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Development of a construct map to describe students' reasoning about introductory quantum mechanics

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We present the development and validation of a construct map addressing introductory quantum mechanics topics at the high school level, as a subset of a larger learning progression on quantum mechanics. Topics include energy quantization, photon absorption and emission, the Heisenberg uncertainty principle, atom stability, orbitals, wave function, and electronic properties of materials. To validate the hypothesized construct map, we designed a multiple-choice questionnaire and a 14-h teaching-learning sequence (TLS) designed in strict relation with the levels of the construct map. Twenty-three classes of Italian students (ages 17–18, $N = 408$) were involved in the study: about half ($N = 200$) were exposed to the TLS, while $N = 208$ students received a typical textbook instruction on the same topics targeted by the construct map. Data were analyzed using a 1D Rasch model. Results show that the proposed construct map consistently describes the increasing abilities of students when exposed to instruction. In particular, when exposed to the TLS activities, students more likely move towards the upper levels of the construct map. Findings have implications for instruction designed to support students' learning in quantum mechanics at the high school level.

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I. INTRODUCTION

The formulation of quantum mechanics (QM) theory is among the greatest achievements of physics. Building on the "big idea" that matter is made up of atoms and interacts with the electromagnetic field by exchanging photons [1,2], QM theory introduces an entirely new description of the microscopic world and of mechanisms underlying all macroscopic phenomena. QM theory not only is essential for our description of the natural world, but it also spawned impressive practical results. Most present technologies are in fact based on QM: for instance, any electronic device, from LED lamps to mobile phones, is strictly speaking a quantum device. Furthermore, several incoming advancements that may deeply change our world are also based on QM (e.g., quantum computing).

Despite its relevance, traditional curricula of physics in secondary school devoted very little space to the microscopic world, leaving topics like the structure and properties of atoms and elements to the chemistry classes. In our opinion, this curriculum organization may contribute to the view of QM as obscure and weird [3] or it may even reinforce misconceptions, like the planetary model of the atom, and alternative ideas like the wave motion of electrons [4,5]. Only since the last ten years, reforms have introduced the basics of QM in physics curricula of many countries [6]. The main reason for this increasing attention is that QM offers students the opportunity to discuss up-to-date technological scenarios and, possibly, may encourage some of them to pursue a STEM-related career. However, besides the above interest-driven arguments, there are more peculiar reasons: (i) QM compels us to reconsider the foundations of physics, such as the concept of measurement and the deterministic interpretation of the world, and, more in general, the wide issue of the nature of science, also in connection with the role of mathematics in physics [7]; (ii) QM demands a critical rethinking of previously learned physical concepts. For instance, force is essential in classical mechanics, but irrelevant in QM; momentum, in turn, becomes crucial. Similarly, the constraints of the Heisenberg principle lead to the dramatic revision of the concept of particle motion [8]; (iii) the difference between classical physics and QM is a paradigmatic example to discuss different scopes of theories and their relationships (i.e., the correspondence principle) [9].

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When analyzing curriculum guidelines and documents of different countries, a *core* and an *extended* quantum curriculum can be identified [6]. The core quantum curriculum features topics such as discrete energy levels, interactions between light and matter, wave-particle duality, matter waves and de Broglie wavelength, technical applications, the uncertainty principle, and probabilistic behavior. The extended quantum curriculum includes, for instance, tunneling, entanglement, and epistemic views of QM. Schematically, these topics can be grouped around three thematic foci [6]: (i) the structure of matter and its interaction with radiation, with emphasis on the physics of atoms and photons; (ii) the wave function, its properties and a qualitative understanding of the Schrödinger equation; (iii) the epistemic aspect, with emphasis on the wave-particle alternative descriptions, the nondeterministic nature of QM, and the entanglement.

In this paper, we will focus on the first theme, which basically represents the most common pathway to quantum mechanics for high school students. Several works made a valuable contribution to the investigation of students' understanding of introductory QM (for an extended review see Ref. [10]; a short review on the topics that are relevant for this study is reported in the following section). However, in spite of such efforts, a complete and coherent description of how students progress in their understanding of these topics is still missing. Such a picture would be relevant not only to interpret students' difficulties in QM at high school, and by extension at the university level, but also to design teaching activities more responsive to students' reasoning.

