactions

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Studying students' problem-solving abilities in physics education research has consistently shown that novices focus on a problem's surface features rather than its physical principles. Previous research has observed that some electricity and magnetism students confuse electricity and magnetism concepts, often presented in parallel problems (or problems with similar surface features). This confusion has been referred to as interference. It is essential to compare students' performance in these problems to evaluate their understanding of these topics. The present work focuses on the students' understanding of interactions between charged particles (i.e., electric force) and electric currents (i.e., magnetic force). We present and compare the findings on students' conceptions when analyzing electric and magnetic interactions for different systems of field sources. We conducted this study with engineering students finishing a calculus-based course on electricity and magnetism. We administered a written, open-ended questionnaire with two sets of three items: one version contained only electricity problems, and the other contained only magnetism problems. Each item in the electricity version of the test had a parallel counterpart in the magnetism version. We used a phenomenographic approach to analyze our data to identify categories that emerged from students' answers. We identified four main ideas in the results: (a) the rule of signs (ROS), which does not evidence a complete conceptual understanding of electric interactions; (b) the force-field confusion due to the similarity of electricity and magnetism contexts; (c) the importance of semiotic representation when answering an electricity and magnetism problem, where the student's choice of representation indicates their understanding, and (d) the interference phenomenon, in which we find evidence of other factors besides those produced by the timing of instruction and administration of the tests. At the end of this work, we provide recommendations for instruction.

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## I. INTRODUCTION

The various studies of conceptual understanding in electricity and magnetism have been substantial [\[1\]](#page-18-0). Many focus on students' understanding of electric circuits or

electric fields [[2\]](#page-18-1). Students' expertise in using physical principles and problem-solving abilities, regardless of any given system's surface features, represents one of the major concerns of electricity and magnetism education research [\[3](#page-18-2)]. Maxwell's equations in electricity and magnetism present some level of symmetry that may induce students to confuse electricity concepts when approaching magnetism problems and vice versa [[4](#page-18-3)[,5](#page-18-4)]. Still, regardless of the importance of Maxwell's equations in electricity and magnetism, in some studies, only 5% of students evoke them when answering items or solving problems; the others attempt versions of other concepts, for example, Ohm's law [\[6](#page-18-5)].

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The objective of the study is to compare students' conceptual understanding of electric and magnetic interactions, its relationship to interaction principles like Newton's laws of motion, and the level of interference when students answer questions about the concepts of electric force and magnetic force that are superficially alike, but conceptually different. We use the tag "parallel" for these types of questions that share similar surface features. We presented these parallel problems to the students and then analyzed their answers through a phenomenographic lens. We focus on the diagrams they produced to answer them and their written explanations in both contexts. This study's relevance lies in comparing students' understanding of electric and magnetic interactions and the critical information detailing their confusion between electricity and magnetism concepts.

There has been quite extensive work regarding the difficulties of understanding electric and magnetic interactions. Most of the studies focus on each interaction separately without emphasizing their similarities. The present work focuses on the students' understanding of interactions between charged particles (i.e., electric force) and electric currents (i.e., magnetic force). We present and compare the findings on students' conceptions when analyzing electric and magnetic interactions for systems of field sources with equal magnitudes. Afterward, we present and compare an analysis of students' conceptions of electric and magnetic interactions for a system with field sources of different magnitudes and observe if students make a connection between electricity and magnetism laws (such as Coulomb's or Biot-Savart laws) and Newton's laws of motion. We then take the results of both scenarios, compare contexts (electricity vs. magnetism for each configuration), and point out the similarities and differences between the most common types of answers. We also present an analysis of how fixed each conception is (naïve or not) by observing the evoked ideas of the same student when moving between questions. Finally, we present our discussion and conclusions based on these findings.

### II. THEORETICAL FRAMEWORK

There are three main theoretical frameworks in students' conceptual understanding studies [\[7\]](#page-18-6). The misconceptions view postulates that students have been learning about the world through experience for all their life [[8](#page-18-7)]. Misconceptions or naïve theories have complex structures in the mind of students that interfere with scientific conceptions students try to learn in the classroom. These misconceptions are deeply attached, resisting a conceptual change that fosters educational strategies.

The second framework is the knowledge-in-pieces view in which students' conceptions consist of pieces of knowledge, phenomenological primitives also called p-prims, instead of structured misconceptions [[9](#page-18-8)]. Students will access different pieces of knowledge (also

called resources [[10](#page-18-9)]) depending on the context of the same physical phenomena. The framework postulates that these p-prims are why students answer with different arguments to the same question in a different context, something is more difficult to explain with the misconception view.

The last framework of students' conceptions is the ontological category view [\[11\]](#page-18-10). In this view, students classify knowledge into ontological categories that are not necessarily correct. Students could categorize electric field lines as a thing that can be transported from one charge to another. This framework postulates that students have well stable ontological categories. Educational strategies that foster conceptual change should work on students to classify the concept into a proper ontological category.

As we will see in this work, we believe students' difficulties are explained consistently with different frameworks. That is, students could have a naïve conception such as that it is hotter during the summer because the Earth is closer to the Sun, something that is easily accessible and context dependent [\[12\]](#page-18-11). Nevertheless, they could also have a well-established and stable misconception that an object in motion has an internal force obtained when placed in motion. This misconception is known as the impetus misconception [[13](#page-18-12)].

#### III. LITERATURE REVIEW

There are common difficulties in understanding electric and magnetic interactions. As some studies have shown, concepts such as electric and magnetic fields are unfamiliar to students because they are intangible, making them complicated to understand [\[7](#page-18-6)[,14](#page-18-13)]. Even though most studies agree that electric and magnetic fields are the spine for understanding electricity and magnetism [\[15](#page-18-14)–[17\]](#page-18-15), less than 20% of students can describe and analyze them properly [\[16](#page-18-16)–[18\]](#page-18-17). Researchers have developed and used several conceptual inventories to conduct these studies, such as the Conceptual Survey of Electricity and Magnetism (CSEM) [\[1](#page-18-0)], the Brief Electricity and Magnetism Assessment (BEMA) [[19](#page-18-18),[20\]](#page-18-19), and the Magnetism Conceptual Survey (MCS) [[21\]](#page-18-20).

Research on students' understanding of electrical fields has highlighted several naïve conceptions. Some students believe that electric field lines are actual physical entities that "transmit" electric force [[22\]](#page-18-21). In some cases, field lines add complexity for students analyzing an electric field and the interactions produced [[23\]](#page-18-22). Some instructors even have problems describing an electric field, basically "reducing" it to an electric force to simplify the concept for students [\[24\]](#page-18-23). Students mimic this behavior, thinking that an electric field and an electric force are interchangeable concepts [[25](#page-18-24)].

Other misunderstandings about electric fields come from the source of the field itself. Students have difficulties identifying the interactions between two electric charges [\[26](#page-18-25)[,27\]](#page-18-26), with most of these difficulties from previous physics courses, for instance, those related to Newton's laws [\[5\]](#page-18-4). Ohm's p-prim misconception is an example of this: the greater the electric charge, the greater the force it will exert on another charge, a typical mistake widely studied [\[1](#page-18-0),[28](#page-18-27)–[30](#page-18-28)]. To be more explicit on the nature of this difficulty, we will be referring to it as the *larger implies* larger p-prim difficulty.

Difficulties in understanding magnetism also have underlying causes related to its electric counterpart. For instance, the notation used to introduce magnetic fields and interpret physical quantities such as magnetic field and magnetic flux could cause students to struggle to understand [[31](#page-19-0)]. Some students believe that a magnetic field is produced by an electric charge, regardless of its state of motion [\[5,](#page-18-4)[29\]](#page-18-29). As with electricity, a functional reduction of each concept may cause some difficulties; in this case, students confuse magnetic force with magnetic field and simplify them both to use the right-hand rule [\[29,](#page-18-29)[32](#page-19-1)], disregarding Lorentz's magnetic force. Thus, in some instances, students would use the right-hand rule to determine the direction of a magnetic field rather than a compass [[33](#page-19-2)], indicating that they are unaware of the phenomenon's real nature. Some difficulties come from teachers, too, like the one mentioned in Ref. [\[34\]](#page-19-3), where instructors cannot describe the effects of a magnetic field produced by an electric current, which is why studies suggest stating the difference between Lorentz's magnetic force and a magnetic field through Biot-Savart's law [[35](#page-19-4)].

