University student conceptual resources for understanding forces

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Existing research identifying common student ideas about forces focuses on students' misunderstandings, misconceptions, and difficulties. In this paper, we characterize student thinking in terms of resources, framing student thinking as continuous with formal physics. Based on our analysis of 2048 written responses to conceptual questions, we identify six common conceptual resources for understanding forces. We document context-sensitive patterns in resource activation, and we discuss limitations of our research based on the demographics of our sample.

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I. INTRODUCTION

Much of the research identifying common student understandings in physics takes a misconceptions, misunderstandings, or difficulties orientation toward student thinking, focusing on what is challenging for students and/or what is incorrect about their thinking. This abundance of misunderstandings-oriented research has the potential to orient instructors' attention toward what is wrong with students' thinking and how to fix it, or toward what is especially challenging for students and how to address this. What an instructor might *miss* from this perspective are ways in which students' thinking is sensible and continuous with formal physics, even if not phrased in language that is immediately recognizable to physicists as physics. Instructors and researchers might also miss opportunities to learn which contexts elicit depth and richness in student thinking or opportunities to build on student thinking in physics courses.

This paper is an empirically driven argument for an orientation toward students' physics ideas as *resources* for learning. This argument is not new; many researchers and practitioners have made it before, and we draw extensively from one framework that emerged from earlier efforts, the resources theoretical framework [1–3]. What is novel about our work is our application of the resources theoretical framework to analyze large numbers of students' written responses to conceptual questions, to identify patterns in student thinking that might serve as starting places for building conceptual understanding. In particular, we report six common conceptual resources for understanding forces, based on our analysis of 2048 written student responses. We look at the relationship between these resources and the categories of student thinking in the literature, usually described as difficulties or misconceptions (e.g., students tend to associate force with velocity, rather than acceleration). We briefly highlight the context sensitivity of these resources. First, though, we provide the context for our work, reviewing existing literature on student ideas about forces, describing our theoretical and instructional motivations, and providing details about our sample and methods. Our hope is that our work can enhance instructors' capacity for engaging in resources-oriented instruction by providing a model for identifying resources in student thinking.

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II. LITERATURE REVIEW

Since the early 1980s, there have been more than fifty papers published about common student ideas about forces. Nearly all of these papers use a misconceptions, alternative conceptions, or difficulties lens, foregrounding patterns in student thinking that diverge from the canon of physics, and emphasizing how physics instruction might address students' incorrect ideas. In this section, we summarize some of the major themes of this existing literature on student ideas about forces.

A. Student ideas about forces: Themes from existing literature

Probably the most common theme in the literature on student thinking about forces is the idea of an impetus force [4–28], which Clement [18] defines as the "belief that there is a force inside a moving object that keeps it going and causes it to have some speed," where this "force can fade away as the object moves along." According to the literature, students use impetus force reasoning to argue that forces are necessary to sustain motion [8–12,25] or that motion implies force [10,12–15,17,25]. One popularly reported example is the "curvilinear impetus principle" [24–26]:

> *"According to this principle, an object constrained to move in a curved path acquires a curvilinear impetus that causes it to continue in a curved trajectory for some time after the constraints on its motion are removed. However, the curvilinear impetus gradually dissipates, and consequently, the object's path gradually straightens." [26]*

Further, the more dramatic the curvature and/or the longer the tube, the more curved the motion of the ball after it emerges. Thus, when asked to predict the trajectories of a ball that emerges from the curved tubes in Fig. 1, some students drew a curved trajectory for problem 1 and an even-more-curved trajectory for problem 2.

A second commonly reported example of impetus force reasoning comes from a study where students were asked to draw the trajectory of a pendulum bob that was released at various locations along its swing. Students drew trajectories where the ball "continue[d] for a short time along its original arc, and then [fell] directly to the ground." The authors of the study connect these responses to the impetus force idea, saying,

> *"these subjects apparently believe that the motion of the pendulum imparts to the ball some sort of impetus that causes it to retain the original arc for some time after the string is cut. However, this impetus gradually dies out, at which point gravity takes over and causes the ball to fall directly to the ground." [27]*

The literature also extensively reports situations in which students associate force with velocity, rather than acceleration [10,12,13,17,25,29–33]. This takes many different forms, including: if there is no motion, there is no force [10,12,25]; if there is no force, there is no motion [17,33]; the magnitude of the force is proportional to the speed [10,30,33]; constant forces produce constant speed [17,33]; constant speed implies a constant net force [29]; diminishing force implies diminishing speed [34] and increasing speed implies increasing net force [29]; and forces and velocity are in the same direction [31,32]. The association of force with velocity is related to the impetus idea in the sense that when force is associated with velocity, forces are needed to sustain motion. However, a relationship between force and speed does not imply the full impetus model, and the impetus model does not specify a direct proportionality between force and speed.

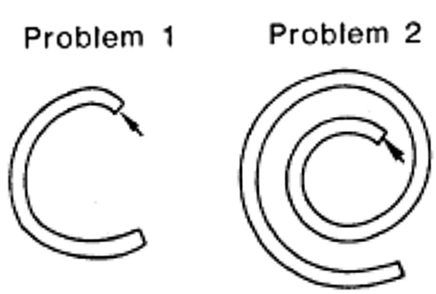

FIG. 1. Curved tubes. From McCloskey and Kohl [26].

The literature also speaks extensively to students misunderstanding Newton's third law. When two objects are interacting, students attribute a greater force to the object with a greater mass [17,25,35–37], to the object that "causes" the interaction [17,35] or the object that is pushing [36] or moving [25], to the object with a larger speed [25,36], to the "stronger" object [25], to the more "active" or "energetic" object [25], or to the object that speeds up [36]. Hestenes, Wells, and Swackhamer [20] make sense of these patterns in these terms:

> *"Students often interpret the term 'interaction' by a 'conflict metaphor.' They see an interaction as a 'struggle between opposing forces.' It follows from the metaphor that 'victory belongs to the stronger.' Hence, students find Newton's third law unreasonable, and they prefer some version of the dominance principle: In a conflict, the 'more forceful' exerts the greater force. Here 'more forceful' can mean, 'bigger,' 'greater mass,' or 'more active.'"*

Other authors focus on how students think about the kind of thing that a force is, as distinct from what physicists think forces are. In particular, students treat forces as things that objects have [12,22,23,25,38] and can transfer [39], rather than as an interaction between two objects. This way of thinking about forces may be related to the impetus idea, which embeds a possession metaphor: "impetus can be imparted by an applied force and transmitted from one object to another" [17]. It may also be related to some of the third law ideas discussed above, e.g., objects *have* force as a result of their qualities—mass, motion, etc.—and so, for example, heavier objects exert more force on lighter objects in a collision [38].

B. Orientations toward student thinking in literature on student ideas about forces

Much of the work we reference in the previous section emerged at a time that the physics and science education research communities were negotiating models of student cognition and learning [7,40–43]. Some of this work, e.g., Ref. [42], used forces and motion as a *context* in which to develop and explore cognitive theory. Other work focused on documenting and characterizing student ideas about forces, drawing on emerging cognitive frameworks more implicitly. Given this history, the literature on "student ideas about forces" is as much an attestation to how researchers were orienting to and thinking about the structure of student thinking as it is an inventory of forces ideas.

Looking back at the studies cited in the previous section, the primary orientation that is reflected there parallels that reflected in early misconceptions research [4,5,7–9,11,15,16,19,22,24,27,31–34,37–39,44]. These papers take an almost-exclusively deficit orientation toward students' ideas. For example, McCloskey, Caramazza, and Green write that "…students do not merely lack such knowledge; they espouse 'laws of motion' that are at variance with formal physical laws" [24].

Other papers [6,10,17,18,20,23,26,35,45] orient toward student ideas as misconceptions in the substance of their work, but also either (i) overtly acknowledge that students' ideas come from legitimate efforts to sense make about the world or (ii) themselves sense make about where students' ideas may come from. For example, Hestenes, Wells, and Swackhamer state that "[students' misconceptions should be] accorded the same respect we give to scientific concepts. The most significant common sense beliefs have been firmly held by some of the greatest intellectuals of the past, including Galileo and even Newton" [20].