In order to give a contribution toward a more general understanding of these issues, we adopted the *construct map* (CM) approach [11] to describe the incremental students' reasoning about QM topics addressed at the high school level. Using the definitions of hypothetical construct map (HCM) levels, we contextually designed and evaluated a teaching sequence to address the topics targeted in the HCM. The next sections are hence devoted to answer the following research questions:

RQ1: *To what extent does the hypothesized construct map (HCM) describe students' reasoning about introductory QM topics?*

RQ2: *To what extent are the instructional activities informed by the HCM effective in improving students' knowledge about introductory QM topics?*

II. BACKGROUND

A. Construct maps

Construct maps are "designed to help conceptualize how assessments can be constructed to relate to theories of cognition" [11] (p. 717). A construct map can be thought as a hypothetical hierarchical structure of qualitatively different levels of performance about a specific topic or a related set of concepts [12]. In other words, each level represents a more sophisticated understanding of the concept in comparison to the previous level [13]. Thus, construct map levels become more sophisticated as students progress in their understanding of the targeted topic. The top level of a construct map represents the understanding expected from students in comparison to the specific targeted school level. Hence, construct maps can inform suitable teaching activities that can support students' progression from one level to the next [14–17]. In this view, construct maps are not only models that describe and predict how students progress in their understanding of a given topic, but also tools to fine-tune the learning goals of specific instructional practices with the expected learning outcomes listed in the curriculum.

Construct maps can be viewed as the organizational structure of learning progressions [12]. In science education research, learning progressions are "empirically derived descriptions of how learners' conceptualizations of big ideas in science increase in sophistication through the mediation of instruction" [12] (p. 3). Being centered around one *big idea* (e.g., celestial motion), learning progressions are often multidimensional and hence can be constituted by several construct maps, each corresponding to a specific dimension of the big idea [18,19].

On such a basis, learning progressions can potentially extend over several school years [20], and have different grain size [21]. The grain size of a learning progression or of a construct map is the shift in reasoning that students need to make to move from a given level to the next one according to a defined scope and learning objectives. Construct maps feature a smaller grain size in the analysis of learning science to focus on a more detailed description of students' reasoning, emphasizing only critical elements of change between students' performances at different levels. As such, construct maps levels should be constructed starting from the empirical evidence about students' reasoning of the targeted topic, in our case, introductory QM.

B. Prior work about introductory QM

Research studies over the past twenty years showed that high school students often do not develop a sound understanding of introductory QM topics [22,23]. For instance, some students think that atoms can release energy in packets with arbitrary values [24]. In other cases, students have difficulties in understanding the concept of ground state, locating it at the zero of the energy scale [25]. Other typical misconceptions include the idea that a photon can be partially absorbed or emitted, the confusion between the energy of a single photon and the energy of a beam, and the difference in energy between two atomic levels is not equal to the energy of the emitted or absorbed photon. In a Spanish study [26], the authors report that high school students are often unable to use Bohr's atomic model to justify the discrete spectral lines of the hydrogen spectrum. In particular, students often have difficulties in recognizing that only stationary states, characterized by fixed energies, are available to electrons and that any change in the electronic state involves a transition from one stationary state to another. Similarly, students often have difficulties when dealing with energy quantization of electromagnetic radiation and when they have to relate the wave to the particle model [10]. Students have also difficulties in relating quantum and classical concepts: for instance, they often assign to a particle a well-defined trajectory or, use the “trajectory” of a particle in combination with the notion of orbital, or transform a well-defined trajectory into a “wave-shaped” one [27–29]. Finally, a typical incorrect reasoning concerns the uncertainty principle, since students confuse uncertainty relationships with inaccurate experimental measurements [8].

To address these misconceptions that may hamper a sound understanding of scientific models about QM introductory topics at secondary school level, many research-based approaches have been proposed. Early studies focused on improving students’ understanding of atomic models. For instance, Kalkanis and colleagues [30] proposed an instructional module focused on the energy levels of the hydrogen atom and the Heisenberg uncertainty relationships. Results report an improved capability of describing the hydrogen atom correctly and appropriately applying Heisenberg uncertainty principle. In other studies [31,32], Niedderer and colleagues addressed students’ incorrect ideas about atomic structure using the *electronium* model. In this model, the electron is depicted as an extended liquid substance, continuously distributed around the nucleus. The *electronium* is hence primarily used to promote the idea of an electron or charge cloud model, and the idea of orbital.

More recently, Savall-Alemany and colleagues [33] proposed a 12-h problem-based teaching-learning sequence focusing on students’ understanding of atomic spectra. Results show that students’ capability to interpret frequencies and intensities of radiation in emission and absorption phenomena improved significantly after the implementation of the TLS. The authors report also that students were able to use energy diagrams to deal with the quantitative aspects of emission and absorption phenomena.