There are difficulties where students confuse concepts in the same context. For example, some students mistake electric or magnetic fields for their respective forces [\[24](#page-18-23)[,32](#page-19-1),[36](#page-19-5)]. However, in some cases, the confusion between concepts is contextual. The literature has referred to this type of difficulty as interference [[32](#page-19-1)]; for instance, a student evokes magnetism concepts to answer an electricity question (or vice versa). Research has shown that some students confuse electric force with magnetic force [\[4](#page-18-3)[,5\]](#page-18-4), a confusion sometimes also found among instructors [[37](#page-19-6)]. As mentioned in Ref. [\[35\]](#page-19-4), these confusions might stem from using analogies when teaching magnetism concepts (often later in the electricity and magnetism instruction).

Finally, semiotic representation also plays a part in magnetism difficulties because students who can switch between representations of the magnetic field have a better understanding of the concept [[38](#page-19-7)]. This finding has a counterpart for the electric field, studied in Ref. [[39](#page-19-8)]. In general, the ability to use multiple representations to describe an electric or magnetic field indicates a deeper understanding of the concept of the field itself. We analyze this concept of semiotic representation and its implications in the following sections.

#### IV. METHODOLOGY

We conducted this study in a large private university in Mexico with 322 participants. All the participants were engineering students finishing a calculus-based course on electricity and magnetism. This course is the last of three introductory physics courses offered in this institution. Students use a widely known textbook [\[40\]](#page-19-9) and tutorials [\[41\]](#page-19-10). The course consists of 3 h of lecture and 1.5 h of laboratory sessions each week. Different instructors implement lectures in sections of 30–35 students each. Lab sessions are in 12-student groups, and students in the lab are not necessarily in the same lecture section. The course is delivered in Spanish, as it is the official language in this institution. By the end of the course, we administered two open-ended questionnaires in Spanish with conceptual electricity or magnetism questions to all students taking the electricity and magnetism course  $(N = 322)$ . Students randomly received one of the two tests; 160 students answered the electricity test and 162 the magnetism test. The randomness of the administered tests and the variety of electricity and magnetism instructors allowed us to consider these two groups statistically comparable [\[42\]](#page-19-11).

### A. Instruments

We present two parallel open-ended questionnaires with three electric or magnetic interactions questions. One test contained only electricity problems, and the other contained only magnetism problems. We designed the questions using the parallelism between electricity and magnetism and used schematic representations with similar surface features to represent this parallelism, done in previous studies by the authors [\[43,](#page-19-12)[44](#page-19-13)]. Each item in the electricity test had a parallel counterpart in the magnetism version. We present the three questions in a parallel manner in Figs. [1](#page-3-0)–[6.](#page-5-0)

The questions prompt the students to analyze the interaction between two electric charges in the electricity test and two electric currents in the magnetism test. The first pair of items (Fig. [1\)](#page-3-0) matched two identical electric charges in the electricity test and two identical electric currents in the magnetism test.

The expected answer is presented in Fig. [2.](#page-3-1) This includes the diagram that students should have presented and the main concepts they should turn to for their answer to be classified as correct. In this case, there is an explicit reference to Coulomb's law for the electricity version and Lorentz's law in the magnetism version.

The second pair of items (Fig. [3\)](#page-3-2) matched two electric charges of opposite signs in the electricity test and two conventional electric currents with opposite directions in the magnetism test, with its corresponding expected answer in Fig. [4.](#page-4-0) These first and second pairs of items allow us to explore students' understanding of electric and magnetic interactions through their drawings and explanations when using Coulomb and Lorentz force.

Similar scenarios have been used in some items created for previous studies to explore student understanding of these concepts, for example, questions 1 to 3 of BEMA [\[20\]](#page-18-19) and 3 to 5 of CSEM [\[1\]](#page-18-0). Given that these questions have

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FIG. 1. Pair No. 1 of *parallel* items. In the electricity version  $(E)$ , two identical, static positive charges  $+q$  are presented. The magnetism version (*M*) presents two long cables with identical conventional electric currents  $I_0$  with direction out of the page.

<span id="page-3-1"></span>

FIG. 2. Expected answer for the pair No. 1 of *parallel* items. On the top, the expected diagram and reasoning for the electricity version  $(E)$ ; on the bottom, the expected diagram and reasoning for the magnetism version  $(M)$ .

<span id="page-3-2"></span>

FIG. 3. Pair No. 2 of *parallel* items. In the electricity version  $(E)$ , two static opposite charges of the same magnitude are presented. The magnetism version ( $M$ ) presents two long cables with opposite conventional electric currents  $I_0$ , one going out of the page and the other going into the page.

<span id="page-4-0"></span>

FIG. 4. Expected answer for the pair No. 2 of *parallel* items. On the top, the expected diagram and reasoning for the electricity version  $(E)$ ; on the bottom, the expected diagram and reasoning for the magnetism version  $(M)$ .

gone through a robust validation process, for instance in Refs. [\[45](#page-19-14)–[47](#page-19-15)], they were taken as the main base for creating this instrument. We made the necessary modifications in the text to convert this multiple-choice scenario into an open-ended question.

The third pair of items (Fig. [5](#page-4-1)) matched two positive electric charges with a 3∶1 magnitude ratio in the electricity test and two outward electric currents with a 3∶1 magnitude ratio in the magnetism test. We present the expected answer for this item in Fig. [6](#page-5-0). These items allow us to explore students' application of Newton's third law of motion in the electricity and magnetism contexts. Similar items have been used in previous studies, with their corresponding validation. Examples of this would be questions 3 to 5 of the CSEM [\[45](#page-19-14)[,46,](#page-19-16)[48\]](#page-19-17) and question 24 of the BEMA [\[20](#page-18-19)[,47\]](#page-19-15).

We want to point out that Figs. [2](#page-3-1), [4,](#page-4-0) and [6](#page-5-0) represent only a guide for the researchers for answer classification and not a strict answer key. In other words, we were not looking for exact matches in students' responses but similar diagrams and correctly used written ideas that match the concepts listed in the expected answers. For instance, in pairs No. 1 and No. 2, we would list as correct an answer where students pointed out the nature of the interaction (attraction or repulsion) as an effect of the electric or magnetic fields, even if they did not explicitly mention the name of Coulomb's law or Lorentz's law. The same would occur for pair No. 3, where pointing out the interaction and the fact that forces are of the same magnitude (supporting the statement only on Newton's third law) would be enough to list the answer as correct.

<span id="page-4-1"></span>

FIG. 5. Pair No. 3 of parallel items. In the electricity version  $(E)$ , the items present two static positive charges with magnitudes  $+3q$ and  $+q$ , while the magnetism version (M) presents two long cables with conventional electric currents  $I_0$  and  $3I_0$  in the same direction.

<span id="page-5-0"></span>

FIG. 6. Expected answer for the pair No. 3 of *parallel* items. On the top, the expected diagram and reasoning for the electricity version  $(E)$ ; on the bottom, the expected diagram and reasoning for the magnetism version  $(M)$ .

