Editors' Suggestion

Identifying students' mental models of the apparent motion of the Sun and stars

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We investigated to what extent secondary school students have insight in the apparent motion of the Sun and stars (AMoSS). We used the AMoSS test instrument, which focuses on distinctions between different aspects of the apparent motion of the Sun and stars. It consists of 12 multiple-choice questions accompanied by explanations. We administered the test to students of the fifth year (16–17 years old) of six Belgian secondary schools ($N = 410$) during a science lesson in school and asked them to explain their choices. We found that, despite instruction, most students only demonstrate a rudimentary understanding of the apparent motion of the Sun and stars for different times during the day, different times during the year, and different locations of the observer. Moreover, we see a clear distinction between the responses to the questions about the Sun and stars. Thanks to the classification system that we developed to categorize the written explanations and a latent class analysis, we are able to identify different mental models that students use to answer questions about the apparent motion of the Sun and stars.

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I. INTRODUCTION AND PROBLEM STATEMENT

The aim of the SLOPE project is to study learning opportunities in a planetarium environment: visualizing the (night) sky throughout the day, the year, and at various locations on Earth is one of the main goals of a planetarium. For this purpose, planetariums might be a powerful setting to support and enhance student learning of astronomical phenomena, which have been shown difficult to understand deeply. We want to explore this setting by developing a research-based planetarium presentation, based on a deep understanding of student thinking and student difficulties. In a first step we designed and validated a new test instrument [1], which makes it possible to study students' understanding of basic astronomical concepts. The instrument focuses on the apparent motion of the Sun and stars (AMoSS), because we want to obtain a deep, systematic insight into students' ideas of the differences and similarities between these celestial motions. This will help us to design a planetarium presentation that supports students to increase their understanding of these phenomena. In this paper we report on findings on student thinking, based on the administration of the AMoSS test to a group of 16–17 year old Belgian students. We are specifically interested in examining the mental models that students use while explaining their answers.

Although we see the Sun rise and set every day and observe the stars during the night, it is still difficult for many people to describe properly the movement of the Sun, Moon, and stars in the sky. Moreover, children, adolescents, and adults seem to have difficulties explaining the cause of the apparent motion of celestial objects [2–7]. Teaching these phenomena seems not that easy. Therefore, detailed insight in student thinking of these apparent motions is needed.

We first summarize different studies about students' understanding of the apparent motion of the Sun and stars. In Sec. III, we describe how we organized the test and how we analyzed the answers of the multiple-choice questions. We elaborate on the development of the classification system of the written explanations and how we found patterns in the answers which lead to the delineation of different mental models. Section IV lists the results of the various steps in the study. The last section concludes with a discussion and ideas for further research.

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II. BACKGROUND

A. Students' difficulties in learning about the apparent motion of the Sun and stars

We highlight several studies about aspects of the understanding of the apparent motion of celestial bodies by students of different ages.

Vosniadou and Brewer [8] report that primary school children have alternative ideas about the cause of the alternation of the day and night. They think that the Sun goes down behind a mountain or is covered by clouds, they explain that the Sun revolves around Earth or that Earth revolves around the Sun in one day. The children use the same arguments to explain the disappearance of the stars during the day: the stars are hidden behind the clouds, they move down on the ground, or they move to the other side of Earth. However, most children think that the stars are fixed and do not move in the sky. The reason we cannot see them during the day is because of the brightness of the sky due to the sunlight.

Plummer [9] describes in her cross-age study of children's knowledge of apparent celestial motion that most first-grade students do not yet understand that all celestial objects appear to move continuously across the sky as they rise and set, in the same direction, and along similar paths. Most third-grade students made a shift to viewing celestial objects as moving slowly across the sky, rather than staying fixed in the sky. The eighth graders in the study know that the Sun rises and sets on opposite sides of the sky. Concerning the apparent motion of the stars or the idea that we see different stars at night throughout the year, there is no significant improvement compared with the third-grade students.

Sadler came to similar conclusions [10] about the ideas of 25 ninth-grade students about the day/night cycle. He found five different explanations for the alternation of day and night: (1) the spinning of Earth, (2) the motion of the Sun around Earth, (3) the blocking of the Sun by the Moon, (4) the Sun goes out at night, and (5) the blocking of the Sun by the atmosphere. He also reports that students who followed a course in Earth sciences did not obtain better results than others. In accordance with these findings, Baxter [11] reports that many students of 15–16 years old believe that the reason for the day/night cycle is that Earth rotates around the Sun, the Sun rotates around Earth, or that the Moon covers the Sun.

Heywood *et al.* [12] found in their study on preservice teachers reasoning about the Sun's apparent motion that all participants attributed the day/night cycle to Earth spinning around its axis. Unfortunately there were no preservice teachers who related this day/night cycle explicitly to the Sun's apparent motion during the day. Most participants in the study mentioned that the Sun always rises in the east and sets in the west, without indicating an awareness of the variation of the exact location where the Sun rises and sets and hence follows a different path throughout the year.

Slater *et al.* [13] reported that a noteworthy percentage of undergraduate students in their study (12%) believe that in the Northern Hemisphere in winter the Sun is higher in the sky than in summer. Most students (38%) preferred the "fixed" notion of stars over "moving" stars at night.

From the above studies, we can summarize that students of all ages often reason alternatively about the apparent motion of the Sun and the apparent motion of the stars. This suggests that there is something inherently difficult in understanding these apparent motions. Since to our knowledge student understanding of both of these apparent motions—and particularly the mental models students have in this domain—has not been studied and compared systematically, we have set up this study.

B. Mental models

For a more detailed understanding of how students explain the phenomena of apparent motion, we try to identify their underlying mental models. Although in the literature there is some discussion about the exact definition of a mental model, in general, the term refers to the internal representations that people form of the outside world through their interaction with it. The notion of a mental model was introduced by the psychologist Craik [14] who postulated that people carry in their minds a small-scale model of how the world works. Johnson-Laird developed this idea in his research on human reasoning. For him a mental model is a reasoning mechanism that exists in a person's working memory [15]. Vosniadou and Brewer [8] use the term "mental model" in a similar way to Craik. They describe a mental model as a particular kind of mental representation with the following characteristics: (1) its structure is an analog to the states of the world it represents, (2) it can be manipulated mentally to make predictions, and (3) it provides explanations of physical phenomena. As Johnson-Laird, they also assume that mental models are dynamic structures which are usually created on the spot in the working memory to meet the demands of specific problem solving situations [8].

Bao [16] put forward his definition of a mental model by considering other descriptions in the literature. He states that mental models are "productive mental structures that can be applied to a variety of different physical contexts to generate explanatory results" (p. 13). Corpuz and Rebello [17] defined a mental model as "students' way of understanding a certain physical phenomenon," which can also be an unseen physical phenomenon. Mental models may contain contradictory elements and are generally different from scientific models, which are accepted as valid if they are coherent, stable, and experimentally validated. We take the definition of Corpuz and Rebello as a guideline in our research. We follow their view that a mental model essentially is inaccessible and that we, as a researcher, only can rely on an expressed version of it. This means that the description of a mental model always refers to what researchers discovered based on the expressed version of the mental model.

Collins and Gentner [18] propose that mental models can be formed through analogical thinking: when a person explains a domain with which they are unfamiliar, they tend to draw on a familiar domain, which they perceive as similar. Studies show that phenomena that cannot be perceived directly are often explained by tapping into an existing mental model and importing its relational structure to another domain [19]. For example, a mental model of water flow may be used to explain electrical current.