III. THE PROPOSED HCM ABOUT INTRODUCTORY QM

A. Instructional context of QM teaching in an Italian secondary school

Since hypothetical construct maps describe the expected understanding of a given topic, their design depends on the specific chosen school level (e.g. primary, middle, or high school) both in terms of underlying themes, and of the relevance given to the different topics [11]. Concerning the teaching of physics in Italy, where students of different high school streams are supposed to deepen the concepts of the same subject in different ways, the mathematical treatment of concepts may greatly vary across high school streams. Because of this horizontal curriculum organization, some physics topics may be taught in earth science and in chemistry classes in a different way, according to the school stream. For this reason, we restricted our attention to a specific high school stream, by proposing a HCM that fits the curriculum guidelines of the *Liceo Scientifico*, which is devoted to broad dissemination of scientific culture and is presently attended by most Italian students oriented toward STEM university careers.

According to the physics curriculum guidelines, before addressing introductory QM topics in their last year (in Italy, 13th grade), students of *Liceo Scientifico* have been familiarized in their physics classes with electricity, magnetism, electrical circuits, optics, and the basic description of electromagnetic waves. The teaching practice usually requires students to solve formula-based exercises with occasional practical activities in the laboratory. From the longitudinal curriculum perspective, at this level, students had been already introduced in previous classes of chemistry (at grades 9–12) to several concepts regarding the structure of matter. Topics include the Mendeleev table of elements; early atomic models based on a historical discussion of the Thomson, Millikan, and Rutherford experiments; the Bohr model of hydrogen; the concept of orbital; electron spin; and the Pauli exclusion principle. We should also mention that the standard teaching of chemistry gives a simple and phenomenological description of the previously listed topics, with little attention to quantitative analysis and problem solving. Finally, before addressing QM, students are also expected to know the basis of algebra, geometry, trigonometry, and elementary functions theory, as prerequisite for an introduction to calculus (derivatives, integrals).

To better describe the extent to which introductory QM is covered, we summarize in Appendix A the list of topics featured in a widespread textbook [34].

B. Design of the construct map levels

The design of our HCM (see Table I) was framed in the more general learning progression that we developed to describe students’ understanding of QM from university entry-level to upper-level physics courses [35]. In particular, we started from the evidence that students develop their knowledge about atomic stability through hierarchically ordered levels: students learn first quasiclassical models and only after they become able to develop more sophisticated models involving probability aspects (atomic orbitals). We hence refined this progression and defined the improvements in sophistication based on whether students are able to (i) develop simple models of matter-radiation interaction, then (ii) make sense of how such models help

TABLE I. Overview of the hypothesized construct map used to describe students' progression when learning introductory topics in QM. Levels are numbered from the simplest (1) to the most advanced (5).

Level	Contents	Description	Assessment items
5	Molecules and condensed matter	Students can use the concepts of chemical bond and of molecular orbital. They distinguish between metals and insulators in terms of energy bands. They know that only the electrons in the conduction band contribute to the electrical current in metals. They know that the number of charge carriers is fixed in metals but can be changed in semiconductors by means of doping processes. They can qualitatively discuss how LED and solar cells work.	(2) (8) (11) (14) (17) (22)[a]
4	Orbitals, wave function, and probability in QM	Students can interpret atomic orbitals as density maps of probability of position, linking the viewpoints of physics and chemistry on the hydrogen atom. They qualitatively know that the ψ function is the mathematical instrument that allows computing the outcome probabilities of the observables' values and associated uncertainties. However, they have non-normative knowledge about materials and many-electron atoms.	(12) (15) (16)[a] (20)
3	Atom stability and emission of radiation	Students know that classical models cannot account for atom stability. They can use the Heisenberg uncertainty principle to qualitatively explain the stability of hydrogen. They correctly account for emission and absorption of photons in terms of energy levels. However, they have non-normative ideas about the structure of atoms and orbitals.	(3) (5)[a] (7) (9) (18)[a] (19) (21)
2	Heisenberg's principle	Students know that the Heisenberg uncertainty principle sets an intrinsic limit to the possibility to determine the particle law of motion and trajectory. They know that it also constrains measurement errors in the quantum limit. They know semiclassical atomic models but they are not able to explain the limitations of such models.	(4) (6) (10)
1	Discrete energy levels of energy and photons	Students know that classical physics cannot fully explain the interaction between matter and radiation. They qualitatively know the concept of photon and they can use Planck's constant h to compute photon energy. They also know that matter exchanges energy with radiation by emitting or absorbing photons.	(1) (13)