Relatedly, a few papers position students' ideas as alternative to or in conflict with the canonically correct answer, but also suggest that teachers use these ideas as a starting place for instruction [14,28,46,47]. For example, Clement [46] describes the role of "anchoring intuitions"—or "belief[s] held by a naïve student which [are] roughly compatible with accepted physical theory"—in helping students to "overcome their misconceptions." In most of these papers, the role of students' productive ideas is to counter other, incorrect ones, rather than to serve as the raw material out of which new understandings can develop.

Very few papers take an overtly or purely resources orientation toward student ideas [21,48,49]. While some of the early work in developing the resources theoretical framework was done in the context of forces and motion (e.g., Ref. [42]), as we say above, to our knowledge there has not yet been a focus in the literature on "reporting common resources for understanding forces." Rather, the authors of the papers we cite [21,48,49] reinterpret student ideas that have historically been treated as deficits as instead reasonable—even productive—ways of thinking about force and motion. Our work extends this line of thinking, by explicitly applying resources theory to identify patterns in student thinking that are continuous with formal physics.

III. THEORETICAL AND INSTRUCTIONAL MOTIVATIONS FOR OUR WORK

Our choice to identify university student resources for understanding forces is not only motivated by existing literature on student ideas about forces; it is also driven by theoretical and instructional considerations that direct our attention to the sensible ideas that students are using as they reason about forces. We discuss these motivations and how they shaped our work in this section.

A. Theoretically grounded: Resources theory of knowledge

In resources theory, a *resource* is a piece of knowledge that gets activated in real time, in context-sensitive ways [1–3,42,50–54]. Researchers have theorized extensively about the development, structure, and role of resources, and have used resources theory to highlight the dynamic, emergent, complex-systems-like nature of student thinking. Our work draws extensively on the following tenets from resources theory:

1. *Resources are fundamentally sensible and continuous with formal physics*, having been derived from a person's physical or sensory experience and then used to make sense of the material world [1,2,42,50,51,53,54]. For example, diSessa [42] says that phenomenological primitives ("p prims," which we consider to be a kind of resource) such as "closer means stronger" are best understood as "serv[ing] individuals well in dealing effectively with the physical world," e.g., in making sense of it, interacting with it, etc. Smith III, diSessa, and Roschelle [2] define resources as "any feature of the learner's present cognitive state that can serve as significant input to the process of conceptual growth," emphasizing the continuity between students' intuitive ideas and formal physics.
2. *The activation of resources is context sensitive* [1,3,42,50,51,53–55], where context includes any aspect of the environment that students notice [53]. This tenet creates an expectation of variability in student thinking. That is, we expect that resources will show up in different forms and at different frequencies in different contexts. Thus, "observing" a student use an idea in one context does not guarantee that we will observe that same student use the same idea in another (even similar) context, nor does "not observing" an idea in one context guarantee that we will not see that idea in another.
3. *Learning involves changing the structure or activation of resources*, by reorganizing, refining, properly activating, increasing the degree of formality of, or changing the role of resources [1–3,42,53,54]. For example, diSessa [42] theorizes that a primary difference between novice and expert cognition in physics is in the structure and connectedness of networks of resources. In this view, the resources that novices activate as they sense make about the natural world become part of the structures that organize expert physics thinking. The idea that

resources are integral to learning is also reflected in the language that researchers use to describe them. That is, resources are often depicted as resources *for* something—for "understanding physical phenomena" [3], for "learning" [2], for "the development of a conceptual understanding of Coulomb's law" [50], for "thinking about physical situations" [1], or for "cognitive growth" [2]—emphasizing their generative role in thinking and learning.

The role of resources reflected in diSessa's work and in the quotes from the previous sentence links back up to the first tenet, which poses resources as continuous with formal physics. In particular, this third tenet frames continuity not only in terms of resources being plausibly derived from students' sense making about the physical world (first tenet), but also in terms of their capacity to develop into canonical understandings.

These three tenets of resources theory[1] shape our work. The expectation of sensibility and continuity with formal physics from the first tenet directs our attention to patterns in student thinking that seem like "seeds" [57] or "conceptual progenitors" [51,58] of force concepts in the introductory physics curriculum and prompts us to ask ourselves why a reasonable person might answer the way we see students doing. The expectation of context sensitivity (second tenet) shapes our interpretation of patterns in student responses. Though we are looking for *common* resources for understanding forces, we do not expect the resources we identify to be used in the same way across questions or students. In fact, in Sec. V, we highlight a diversity of ways that students use each resource, and in Sec. VII, we track context-dependent patterns in resource activation. Our work draws more from the orientation and context sensitivity aspects of the resources theoretical framework, and less from the structural aspects, which shapes the instructional relevance of our work; we turn to this next.

B. Instructionally significant

We want to produce research that is instructionally significant, by which we mean that we want our work to have the potential to shape what instructors do in the classroom. Sherin and Star [59], quoting William James [60], describe the classroom as a site of a "blooming, buzzing confusion of sensory data." We expect our work will serve as *one input* to a complex process of both emergent and planned decision making. Building on both resources theory and the focus of existing research on student ideas about forces, our hope is to "shape instructors' judgment of student thinking," toward believing that that thinking has the potential to be generative for learning. Broadly speaking, our position is that by identifying specific resources for understanding forces, our research has the potential to shape instructors' (i) believing that students have resources for understanding forces, (ii) knowing what some of those resources might be, (iii) planning instruction to elicit and build from these resources, and then (iv) noticing these resources as they are deployed by students in real time.

The specific questions and methods we chose draw on a model of generalizability that emphasizes reproducibility across heterogeneous data sources [61]. In this model, the commonality of a resource, or the extent to which we observe the same resource being used by many students in many contexts, enhances the predictive capacity of our work—i.e., whether instructors might expect that resource to come up in similar (though not yet tested) contexts. This predictive capacity is limited by the representativeness of our sample, as we discuss in Sec. IV.

Our research questions and methods also foreground resources that we perceive to be continuous with conceptual instructional targets from introductory physics textbooks, making visible that instruction can build from students' ideas toward canonical understandings. This choice means that our analysis highlights concrete, larger-grain-sized resources, over and above resources that are smaller in grain size and articulated in more abstract language (e.g., closer means stronger [42]). This is consistent with resources theory,[2] but it is certainly not the only resources-oriented analysis one could have performed on our data.

IV. RESEARCH CONTEXT AND METHODS

In this section, we describe the conceptual questions we used, the samples we drew from, and our data analysis methods.

[1]Resources theory is often contrasted with misconceptions as a way of thinking about student thinking. For example, in contrast with the three tenets of resources theory that we name above, early misconceptions theory framed student thinking as *inconsistent with* formal physics, depicted student knowledge as theorylike and/or brought to bear consistently across contexts, and represented learning as replacing or addressing students' incorrect ideas [2,6,7,40,52,56]. Our project responds to calls for resources research to "develop explicit accounts of student resources" [1] and to conduct "complementary research to identify possible conceptual progenitors of expert understanding in students' intuitions" [51].

[2]The literature identifies resources at multiple grain-sizes, from basic cognitive elements to statements that sound more like conceptions or ideas, such as "the [less massive] car reacts twice as much" [1] in a collision. The literature also identifies resources at multiple levels of abstraction, from "general rules and relationships that become concrete statements about a particular system as a result of a *mapping*" [53] to concrete statements about forces and other physics topics (e.g., facets [25]).