In the literature several descriptions of student mental models in the context of a broad range of phenomena [20–22] are documented. In the context of the apparent motion of celestial bodies we refer to the work of Vosniadou and Brewer. In their study about the mental models of the day/night cycle [8], they report that the majority of the elementary school children used a well-defined mental model of this cycle to answer questions and that these models seem to be logically consistent. Vosniadou and Brewer make a distinction between initial, synthetic, and scientific mental models. The synthetic models are generated by children as a solution to problems arising from the inconsistency between their initial model and the cultural accepted, scientific model [23].

There are, however, also some critical comments to the conclusions of Vosniadou and Brewer [24–26]. The main critic is that their conclusions are strongly related to the instructions they gave to the children, arguing that if you give the same instructions to educated adults, you get the same results. Nobes and Panagiotaki [24] conclude that there is little or no evidence for the consistent mental models described by Vosniadou and colleagues. According to these authors, children have neither conceptions nor misconceptions before they have any scientific knowledge about a specific topic: they simply do not know. Second, they argue that any information that children have is not coherent or theorylike, but instead is fragmented. They refer to diSessa's idea of "knowledge in pieces" [27]. According to diSessa and colleagues, the intuitive physics of a novice "does not come close to the expert's in depth and systematicity," and "the development of scientific knowledge about the physical world is possible only through reorganized intuitive knowledge" [28]. diSessa understands intuitive knowledge in terms of cognitive building blocks he calls "phenomenological primitives" or "p-prims." They are "phenomenological" in the sense that they are minimal abstractions from experience; they are closely tied to familiar phenomena. And they are "primitive" both in how people use them, as the obviously true ideas at the bottom level of explanation, and in their role as "nearly minimal memory elements, evoked as a whole …." The fact that any p-prim may or may not be activated, while building up a reasoning, can explain the incoherence in student answers to similar questions asked in slightly different ways [28]. However, it does not reject robust patterns of reasoning, such as so-called misconceptions.

Brown and Hammer [25] propose that the apparently conflicting views on the nature of students' mental models can be overcome by what they call "the complex system perspective." By considering students' conceptual thoughts as a complex dynamic system, both the existence of stabilities in student reasoning and the possibility of students answering one way in one context and a different way in another context can be understood [25]. In our study we follow their arguments and we will look for both consistency and inconsistency in student ideas about the apparent motions of the Sun and stars.

C. Different frames of reference

Several studies [2,3,29,30] suggest that students can only achieve insight in the underlying mechanisms of the apparent motion of celestial bodies by studying both the observations from Earth (geocentric frame of reference) and the actual motions as observed from an allocentric frame of reference, the view from space. It seems to be essential that students learn to think and alternate between these two frames of reference in order to understand the apparent motion of the Sun and stars and link these to the actual motion of Earth. Probably, specific instructional strategies are needed and students must be trained to switch between different frames of reference, to be able to really understand apparent celestial motions.

Cole *et al.* [31] argue that spatial thinking skills are needed to create an accurate mental model of these complex phenomena. Testa *et al.* [3] propose that teaching celestial motions by using a learning approach, that integrates causal reasoning with spatial thinking about the phenomena related to these motions, may help students progress from qualitative to more quantitative explanatory models of these motions.

In the context of our research project, we are interested in investigating how a planetarium can scaffold this process. Given their possibilities to visualize the (night) sky, planetariums might be a powerful setting to support students learning to switch between different frames of reference and enhance spatial thinking. While the traditional planetarium essentially provides a geocentric frame of reference, digital planetariums also use the available technology to contrast this geocentric frame with an allocentric view from space.

III. METHODOLOGY

A. AMoSS test: Multiple-choice questions

To investigate to what extent students have insight in the apparent motion of the Sun and stars, we developed and validated the AMoSS test instrument with 12 multiple-choice questions [32], which focus on distinctions between different aspects of the apparent motion of the Sun and stars. We reported on this process in a previous article [1]. We synthesize the test here in Table I. The six categories (rows) in this table give a symmetrical overview of elements related to the apparent motion of the Sun in the left-hand column and the corresponding elements related to the apparent motion of stars in the right-hand column. For each of these categories, we designed a parallel question for the Sun and the stars. For an example, we refer to Fig. 3.

We administered the AMoSS test to students of the fifth year (16–17 years old) of six Belgian secondary schools ($N = 410$) during a science lesson at school. All students attended general secondary education, which means that they are expected to go to college or university after their secondary school studies. Most students (94%) were following a science, technology, engineering, and mathematics (STEM) curriculum with extensive sciences and/or math courses. An introduction to astronomy was part of the geography curriculum of all students. The rotation of Earth, Earth's revolution around the Sun, and the cause of seasons were discussed. All elements of the test were addressed in one way or another during the lessons, but the apparent celestial motions were not all explicitly and systematically discussed during lectures. Most teachers use a beamer to project short movies or simulations and students use their textbook to consult figures on these phenomena. They have a globe in most classrooms, but more advanced 3D models such as a tellurium are rather rare. Thus, teachers have the basic tools to teach students the underlying spatial models of the apparent motion of celestial bodies, but because the number of lessons available is so small, the actual time spent on this subject is rather limited. The test was taken just after the Christmas break. During the weeks before this break, the students attended geography classes and also took exams of the material covered.

The 12 questions were asked to the students in a random order. To exclude a bias in the results due to this order, four different series of questions were created, each with a different order.

The students were free to decide whether or not to participate. Only the students who signed the informed consent form are included in the study. No incentive was given to the students. All students delivered the test before the science class, which lasted 45 minutes, ended. Fourteen students left a few multiple-choice answers (1, 2, or 3) blank.

During the analysis of the multiple-choice answers a score of 1 was given if the correct alternative was chosen and 0 if an incorrect alternative was chosen or if no answer was given.

B. AMoSS test: Written explanations

We also asked the students to explain their answers in order to get insight in the mental models they use. Because time for test administration was limited to 45 minutes, we limited the number of written explanations to three pairs of two questions. For each selected question about the Sun, we also asked an explanation for the parallel star question. For each of the four series of questions, different pairs were selected, such that enough written explanations were obtained for the 12 questions of the AMoSS test. To analyze these written explanations, we built a categorization scheme bottom up from the data. In the first step we classified the written explanations into five groups (see Fig. 1) as follows.

1. Daily motion (D): the explanation refers to the daily apparent motion of the Sun or star;
2. Yearly motion (Y): the explanation refers to the yearly apparent motion of the Sun or star;
3. Globe (G): the explanation refers to the shape of the Earth;
4. Incomprehensible (Z): it is not clear what the student means;
5. No explanation (X): the student has not written an explanation.

The literature indicates that specific instruction is needed to make students understand the basics of the celestial motion of the Sun, Moon, and stars. This dedicated instruction should consider the motion of celestial objects from both a

TABLE I. Framework of the AMoSS test. Similarities and differences between the apparent motion of the Sun and stars.