[a] Items that were removed from the analysis.

explain the atomic structure, and finally (iii) provide explanations for the behavior of metals, insulators, and semiconductors on the basis of the microscopic properties of matter. Our approach to the definition of the HCM levels was also informed by research in chemistry education on students' difficulties about atomic and molecular structure [28,29]. First, we took into account that students can progress across the levels of the construct map only after the clarification of well-known misconceptions. For instance, students cannot reach level 3 of the HCM if they still rely on models based on classical reasoning. Similarly, students cannot reach level 4 if they have incorrect ideas about orbitals. Second, the progression across the HCM levels can be fostered by emphasizing the simpler aspects of atomic structure in relation to the properties of solids, such as conductors and insulators [29]. With such a tight connection between chemistry and physics, students are likely to grasp the physical meaning of the targeted topics also in more advanced QM courses [8].

The HCM levels are reported in Table I. The description of each level reports the unpacked goals, specifying what students should know and what they are expected to do with this knowledge [36].

IV. METHODS

A. Design of the assessment instrument related to the HCM

To answer our first research question, we designed a suitable questionnaire linked to the hypothesized levels of the map [37–39]. The questionnaire consisted of 22 items building on existing literature [5,40–42] and our previous studies about QM [35,43]. Each item has at least two incorrect answer choices that correspond to known misconceptions, while one option was chosen as "correct" on the base of a scientifically sound idea.

During the design process, the items were iteratively mapped onto the levels of the hypothetical construct map of Table I as follows. First, to minimize issues related to different interpretation of the item wording and levels' description, three of the authors independently mapped the items, reaching an initial interrater reliability of 0.7. Different categorizations of specific items were then discussed, and a final mapping was agreed (see Table I). For instance, item 3 was initially coded as level 3 by two researchers and level 1 by the third researcher. The first two researchers argued that this item required a more sophisticated explanation of photons' emission by a material, so it could not fit level 1, which concerned only the notion of the photon and its role in the energy-matter interaction. Therefore, it was finally assigned to level 3. Similarly, item 15 was initially coded as level 3 by one researcher and level 4 by two researchers. The first researcher justified their categorization with the argument that level 3 could also include electronic levels. The other two researchers argued that level 3 entails the knowledge about emission and absorption of photons, while the item required the more advanced notion of energy level. Therefore, the item was finally assigned to level 4. Face validity of the items was then established by involving undergraduate students, university instructors and high school teachers in different phases. The reason to include university students and instructors in the process was to establish mainly the correctness of the answer choices, which required at least some degree of expertise about the targeted topics. On the other hand, high school teachers were involved in the process to assess if the items were suitable for students of *Liceo Scientifico*.

We first administered an initial version of the questionnaire to 52 undergraduate physics students who attended a method course in physics education. This pilot version featured, for each item, a 5-point Likert scale to assess perceived ambiguity ($1 =$ not at all ambiguous; $5 =$ very ambiguous). Values of average perceived ambiguity greater than 3.5 signaled items that needed to be reformulated. Average perceived ambiguity was 3.0 ± 0.6, with only four items with a score greater than 3.5. An inspection revealed that these specific items concerned properties of metals, semiconductors, and insulators, topics which the students had not yet addressed in their courses. After reformulating these items, we submitted the new version to a new group of 22 undergraduate physics students attending a laboratory course in physics education. The average perceived ambiguity was in this case 2.7 ± 0.6, with no item with a score greater than 3.5.

The revised version of the questionnaire was discussed with five academic instructors of introductory QM and structure of matter working at the university located in the same region of the schools involved in this study. All agreed that the items suitably probed students' knowledge about atomic structure, energy quantization and the role of the Planck constant h. They also manifested some criticism: three of them pointed out that the questionnaire did not probe some relevant topics, addressed in usual curriculum, such as the photoelectric effect and the blackbody radiation. However, we decided not to include questions on these specific topics since Italian high school teaching mostly focuses on their historical role in the development of QM theory [34], rather than on their conceptual aspects (that anyway would have been too advanced for the chosen theme).

Finally, before the class administration, the items were discussed with the teachers enrolled in the study (see Sec. IV D for details) and further refined to improve readability for high school students.