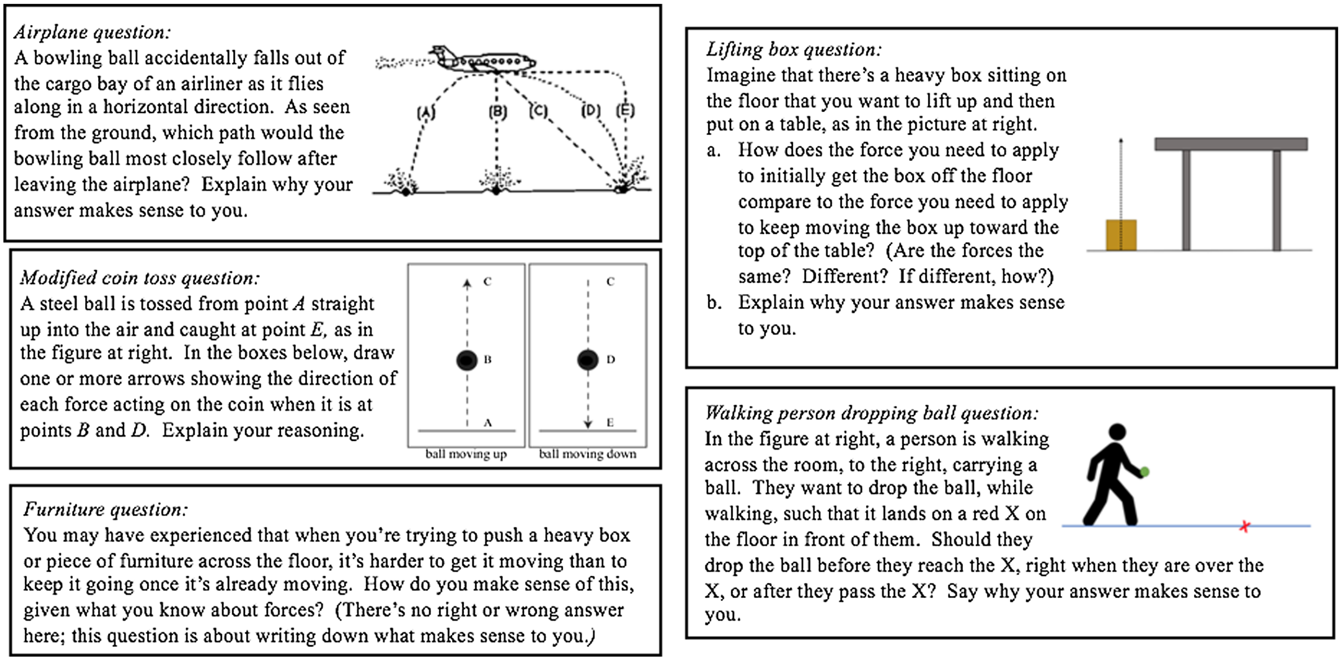

FIG. 2. Conceptual questions used in forces study. From Hestenes, Wells, and Swackhamer [20].

A. Conceptual questions

Similar to misconceptions-oriented or difficulties-oriented research, our study elicited student reasoning about forces by asking students to write down their answers to conceptual questions. For this study, we coded student responses to the five conceptual questions shown in Fig. 2: the *airplane* question, the *modified coin toss* question, the *furniture* question, the *lifting box* question, and the "walking person dropping ball" question.

We chose these five questions for a variety of reasons. The airplane question and the modified coin toss question have been used to identify student misunderstandings or misconceptions in previous studies [6,20], and we were curious as to whether a resources lens would yield different insights about student thinking. The furniture question was written to tap into students' intuitions about forces in an everyday scenario involving unbalanced forces; the lifting box question was constructed to further explore these intuitions. The walking person dropping ball question was one that first-author ADR observed students having a lively discussion about in video from a K–12 classroom [62]; it is similar to the airplane question but also distinctly different (e.g., the ball is moving at much smaller speed in the walking person dropping ball question, and students may have more lived experiences to draw on in that question). Five validation interviews were conducted per question, with students from local universities, to check that students interpreted the questions as intended. No significant modifications were made on the basis of these interviews.

These were not the only five conceptual questions about forces that we asked students to answer in our study. We chose to highlight these five in this paper because they were the ones used most frequently by instructors, and so we had a lot of data from multiple universities for each of these questions. Thus, the patterns we observe in students' responses to these questions are reproduced across multiple sources of heterogeneity [61], and so have the potential to enhance the generalizability—and thus predictive capacity—of our results.

B. Samples and data collection

For this study, we coded a total of 2048 responses from students in algebra- and calculus-based introductory physics courses at nine different universities across the U.S.: Baylor University, Bellevue College, DePaul University, George Mason University (GMU), Michigan State University (MSU), San José State University (SJSU), Seattle Pacific University (SPU), University of Washington (UW), and Western Washington University (WWU). We only included samples in which more than 70% of the students provided a response for a given question, to mitigate concerns of a skewed sample. Students answered the questions on homework, quizzes, or exams, and were awarded either a grade or participation credit based on their answers. All questions were given after instruction; because of this, patterns in student responses represent what an instructor of a similar course might expect to elicit as students learn about forces. Table I details which samples received each question.

The demographic composition of our study sample may limit the extent to which our results apply to a representative introductory physics course. As Kanim and Cid point

TABLE I. Study samples, by question.

	Samples		
Conceptual question	University	N	Response rate
Airplane question	MSU	84	84%
	WWU	234	86%
Modified coin toss question	Baylor	114	78%
	DePaul	84	94%
	UW	361	84%
Furniture question	GMU	171	83%
	MSU	84	84%
	WWU	251	92%
Lifting box question	Baylor	112	79%
	Bellevue	34	91%
	SPU	47	77%
Walking person dropping ball question	DePaul	88	91%
	GMU	171	81%
	SJSU	213	98%

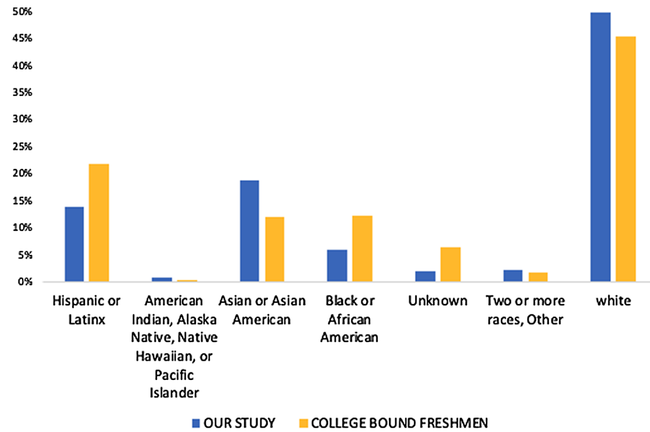

FIG. 3. Estimated racial and/or ethnic demographics of samples from our study (blue) versus all students taking the SAT in 2015 (orange). Blue bars are constructed from university-level demographic data, weighted by sample size. Orange bars are constructed from percentages cited in Ref. [65].

out [63], PER has historically oversampled from white, wealthy, mathematically prepared populations of students. They write,

> *"While Latino/Latina, Black, and Indigenous American students are 34.5% of the college-bound students taking the SAT, only 15.2% of the [physics education] research student population are from these groups."*

This is compared to white students comprising 62.9% of the PER student population and only 45.3% of college-bound students taking the SAT. In spite of this disproportionality, the PER community has historically treated the results of PER as indicative of "typical" university students, as though our samples are representative of the population of introductory physics students at large. This unexamined assumption means not only that the benefits of decades of research are disproportionately skewed toward mathematically prepared, white, wealthy populations of students but also that there is a higher likelihood of racialized bias in research-informed physics teaching and learning, as students are being implicitly assessed against a "norm" that was constructed from largely white, wealthy students [64,65].

Following the methods in Kanim and Cid, we approximated the demographics of the introductory physics courses in our study using university-level data, and then constructed a weighted sample based on the numbers of students in each course. Ideally, we would compare (a) the demographics of our study to (b) the demographics of a representative introductory physics course. However, we know neither (a) nor (b)—i.e., we did not collect sample-level demographic data, and, as of the writing of this manuscript, we (as a PER or physics community) have not done the work to identify what counts as a representative sample of introductory physics students [66]. What we have done here allows for a coarse comparison similar to the one done by Kanim and Cid.