(I)	Apparent motion of the Sun	(II)	Apparent motion of a star
	(A) Daily Sun position changes: Sun's path		(A) Nightly star position changes: Star trail
	(B) Sun culmination changes during a year		(B) Star culmination does not change during a year
	(C) Sunrise and sunset position change during a year		(C) Star rise and star set position do not change during a year
	(D) Sun culmination depends on observer position		(D) Star culmination depends on observer position.
	(E) Sunrise and sunset position depend on observer position		(E) Star rise and star set position depend on observer position
(III)	Seasons: Colder and warmer periods on a specific location during a year, due to Earth's revolution	(IV)	Sky map changes on a specific location during a year, due to Earth's revolution

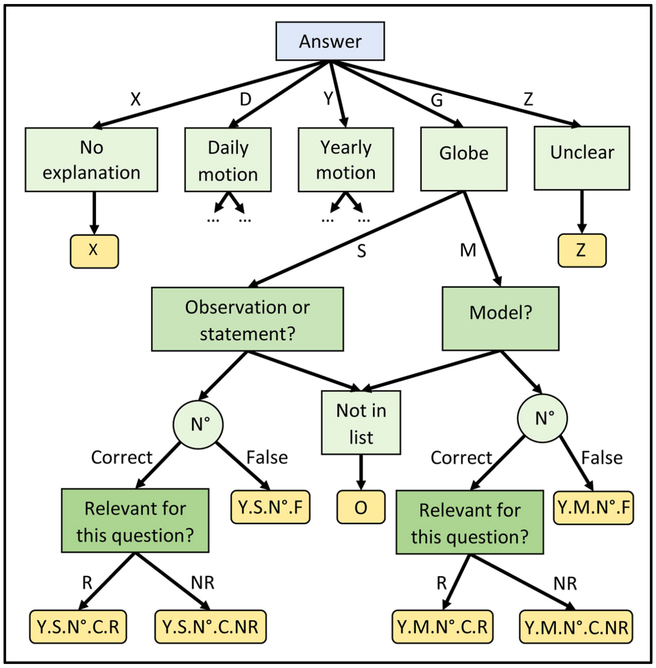
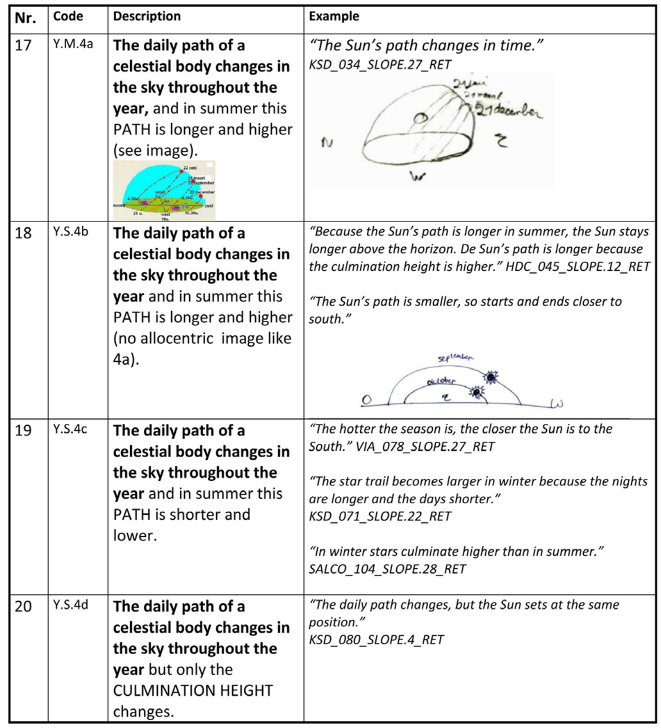


FIG. 1. Categorization scheme for the written explanations of the students.



Nr.	Code	Description	Example
17	Y.M.4a	**The daily path of a celestial body changes in the sky throughout the year,** and in summer this PATH is longer and higher (see image).	*"The Sun's path changes in time."* *KSD_034_SLOPE.27_RET*
18	Y.S.4b	**The daily path of a celestial body changes in the sky throughout the year** and in summer this PATH is longer and higher (no allocentric image like 4a).	*"Because the Sun's path is longer in summer, the Sun stays longer above the horizon. De Sun's path is longer because the culmination height is higher." HDC_045_SLOPE.12_RET* *"The Sun's path is smaller, so starts and ends closer to south."*
19	Y.S.4c	**The daily path of a celestial body changes in the sky throughout the year** and in summer this PATH is shorter and lower.	*"The hotter the season is, the closer the Sun is to the South." VIA_078_SLOPE.27_RET* *"The star trail becomes larger in winter because the nights are longer and the days shorter." KSD_071_SLOPE.22_RET* *"In winter stars culminate higher than in summer." SALCO_104_SLOPE.28_RET*
20	Y.S.4d	**The daily path of a celestial body changes in the sky throughout the year** but only the CULMINATION HEIGHT changes.	*"The daily path changes, but the Sun sets at the same position." KSD_080_SLOPE.4_RET*

FIG. 2. Part of the numbered list with possible written explanations.

geocentric and an allocentric frame of reference [3,33,34]. Based on this idea, in the second step of the classification process, we distinguish two types of explanations.

1. Statement (S): the explanation is based on an observation from the point of view of an observer on Earth or on something the student knows;
2. Model (M): the explanation shows at least one element of an allocentric point of view.

For each written explanation we also indicated whether the written explanation was correct (C) or false (F) and whether the explanation was relevant (R) or not (NR) for the question. Moreover, we created a numbered list of common answers and assigned a number from the list to each written explanation (see Fig. 2). As a result, each answer is characterized by a code of several letters and a number (for an example, see Table II). Answers that did not appear in the list were given code O. The steps taken to classify an answer are schematically represented in Fig. 1.

Developing the classification system was a cyclic process. After the first reading of the students' responses and listing the written explanations, the list of answers was far too long. In several rounds, we combined different answers and shortened the list. This resulted in a coherent classification system which we checked for interrater reliability with two independent raters and satisfactory results (overall Cohen's $\kappa = 0.75$). We also checked the validity of the classification system by interviewing 11 students, who were part of the group ($N = 410$) who filled in the AMoSS test. In these interviews, we checked if our interpretation of the written responses matched the student's understanding as expressed during the interview. For 47 of the 64 responses (73%) checked, the student stuck to their original explanation and the assigned code was consistent with what we learned from the interview. Seventeen answers could be refined, mostly related to clarifying an incomprehensible answer (code Z) or related to question III.

C. Mental models

Based on the classification of all student answers and by studying answer patterns over different questions, we have identified specific mental models students use to explain different aspects of the apparent motion of the Sun and

TABLE II. Example of the classification of students' written explanations of the questions of the AMoSS test about the daily motion (questions I.A and II.A; see Fig. 3).

Statement (S)	Model (M)
"The Sun rises in the east, culminates in the south and sets in the west." (D.S.2a.C.R).	"Since Earth rotates on its axis from the west to the east, the Sun moves in the sky from the east to the west." (D.M.1a.C.R)
"Stars move from the west to the east." (D.S.2b.F).	"Earth rotates but the star does not move in the sky." (D.M.1b.F)

stars. Some students seem to be very consistent in their way of answering the questions of the AMoSS test.

We also identified these homogeneous groups within the tested sample group by running a latent class analysis (LCA) on the multiple-choice answers. This statistical tool is an appropriate technique, because we assume that latent classes exist and can explain the patterns found in observed answers [35]. We used the poLCA package of software R and entered a data file with the answers of the multiple-choice questions (a, b, c, …) of the students. We excluded question III because this question allowed for multiple answers. Because we study pairs of questions to find differences and similarities between the Sun and the star questions, we also excluded question IV.

IV. RESULTS

A. Descriptive results of the multiple-choice answers

The mean score of the results of the AMoSS test for the analyzed student population in Belgium ($N = 410$) is 5.5 out of 12 possible, or 46%, while the median is 6 and the standard deviation is 2.2. For the questions about the Sun the mean score is 3.4 out of 6 (57%), with a median of 3 and standard deviation 1.4. For the star questions the mean score is 2.1 out of 6 (35%), with a median of 2 and a standard deviation of 1.2 (see Table III). A paired-samples t-test was conducted to compare the scores on the six Sun and the six star questions. There was a significant difference in the scores for the Sun and the star questions, $t(409) = 15.8,\ p < 0.001$.