After class administration, the items were examined for the fit to the Rasch model (see Sec. IV E for details). We report in Appendix B the 18 items that were finally retained. We adopted a dichotomous approach to score students' responses, giving 1 point if the student picks the "correct" answer choice and 0 points for wrong or blank answers. The number of respondents for each question is also reported in Appendix B.

B. Design of instruction related to the HCM

To answer our second research question, we used the definition of the hypothesized construct map levels to inform the design of a TLS about introductory QM. TLSs are structured sequences of instructional activities, with "well-documented teaching suggestions and expected student reactions" [44] (p. 12). They have been extensively used in physics education research to introduce innovative contents and practices and to promote active construction of student scientific understanding in a variety of content areas, including relativity and QM [33,45–49]. Given their flexibility, TLSs can easily be organized in subsequent "phases" that allow the students to move from the lower levels of the construct map to the upper levels.

The TLS developed for this study is based on the following conceptual sequence: energy exchange between matter and radiation → role of Planck constant h → Heisenberg principle → atom stability → atomic energy levels → orbitals → energy band in solids. In particular, students build increasingly sophisticated explanations that begin with constructing simple models of energy exchanges between radiation and matter, then construct an explanation of the atom stability by resorting to the Heisenberg principle, and finally develop more complex models of the electronic structure in terms of orbitals, energy levels, and energy bands. The TLS largely builds on the help of students' previous knowledge from chemistry classes, with the aim of connecting the language of chemistry and physics. In particular, we help students recall that atomic orbitals are mathematical functions that describe the behavior of the electron in the atom from the probabilistic viewpoint, but we avoid the emphasis, given in chemistry classes, on orbital shapes [28].

Through the proposed activity sequence, students achieve a basic knowledge of the phenomenological quantum properties of matter and radiation. This implies that the proposed activities are more devoted to the description and conceptualization of experimental facts, rather than to the solution of quantitative problems. Furthermore, the proposed TLS aims at helping learners understand the social and technological implications of QM in their life. By analyzing familiar experimental evidence, first students can reinforce the idea that QM reasoning is required to explain everyday phenomena. Second, they can integrate the physics and chemistry approach to scientific problems. As a result, our focus is also on content that is related to the current scientific practises and technologies, with only minor emphasis on historical perspectives.

Overall, the TLS duration spans a range of 12–16 h of activity. In a forthcoming paper, we describe in detail the phases of the TLS and its further evolution. A synthesis of the version used in this study can be found in the Supplemental Material [50]. In Appendix A, we report a schematic comparison of the contents addressed in the TLS and in the textbook [34].

C. Pedagogical approach

The adopted pedagogical approach followed a guided inquiry approach [51]. In this approach, students develop their own investigation to respond to a stimulus or a guiding question posed by the teacher. We chose this approach since prior work suggests that it can foster the acquisition of enduring scientific skills [52–54]. Moreover, we had already adopted it in past proposals [55,56]. According to the chosen approach, during the small group activities, the students are asked to propose and implement experimental setups (for instance, to design a basic experimental setup to measure the voltage at which an LED lights up) or solutions to theoretical problems (for instance, asking how to drive an insulator to the conductive state) and then they are given a "critical feedback" to their solutions. Critical feedback consists of providing the students an outside point of view to the proposed solution to the posed problem, focusing on strengths and weaknesses. As common to inquiry-based approaches, the teacher hence acts as a knowledgeable person who is charge of confronting the students with the possible outcomes of their solutions [57]. During the activities, the teacher provides also a "regulatory feedback" on what the students have learned in classical electromagnetism and chemistry, so that they can critically reflect on their own acquired knowledge. Finally, our inquiry approach included the "think the opposite" strategy. In the adopted version of this pedagogical strategy, the students, working in small groups, are prompted to assess how and why their own constructed models might fail to account for the observed behavior.