As Figs. 3[3] and 4 show, our forces research likely oversamples from white, Asian, and wealthy student groups and undersamples from Latinx, Indigenous, Black, and low-income student groups. We say "likely" because results from early-stage research being conducted by our team suggests that institution-level demographics do not map onto the demographics of introductory physics courses [68]. However, whether or not our research over- or undersamples from specific racial and/or ethnic groups, or from specific parental income quintiles, as compared to a representative "introductory physics course," we can say this: Figs. 3 and 4 estimate that our sample overrepresents the physics ideas of students at universities disproportionately serving white, Asian, and wealthy students, as compared to the college-aged population, and underrepresents the physics ideas of Latinx, Indigenous, Black, and low-income student groups. Whether this is a matter of who is being recruited into physics courses or a matter of sampling does not change that our results are limited in their generalizability in this way, and thus contribute to the systemic problems that Kanim and Cid highlight. This is a community-level concern, not a manuscript-level one, and we raise it here, while also pointing to recent work—such as that done by Rosa and Mensah [69], Hyater-Adams et al. [70], Hazari [71], Rodriguez and Zamarripa-Roman [72],

[3] As explained by Lazo, neither Hispanic nor Latinx is a racial group, and these two identities are not the same. "Hispanic" is a descriptor for people of Spanish-speaking origins, and "Latinx" is a descriptor for people with origins in Latin America. The former focuses on language, the latter on geographic location. For more on this, see: Ref. [67].

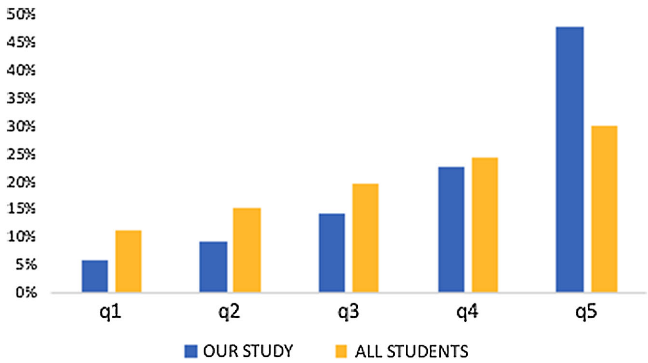

FIG. 4. Estimated parental income of samples from our study (blue) versus overall college student population (orange), by quintiles. Blue bars are constructed from university-level data pulled from the Equality of Opportunity Project [75,76], weighted by sample size. Orange bars are constructed from percentages cited in Ref. [65].

Quichocho, Schipull, and Close [73], the PoC in PER group [74], and others—that can guide our conversations.

C. Data analysis

Preliminary analysis of student responses to each of the questions in Fig. 2 focused on the resources we saw students using[4] in their responses. Consider the following student response to the airplane question:

> *"The bowling ball would most likely follow the path A because the ball will experience air resistance as it is dropped. The air resistance will pull the ball back making it follow this trajectory."*

A misconceptions-oriented analysis of this question may focus on the incorrectness of the student's answer, and possibly take this response as evidence that this student does not recognize the forward motion of the ball, or does not know that moving objects keep moving. Our resources-oriented analysis, on the other hand, seeks continuities between student responses and canonical understandings. When the student says, "The air resistance will pull the ball back making it follow this trajectory," we see them identifying a force (air resistance), and then attributing a change in motion of the ball to that force ("making it follow…"). We would then say that this student is using the resource "forces influence the motion of objects."

Our identification of resources usually proceeds from concrete to general; we first name what we consider to be concrete "good ideas" at the level of the question, and then we thematically group these ideas across questions, looking for and naming more generalized resources that are being deployed in individual questions. For example, in the airplane question, we identified the ideas "the bowling ball still has some of the forward motion of the plane," "the vertical velocity of the bowling ball increases as it falls because of gravity," and "air resistance will pull the ball back," among others. We then looked for themes across questions, collapsing the question-specific ideas we identified into a set of more generalized resources that highlight the productive substance of student thinking. For example, we collapsed the following question-specific ideas (and more) into the generalized resource "forces influence the motion of objects":

- Air resistance will pull the bowling ball back, causing it to follow path A. (airplane question)
- The plane exerted a force on the bowling ball, which is what gave it horizontal velocity in the first place. (airplane question)
- It takes a lot of force to get the box moving (and less force to keep it moving). (lifting box question)
- It takes more force to keep the box moving than to get it moving, because gravity will slow the box's motion. (lifting box question)

These more generalized resources that we identified—like "forces influence the motion of objects"—became the codes in an emergent coding scheme [77]. Two independent coders (ADR and LMG) then used this scheme to code student responses. A single student response could receive more than one code if it used more than one resource; i.e., the resources are not independent. The frequencies reported in Secs. V and VII represent the percentage of responses that both coders agreed were using each resource; that is, a response kept a code only if both coders assigned it that code.

Standard statistical measures of agreement between coders, such as Cohen's kappa, require that individual codes are independent or mutually exclusive [78]. Since our codes are neither independent nor mutually exclusive, such statistical measures are inappropriate for our analysis. As a measure of agreement between coders, we took the normalized difference between the total number of codes possible and the total number of disagreements between the codes:

$$\frac{(n_{\text{possible codes}})(n_{\text{coded responses}}) - (n_{\text{total disagreements}})}{(n_{\text{possible codes}})(n_{\text{coded responses}})}.$$

Our percentage agreement for the entire dataset was 94%.

V. CONCEPTUAL RESOURCES FOR UNDERSTANDING FORCES

In this section, we describe six common conceptual resources for understanding forces that we identified in student responses to the airplane, modified coin toss, furniture, lifting box, and walking person dropping ball questions. To be considered "common," a resource had to

[4] Although we frequently refer to students "using" this or that resource, we simply mean that their responses are *consistent with* the use of that resource. That is, we acknowledge that the existence of these resources is hypothetical, as is their use in specific cases.

have been used (a) in responses to *multiple questions* in our study and (b) in *at least 10%* of student responses for at least one sample. In many cases, the resources we report were used much more frequently than this. These common resources include the following:

a. Moving objects keep moving.
b. Forces influence the motion of objects.
c. Imbalanced forces change the motion of objects (and balanced forces do not).
d. It takes more effort to overcome a given force than to match it.
e. It takes more effort to overcome a bigger (net) force (and less effort to overcome a smaller one).
f. It takes more effort to change the motion of an object than to sustain it.

We will argue that each of these resources is continuous with Newton's first or second laws, and we will illustrate ways in which student responses use these resources to answer our five conceptual questions.

A. Moving objects keep moving

Students used the resource "moving objects keep moving" to justify their answers to all five conceptual questions in our study. It was most common in the airplane and walking person dropping ball questions, with between 64% and 76% of responses using this resource in the former and between 34% and 48% using this resource in the latter. It was least common in the modified coin toss question, with <5% of responses using this resource there.

The ways in which student responses relied on this resource varied across questions. In the airplane, modified coin toss, and walking person dropping ball questions, students often used this resource to explain the trajectory of the ball. For example,

Airplane question: "*Since the plane has forward momentum, the bowling ball also has forward momentum and as it is released, the force of gravity pushes down on the ball, causing a curved downward motion*."

Airplane question: "*A. The ball will be traveling at a slower velocity than the plane while in it. It will then roll out of the plane and continue in the same direction until the velocity slows to 0*."

Walking person dropping ball question: "*They should drop the ball before they get to the red x. This is because the ball is traveling at the same velocity that the person is walking and therefore will continue to travel in that direction*."

In these three quotes and others like them, students justified the continued forward or backward motion of the ball in terms of it "having" momentum (or speed) already, which keeps it moving (first quote), or in terms of it continuing its motion (second and third quotes). In all of these cases, students implicitly invoke the resource moving-objects-keep-moving. Notably, students sometimes used this resource to justify incorrect answers, such as the second quote here, in which a student selects trajectory A, the trajectory that shows the ball moving in the opposite direction of the plane's motion as it falls to the ground. In this case, the student may be thinking that the ball is rolling backwards out of the plane, such that continued motion (in the plane's reference frame) would be to the left. Though this response is canonically incorrect, we argue that it draws on a resource that is continuous with formal physics, and that instruction could *build on* this student's thinking.