#### B. Data analysis

We used a phenomenographic approach to analyze our results because it allows analyzing how people understand the physical world [[49](#page-19-18)]. Phenomenography recognizes each person's unique experience and understanding of the physical world while assuming that commonly shared experiences and conceptions exist. The shared experiences and conceptions can be grouped into categories that describe a collective intellect [\[49\]](#page-19-18). The main interest of phenomenography is the variations between the categories that emerge from qualitative data, which may result from interviews or other instruments, as long as the questions are open ended [\[50\]](#page-19-19). The method has been used in physics education research to analyze open-ended questions and derive students' understanding and difficulties when learning physical concepts [[39](#page-19-8),[44](#page-19-13),[51](#page-19-20)–[54\]](#page-19-21). In our experience using this method, we have found that using open-ended questionnaires instead of interviews has provided the advantage to find the collective intellect of many participants on different topics of electricity and magnetism [\[39](#page-19-8)[,44](#page-19-13)].

The procedure involved two of the authors: First, we identified emerging categories from the answers of a 20-student random sample, reaching a consensus between the experts. Next, the two authors individually analyzed the answers from all the participants and classified them into emerging categories. We compared the classifications done by the two authors. If more categories emerged, we included more categories in the analysis in an iterative process. We used Cohen's kappa to measure the interrater reliability of our analysis, attaining an average of 0.95 for the electricity test and 0.94 for the magnetism test [[44,](#page-19-13)[55](#page-19-22),[56\]](#page-19-23).

We analyzed the students' diagrams for each question and their written explanations to identify the emerging categories. We analyzed each question of each test independently. Once all answers were classified, we selected the categories that collected 5% of the sample's answers or more. Any category that did not fulfill the 5% criterion was allocated to the "Other" category. The emerging categories are the main result of phenomenographic research [\[49,](#page-19-18)[50](#page-19-19)]. Section [V](#page-5-1) presents the results about students' understanding of electric and magnetic interactions. Sections [V. A](#page-5-2) and [V. B](#page-7-0) describe the emerging categories for the first and second pairs of items. Section [V. C](#page-8-0) compares the electricity and magnetism contexts, which was possible because of the parallelism between the questions. Section [V. D](#page-10-0) presents a consistency analysis of how students had the same emerging categories for the first and second pairs of items. Section [VI](#page-11-0) presents the results about students' application of Newton's third law of motion in electricity and magnetism scenarios, with a similar structure. Sections [VI. A](#page-11-1) and [VI. B](#page-12-0) describe the emerging categories for the third pair of items, followed by a comparison between the context in Sec. [VI. C](#page-13-0) and a consistency analysis in Sec. [VI. D](#page-13-1). It is important to note that the whole study (instrument, data collection, and analysis) was performed in Spanish. All the examples presented in the results section are translated from Spanish.

## <span id="page-5-1"></span>V. RESULTS ANALYSIS OF ELECTRIC AND MAGNETIC INTERACTIONS PROBLEMS

### <span id="page-5-2"></span>A. Students' understanding of electric interactions

Given the conceptual similarity between the pairs of items 1 and 2, the emergent categories were the same.

<span id="page-6-0"></span>

FIG. 7. Examples of the diagrams drawn by students for each category in the electricity version of the test. Examples a, b, d, and e were answers for item 1, and example c was a response for item 2.

In Fig. [7](#page-6-0), we present the categories that emerged for the electricity test, items 1 and 2.

First, we find the physics principles category, where we classified the answers given by students who explicitly wrote about the interaction of the presented charges based on Coulomb's law. Also, students must have included a diagram as shown in Fig. [7\(a\)](#page-6-0) (taken from an answer given to item 1 with reasoning such as, "Since both charges are positive, they experience a repelling force between them because of the effect of their electric fields." Furthermore, we observe that the interaction (force) is represented as a single vector that starts at the charge exerted by said force. We classified this combination (diagram  $+$  explanation) as the "correct" or "expected" answer for each item because students recur to fundamental laws or principles to justify their answers.

Next, we have the sign-based category. We grouped answers given by students that may have similar diagrams to the previous category but explicitly mentioned that the interaction was due to the signs of the charges without explaining further. Figure [7\(b\),](#page-6-0) taken from an answer given to item 1, shows a diagram quite similar to the one in the physics principles category, which means that the sign-based category is based on the students' written explanations. The student who presented the diagram of Fig. [7\(b\)](#page-6-0) wrote, "They repel each other because they are both positive".

There was also a category for students' answers that had a diagram consistent with the expected diagram for the physics principles category but lacked further explanation other than (maybe) declaring the interaction's name. We grouped these cases in the *unexplained* category. Figure [7\(c\)](#page-6-0) presents an example of an answer to item 2 under this category. The explanation following the said diagram was, "There is an interaction. Each charge acts on the other. In this case, the interaction is called attraction". As we can see, the student describes the interaction but does not explain anything beyond that.

The fourth category was force-field confusion. This category groups all answers where students showed confusion between electric force and electric field in their diagrams or explanations. It is common to describe force lines when referring to field lines, which may be the underlying cause of this confusion. This confusion leads to electric field lines representing an electric repulsion, as shown in Fig. [7\(d\)](#page-6-0). The answers were classified mainly depending on the diagram rather than the given explanation in this category. The referred sample in Fig. [7\(d\)](#page-6-0) was given as an answer for item 1 and had the following description: "Yes, they repel each other. Since they are both the same, they repel each other". This reasoning is consistent with the sign-based category, but the diagram implies that the student may not differentiate between an electric force and an electric field.

Then, the electricity and magnetism interference category groups the students' answers that used magnetism concepts even though this was an electricity problem. For example, in an attempt to answer item 1, a student drew the diagram shown in Fig. [7\(e\),](#page-6-0) emulating a magnetic field. Moreover, the explanation written by the student was, "No. Both charges go to the same side and do not ever intersect". We infer that going "to the same side" means the student thinks about electric currents rather than electric charges. We also classified a few answers in this category that used a pole analogy as part of the explanations (attaching a magnet sketch). An example found for this case was "There is interaction. Charges attract each other because they are opposite poles".

<span id="page-7-1"></span>TABLE I. Comparison of the results for items 1 and 2 per category in the electricity and the magnetism versions.

Category	Item 1		Item 2	
	Electricity	Magnetism	Electricity	Magnetism
Physics principles	9%	19%	$5\%$	17%
Sign-based	49%	10%	51%	$10\%$
Unexplained	$7\%$	$2\%$	$8\%$	$4\%$
Force-field confusion	18%	$12\%$	21%	$16\%$
Electricity and magnetism interference	6%	25%	$4\%$	22%
Zero force	$0\%$	$6\%$	$0\%$	$3\%$
Other	$10\%$	24%	10%	27%
No answer	$1\%$	$2\%$	$1\%$	$1\%$
Total	100%	100%	100%	100%

Finally, we listed any answer that did not fit into these six categories into the other category. We grouped blank responses in the no answer category. Table [I](#page-7-1) shows the frequency of all of these categories for items 1 and 2 of the electricity set.

#### <span id="page-7-0"></span>B. Students' understanding of magnetic interactions

Like the electricity version, the categories emerging from items 1 and 2 of the magnetism version were the same, given the items' similarity. The main difference was a recurrent category named *zero force*, which did not appear in the electricity set. In Fig. [8,](#page-7-2) we present diagram examples for all the categories.

The physics principles category gathers students' answers whose reasoning was based on Lorentz's law and included a diagram like the one shown in Fig. [8\(a\)](#page-7-2), an answer to item 1. The written explanation included with this diagram was, "There is. They attract because of the current cross-product with the magnetic field  $(I \times B)$ ."

The sign-based category diagram example is shown in Fig. [8\(b\),](#page-7-2) also as an answer for item 1, and its corresponding explanation was, "Yes, there is an interaction between the wires. The wires would feel an attractive force towards each other because the currents are going in the same direction."