D. Participants

Two parallel experimental groups from five different schools for a total of 23 intact classes in a large town in Southern Italy were involved in the study. Twelve randomly chosen classes ($N = 200$ students, average age: 18.1 ± 0.2 y) of the last year of *Liceo Scientifico* received instruction through the TLS ("TLS" group). The TLS activities were proposed during curricular hours, for a total of 14 in-person hours (about 4 weeks), part of them was spent in the laboratory (4 h) and part in the classroom. The remaining eleven classes ($N = 208$ students, average age: 18.0 ± 0.2 y) were taught using the textbook sequence of introductory QM reported in Appendix A ("textbook" group). The duration of the instruction was about 4 weeks also for this group. To match the syllabus of the TLS group, instruction of the textbook group included a 2 h lecture about semiconductors in extracurricular activities. Moreover, guided inquiry practical activities were substituted by time-equivalent nonconsecutive sessions of paper-and-pencil exercises. Each class had a different teacher. To rule out spurious effects, all the participating teachers were purposely chosen to be greatly experienced in teaching physics at the high school level (overall experience: 20 ± 6 yr; TLS group: 19.2 yr; textbook group: 20.5 yr; physics graduates $= 4$; mathematics $= 16$; biology $= 1$; engineering $= 2$), familiar with a student-oriented pedagogical approach and with at least three years of experience in teaching introductory quantum mechanics using textbooks and curriculum materials. Moreover, before participating to the study, all the participating teachers were trained for about 40 h (20 h of in presence interactive lectures and 20 h of individual homework) to implement the TLS activities, receiving notes and scripts as background materials. During the training sessions, the designed items of the assessment questionnaire, described in Sec. IV A, were discussed and, as pointed out above, refined whenever needed. The 22 items at this stage were all retained with slight modifications. Refinements consisted mainly in the reformulation of some answer choices and the shortening of the text of some items. At the beginning of the study, the TLS and textbook group students' school grades in math, physics, and chemistry were screened for homogeneity and no significant differences were found.

E. Data analysis

We used a 1D Rasch model to analyze students' responses to the questionnaire [58]. Estimates were obtained using Winsteps 3.98 [59]. In order to establish reliability of the questionnaire, we considered the following indices: person reliability, item separation, person separation, infit and outfit mean square (MNSQ), point-measure correlation and point-biserial correlation. Person reliability is analogous to classic Cronbach's alpha, with acceptable values above 0.5. Item separation indicates whether the sample was able to distinguish items according to their difficulty. Acceptable values are above 3. Person separation, which ranges from 0 to infinity, indicates the distribution of person abilities across the questionnaire's items, so it can be used to investigate if the sample can be divided into levels of increasing ability. Acceptable values are above 2. MNSQ outfit and infit can be used to investigate the goodness of the model fit. They indicate whether students' responses showed more or less randomness than expected. Acceptable MNSQ infit values are between 0.7 and 1.3. Items with MNSQ infit greater than 1.4 have a variability that is 40% greater than expected (unpredictable). Similarly, items with MNSQ infit of 0.6 have a predictability that is 40% greater than expected (too predictable). Point measure correlation measures the Rasch construct validity and should be as high as possible, with negative or very small values (around zero) indicating a wrong scoring, guessing, or unexpected randomness in the data. Point-biserial correlation is the Pearson correlation between the observations on an item and the person measures, thus it gives an indication of the discriminant power of an item. Acceptable values are greater than 0.2. Finally, we checked multidimensionality through a principal component analysis (PCA) of residuals [60] and performed a differential item functioning (DIF) analysis. Results from PCA showed that the raw variance explained by measures was 25.3% with about 14% of the raw variance explained by items. Eigenvalues of the first three contrasts were, respectively: 2.1 (7.4% of unexplained variance); 1.67 (5.7%); 2.0 (2.9%). The eigenvalue of the first contrast suggested a second latent dimension in the questionnaire, with three clusters of items. We inspected the disattenuated correlation values between the first and second cluster and between the first and third cluster, obtaining, respectively, 0.95 and 0.37. While correlations between 0.30 and 0.70 are acceptable, multidimensionality could potentially exist [60]. Hence, we performed a DIF analysis using as grouping variable the assignment to one of the experimental groups. While differences between groups should be expected, DIF analysis can identify interactions between individual items and types of persons different from the instructional variable. We found four items (Q5, Q16, Q18, Q22) with a significant DIF contrast (>0.64 and $p < 0.05$, see Ref. [59]). Since the number of items for each level would have not decreased significantly, we decided to remove these four items from the analysis. After running again the PCA of residuals, we obtained an increase in the raw variance explained by measures (27%), with 14% of variance explained by items. More interestingly, the unexplained variance in the first contrast dropped to 1.9, which is lower than the minimum value of item strength necessary to violate the unidimensionality assumption in the Rasch model. We therefore concluded that the items showing DIF concurred in forming a second unwanted dimension in the questionnaire. In the following, we report the Rasch statistics and subsequent analysis for the remaining 18 items.