A detailed report for each pair of questions of the different categories of the AMoSS questionnaire (see Table I) is given below. For each pair, we present a cross table with the answers of the students. This table gives a complete view on how student answers differ between the questions about the Sun and the stars. We will only highlight the most important findings, which are relevant for the description of student mental models about the apparent motion of the Sun and stars, which is the aim of this paper. The numbers that are called out in the text are underlined in the cross tables.

1. Question I.A and question II.A

A large proportion of the students (62%) answered both questions I.A and II.A about the daily apparent motion (see Fig. 3) correctly. Out of 365 students (89%) who knew that the Sun apparently moves from east to west during the day, 254 students (62%) knew that also the stars apparently move from east to west during the night, 46 students (11%) thought that the stars move opposite to the Sun, while 25 students (6%) indicated that the stars have not moved in one hour (see Table IV).

TABLE III. Student scores on the AMoSS test ($N = 410$).

	All questions (12)	Sun questions (6)	Star questions (6)
Mean	5.5	3.4	2.1
Median	6	3	2
Standard deviation	2.2	1.4	1.2

2. Question I.B and question II.B

Most students answered questions I.B and II.B (see Fig. 4) about how the Sun's culmination height changes in a one month period correctly, but only a small minority knew that the culmination heights of the stars do not change throughout the year. Out of 299 students (73%) who knew that the culmination height of the Sun increases as we approach summer, 100 students (24%) thought that this is also the case for the stars, while 36 students (9%) thought that the culmination heights of the stars decrease. Only 27 students of this group (7%) answered correctly that the culmination heights of the stars stay fixed throughout the year (see Table V).

3. Question I.C and question II.C

With questions I.C and II.C (see Fig. 5) about the position of the Sun and star set, we observe something similar as with the previous question: most students answered the question about the Sun correctly, but the question about the stars incorrectly. Out of 237 students (58%) who knew that for an observer in the Northern Hemisphere the sunset shifts to the south as we approach winter, 113 students (28%) thought that this is also the case for the stars and 60 students (15%) thought that the stars set more to the north. Only 28 students of this group (7%) answered that the position of the star setting does not change throughout the year. It is also remarkable that 80

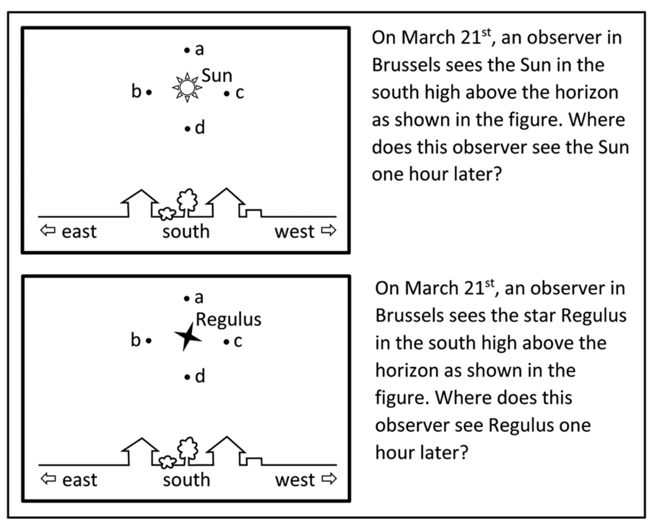


FIG. 3. Shortened version of questions I.A and II.A. For the complete versions, see Ref. [32].

TABLE IV. Cross table with the multiple-choice answers of questions I.A and II.A. The correct alternatives appear in bold. Blank answers are marked by "x". The underlined numbers are called out in the text.

	Question II.A (Stars)							
Question I.A (Sun)	a	b	**c**	d	e	f	x	Total
a		1	2					3
b		8	4	1	2	2		17
c	6	46	254	11	25	21	2	365
d	1	2	6	1	1			11
e		2	1		4	1		8
f		1	1		2	1		5
x			1					1
Total	7	60	269	13	34	25	2	410

TABLE V. Cross table with the multiple-choice answers of questions I.B and II.B. The correct alternatives appear in bold. Blank answers are marked by "x".

	Question II.B (Stars)							
Question I.B (Sun)	a	b	c	d	**e**	f	x	Total
a	100	30	51	36	27	50	5	299
b	2	11	7	2	3	4		29
c	2	5	4		3	5		19
d	6	3	5	9	1	3		27
e	7	1	4		8	3		23
f		1	3			8		12
x						1		1
Total	117	51	74	47	42	74	5	410

students (20%) thought that the position of the sunset does not change throughout the year (see Table VI).

4. Question I.D and question II.D

Other then the previous six questions which were time related, questions I.D and II.D are about the position of the observer (see Fig. 6). Out of 174 students (42%) who answered that the culmination height of the Sun increases when the observers's latitude decreases, 27 students (7%) thought that for the stars the maximum altitude decreases when the observer's latitude decreases. Only 90 students (22%) answered the parallel question for the stars correctly. It is remarkable that 135 students (33%) thought that the culmination height of the Sun decreases when the observer's latitude decreases, and that 55 students of this group (13%) thought that the maximum altitude of a star increases when the observer's latitude increases (see Table VII).

5. Question I.E and question II.E

Like the previous two questions, questions I.E and II.E are about the position of the observer (see Fig. 7). Other than the previous pairs of questions, question II.E about the stars was better answered than question I.E about the Sun. Out of 161 students (39%) who realized that when the observer's latitude decreases, not only the culmination height but also the position of sunrise and sunset change, 82 students (20%) answered the parallel question about the stars correctly. Most students (42%) thought that the position of sunrise and sunset stay fixed, when the observer's latitude decreases (see Table VIII).

6. Question III and question IV

Questions III and IV are about the seasons and seasonal stellar constellations (see Fig. 8). Although 151 students (37%) ticked the correct alternatives about the cause of

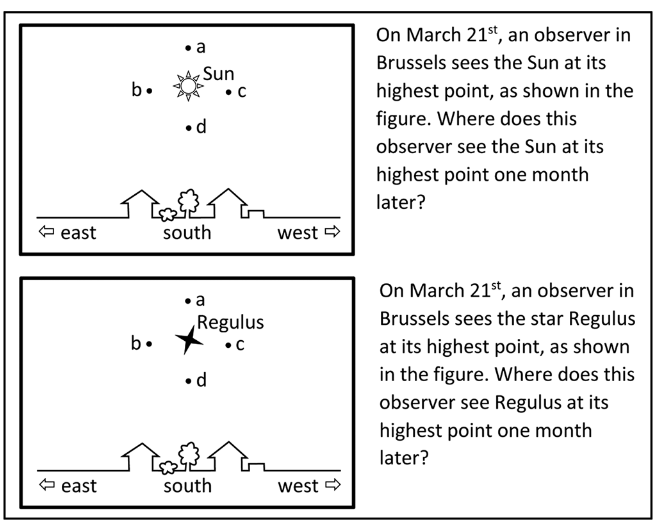


FIG. 4. Shortened version of questions I.B and II.B. For the complete versions, see Ref. [32].