The only category in this version of the test that did not have a parallel answer to the electricity version was the zero force category. It can be observed in Fig. [8\(f\)](#page-7-2) that the student is probably mistaking the magnetic field for the magnetic force, applying the superposition principle between the two wires, which we could categorize as the force-field confusion category. We decided to keep this

<span id="page-7-2"></span>

$\sqrt{ }$	$F_{M}$ nn! $\left( \cdot \right)$ $+I_{\theta}$ $+I_{\theta}$	⊗ -----> $\leftarrow \circ$ $H_{\boldsymbol{\theta}}$ $+I_{\rho}$	
(a) Physics principles	(b) Sign-based	(c) Unexplained	
<b>Service</b> ᠬ  $+I_{\rho}$ (d) Force-field confusion	$\odot$ $\bullet$ 0 $+Io$ $+I_{o}$ (e) E&M interference	$\odot$ $_{\odot}$ $+I_{q}$ (f) Zero force model	

FIG. 8. Diagram examples given for each category in the magnetism version of the test. Examples a, b, d, e, and f were taken from item 1, and example c was taken from item 2.

category separated because the central answers are different. Students explicitly stated that there is no interaction for the zero force category because the magnetic field is zero in the analyzed position, explained in the example diagram, "No. The magnetic fields interact and cancel out because of their direction".

Finally, we listed any answers that did not fit these seven categories in the other category. We grouped any blank responses in the no answer category.

## <span id="page-8-0"></span>C. Comparison between electricity and magnetism results per category

This section presents a parallel analysis of the results obtained after implementing both sets of items. In Table [I](#page-7-1), we show the percentages of students under each category. All the categories collected at least 5% of the sample answers in at least one of the contexts (electricity or magnetism). We merged it into the other category if a category did not attain 5%. We compared each category separately based on these results, except for the zero force category, which did not exist in the electricity set.

## 1. Physics principles category

The physics principles category represents the complete answers to each item. Students referred to physical principles more in the magnetism test for items 1 and 2 (19% and 17%, respectively) than the electricity test (9% and 5%). The difference is noteworthy. More than twice as many students based their answers to the magnetism problems on a physics law of interactions compared to the number of students answering the electricity problems. It is also remarkable that the physics principles category is the second largest magnetism category for items 1 and 2. The opposite occurs with the electricity items, where Coulomb's law is the next-to-last option in the students' reasoning.

Furthermore, in both cases, the percentage of students using physics principle reasoning to answer item 1 is higher than those using it for item 2, which might indicate that, even though this is correct reasoning, not all students are confident of their first answer. Thus, they change the reasoning from one item to another. This result could be related to some students' familiarity with the sign-based reasoning for electric interactions, which we discuss further ahead.

It is also notable that if we only account for classified answers (with written explanations), the results show that Coulomb's law is the next-to-last option in the students' analyses of electric interactions. We may explain this finding by some students' familiarity with the sign-based reasoning for electric interactions. Evoking Coulomb's law to answer this type of item might not be top-of-mind because it provides no clear advantage over sign-based rationale, which is more straightforward, more familiar, and leads to a correct answer. It occurs the other way around for the magnetism pair of items. Lorentz's force is the second

most evoked concept, which would indicate that the need to calculate a cross-product requires the student to conduct a deeper analysis from the first moment of instruction (which might be the first time they study magnetism in a formal setting).

## 2. Sign-based category

As observed in the electricity columns in Table [I,](#page-7-1) 49% of students in item 1 and 51% of students in item 2 used a sign-based explanation. On the other hand, for the magnetism pair of items in this set, only 10% of the sample used the sign-based category explanations for item 1 and 10% for item 2. We want to state that, for this study, we do not consider sign-based reasoning as a mistake because it eventually leads to a correct answer. However, it does not present proof that the student understands the phenomenon. We consider this to be only an indication of students evoking the common knowledge of electric attraction or repulsion that most of them learned in previous stages of their education mnemonically. We coined the term "Rule of Signs" (ROS) to refer to this previously learned mnemonic that students use to explain electric interactions. We interpret the ROS as a very naïve piece of knowledge easily accessible to students. Since it is effortless to get, some students understand the concept well and use it. Other students accessing it do not have another way to explain as we will analyze it later.

Our findings support the assumption that students turn to the ROS more often when analyzing the interaction between electric charges than electric currents. As we suggested before, this implies that the ROS is so deeply rooted in students that they recur to mnemonics instead of the physical principle when analyzing these cases. Then, it might not be surprising that this was the most recurrent reasoning in the electricity set of items, doubling the frequency of the most common difficulty and far above the use of Coulomb's law. We also point out that the percentage of students who evoke the ROS increases when answering item 2 compared to item 1 of the set. We interpret this as some students remembering and using the ROS while moving to the second item after using different reasoning for the first one.

Moreover, for magnetism, if students try to use something similar to the ROS when analyzing the interaction of electric currents, they will need to invert it (i.e., if they have learned that charges of the same sign repel each other, they need to relearn that currents with the same direction attract each other). In this case, we observed that the percentage of students who used this reasoning to answer the magnetism items stayed the same and was well below its electricity counterpart. Overall, the ROS is a mnemonic device that students primarily use for electricity items. Since most students probably are learning magnetism formally for the first time, memorizing the interactions between parallel and antiparallel currents does not become a handy option; it is

contrary to the ROS, which could confuse them. Also, as stated before, the complexity of Lorentz's law for magnetic interactions may bound students to it, requiring more profound thinking before answering (thus, becoming a first go-to resource for some of them).

#### 3. Force-field confusion category

Our results may be yet another indicator of the recurrence of this confusion. As shown in Table [I,](#page-7-1) in the electricity columns, the most common difficulty can be found in students' answers under the category force-field confusion, with 18% of the sample for item 1 and 21% for item 2. On the other hand, this naive idea was not the most frequent difficulty for the magnetism set. Still, it accumulated many answers, with 12% for item 1 and 16% for item 2. In other words, it was more common to find this difficulty in the electricity set of items than in the magnetism set, but the difference is relatively small.

Again, as it happened with the sign-based category, the force-field confusion shows a similar frequency increase (3% and 4%), implying that the confusion accentuates. Still, it seems necessary to state that, in both contexts, item 2 has opposite field sources interacting. In the first case, we presented students with two static electric charges, one positive and one negative, and asked them to analyze the interaction. In the second case, there is a conventional ingoing electric current and conventional outgoing electric current set to analyze their interaction.

Another important observation in these results comes from the students' visual representations in any category. Figure [9](#page-9-0) shows a side-by-side comparison of the two types of drawings made in each pair of items, field lines or field vectors. This observation becomes relevant in this section because, for the categories physics principles, sign-based and unexplained, the diagrams used were field vectors. However, in the force-field confusion category, the possible visual representations for an electric field diversify.

Figure [9\(a\)](#page-9-0) contains an electric field around the charges, represented by electric field lines, which deflect at some point to show the interaction. This drawing corresponds to most of the students taking the electricity test and shows this confusion, which was sketched by four out of five of the students in this category; the others used a drawing consistent with Fig. [9\(b\).](#page-9-0) We interpret sketches like Fig. [9\(b\)](#page-9-0) as the student's attempt to draw an electric field vector for each of the charges that go on the same line (axis) of the position of the charges. For instance, we interpret the vector drawn on the positive charge as the electric field that the negative charge produces in this position. Similarly, we interpret the vector next to the negative charge as the electric field that the positive charge produces in this position, but it is incorrectly drawn because it should be on the right of the charge. However, the difference is slight in the magnetism items. Figure  $9(c)$  presents what we consider a hybrid of field lines and field vector representations, similar to what [[57](#page-19-24)] reported. The reason we believe this to be an example of magnetic field lines is that this student explicitly symbolized the magnetic field around the currents  $(B)$  and placed dotted lines around them. Figure [9\(d\)](#page-9-0) presents the field vector version, both of them have around half of the students with answers in this category for items 1 or 2. We consider that the difference between using these two visual representation options may come from instruction: it may be more common to sketch and analyze electric fields through field lines in the space around the source.

<span id="page-9-0"></span>

FIG. 9. Student examples of visual representations for the force-field confusion category for items 1 and 2 in each context.