To answer RQ1, we calculated the average difficulty for each of the five levels of our construct map by averaging the difficulties of the corresponding items and performed a one-way ANOVA to see if the levels' difficulties were statistically different. To answer RQ2, we first performed a t test on the abilities of the TLS and textbook groups. Then, we subdivided the sample according to their abilities on knowledge items, using as cut values the average difficulty of each level, thus obtaining six groups: in the first group, students have an ability lower than the first level difficulty, i.e., they have less than 50% probability of reaching level 1 of the construct map; in the sixth group, students have an ability that is greater than the difficulty of the fifth level of the construct map, hence they have more than 50% probability to reach level 5 of the construct map. Students in the remaining four groups have more than 50% probability to reach the $(i+1)$th ($i = 0,1,2,3$) level of the construct map. Finally, a chi-square analysis was carried out to determine if a relationship existed between the six groups obtained through the ability measures and the TLS and textbook groups.

All statistical analyses were carried out with SPSS 22.

V. RESULTS

A. Rasch item statistics

Person reliability of the 18-item questionnaire is 0.75. Initially, Cronbach's alpha was 0.81 but after removal of the four items showing DIF it decreased to 0.78. In both cases, its value is well above the 0.7 threshold usually adopted to assess the reliability of this type of questionnaires [61]. Person separation is 1.73, which suggests that the instrument is able to discriminate the students in the sample in at least two groups according to their abilities. Item reliability is 0.97, while item separation is 5.74. Both can be considered satisfactory. Mean difficulty, infit and outfit statistics, and point-measure correlation for the retained 18 items are reported in Table II. All items have acceptable infit and outfit MNSQ (namely, between 0.7 and 1.3). Point measure correlation is acceptable (i.e., greater than 0.3) for all items. Similarly, also point-biserial correlation is acceptable for all items (i.e., greater than 0.2).

B. Validation and revision of the construct map levels

Figure 1 reports the Wright map of our dataset. The map shows both the TLS and textbook group students' abilities on the two left-hand panels and the estimated item difficulty on the right-hand side. As usual in Rasch

TABLE II. Rasch analysis statistics of the questionnaire.

Item	Difficulty (logit)	Model SE.	Infit MNSQ	Infit Zstd	Outfit MNSQ	Outfit Zstd	Point-measure	Point-biserial
1	−1.33	0.12	0.8901	−2.1491	0.7982	−2.2692	0.55	0.45
2	1.26	0.13	1.0479	0.6810	1.0628	0.4611	0.35	0.26
3	−0.10	0.11	0.9845	−0.3090	0.9725	−0.3590	0.48	0.39
4	−0.25	0.11	0.9059	−2.0691	0.8974	−1.5091	0.54	0.46
6	−0.95	0.11	1.0305	0.6510	1.0567	0.7411	0.45	0.33
7	0.35	0.12	1.0696	1.3211	1.0847	0.9711	0.40	0.30
8	0.63	0.12	1.0828	1.4411	1.1338	1.2811	0.37	0.28
9	0.26	0.12	1.0998	1.9111	1.2046	2.3112	0.38	0.27
10	−0.23	0.11	0.9580	−0.8990	0.9253	−1.0791	0.51	0.41
11	0.92	0.13	0.9788	−0.3190	0.9479	−0.3891	0.43	0.35
12	0.92	0.13	1.0274	0.4510	1.2765	2.1313	0.37	0.29
13	−1.14	0.12	0.7894	−4.5492	0.7245	−3.5693	0.62	0.53
14	0.49	0.12	1.1133	2.0311	1.1988	2.0012	0.36	0.26
15	−0.22	0.11	0.9560	−0.9390	1.0400	0.5910	0.49	0.41
17	−0.13	0.11	1.0199	0.4310	0.9878	−0.1490	0.46	0.36
19	−0.10	0.11	0.8819	−2.5791	0.8308	−2.4692	0.56	0.48
20	0.09	0.11	1.0271	0.5610	1.0452	0.6010	0.44	0.35
21	−0.45	0.11	1.0577	1.2511	1.1868	2.6012	0.42	0.32

analysis, the mean difficulty of the items is set to zero logits. The overall mean ability of the students is −0.45 ± 1.22 (st.dev) logit, which suggests that the QM items were difficult for the sample as a whole. The average difficulty of each of the five levels is also reported in Fig. 1. We note by inspection that on average, items corresponding to the level *molecules and condensed matter* (level 5) are more difficult than items corresponding to *orbitals, wave function, and probability in QM* (level 4) and so on. There is a significant effect of item difficulty on the HCM levels ($F = 7.615$, $df = 4$, $p = 0.002$, partial $\eta^2 = 0.70$). Table III reports the planned contrasts between each of the five levels. Only five out of the 10 contrasts are statistically significant, in particular those between the lowest and upper levels, while there are not significant differences among the two upper levels.