B. Forces influence the motion of objects

We identified the resource "forces influence the motion of objects" in responses where students characterized the effect of an individual force on the motion of an object. Students sometimes did this for multiple individual forces—e.g., they identified the effect of gravity on the motion of a ball and the effect of air resistance on the same ball. So long as they did this for each force individually, rather than for the effect of an imbalance or net force, we considered them to be using the resource forces-influence-the-motion-of-objects. Students need not have used the word "force"; they may instead have named the effect of something that we identified as a force, e.g., gravity, air resistance, friction.

As with moving-objects-keep-moving, this resource was used by students in response to all five questions. Between 41% and 49% of students used this resource in response to the airplane question, 29%–38% in response to the modified coin toss question, 1%–12% in response to the furniture question, 2%–8% in response to the lifting box question, and 14%–23% in response to the walking person dropping ball question.

Examples of responses that used the resource forces-influence-the-motion-of-objects include the following:

Walking person dropping ball question: *Student drew Fig. 5 and wrote "…before they reach the x. Because there is a force exerted from when the person begins to walk. Along with the force of the earth, the force of the person walking will interfere with the ball, changing its trajectory downwards to the right."*

Airplane question: "*The bowling ball would most likely follow the path A because the ball will experience air resistance as it is dropped. The air resistance will pull the ball back making it follow this trajectory*."

Coin toss question: "*The force applied prope[l]s the ball into the air, it eventually comes to a stop by gravity, and then it falls to the ground because gravity pulls it down*."

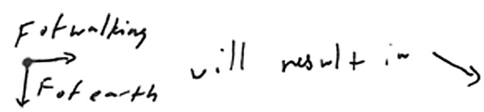

FIG. 5. Diagram accompanying student response to walking person dropping ball question.

Lifting box question: "*It would take more force to keep it going since the object will be affected by gravity. Gravity will slow down its velocity…*"

In all of these responses, students identify an individual force or forces and state how each force influences or changes the motion of an object. In the first response, gravity "changes the trajectory of the ball" (making it move downwards); in the second, air resistance "pulls the ball back" and "makes it follow" trajectory (a); in the third, an applied force "propels the ball into the air" and gravity makes the ball stop and change direction (toward the ground); and in the fourth, gravity "slows down the box's velocity."

As with the resource moving-objects-keep-moving, not all of these responses are canonically correct. For example, the second response uses the resource forces-influence-the-motion-of-objects to justify trajectory (a) in the airplane question, where the ball moves opposite the direction of the plane's motion. (The correct answer is trajectory (d).) However, as we argued above, such responses are drawing on ideas that are continuous with formal physics. In this case, the student is drawing on the resource forces-(here, air resistance)-influence-the-motion-of-objects, which is continuous with both Newton's first and second laws.

C. Imbalanced forces change the motion of objects (and balanced forces do not)

Students also commonly characterized, discussed, or described the effect of an *imbalance* of forces on the motion of objects in the modified coin toss, furniture, and lifting box questions, with between 6% and 13% of responses using the resource in the first question, between 5% and 22% in the second, and between 18% and 36% in the third. These responses often articulated a need for a particular force to be "overcome" by another in order for motion to change, rather than assessing the effects of individual forces. In some cases, students would say that an object's motion does *not* change because the forces are balanced, also a case of the use of this resource.

Some responses limited their analysis to a moment in time: the forces need to be imbalanced to initiate or change the motion of an object at that instant. For example,

Lifting box question: "*The force needed to initially pick up the box from the ground is greater than the force needed to move the box to the top of the table. This is because you have to overcome the force that keeps the box on the ground and stationary…*"

Furniture question: "*It's hard to initially move it due to the static friction holding in that place. You have to exert a larger force than the static friction force. You will have to apply a lot of force to move it from rest.*"

Coin toss question: *Student drew Fig. 6 and wrote, "Gravity is acting the same on both points, the only difference is that on the ball at point B, the force of the throw has a magnitude big enough to overcome the gravity trying to force the ball down."*

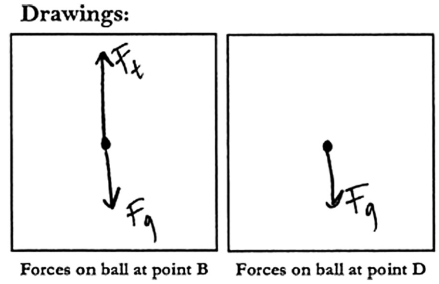

FIG. 6. Free-body diagram accompanying student response to coin toss question.

In all three of these examples, students suggest that an imbalance is necessary to get an object moving (i.e., change its motion): in the first, one must exert a force bigger than the gravitational force to get the box to start moving upward; in the second, one must exert a force bigger than the frictional force to get the furniture to start moving; and in the third, the person must have exerted a "throw force" bigger than gravity in order to move the ball upward.

Other (less common) responses explain phases in an object's motion in terms of imbalances of forces. For example, one student drew the free body diagram in Fig. 7 and wrote:

Coin toss question: "*At point B, the force of the throw is greater than the weight of the ball, causing it to move up. When the weight of the ball is more than the force of the throw at point D, the ball falls down.*"

This student identified two different phases in the ball's motion—moving up and then falling down—and justified the difference in motion in terms of the imbalance of forces in the two phases. In this case, the ball *changed its motion* because of a "change in the imbalance of forces"; the imbalance favored upward movement at point B and downward movement at point D. (In the figure the F_{throw} was crossed out by the instructor, not by the student.)

The resource "imbalanced forces change the motion of objects" (and balanced forces do not) is continuous with Newton's second law, which identifies the net force on an object as the cause of its acceleration. When there is an

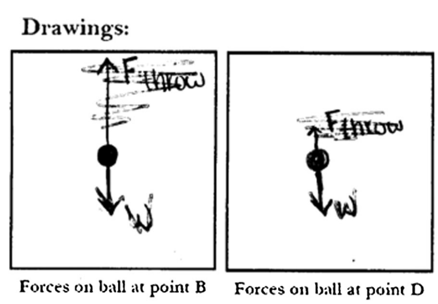

FIG. 7. Free-body diagram accompanying student response to coin toss question. Note that the crossing out of F_{throw} was done by the instructor who graded the response, not by the student.

imbalance of forces, there is a net force, and the object accelerates in the direction of the net force. When there is a balance of forces, the acceleration is zero and the object has a constant velocity. Though some of these responses identify or characterize forces in ways that physicists would call incorrect, such as including a throw force after the ball has left the hand in the modified coin toss question, they all explicitly attribute changes in the motion of objects to an imbalance of forces.

D. It takes more effort to overcome a given force than to match it

The resource we just discussed, imbalanced-forces-change-the-motion-of-objects, focuses on the presence of an imbalance in forces, but not on its amount. The next several resources embed a more quantitative sense of imbalance, asking not just, "Are the forces imbalanced?," but also, "By how much?" For example, in the furniture and lifting box questions, students were asked to compare two phases in an object's motion: (1) starting and (2) maintaining movement. In doing so, students often invoked the resource "it takes more effort to overcome a given force than to match it." We saw this frequently among responses to the lifting box question, with between 6% and 18% of responses using this resource, and less frequently in the furniture question, in 0%–2% of those responses. For example,

Furniture question: "*When you start pushing you must produce enough force to match the force of the object's friction, plus enough additional force to accelerate the object to a comfortable moving speed. Once the object is moving you only need enough force to cancel out friction. No additional acceleration is required.*"

Lifting box question: "*The forces are different. An initial force to pick up the box must be greater than its weight to get the object moving. Once the object is moving the force applied to carry the box up to the table can be less than the initial force, which is also a force equal to its weight. This answer makes sense to me because I know that I have to exceed a threshold of force to actually pick up the box before I can bring it up to my height.*"

Lifting box question: "*They are different because a large imbalance of forces is need[ed] to accelerate the box from no motion. A balance would be used to keep the object in constant velocity upward. Because one would be trying to super[s]ede the force due to gravity of the object just to lift it up. After that, there is no need to apply that same force because one can move the box in constant velocity (balance of forces).*"

In all three of these responses, students considered a single force, typically friction or gravity, and argued that it is more difficult to overcome that force to get the object moving than to match the force to keep it moving. Such responses embed a particular image of balance: there is a force (gravity or friction) that is in a sense "already there"—on one side of a balance—and we (the agent) are deciding what to do to the other side—match it or exceed it. Whether to match or exceed the force depends on whether you want an acceleration or change in motion or not.