<span id="page-10-1"></span>

FIG. 10. Adapted from Fig. [1](#page-3-0) used by Ref. [[4\]](#page-18-3). On the left (a) is the electricity scenario; on the right (b) is the magnetism scenario.

#### 4. Electricity and magnetism interference category

Interference is the use of magnetism concepts when answering electricity items and vice versa [[32](#page-19-1)]. For example, the students who use magnetism concepts to answer electricity items mostly do it because it would lead to a correct answer if the problem were in the magnetism context. There might not be a lack of understanding of the concept itself but an inability to tell when this concept becomes relevant and valuable. Studies, such as Refs. [[4](#page-18-3),[5](#page-18-4),[32](#page-19-1)], have shown that interference is a phenomenon that appears in both electricity and magnetism contexts. However, it is usually more evident in one of them, depending on the concept of the question and the time of instruction. An example of this idea is reported in Ref. [[4](#page-18-3)]. The authors presented the students with a moving, positive electric charge in two situations: inside an electric field that points into the page and inside a magnetic field that points upwards [see Figs. [10\(a\)](#page-10-1) and [10\(b\)](#page-10-1), respectively].

They tested the students in two moments: before and after instruction on electricity and magnetism. They found that, after instruction, around 30% of the students answering the electric force question [Fig. [10\(a\)\]](#page-10-1) did so if it was a magnetic force question, which is very close to the 40% of the students who correctly answered that question. Still, this situation was not so evident in the magnetism question since 10% or less of the sample answered the magnetic force question as if it was an electric force question. In other words, although the interference presented itself in both contexts, it was not as frequent in the magnetism question as it was in the electric question.

However, what supports the idea of the time of instruction as the cause of interference is that this phenomenon appeared less than 10% in both contexts in the test administered before instruction, which means the number of students presenting an interference difficulty increased after instruction. The most common explanation for interference found in these studies is instruction time. However, our results provide a different behavior that does not necessarily fit this last idea. We will elaborate on this in the discussion section.

For this pair of items, the electricity and magnetism interference category gathered a low number of students in the electricity version, with 6% for item 1 and 4% for item 2. However, the same category garnered 25% of the sample for item 1 and 22% for item 2 in the magnetism version. Although bidirectional, the interference, in this case, is strongly opposed to the usual direction. Another indicator supporting this fact is that electricity and magnetism interference is the least frequent difficulty in electricity items, but it is the most frequent difficulty in magnetism. In other words, when it comes to interactions, interference is more likely to appear when answering magnetism items using electricity concepts than the other way around.

### D. Consistency analysis of electric and magnetic interactions

<span id="page-10-0"></span>We present Table [II](#page-10-2) as a *consistency* analysis, showing how students changed or maintained an idea when moving through items of the same nature. Table [II](#page-10-2) shows the percentage of students who used the reasoning that

<span id="page-10-2"></span>TABLE II. Consistency analysis for each pair of items in the four main result categories.

	Electricity		Magnetism	
	Used reasoning once	Consistent reasoning	Used reasoning once	Consistent reasoning
Physics principles	$11\%$	$4\%$	$20\%$	$16\%$
Sign-based	59%	41%	$10\%$	5%
Force-field confusion	23%	$16\%$	18%	$10\%$
Electricity and magnetism interference	6%	$3\%$	28%	19%

differentiates each of the four major result categories for each context. The column used reasoning once accounts for students who answered items 1 and 2 with different reasoning in each. In contrast, the column consistent reasoning accounts for students who used the same reasoning in both items of the set. The numbers shown are percentages of the total sample and do not sum to 100% because we did not include the unexplained, other, and no answer categories in this analysis. These results provide some notable contextual tendencies.

In the electricity context, it is clear that the students had a proclivity to use memorized knowledge, such as the ROS. The majority of students who resorted to the previously learned rule that equal charges repel, opposite charges attract, consistently did it in both items. Even though this cannot be considered an error, this result shows that most students might not think of Coulomb's law, implying that they do not understand the interaction itself. We observed a similar tendency in the force-field confusion category. More than half of the students who presented answers following this naive idea did so in both items.

In the magnetism context, the tendency to analyze the magnetic interaction through Lorentz's law seems to prevail in most cases if evoked at least once. Students are probably bound by the complexity of this law (specifically, the cross product required to answer the problem), which may discourage their use of memorized ideas. Also, the force-field confusion is consistently evoked. We observed that more than half of the students experienced this difficulty while answering both items in the set. This result is similar to the electricity counterpart, supporting the idea that students might be unable to distinguish a force from a field, regardless of its nature (electric or magnetic).

<span id="page-11-2"></span>Moreover, we observed that most students who presented some interference did so in both items, proving that, when it comes to interactions, the interference is more substantial and consistent in answering magnetism items as if they were electricity items. An explanation for this is ROS, which is so easily accessible that students use it in the magnetism concept, mixing ideas and causing interference.

Finally, we would like to point out that adding up the percentages of students who used the reasonings of the four main categories for these two items results in a different fraction of the sample for each test version. While 99% of the students evoked these ideas at least once in the electricity set, only 76% did so in the magnetism set. The magnetism results had many unanswered or unclassifiable responses, probably due to the context's difficulty.

## <span id="page-11-1"></span><span id="page-11-0"></span>VI. RESULTS ANALYSIS IN NEWTON'S THIRD LAW PROBLEMS

### A. Newton's third law in electric interactions

Just as we did with items 1 and 2, for item 3, we established categories to classify students' answers based on their diagrams and associated explanations. The same four categories emerged for both the electricity and magnetism versions of the test. We present examples from the electricity test in Fig. [11](#page-11-2).

The expected correct answer for any interaction item is that both charges exert a force of the same magnitude on each other, just in opposite directions. Students who acknowledged this by drawing vectors of similar length, such as shown in Fig. [11\(a\)](#page-11-2), and explicitly mentioned either Newton's third law or a comparison of the forces' magnitudes using Coulomb's law, were listed in the physics principles category. The explanation given by the student in the example was, "They are of the same magnitude.



FIG. 11. Student diagram examples for each category of item 3 in the electricity test.

Both charges should feel the same force because of Newton's third law".

The following two categories are very similar. Both mention that the greater charge will exert a greater force on the smaller one than vice versa. What makes these two categories different is the reasoning behind them. The category magnitude of the source groups answers that state the reason for this greater force is exclusively the magni-tude of the charge producing it. Figure [11\(b\)](#page-11-2) shows an example diagram for an answer classified under this category, represented by the following statement: " $+3q$ exerts a bigger force on  $+q$ .  $+3q$  is bigger and repels  $+q$ with greater magnitude".

On the other hand, the category magnitude of the field groups the answers of students who claimed that the reason for the greater force is the magnitude of the electric fields produced by the charges. Figure [11\(c\)](#page-11-2) shows an example diagram for this category with this explanation: "The force exerted on the charge on the right is three times larger than the other one. The value of  $+q$  and the value of  $+3q$  means that it is three times  $+q$ , for which we know that the magnitude of the electric field is three times stronger." If a student mentioned the charge and electric field magnitude being greater than in the previous case, we classified their answer in the same category. Students who mentioned both characteristics used the charge's magnitude to explain the field's magnitude and consequently justified the exerted force.

The *inverse* category was the name for those answers claiming that the smaller charge exerts a greater force on the bigger one than vice versa. All responses with a similar claim were classified in this category regardless of the associated explanation (charge's magnitude or field's magnitude). For example, Fig. [11\(d\)](#page-11-2) shows a diagram for this category, and the explanation was, "The bigger the charge, the bigger the magnitude of the force. If a charge is bigger than the other one, the force that it experiences concerning this one will be greater".

On this occasion, the category other groups all of the answers that did not fit any other category, were not frequent enough to have a class of their own, or were (majorly) unanswered items.