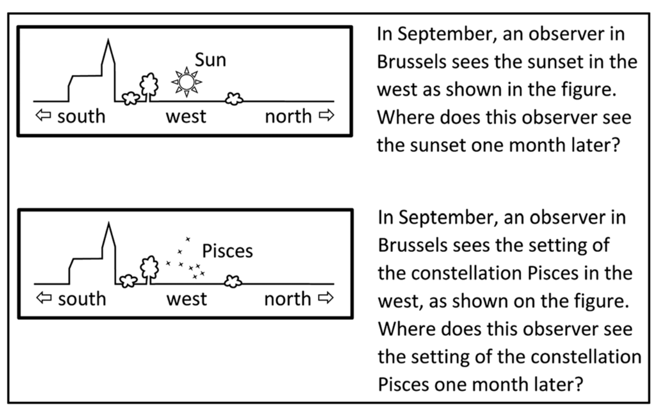


FIG. 5. Shortened version of questions I.C and II.C. For the complete versions, see Ref. [32].

TABLE VI. Cross table with the multiple-choice answers of questions I.C and II.C. The correct alternatives appear in bold. Blank answers are marked by “x”.

	Question II.C (Stars)					
Question I.C (Sun)	a	**b**	c	d	x	Total
a	113	28	60	35	1	237
b	16	31	19	12	2	80
c	19	8	23	13	1	64
d	7	2	4	15		28
x		1				1
Total	155	70	106	75	4	410

TABLE VII. Cross table with the multiple-choice answers of questions I.D and II.D. The correct alternatives appear in bold. Blank answers are marked by “x”.

	Question II.D (Stars)							
Question I.D (Sun)	a	**b**	c	d	e	f	x	Total
a	27	90	3	4	28	20	2	174
b	55	40	2	2	18	18		135
c	4	3	5	7	2	4		25
d	1	5	10	4	1	4		25
e	1	2	1	2	12	2		20
f		5	2	1	2	20		30
x		1						1
Total	88	146	23	20	63	68	2	410

seasons, a non-negligible group of 123 students (30%) attributed the existence of seasons to the varying distance between the observer and the Sun. As for the stars, while 164 students (40%) correctly understood why the constellation of Gemini is not visible in summer for an observer in Leuven, most students seem to be confused about the cause of the change of the sky map during the year (see Table IX).

B. Descriptive results of the written explanations

Since we only asked to explain six out of 12 questions, due to time constraints, 2460 written explanations were expected. Since some students wrote an explanation, while it was not asked for, 2462 responses were taken into account. From these answers, 365 (15%) were left blank (code X) and 243 (10%) explanations were incomprehensible (code Z). Table X gives a general overview of how the remaining responses ($N = 1854$) were classified in the different categories.

This categorization reveals that for the questions about the Sun students tend to use a statement as an explanation of their answer of the multiple-choice question more often than for the star questions. A paired-samples t-test was conducted to compare the number of explanations classified as statement (S) between the six Sun and the six star questions. There was a significant difference in the number of statements for the Sun ($M = 1.70$, $\mathrm{SD} = 0.92$) and the star ($M = 1.00$, $\mathrm{SD} = 0.93$) questions, $t(409) = 14.1$, $p < 0.001$. Thus, of the three Sun questions to be explained, an average of 1.70 are coded as statement (S), while for the three star questions, this is only 1.00. In the written explanations categorized as model (M), where students take the view from space, we see for the star questions ($M = 1.00$, $\mathrm{SD} = 0.94$) more M coded responses than for the Sun questions ($M = 0.80$, $\mathrm{SD} = 0.77$). This difference is also

On the first day of summer, the Sun rises to a maximum altitude of 62 degrees in Leuven. In city X, the maximum altitude of the Sun on the same day is 58 degrees. What can you conclude from this?

a) Leuven is situated 4 degrees south of city X.
b) Leuven is situated 4 degrees north of city X.
c) Leuven is situated 4 degrees west of city X.
d) Leuven is situated 4 degrees east of city X.
e) From the position of the Sun, you can’t make decisions about the location of city X.
f) I really don't know.

On the first night of winter, the star Mintaka as seen from Leuven reaches a maximum altitude of 39 degrees. In city Y, the maximum altitude of Mintaka during the same night is 44 degrees. What can you conclude from this?

a) Leuven is situated 5 degrees south of city Y.
b) Leuven is situated 5 degrees north of city Y.
c) Leuven is situated 5 degrees west of city Y.
d) Leuven is situated 5 degrees east of city Y.
e) From the position of Mintaka, you can’t make decisions about the location of city Y.
f) I really don't know.

FIG. 6. Questions I.D and II.D.

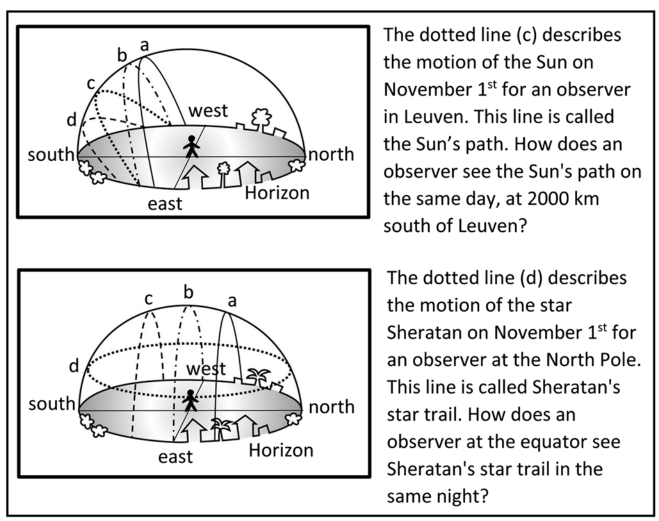


FIG. 7. Shortened version of questions I.E and II.E. For the complete versions, see Ref. [32].

TABLE VIII. Cross table with the multiple-choice answers of questions I.E and II.E. The correct alternatives appear in bold. Blank answers are marked by "x".

	Question II.E (Stars)							
Question I.E (Sun)	**a**	b	c	d	e	f	x	Total
a	82	21	6	13	23	15	1	161
b	87	26	11	15	27	6		172
c	4	4		6	4	2		20
d	6	3	3	5	4	4	1	26
e	2	1		1	1			5
f	6	5	2	4	1	7		25
x				1				1
Total	187	60	22	45	60	34	2	410

significant, $t(409) = -4.21$, $p < 0.001$. For the classification of correct (C) explanations, there is also a significant difference in the number of correct explanations for the Sun ($M = 1.91$, $\mathrm{SD} = 0.94$) and the star questions ($M = 1.14$, $\mathrm{SD} = 0.86$) questions, $t(409) = 14.7$, $p < 0.001$. For the classification of relevant (R) explanations, we also find a significant difference in the number of relevant explanations for the Sun ($M = 1.89$, $\mathrm{SD} = 0.95$) and the star questions ($M = 1.01$, $\mathrm{SD} = 0.83$), $t(409) = 17.1$, $p < 0.001$.

C. Latent class analysis

1. Choosing the number of classes

In the first step of the latent class analysis, we determined the number of classes. Based on the following considerations, we selected a five-class solution. First, we calculated two fit indices to decide which model suits our data best: the Akaike information criterion (AIC) [36] and the Bayesian information criterion (BIC) [37]. As a rule, the model with the lowest AIC and BIC values corresponds to the one with the best model fit. Here the model with four classes and the model with two classes (see Table XI) have, respectively, the lowest AIC and BIC. Second, we searched for the most preferable model from a theoretical interpretation point of view. The profiles found in the four-class

Why is summer in Belgium warmer than winter? Several students give their opinion. Mark every correct claim.