E. It takes more effort to overcome a bigger (net) force (and less effort to overcome a smaller one)

Similar to it-takes-more-effort-to-overcome-a-given-force-than-to-match-it, the resource "it takes more effort to overcome a bigger (net) force" also embeds imagery of imbalance. However, students using the former, it-takes-more-effort-to-overcome-a-given-force-than-to-match-it, are, in effect, imagining a fixed force on one side and deciding what to do with the other side (exceed or match it). Students using it-takes-more-effort-to-overcome-a-bigger-(net)-force have determined that they need an imbalance, and are figuring how much force would need to be applied to achieve that imbalance: more to overcome a bigger force and less to overcome a smaller one. This resource was used in 22%–62% of responses to the furniture question and in 12%–26% of responses to the lifting box question.

In response to the furniture question, some students reasoned that it is easier to continue moving the furniture than to start its movement because of the relative magnitudes of static and kinetic friction. For example,

> *"Because the coefficient of static friction is larger than the coefficient of kinetic friction this makes sense. I imagine it like the force of friction is "grabbing" the object. Once it starts to lose its "grip" (i.e., the object begins to move) then that grip becomes weaker and therefore the object becomes easier to move."*

> *"Because of the friction. At the beginning you have static friction, and after you start moving the box you have kinetic friction, which is lower in value and makes moving the box easier. (kinetic friction < static friction)"*

> *"$\mu_k < \mu_s$ where μ_s is the coefficient of static friction."*

Students used similar reasoning in the lifting box question, saying that because the weight or gravitational force on the box—which opposes the box's motion—is the same before and after the box is lifted off the ground, the force it takes to get the box moving is the same as the force that keeps it moving. For example:

> *"The force has to be at least equal to the box's weight. Since the weight of the box does not change the force will stay the same."*

> *"The forces are the same. This is because the box does not change mass; thus, the force needed is only to fight the box's weight (gravity times mass), which does not*

change. This is because, assuming gravity and mass remains constant (and there is no evidence that says gravity and mass are not constant), the force needed to overcome the box's weight remains constant whether beginning to lift it or continuing."

"The force you need initially is the same as the force you need to keep moving the box upward toward the top of the table because the only force you are acting against is gravity. Since you are only lifting the box to the height of the table and not actually placing it on top of the table no other forces will be involved."

In all of these responses, students consider the magnitude of the forces that oppose an object's motion at two different points, and they compare the forces they would need to apply to unbalance those forces.

Some students identified that the sum of two forces changes as the object goes from rest to moving, and then suggested that this change matters for how much force needs to be applied to keep an imbalance. For example, in the lifting box question, students noted that the presence or absence of a normal force exerted on the box by the floor would have consequences for the force one would have to apply in the two phases of the box's motion:

"I think the initial force [to get the box moving] is less because you have the assistance of the floor pushing the box up already so when you apply your force in addition to that the net force on the box is no longer zero allowing it to move upwards. Once you have the full weight of the box in your hands however you have no assistance f[ro]m the ground. Therefore the box's mass is pulling it down with gravity and depending how heavy it is the box is much harder to hold up. You are pushing the box up while the force of gravity pushes it down."

Our interpretation of this student's response is illustrated in Fig. 8. When the box is stationary and on the ground, there is a weight force exerted on the box by the Earth and a normal force exerted on the box by the floor. Only a small applied (push) force is required to imbalance the weight force to get the box to move upward under these circumstances. However, once the box is no longer in contact with the floor, the normal force on the box from the floor goes away, and the applied (push) force must therefore become much bigger to create an imbalance.

Students used similar reasoning in the furniture question, arguing that when the furniture is moving, there is a "movement" force in the direction of motion, such that a smaller applied force is necessary. For example,

"When I am pushing a heavy object or piece of furniture across the floor, I know it is harder for me to get it moving because of friction and there is no other force assisting me in moving the object. When getting it to move, I am the only force acting on the object so it is more difficult to start moving the object. While when keeping an object moving, it already has a force of momentum on it and any more force I push on the object makes it keep moving or moves faster depending on the amount of friction there is."

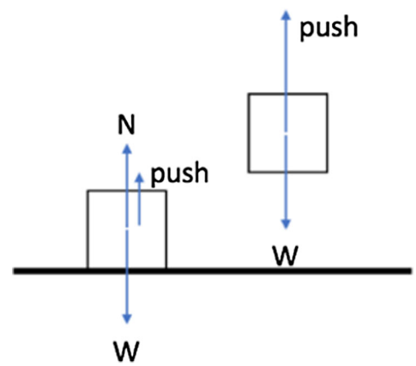

FIG. 8. Our interpretation of student reasoning that it takes more force to keep the box moving, since you no longer have "assistance f[ro]m the ground."

Similar to above, our interpretation of this student's reasoning is shown in Fig. 9. When the box is not moving, the applied push force is the only force to the right and must overcome the frictional force all by itself. Whereas when the box has started moving, there is a motion force also to the right, which "assists" the applied force in moving the box, such that the applied force need not be so big for the friction force to be overcome.

As with it-takes-more-effort-to-overcome-a-given-force-than-to-match-it, the resource it-takes-more-effort-to-overcome-a-bigger-(net)-force embeds intuitive notions of imbalance and is continuous with mathematical formulations of Newton's second law. In particular, students are recognizing (a) the relationship between an imbalance of forces and the changing motion of an object, and (b) interpreting $\boldsymbol{F}_{\text{net}}$ as a vector quantity, such that the amount of applied force needed to achieve imbalance depends on the other forces on the object. Though students sometimes use this resource in ways that physicists might call canonically incorrect—e.g., marshalling a "motion" force to justify a smaller push force—we see their ideas as nonetheless continuous with formal physics in the ways we have articulated here.

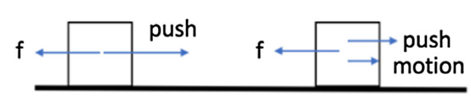

FIG. 9. Our interpretation of student reasoning that it takes less force to keep the furniture moving because it "has a force of momentum on it" that "assist[s]" the pusher.

F. It takes more effort to change the motion of an object than to sustain it

Still other students compared the "effortfulness" of two different scenarios, but instead of considering the forces at play, or focusing on balance/imbalance, they considered whether the motion was being changed or maintained. These responses used the resource "it takes more effort to change the motion of an object than to sustain it," an intuitive version of Newton's first law. As with the last two resources, this resource was elicited in the furniture and lifting box questions with between 6% and 15% of responses using it in the former and between 12% and 18% using it in the latter.

Some students combined the it-takes-more-effort-to-change-the-motion-of-an-object-than-to-sustain-it resource with another resource, moving-objects-keep-moving, to construct an explanation for why it is easier to keep the furniture or box moving than to start its motion. For example,

Furniture question: "*This phenomenon is known as inertia, the tendency of an object to continue its path of motion. Once the box is already moving, it is less difficult to move it, but in order to get it going, you have to change its state of motion from at rest to in motion.*"

Furniture question: "*An object in motion will tend to remain in motion unless acted on by an outside force is why it's easier to keep something moving. The inertia of a non-moving object must be overcome, but after that, it's easy to keep moving.*"

Lifting box question: "*The force required initially is greater than the force needed to keep it moving. This is because it is easier to keep something in its current state, like keeping the box moving, than it is to change its state, like lifting the box. This is shown by Newton's Law of Inertia.*"

These quotes helped us to see the relationship between the resource it-takes-more-effort-to-change-the-motion-of-an-object-than-to-sustain-it and Newton's first law: in these quotes, it is *because* moving objects tend to keep moving that it is easier to sustain an object's motion, be it furniture moving horizontally or a box moving vertically.