#### <span id="page-12-0"></span>B. Newton's third law in magnetic interactions

After analyzing the magnetism version, the same four categories emerged for item 3 in the electricity test. Figure [12](#page-12-1) presents the example diagrams for each category.

For the physics principles category, represented in Fig. [12\(a\),](#page-12-1) the student's explanation was, "They are the same. Because of Newton's laws, action reaction of the same magnitude."

In the magnitude of the source category  $[Fig. 12(b)]$  $[Fig. 12(b)]$ , the student wrote: "The force in B produced by A is greater than the one on A produced by B. A has a greater current, for which the force increases."

Figure [12\(c\)](#page-12-1) shows the diagram for the category magnitude of the field, explained by, "The magnitude of the force with which  $(1)$  attracts  $(2)$  is three times larger than the one with which (2) attracts (1). Using the formula  $F = v \times B$ , if the field of a wire is greater [than the other], then it will

<span id="page-12-1"></span>

FIG. 12. Student example diagrams for each category of item 3 in the magnetism test.

attract the other wire more potently." In this case, the student uses an incomplete version of the Lorentz equation for a charge moving in a field, disregarding that the item refers to an electric current.

The *Inverse* category is also present for this test version. The written reasoning for this diagram [\(11d\)](#page-11-2) was, "One is three times larger than the other one".

<span id="page-13-0"></span>Again, we grouped unanswered items and answers that did not fit any other categories under the other category.

## C. Newton's third law in electricity and magnetism: Comparison

Item 3 presented similar categories in both versions of the test, which allows a direct comparison of students' answers in both contexts. In this case, we center the comparison in two sections: the use of physical principles, such as Newton's third law, Coulomb's law, or Lorentz's law, and the *larger implies larger p-prim*, which encompasses the remaining three main categories. We present the percentages of answers for each category in Table [III.](#page-13-2)

## 1. Physics principles category

As we observe in Table [III,](#page-13-2) only 19% of students answered this item correctly in the electricity test, of which 10% evoked Coulomb's law and the others Newton's third law. This result is similar in the magnetism context, where only 15% of students correctly answered this item, of which half referred to Newton's third law and the other half evoked Lorentz's law. Since most sample students took the mechanics course about a year before the present study, this result shows that many students take universal concepts and apply them in different contexts. Still, the idea of an incomplete understanding of Coulomb's law or Lorentz's law cannot be ruled out, given the context in which the students answered the test. In other words, there could be a chance that students do not fully understand why the forces exerted are of the same magnitude based on either of the mentioned electricity and magnetism laws. However, they remember the universality of Newton's third law and turn to it.

<span id="page-13-2"></span>TABLE III. Comparison of the results for item 3 per category in the electricity and the magnetism versions.

	Electricity	Magnetism Item 3	
Category	Item 3		
Physics principles	19%	15%	
The magnitude of the source	38%	36%	
The magnitude of the field	13%	17%	
<b>Inverse</b>	18%	16%	
Other	12%	16%	
Total	$100\%$	$100\%$	

#### 2. Larger implies larger p-prim

It is easy to detect that the difficulty "the bigger the charge, the bigger the force it will exert" is the strongest one in both contexts. Fifty-one percent of the sample stated it in the electricity test and 53% in the magnetism test (combining the categories magnitude of the source and magnitude of the field).

The category magnitude of the source garnered 38% of the students' answers in the electricity test, which shows the most common larger implies larger p-prim because they focus exclusively on the amount of electric charge. This same category received 36% of students' answers concentrating directly on the magnitude of the electric current carried along the wire in the magnetism counterpart. This result is very similar to other results in electricity and magnetism, as well as in mechanics.

As for the magnitude of the field category, 13% of students who took the electricity test provided consistent answers trying to use Coulomb's law, but they focused only on the electric field produced by the charge exerting the force without considering the magnitude of the charge receiving the force—the opposite of the responses in the Inverse category (18%). In the magnetism version, 17% of the responses fell in the category magnitude of the field. Again, these students might have been trying to focus on the Lorentz force concept but only paying attention to the magnetic field and not on the magnitude of the current on which the field exerts force. The inverse category for this same context showed that 16% of the students analyzed the Lorentz force equation incompletely by focusing on the magnitude of the current in the wire that receives the force but not on the magnetic field that exerts it. In both cases, the categories magnitude of the field and inverse make it more evident that students try to work the math in the equation, leaving aside any physics principle such as Newton's third law, implying a lack of understanding of the topic.

Our results generally showed very consistent behavior in all the categories in each test version, with similar numbers of students. This finding could imply that, regardless of the topic (electricity or magnetism), students could not relate Newton's third law (learned in the year before the electricity and magnetism course) to electric or magnetic interactions or more recently taught physical principles.

## <span id="page-13-1"></span>D. Consistency analysis of Newton's third law in electric and magnetic interactions

Some information was not immediately available through these results and required follow-up on the students' answers to the three items. Table [IV](#page-14-0) presents an analysis of the follow-up to students' responses through the three items, considering the four main categories of items 1 and 2, and the categories of item 3 separated as physics principles and p-prims. The table presents the students' percentages for each reasoning in items 1 and 2 and consistency with their answers in item 3.

Categories for items 1 and 2	Electricity item 3		Magnetism item 3	
	Physical principles	p-prim	Physical principles	p-prim
Physics principles	$7\%$	4%	10%	$10\%$
Sign-based	11%	45%	$2\%$	8%
Force-field confusion	$3\%$	19%	$0\%$	15%
Electricity and magnetism interference	1%	$2\%$	$2\%$	22%

<span id="page-14-0"></span>TABLE IV. Consistency analysis of the total test, comparing answers in items 1 and 2 with responses in item 3 for each test version.

Our first finding in this analysis suggests that when the students evoke the physical concepts, such as Coulomb's law or Lorentz's law in items 1 or 2, they are more likely to evoke physics principles for interaction item number 3. As shown in Table [II,](#page-10-2) 11% of students were classified at least once in the physics principles category for either item 1 or 2 in the electricity test; as shown in Table [IV,](#page-14-0) 7% of the sample (more than a half of these students) also used physics principles to answer item 3 correctly. This proportion, close to 2 out of 3, could imply that only students who truly understand basic physics concepts, such as Newton's laws, can understand Coulomb's law, or at least can evoke physics principles to analyze items regarding interactions. Something similar occurs in the magnetism context, where 20% of the students evoked at least once an idea related to Lorentz's law to answer items 1 or 2, and half of them correctly answered item 3 (10% of the sample). Moreover, comparing the results of Tables [II](#page-10-2) and [IV,](#page-14-0) we can observe that all of the students who used Coulomb's law (11%) or Lorentz's law (20%) to answer either item 1 or 2 answered item 3 using Newton's third law or larger implies larger p-prim answer, rather than responses that would be unclassifiable or no answer. This result differs, for example, with 56% of students who used sign-based reasoning for items 1 or 2 (which, in total, were 59% according to Table [II](#page-10-2)), and then a classifiable answer for item 3.

The number of students who correctly answered item 3 after not using reasoning corresponding to the physics principles decreased significantly compared to those guiding their answers by a larger implies larger p-prim. Naturally, the sign-based category being the most frequent in the electricity test showed that most responses grouped in that category did not relate electrical interactions with other physics principles. Around 4 of 5 students with Signbased answers for items 1 or 2 based their response to item 3 with a larger implies larger p-prim. The same proportion resulted when the magnetism version was analyzed, except that sign-based reasoning was not as frequent.

The tendency continues in the force-field confusion category and is even more evident than in the previous one. In the electricity test, most students who presented this difficulty at least once on the first pair of items followed a larger implies larger p-prim when answering item 3. However, the most substantial evidence emerged from the magnetism test analysis, where none of the students who presented a force-field confusion (18%, see Table [II\)](#page-10-2) gave an answer based on physics principles (15% evoked a p-prim-like answer, and 3% provided an unclassifiable or unanswered question). This result may imply that students who cannot distinguish between a force and a field might lack understanding of interaction (allegedly learned during the mechanics course a year earlier). This lack of comprehension of forces and interactions might be one of the causes of the force-field confusion.