- ☐ Student A says, "The Earth is closer to the Sun in summer."
- ☐ Student B says: "Due to the tilt of the Earth's axis, Belgium is closer to the Sun in summer."
- ☐ Student C says: "In summer, the Sun stays longer above the horizon in Belgium."
- ☐ Student D says: "In summer, the Sun rises higher above the horizon in Belgium."

The constellation Gemini is visible in Leuven in February during the night, but not in July. Why is this?

a) In July, the constellation Gemini doesn't rise above the horizon for an observer in Leuven.
b) When the constellation Gemini is above the horizon in July for an observer in Leuven, the Sun is also above the horizon.
c) In July, the constellation Gemini is only visible in the southern hemisphere of the Earth.
d) Because the Earth rotates around its axis, you see different stars in the sky in July than in February.
e) I really don't know.

FIG. 8. Questions III and IV.

TABLE IX. Cross table with the multiple-choice answers of questions III and IV. The correct alternatives appear in bold. Blank answers are marked by "x".

	Question IV (Stars)									
Question III (Sun)	a	ac	ad	**b**	c	cd	d	e	x	Total
a				3			1			4
ab				2	1					3
abc				1				1		2
ac	1			2			1	1		5
acd				2	1					3
b	7			7	7		15	7		43
bc	9		1	8	4		3	1		26
bcd	8			11	1		2	4	2	28
bd				6			2	1		9
c	16			17	6		14	7		60
cd	25	2	1	74	19	1	11	18		151
d	12		1	31	9		12	10	1	76
Total	78	2	3	164	48	1	61	50	3	410

TABLE X. General overview of the classification of the written explanations ($N = 1854$).

	Sun questions (%)	Star questions (%)
Statement (S)	38	22
Model (M)	18	22
Correct (C)	43	25
False (F)	12	19
Relevant (R)	62	32
Not relevant (NR)	1	5

TABLE XI. AIC and BIC values for different number of classes.

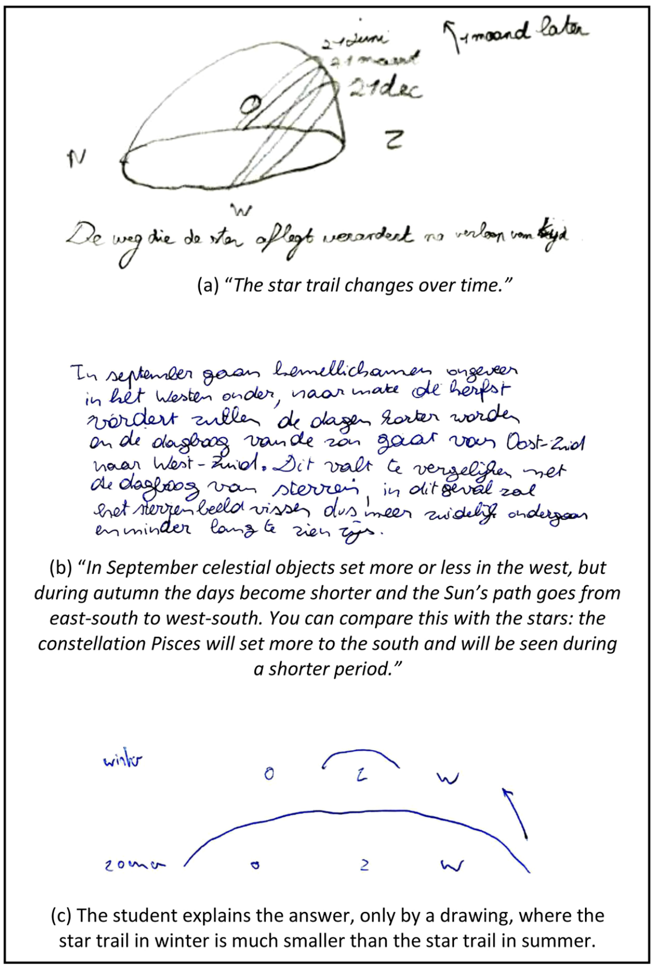
Number of classes	AIC	BIC
1	10 505.08	10 705.89
2	10 241.05	10 646.68
3	10 221.37	10 831.82
4	10 201.37	11 016.65
5	10 220.13	11 240.24
6	10 221.38	11 446.31

TABLE XII. Predicted memberships of the latent classes.

Class	Probability (%)
Class 1a	35
Class 1b	7
Class 1c	19
Class 2	20
Class 3	19

TABLE XIII. Example of the output of software R for question II.B.

Class	a (%)	b (%)	c (%)	d (%)	e (%)	f (%)	x (%)
Class 1a	59	7	16	12	3	3	0
Class 1b	26	3	18	31	0	19	3
Class 1c	3	15	17	17	2	4	1
Class 2	20	9	24	9	2	7	3
Class 3	15	26	16	0	15	28	0



(a) *"The star trail changes over time."*

(b) *"In September celestial objects set more or less in the west, but during autumn the days become shorter and the Sun's path goes from east-south to west-south. You can compare this with the stars: the constellation Pisces will set more to the south and will be seen during a shorter period."*

(c) The student explains the answer, only by a drawing, where the star trail in winter is much smaller than the star trail in summer.

FIG. 9. Examples of student answers of class 1a.

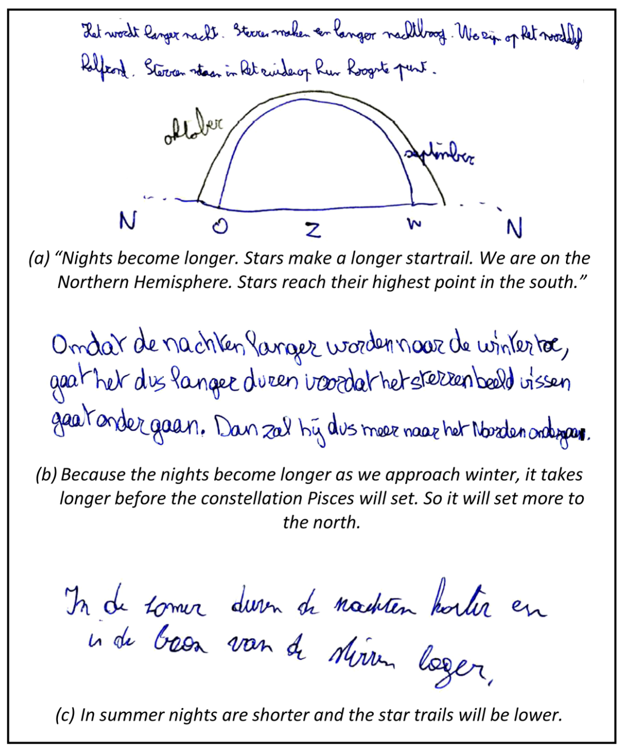
solution and the five-class solution are highly comparable. In the five-class solution, the first class of the four-class solution is subdivided in an extra class. This additional profile in the five-class solution is theoretically interesting because it is in line with what we learned from the categorization of the written explanations. For all above reasons, we choose the five-class solution.

2. Description of the classes

In the output of the latent class analysis, conducted by the software R, the size of the five classes is indicated by the probability that a respondent belongs to a certain class (see Table XII). These posterior probabilities are calculated based on the respondent answers on the multiple-choice questions.

For each question a table is generated with the probabilities that a certain answer is given by a member of a certain class. Table XIII presents an example for question II.B and can be interpreted as follows: there is a 59% chance of a respondent in latent class 1a answering alternative a to the multiple-choice question, a 7% chance of answering alternative b, a 16% chance of answering alternative c, etc.