Similarly, some students suggested that there is something in or on an object that is aiding its motion when it is already moving. That is, it is less effortful to sustain an object's motion because something—often "momentum"—is helping it move. For example,

Furniture question: "*You are initially pushing against friction and feeling the full force of the weight of the object on your body, but once the object gets moving you are being aided by its own momentum.*"

Furniture question: "*I have always thought that momentum was the reason for something moving easier once it starts. As when an object is at rest it is settled in [a] way and when you first start to move it that settling force must first be broken and then the weight or the mass of the boxes helps it to keep moving. I also think that friction has a settling force too. Once you get the weight moving you should never stop in the middle as it is then harder to start it again…*"

Furniture question: "*It is harder to overcome the static friction that is first pushing back on to you, but the momentum once moving, helps overcome the kinetic friction.*"

In each of these quotes, students treat momentum as something the object has that sustains its motion; when an object does not have momentum, it takes more effort to move the object, but once the object has momentum, it is less effortful to keep the object moving.

Finally, some students suggested that it is more effortful to accelerate an object than to keep the motion the same, since the former requires a (net) force. For example:

Lifting box question: "*Initially you need greater force to accelerate the box than you do to keep the box moving. [This made sense to me because] once the box is moving upward it doesn't need to accelerate any more, but to get it from not moving to moving upward it requires acceleration, which requires greater force.*"

Lifting box question: "*The forces are different. When you initially lift the box off the floor, you must accelerate it upward (speed it up); the velocity starts at zero and then must change velocity as you lift it. But when you keep moving it toward the top of the table, it is no longer necessary to (positively) accelerate it. In other words, you could maintain a constant speed or even slow it down (up to a certain rate) and the box would still reach the table. In summary, the force required to initially lift the box will likely be greater than the force required to maintain the box's motion because of the necessary positive upward acceleration as the box begins moving.*"

In both of these quotes, students are comparing the force needed to initially lift the box off of the floor and the force needed to keep the box moving upward toward the top of a table. These students answer that the force needed to initially lift the box is greater, because one has to accelerate the object from rest to some non-zero speed, whereas continuing to move the box upward does not necessitate a change in the box's speed.

The resource it-takes-more-effort-to-change-the-motion-of-an-object-than-to-sustain-it is continuous with Newton's first law, which says that an object in motion will stay in motion unless acted upon by an external force. Whereas moving-objects-keep-moving emphasizes the first part of this law—that objects in motion stay in motion—it-takes-more-effort-to-change-the-motion-of-an-object-than-to-sustain-it captures the essence of the law in its entirety. Further, it points us to an intuitive formulation of that law; students' sensory and lived experiences suggest that changing motion is harder than sustaining it, and in the quotes in this section we see them connecting these experiences to physics ideas like acceleration and momentum.

VI. COMPARISON OF RESOURCES WE REPORT AND STUDENT IDEAS DISCUSSED IN THE LITERATURE

A. Resources related to student ideas represented in the literature

As noted in Sec. II, there have been more than fifty journal articles and conference proceedings papers about student ideas about forces, most of which take a misconceptions- or difficulties-oriented perspective on student thinking. Two of the student ideas heavily represented in the literature are (i) impetus-force-like reasoning and (ii) an association between force and velocity, rather than between force and acceleration. We see our first two resources, moving-objects-keep-moving and forces-influence-the-motion-of-objects, as overlapping with these two themes. Many student responses that we characterized as moving-objects-keep-moving share with the impetus force idea the notion that *something* keeps moving objects moving. In the impetus force idea, the thing that keeps objects moving is a *force*; in our data, when students attributed the continued motion to something in the object, it was often momentum. (However, we do not know whether the notions of force and momentum are conceptually distinct for such students.) Likewise, the resource forces-influence-the-motion-of-objects shares elements of the "impetus force" and "associating force with velocity" categories in the literature; all three focus on the relationship between force and motion.

In this sense, these two resources are not *new*, *per se*, and may in fact reflect what is sensible about the impetus force and force-associated-with-velocity ideas reported in the literature. (We could even think of them as instantiations of the same resource in different contexts.) However, we do think our analysis offers more than a reinterpretation of existing literature. Even if we could have articulated these resources from existing literature, our work empirically affirms the legitimacy of these interpretations. Rather than layering interpretation onto interpretation—"we interpret these researchers' interpretation of student ideas in a different way"—we went back to the data with a resources-oriented lens, seeking continuities between student thinking and formal physics. That our interpretations are consistent with the literature while being rooted in new analysis is, in our mind, a feature, not a bug. Importantly, our analysis also yielded new insights not reported in the literature, which we will turn to next.

B. Resources distinct from student ideas represented in the literature

By training our analytic vision on resources, we identified categories of student thinking that to our knowledge have not been reported elsewhere; namely, the four imbalance and effort resources in our list: imbalanced-forces-change-the-motion-of-objects; it-takes-more-effort-to-overcome-a-given-force-than-to-match-it; it-takes-more-effort-to-overcome-a-bigger-(net)-force; and it-takes-more-effort-to-change-the-motion-of-an-object-than-to-sustain-it. These resources imply intuitive notions of balance or effort, or of the relationship between imbalance and changes in motion. In this way and in the ways depicted above, using a resources framework to analyze student written data has yielded new insights into student thinking about forces. We expect the same would be true of instructional practice if instructors chose to use such a framework for interpreting their students' thinking.

VII. QUESTION DEPENDENCE OF RESOURCES

The context sensitivity of resource activation has been extensively illustrated and theorized about (see Sec. III) [1,3,42,50,51,53–55]. The case studies in this literature help to demonstrate *that* resource activation is context dependent and to propose mechanisms by which this context sensitivity unfolds, both of which have been important contributions to theory development. What they do not do is to demonstrate patterns in this context sensitivity, or to identify patterns in resource-context pairings; i.e., they do not answer questions about which resources are reliably elicited in which contexts. Our research makes progress along this dimension, by examining patterns in resource activation across questions. Such patterns may support instructors in predicting which questions may elicit specific resources, or which questions might elicit a diversity of resources.

Figure 10 shows the percentages of responses using each of our six resources, broken down by question. Differently patterned bars indicate samples from different universities. Looking across questions, we can see two instructionally relevant patterns:

(i) Some questions more reliably elicit specific resources than others. That is, if an instructor wants to elicit the resource moving-objects-keep-moving, the airplane question or walking person dropping ball question may be the best choices. If an instructor wants to elicit the resource imbalanced-forces-change-the-motion-of-objects, they may not want to use the airplane or walking person dropping ball questions and may instead want to choose one of the other three. Notably, questions that elicit a particular resource at high frequency tended to do so consistently across the samples in our study. For example, both the airplane and walking person dropping ball questions elicited resource (a) at higher frequencies than resource (b), for all of the universities sampled.

(ii) Some questions more reliably elicit a variety of resources, and others more reliably elicit a subset. For example, the airplane, coin toss, and walking person dropping ball questions reproducibly elicit