The interference category showed a similar result to the force-field confusion category. Although it was uncommon to detect interference in the electricity test (6%, see Table [II](#page-10-2)), we observed that half of these students did not give a classifiable answer for item 3 (or did not answer). However, the magnetism test results showed that 9 out of 10 students who presented an interference while answering the first pair of items also gave a larger implies larger pprim when answering item 3.

In short, these findings may imply that the conceptual understanding of Newton's laws of motion to analyze interactions is crucial to understanding any interaction, such as with electric and magnetic forces. We elaborate on this idea in Sec. [VII.](#page-14-1)

## VII. DISCUSSION

#### A. Electric and magnetic interactions

<span id="page-14-1"></span>Several studies confirm that students perceive electricity and magnetism as two intangible, complex subjects [\[7,](#page-18-6)[14](#page-18-13)[,31](#page-19-0)[,33\]](#page-19-2). The lack of daily proximity to these phenomena can produce difficulties when students try to learn these topics (for instance, electric and magnetic interactions). This explanation relates to our first finding: a small number of students can formulate a description of an electric or a magnetic interaction based on a physics concept, such as Coulomb's law or Lorentz's law. Our findings make sense that students who reason correctly in physics principles are more consistent in the magnetism context than electricity. Most students learn magnetism for the first time, while they could have learned about electric interactions in previous educational stages.

Our results presented another essential fact related to students' diverse visual representations of a field, even when incorrect. As shown in Table [II](#page-10-2), only 11% of the students in the sample evoked Coulomb's law to answer either items 1 or 2 of the electricity test, while 20% turned to Lorentz's law in the magnetism test. Also, Fig. [9](#page-9-0) shows students' types of visual representations under the forcefield confusion category. We found the same tendency after looking at the other answers: around three-quarters of electricity sketches display field lines. In contrast, magnetism sketches are equally distributed, with half of the students drawing field lines and the other half drawing only a vector. Studies like Ref. [[39](#page-19-8)] for electricity and Ref. [\[38\]](#page-19-7) for magnetism found that students with a deeper understanding of the concept of a field represent it through field vectors and use those vectors for analysis. The slight difference in the number of students correctly answering item 1 or 2 supports this idea: more students can use physics concepts to answer magnetic interaction items than electric interaction items. As we mentioned before, this incidence might come from teaching: instructors use different visual representations when talking about magnetism, more than in electricity, thereby developing abilities for magnetism in the students.

We highlight that the ROS statement "The same signed charges repel each other, while opposite sign charges attract each other" is deeply ingrained in students' mindsets. However, previous studies that use multiple-choice items, such as Refs. [[1](#page-18-0)[,4,](#page-18-3)[5](#page-18-4)[,32,](#page-19-1)[58\]](#page-19-25) could have overlooked this idea. A vast amount of literature points to the disadvantages of multiple-choice questions such as Refs. [[59](#page-19-26),[60](#page-19-27)]. Moreover, the fact that this "rule" is, technically, not wrong decreases its possibility of drawing much attention. To clarify this last affirmation, we observe that the reasoning, "Both particles repel each other because of Coulomb's force vector nature," differs from the ROS statement, "Particles repel each other because they are both positive." Still, both are correct answers. However, given the phenomenographic nature of our study based on open-ended items, we show that most students who refer to the ROS do not necessarily understand the nature and physical background of electric interactions. This is true, also, when talking about magnetism, except with a considerably lower frequency. We propose to address this problem during instruction. On the one hand, the students learn magnetism for the first time, and on the other hand, if there were a rule for magnetic interactions equivalent to the ROS, it would be the exact opposite of the ROS, which would then lead to confusion. This difficulty may be why most students turn to other ideas when trying to answer magnetic interaction items.

The literature documents the confusion between force and field (electric and magnetic). Our results showed that around one-fifth of students have difficulties differentiating between a force and a field, whether electric or magnetic. This finding is consistent with previous findings in other studies, like Refs. [\[22](#page-18-21)–[25](#page-18-24)] for electricity and Refs. [[29](#page-18-29),[31](#page-19-0),[32](#page-19-1),[34](#page-19-3),[35](#page-19-4)] for magnetism. This confusion is a very well-studied difficulty and has a significant presence in most related research. However, our results showed this confusion was the most common difficulty found in the electricity test, but not in the magnetism test, just as in the CSEM item 24 [[1\]](#page-18-0). This item asked the students for the force exerted between two wires carrying parallel currents I and 3I. In this item of the CSEM, 7% of the sample selected an option that stated no force, as if the forces canceled each other due to the currents' direction (just like what would happen with the resulting magnetic field produced by equal currents). In our results, 18% of the answers for either items 1 or 2 followed a force-field confusion.

We presented evidence that could hold the key to understanding this naive idea: the force-field confusion slightly increases when students face an electric or magnetic interaction exerted by opposite sources of field (positive and negative electric charges, or ingoing and outgoing electric currents). Students who answered the electricity test and could not differentiate a force from a field presented fewer visual representations to support their answers than their counterparts on the magnetism test. This difference could explain why the percentage of students under the force-field confusion category is more significant for the electricity test than for the magnetism test (again, supporting the ideas presented in Refs. [\[38,](#page-19-7)[39\]](#page-19-8)). It is noteworthy that field vector sketches have been previously related to understanding electric and magnetic interactions [\[23\]](#page-18-22). According to our results, this form of representation was more common in the magnetism test, where physicsprinciples answers were frequent. The instruction and the complexity of the magnetic phenomena are two plausible explanations for this result. Since magnetic force items need complex answers, students turn to sophisticated solutions, thus, including field vector sketches.

As for the interference phenomenon, previous research found that interference is more common in answers based on magnetism concepts provided to electricity items [\[4,](#page-18-3)[5](#page-18-4)[,32\]](#page-19-1), with the timing of implementing the test being the most common cause cited. Usually, students answer these items after magnetism instruction, which could be a couple of months after the electricity instruction ended, making the magnetism concepts the most recallable at the test time. This timing has reportedly led students to answer electricity questions using magnetism concepts. Our results suggest that test administration time as a cause for interference might not be the only one: other factors might produce interference when students answer magnetism questions with electricity concepts. In our study, we administered the test after magnetism instruction, just like the studies in the literature; however, our interference results appeared to be the opposite of previous studies. Our results show that it is far more common for students to answer magnetic interaction items with electricity concepts (such as the ROS) than vice versa. The ROS might be one of the reasons for this interference. For example, item 24 of the CSEM shows results very similar to ours. One of the options states that the exerted force among the wires has equal and repulsive magnitude, and 23% of the sample selected this option [[1\]](#page-18-0). The same happened when we analyzed the MCS item 22 [\[21\]](#page-18-20), which asks for the direction of the interaction between two currents of the same magnitude, in opposite directions, and uses the exact visual representation we do in our test. One of the options implies that the exerted force between the currents is attractive; 29% of the sample selected that option. As shown in Table [II](#page-10-2), 28% of our sample gave similar answers to either items 1 or 2 of the magnetism test. These two similarities may indicate that this interference produced by the electricity ROS in magnetic interactions is a phenomenon that has been occurring for some time and may have been overlooked in previous studies. Thus, electricity and magnetism interference could be caused by multiple reasons other than test administration time, depending on the concept that students are learning.