Based on these tables we describe the profiles of the students in the five classes, supplemented with what we learned from the categorization of the written explanations. Examples of these written explanations illustrate the reasoning of the students in the different latent classes.



(a) *"Nights become longer. Stars make a longer startrail. We are on the Northern Hemisphere. Stars reach their highest point in the south."*

(b) *Because the nights become longer as we approach winter, it takes longer before the constellation Pisces will set. So it will set more to the north.*

(c) *In summer nights are shorter and the star trails will be lower.*

FIG. 10. Examples of student answers of class 1b.

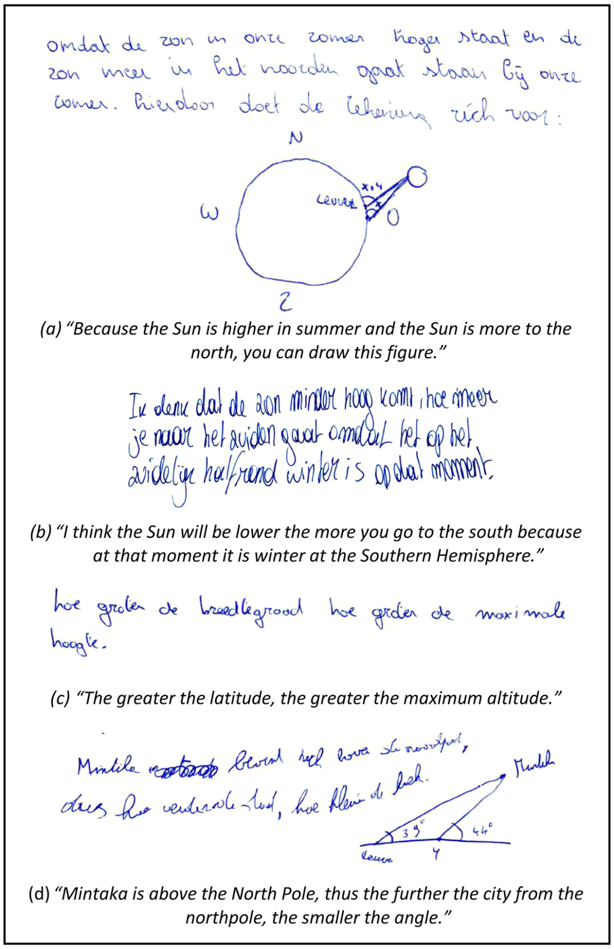


FIG. 11. Examples of student answers of class 2.

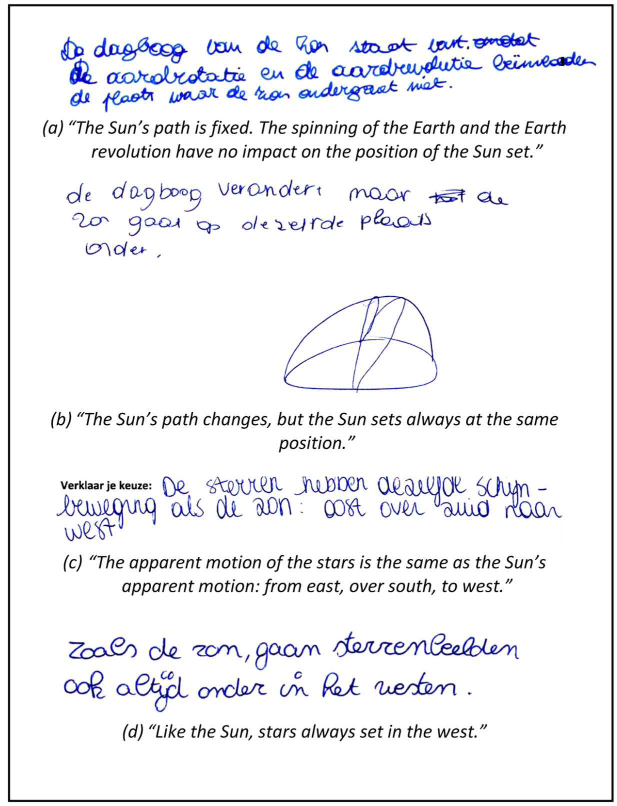


FIG. 12. Examples of student answers of class 3.

Class 1.—The students grouped in class 1 answer almost all questions about the Sun correctly. Concerning the stars, this class is subdivided in three subclasses.

Class 1a: The stars act like the Sun.—The questions about the stars are answered in such a way that the stars seem to act perfectly like the Sun. This group knows that the Sun and stars apparently move from east to west during the day and night. For both the Sun and the stars in summer the path of the apparent motion is higher and wider than in winter. When the observer's latitude decreases, this path becomes higher and wider both for the Sun and the stars. In Fig. 9 we show how the students explain their answers on question II.B about the culmination height of the stars by using arguments that only apply to the Sun.

Class 1b: Longer nights-higher stars.—In this class the answers of the star questions are characterized by the fact that the star trails behave opposite to the Sun's path. For an observer in the Northern Hemisphere in summer the Sun's path is higher and wider than in winter, because the days are longer. The students in this class use this idea also for the stars: in winter the star trails are higher and wider, because the nights are longer (see Fig. 10). When the observer's latitude decreases, the Sun's path becomes higher and wider. When the observer's latitude increases, the star trail becomes higher and wider.

Class 1c: The stars are difficult to grasp.—This group of students knows that the apparent motion of the Sun and the stars is from east to west. Most of the Sun questions are answered correctly, while for the questions about the changes of star trails throughout the year and when the observer changes its position, this group of students selects a wrong alternative or the answer "I don't know!."

Class 2.—The students grouped in class 2 also have good scores on the Sun questions, but they distinguish themselves from class 1 because they think that the culmination height is proportional to the observer's latitude. For these students the maximum altitude of the Sun increases when the observer's latitude increases (see Fig. 11). The students in this class have better scores on the star questions than the class 1 students.

Class 3.—In class 3 the student answers are characterized by the fact that the positions of Sun or star rise and set do not change throughout the year, and also do not change when the observer's position changes. In all conditions the Sun and the stars rise exactly in the east and set exactly in the west (see Fig. 12). Concerning the daily apparent motion, these students are confused about the stars: they think that one hour time is not enough to see changes in the night sky or they think that stars apparently move opposite to the Sun.

V. DISCUSSION

We have administered the AMoSS test [1] with 12 multiple-choice questions to 410 students of the fifth year of secondary education (16–17 year olds) in Belgium to investigate to what extent they have insight in the apparent motion of the Sun and stars. Since the AMoSS test consists of six questions about the Sun and six parallel questions about the stars, the results of the test make it possible to compare students' understanding of the Sun's apparent motion with their understanding of the apparent motion of the stars. Thanks to the written explanations, we were able to identify five mental models that students have about the apparent motion of the Sun and stars, by classifying the responses of the students and using the latent class analysis technique.

In line with previous research [8,9,38], we find that students score better on the Sun questions than on the star questions. At first glance, this does not surprise us because during the day we witness and experience the apparent movement of the Sun, see different places in sunlight and shadow, etc. However, we only observe the night for a more limited time, we may be overwhelmed by the number of stars, which prevents us from distinguishing individual stars, and we do not experience any difference depending on the apparent position of the star in relation to the pole. The fact that the apparent motion of the stars is not taught explicitly in the Belgian curriculum does not help the students. On the other hand, it is remarkable that students who have learned about the apparent motion of the Sun, Earth's spinning, and Earth revolution during the astronomy lessons at school do not seem to apply this knowledge to the stars or even think that the stars make an opposite movement. We confirm what we found in the literature [8,10,39] following the astronomy lessons has little impact on the test results.