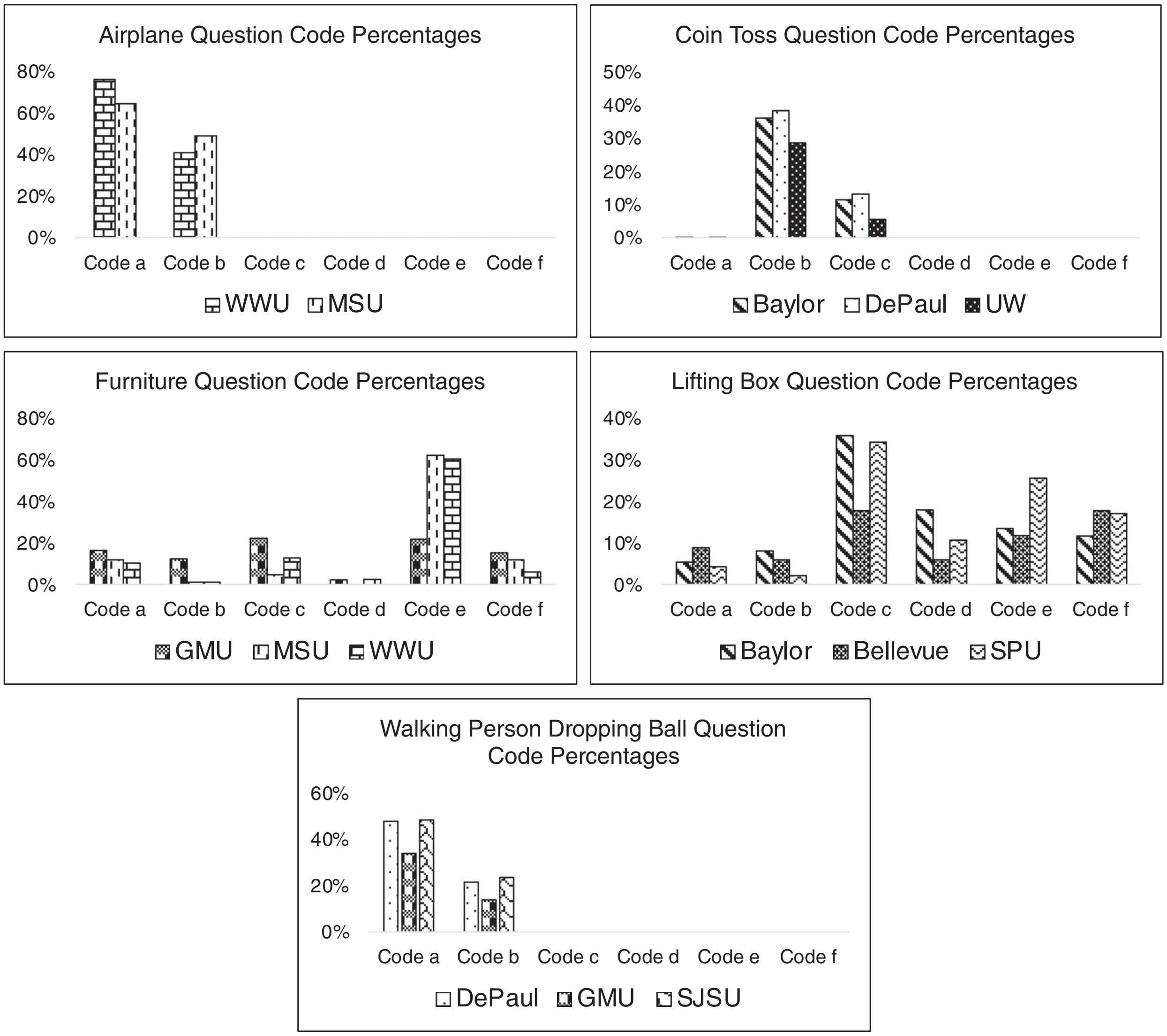

FIG. 10. Fraction of student responses using resources (a) through (f), by question.

two resources in our study, whereas the furniture and lifting box questions elicit all six. A physics instructor who wants to elicit and build on a particular resource might choose a question that elicits it at high frequency. On the other hand, a physics instructor that wants to elicit a range of resources, perhaps for the purposes of collective discussion and argumentation, might choose a question that elicits a variety of resources.

Though “context” in the resources theoretical framework includes much more than the question being asked, here we have focused on the most salient pattern: the relationship between the question and resource activation. (Other relationships are more spurious or less visible in our data.) We can tentatively make sense of these question-resource pairings by considering what ideas may be useful for answering each question. For example, the airplane and walking person dropping ball questions ask students to choose or draw the trajectory of an object dropped from a moving container (an airplane or a hand). It makes sense that in explaining the trajectories they drew or chose, students would rely on moving-objects-keep-moving [resource (a)] and forces-influence-the-motion-of-objects [resource (b)]; students often attribute continued motion to existing movement and changes in motion to forces like gravity or air resistance. It also makes sense that the furniture and lifting box questions would elicit more imbalance resources than the other questions, since students are being asked if (or why) it is harder to get an object moving than to keep it moving. These patterns and hypotheses are the subject of ongoing work by our team [75].

VIII. CONCLUSION

Our purpose in conducting this research has been to contribute to the literature documenting common student ideas in physics, adding conceptual resources to the existing themes of misunderstandings, misconceptions and difficulties. In this paper, we identified six common conceptual resources for understanding forces, based on our analysis of 2048 written student responses to five conceptual questions. These questions were asked after instruction at nine different universities across the U.S. These resources include the following: (a) moving objects keep moving, (b) forces influence the motion of objects, (c) imbalanced forces change the motion of objects (and balanced forces do not), (d) it takes more effort to overcome a given force than to match it, (e) it takes more (less) effort to overcome a bigger (smaller) net force, and (f) it takes more effort to change an object's motion than to sustain it.

In Sec. V, we offered interpretations of how these ideas are continuous with formal physics, particularly Newton's first and second laws. In particular, we think that a number of these resources represent intuitive formulations of these laws. For example, imbalanced-forces-change-the-motion-of-objects may be an intuitive—rather than mathematical—formulation of Newton's second law, drawing on notions of imbalance and overcoming. It-takes-more-effort-to-overcome-a-bigger-net-force is also continuous with (though distinct from) mathematical formulations of Newton's second law. It not only recognizes the relationship between an imbalance of forces and changes in motion but also interprets $\boldsymbol{F}_{\text{net}}$ as a vector quantity, such that the amount of applied force needed to achieve imbalance depends on other forces on an object. Resources theory poses learning as building on students' resources, and these intuitive formulations, elicited by the five conceptual questions used in our study, seem like relevant starting places for such building.

Resources theory also depicts learning in terms of reorganizing, refining, properly activating, increasing the degree of formality of, or changing the role of resources. In Sec. III, we noted that diSessa suggests that a primary difference between novice and expert cognition in physics is in the structure and connectedness of networks of resources; that is, the resources used by novices become part of the structures that organize expert physics thinking. Many of the examples we gave throughout the paper are concrete instantiations of more general principles or physical laws. For example, "the bowling ball will keep moving in the direction it was already moving when it leaves the plane" is a concrete instantiation of Newton's first law. "…a[n] imbalance of forces is need[ed] to accelerate the box from rest" is a concrete instantiation of Newton's second law. Instructors might support learning, from a resources perspective, by scaffolding an "organizing" and "increase in the degree of formality" of these concrete instantiations, e.g., by eliciting ideas in a few different scenarios and inviting students to identify patterns in their answers. This instructional strategy acknowledges that students have the raw material out of which to build formal physics understandings.

Finally, some questions more reliably elicited particular resources than others, and some questions reliably elicited a variety of resources (whereas others reliably elicit a subset). This finding adds empirical weight to the context-dependence tenet of resources theory and has the potential to inform instructional planning. For example, physics instructors who wish to elicit and build on a specific resource may choose a question that tends to elicit a narrow range of resources, with the targeted resource being elicited at high frequency. An instructor who wishes to elicit a diversity of resources may instead choose a question that tends to elicit a range of resources.

The resources we identify are meant to support instructors in a flexible orientation toward student thinking. They are not meant to serve as static categories or deterministic associations. That is, in light of the abundance of research directing instructors' attention toward student misconceptions or misunderstandings, we hope to make plausible that students bring a wealth of knowledge to their learning that is continuous with formal physics. Our characterizations, first and foremost, are meant to illustrate that this is the case, and then to provide concrete examples of what this looks like, in order to promote an orientation of openness toward and sense making about student thinking about forces.

In inviting instructors to recognize and celebrate students' ideas, though, we reelevate the considerations brought to our attention by Kanim and Cid [63]. In particular, *whose* ideas are we foregrounding, and how does this foregrounding construct an implicit norm against which all students are assessed? Further, as our emerging demographic analyses [68] are highlighting, what is a representative introductory physics course? If we find that a representative physics course oversamples from, e.g., white and Asian student groups, and undersamples from Black, Latinx, and Indigenous groups, such that representative sampling will *still* emphasize the physics ideas of "white, wealthy, mathematically prepared students" [63], what will be our response? As above, these are *community* questions, but this manuscript (re-)raises them for us, the authors, and so we also raise them for you.

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