## B. Newton's third law in electric and magnetic interactions

Mainly, when analyzing the capacity to relate electric and magnetic interactions with Newton's laws of motion, we found out that explicitly involving physical concepts (demonstrating proper conceptual understanding) is an ability that shows up in any electricity and magnetism interaction item. The literature shows that students have difficulties identifying and correctly determining the interaction between electric charges or electric currents [\[5,](#page-18-4)[26](#page-18-25)[,27\]](#page-18-26). The larger implies larger  $p$ -prim is extensive in our results, just as many previous investigations in electricity and magnetism contexts [\[1](#page-18-0),[5](#page-18-4),[28](#page-18-27)–[30\]](#page-18-28) and others, like mechanics [\[61\]](#page-19-28), have reported. For instance, 85% of the students can correctly identify the force exerted by a  $+4Q$  charge on a  $+Q$  charge in item 25 of the BEMA. However, when exchanging the charges' magnitudes, the correct answer percentage drops to 45% [[19](#page-18-18),[20](#page-18-19)]. The same happens in the CSEM items 3 and 4 (which present the same  $+4Q$  and  $+Q$  scenario), with 84% and 56%, respectively [\[1](#page-18-0)]. Moreover, the data in this study show that p-prim-like answers spike from 10% to 41%.

Our results in Table [III](#page-13-2) show that only 19% of the students use physics principles to answer item 3, and 69% use larger implies larger p-prim reasoning. We propose a simple explanation for this. A multiple-choice test like CSEM, BEMA, or MCS may limit the student's options and lead them to the correct answer by listing it among the possibilities. However, in an open-ended item, students are not given further information apart from the problem description, requiring them to answer from scratch, resulting in a wide variety of answers (which would also explain the high number of unclassifiable responses that we obtained in all test items.)

Also, our results suggest that a student who turns to physical principles such as Coulomb's law and Lorentz's law to answer electric or magnetic interaction items will most likely turn to Newton's third law when the interaction is between field sources of different magnitude. This finding may have several explanations, but most studies agree that only students with a true conceptual understanding of physics principles could relate to these concepts due to the abstraction of electricity and magnetism concepts and the impracticality to witness them. Thus, most students think these concepts are unrelated to other physical laws that might be more familiar [\[7](#page-18-6)[,14,](#page-18-13)[22,](#page-18-21)[58\]](#page-19-25). They also show that most students cannot let go of some preconceptions even after instructions. For example, the ROS allows them to correctly determine the type of electric interaction between two charges (attraction or repulsion). However, it is not helpful to determine the magnitude of this interaction when the field sources have different magnitudes. Since the ROS does not help students identify the magnitude of the interaction, they turn to the larger implies larger *p*-prim when the magnitude of the charge increases.

#### VIII. CONCLUSIONS

This investigation has contributed to a better view of student understanding of electric and magnetic interactions. When facing problems regarding interactions between identical charges, the same magnitude but opposite charges, and with different magnitudes compared to problems of interactions between identical currents, currents of the same magnitude but opposite directions, and currents of different magnitudes, the difficulties that students manifested were parallel, although not in the same proportion. These results suggest that, in general, students perceive electric and magnetic interactions as equally challenging, and they have the same difficulties regardless of the field source (electric or magnetic).

We identified four main ideas from these results: (a) the easily accessible piece of knowledge rule of signs, which does not provide enough evidence of students' complete conceptual understanding of electric interactions; (b) the force-field confusion and its similarity in the two contexts—as the most parallel category of our results; (c) the importance of semiotic representation when answering an electricity and magnetism problem; and (d) the interference phenomenon, which did not manifest in the same way as in previous studies.

We have identified that many students turn to a previously learned mnemonic resource such as the rule of signs to analyze electric interactions and produce a correct answer. The sign-based answers dominated the electricity version (59%) and garnered 10% of the answers in the magnetism version. This data, by itself, does not necessarily imply that these students misunderstand the electric or magnetic interactions. However, the consistency analysis we conducted revealed that many students in our sample (around 45% in electricity and 8% in magnetism) evoked a larger implies larger p-prim when analyzing interactions of field sources with different magnitudes after turning to the ROS in questions 1 or 2. A small number of students who even evoked the ROS in items 1 and 2 could answer question 3 correctly (evoking a physics principle). These two results together suggest that these students do not actually understand electric and magnetic interactions.

The force-field confusion emerged as the most consistent category between contexts. It was the most frequent difficulty (23% electricity and 18% magnetism) of each sample evoked at least once during the test. Although this was not unexpected, we would like to point out that the consistency analysis revealed this confusion is quite challenging for students. Almost all of the students with forcefield confusion gave a larger implies larger p-prim answer to question 3. Notably, this category showed the most parallel between contexts, suggesting that students do not understand the difference between a force and a field, regardless of context.

Our data support the idea that the different visual representation options can predict student understanding of electric and magnetic phenomena. Field vector diagrams were more frequent in the magnetism test, which had more students analyzing interactions using, at least once, physics principles than in the electricity sample (20% in magnetism vs 11% in electricity), and with fewer Force-field confusion responses (18% in magnetism vs 23% in electricity). Also relevant, three-quarters of the students who presented force-field confusion used field lines in their diagrams to support their answers (as did half of the magnetism students), providing additional evidence that supports the relevance of the selected semiotic representation.

We also presented evidence that could indicate that the interference phenomenon is caused not only by the timing of evaluation versus the instruction but also by other factors. So far, the literature has shown that interference could be produced by the instruction's timing and the administration of the tests. Our data suggest that the most common interference is to analyze magnetic interactions as if they were electric when it comes to interactions. This result contrasts with previous studies in other electricity and magnetism topics, such as field analysis or charge-field interactions [[5](#page-18-4)], in which the electricity questions are answered as if they were magnetism problems.

From these findings, we make five recommendations that could help during instruction. First, although the larger implies larger p-prim is a well known and deeply studied difficulty, we suggest emphasizing the electricity and magnetism principles underlying the interaction analysis of two field sources, possibly reducing the larger implies larger p-prim frequency in students' answers. Second, the force-field confusion emerges as a persistent difficulty for electricity and magnetism students, especially when the interaction is between two opposite-field sources. We

encourage instructors to explore different scenarios and combinations of charges or currents that prompt students to deeper reasoning, allowing them to distinguish a force from a field before answering. Our third recommendation considers the use of different visual representations and the time instructors spend explaining the uses and equivalencies of these representations. Our results (like previous studies in the literature) showed that understanding and producing correct field vectors significantly relates to the conceptual understanding of electric and magnetic interactions. Our fourth recommendation is in the matter of the use of the electricity and magnetism rule of signs. As stated before, using this "rule" is not properly an incorrect concept because it works for any electric interaction (thus resulting in an understudied idea). Still, we observe from our results that the great majority of students who evoke this ROS do not provide evidence of understanding the physics principles behind these electric and magnetic interactions. There probably is an enormous variety of actions that could be taken to address this. One of them could be replacing ROS use during instruction and consciously analyzing any interaction through Coulomb's or Lorentz's law, regardless of how easy it may seem. Fifth, we point out the interference phenomenon presented in our results could be a consequence of the ROS or may have very different causes. Although this type of difficulty has not yet been studied with frequency, there could be some actions to address it, such as pointing out the confusion to bring awareness of it so it can be corrected.

Since the instrument is an open-ended questionnaire, there is always room for authors' interpretation and diversity of student answers that may be hard to interpret and classify. We tried to minimize it using the phenomenographic methodology described previously. However, we recognize that this effect plays a role in the limitations of this study.

We encourage further studies in different contexts to verify and further explore our findings. With this information about the different difficulties in electric and magnetic interactions, we are taking the opportunity to generate a parallel pair of multiple-choice concept inventories that leave the authors' interpretations out of the equation. This would also allow statistical analysis, reducing the inherent noise that comes with open-ended items' analysis.

Lastly, we would like to highlight the usefulness of openended items to uncover details that might have been overlooked due to the massive use of multiple-choice items during evaluation. The phenomenographic methodology allowed more profound insight into students' thinking, and it provided enough information to discern these missing details, hopefully leading to small but significant adjustments to electricity and magnetism instruction. Finally, the information provided from this study when analyzing parallel scenarios in electricity and magnetism will significantly contribute to further research.

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