This is immediately evident in the first two questions about the daily motion of the Sun and stars. While most students know that the apparent motion of the Sun is from east to west, there is a lot of confusion about the stars. A significant part of the students do not really understand how the spinning of Earth is responsible for the apparent motion of all celestial objects. We agree with the literature [40,41] that special instructions are needed for students to fully understand the relativity of motion and to apply this knowledge to the apparent motion of the Sun and stars.

From the questions about the culmination height and the position of rising and setting of the Sun and the stars, it also becomes clear that students do not fully understand the impact of Earth's revolution on these phenomena. Why the tilt of Earth's axis combined with Earth's revolution affects the apparent motion of the Sun and does not affect that of the stars seems to be too complex for most students. They (35%) do not distinguish between the Sun and the stars in this regard. For this group of students the star trail becomes higher and wider as we approach summer, just like this is the case with the Sun's path. They treat the stars as copies of the Sun (class 1a). We recognize here the idea of Collins and Gentner that mental models are formed through analogical thinking [18]: since students are less familiar with the apparent motion of the stars, they tend to perceive these motions as similar to the Sun's apparent motion.

A second group of students (7%) think that the star trails become higher and wider, when nights become longer, as winter approaches (class 1b). They seem to apply the reasoning that the Sun's path gets higher and wider as the days get longer in summer to the stars in winter. Some students are very persistent in using this alternative idea, because they also use this argument as the observers latitude increases: because the nights become longer if you get closer to the North Pole, the star trails will be higher and wider.

For a third group of students (19%), although the culmination height of the Sun changes throughout the year, the position of the rising and setting of the Sun and stars is always exactly in the east and exactly in the west (class 3). It seems that this alternative idea, which has been reported in the literature concerning younger children [9,33,42], is also strongly present with 16–17 year olds, even if they have taken astronomy classes in school.

Concerning the culmination height, which is expressed as an angle, a group of students (20%) is confused about the meaning of this angle. They think this angle is proportional to the latitude of the observer (class 2). When they draw a picture to explain their ideas, they either designate the angle incorrectly on a figure of Earth or they do not take into account the spherical shape of Earth (see Fig. 11). As reported in the literature [43–46], a significant group uses the idea of a flat Earth while reasoning about the culmination height for different positions of the observer.

A last group of students (class 1c) gets good scores on the Sun questions, but on the questions about the stars, they often report not knowing the answer. It seems that they are not able to transfer the ideas they have about the Sun to the situation of the stars.

By combining the answers on the Sun and star questions in a latent class analysis, we also found statistical arguments for these different groups of students with alternative ideas. The fact that the students in the classes described above use the same type of reasoning in their way of answering different questions of the AMoSS test makes us believe that these alternative ideas can be considered as expressed versions of mental models that students use when answering the questions. Like Corpuz and Rebello [17], we speak of expressed versions to emphasize that we can only rely on the student answers we collected by the administration of a limited questionnaire.

In addition to the five classes described, it is remarkable that the LCA did not identify a class of students who scored well on both the Sun and star questions, nor did the analyses identify students who scored better on the star questions than on the Sun questions. This may be due to the fact that only a very small group of students scored well on most star questions.

Although we found arguments for the existence of these mental models, while classifying the students' written explanations of the answers to the multiple-choice questions, we also found that most students are not coherent in the way they answer the different questions about the Sun and stars. This can be due to the fact that most students do not rely on elements of an allocentric view while reasoning about the Sun and stars and trying to explain their answers. We found that most students use arguments based on their observations or knowledge. However, for the questions about the stars, which seem to be more difficult for most students, we found more elements of an allocentric view than with the Sun questions in the written explanations of the students. It seems that if students cannot rely on their prior knowledge or previous experience by observation, they are stimulated to really reason about the answer and use a view from space to find the answer. At first glance, this seems to be contradictory. From the literature [31] we learned that students who are able to think in different frames of reference are more likely to develop a better insight in basic astronomical phenomena, like the apparent motion of celestial bodies. So we would expect better results on the star questions, since students tend to use more arguments of an allocentric point of view in their explanations. This reveals that thinking in different frames of reference is not enough to completely understand the motion of celestial bodies. It seems to be necessary to learn and think about how the celestial motions in different frames of reference are linked. Since the literature [2,3,30,34,47] indicates that spatial skills have to be trained specifically to be able to switch between different frames of reference, we suggest that this has to be trained during the astronomy lessons about these apparent motions.

VI. LIMITATIONS

Since the AMoSS test was administered during a science lesson in school, students had only 45 minutes to answer the 12 multiple-choice questions and explain their answers. During the validation of the AMoSS test we learned that one lesson of 45 minutes is not enough to let the students explain all the answers. Therefore we decided to ask the students to explain only six questions out of 12. Because of this decision and the fact that many students have not explained the requested answers, we only get a partial view on what each respondent really thinks. We solved this by combining what we learned of the classification of the written explanations with the statistical technique LCA that uses the multiple-choice answers (a, b, c, …) to look for hidden classes in the sample group.

The fact that question III about the seasons is formulated in such a way that multiple answers are possible makes it difficult to analyze the results in the same way as the other 11 questions. For this reason, we also excluded this question (together with question IV) from the LCA. Moreover, during the interviews of a subgroup of students we observed that with question III most students adjusted their original answer. Therefore, we suggest to reformulate this question in future research.

VII. CONCLUSIONS AND FURTHER RESEARCH

In this study we have identified five different mental models of the apparent motion of the Sun and stars. We distinguish three major classes. For the three models in the first class the distinction between how students think about the Sun and stars is remarkable. The main group of students thinks that the star trails change throughout the year in the same way as the Sun's path changes. A smaller group thinks that the star trails behave opposite to the Sun's path: while in winter the Sun's path becomes lower and smaller, the star trails become higher and wider. Most typical for the model in the second class is that the culmination height of the Sun and the stars is taken to be proportional to the observer's latitude. The model in the third class is characterized by the fact that the positions of the Sun or star rise and set do not change throughout the year, and also do not change when the observer's position changes: the apparent motions of the Sun and stars seem to start always exactly in the east and finish exactly in the west.

We conclude that clearly specific instructions are needed to deeply understand the apparent motion of the Sun and stars. Our structured analyses and the identification of mental models which are apparent in the student population helps us in our next step of our research, namely, the development of research-based learning materials, both for school and the planetarium, which would lead to a better understanding of all the aspects of the apparent motion of the Sun and stars.

For an alternative planetarium presentation, we suggest to compare systematically the apparent motion of the Sun with that of the stars, both for different times of the year and for different positions of the observer. To explain the similarities and differences, we think it is necessary to visualize the motions each time from a geocentric and an allocentric perspective. The regular planetarium setting is ideal for the geocentric point of view, and thanks to the presence of digital projection systems in most planetariums, it is possible to use short videos to display the allocentric perspective. In addition to these projections, we plan to use physical spatial models of Earth, the celestial sphere, the plane of the horizon, etc., that participants can manipulate during the presentation to gain a better insight in the imagery being used. Since spatial reasoning plays an important role in understanding the impact of Earth's rotational movements on the apparent motion of celestial bodies, we believe that each step in the reasoning should be adequately explained and shown, and we suggest not to offer too many topics in one planetarium presentation.

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