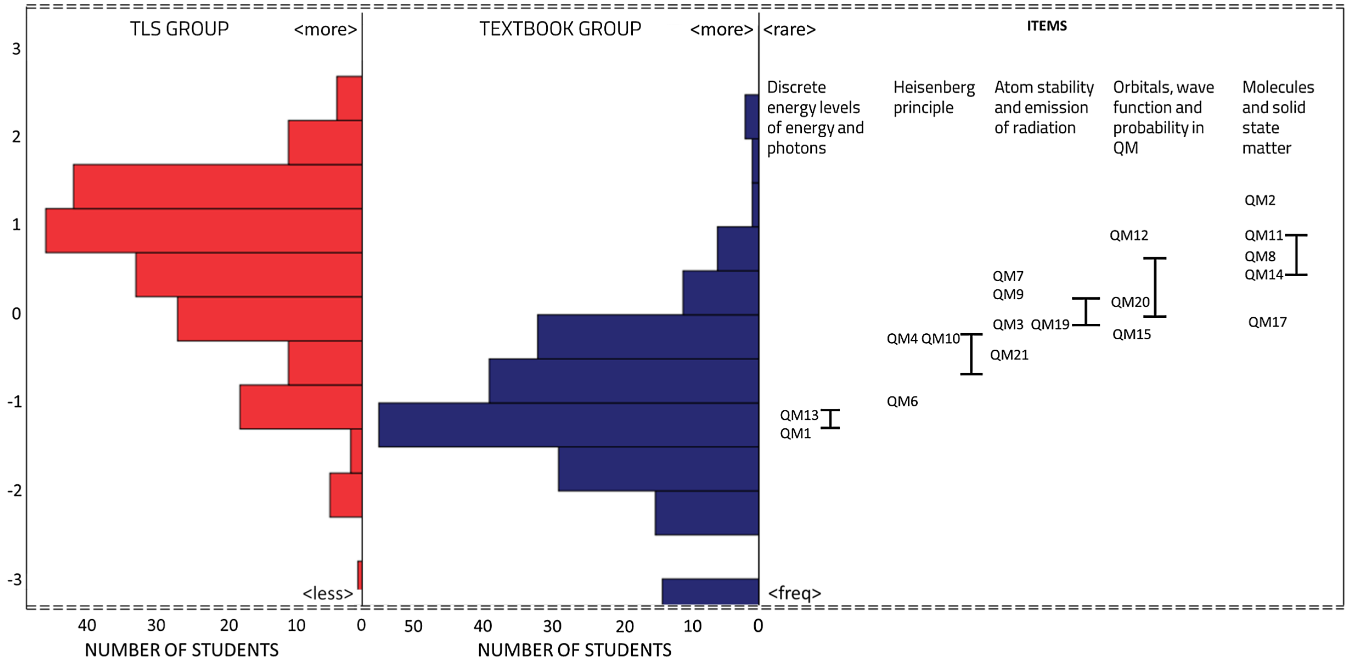

FIG. 1. Wright map of the questionnaire used in this study. Items are labeled as QM1, QM2, etc. Removed items (QM5, QM16, QM18, and QM 22) are not displayed. Items are grouped according to the levels of the construct map in Table I. Estimated difficulty of each of the five levels is indicated by whiskers. The error bar represents 1 standard deviation.

TABLE III. Contrasts between the average difficulty of the construct map levels.

Contrast	Average difficulty of the contrasting levels (logit)		t	df	p
Level 1–Level 2	-1.23 ± 0.09	-0.48 ± 0.24	1.879	13	0.083
Level 1–Level 3	-1.23 ± 0.09	-0.00 ± 0.14	3.318	13	0.006^{**}
Level 1–Level 4	-1.23 ± 0.09	0.26 ± 0.34	3.714	13	0.003^{**}
Level 1–Level 5	-1.23 ± 0.09	0.63 ± 0.23	5.054	13	0.000^{**}
Level 2–Level 3	-0.48 ± 0.24	-0.00 ± 0.14	1.452	13	0.170
Level 2–Level 4	-0.48 ± 0.24	0.26 ± 0.34	2.050	13	0.061
Level 2–Level 5	-0.48 ± 0.24	0.63 ± 0.23	3.441	13	0.004^{**}
Level 3–Level 4	-0.00 ± 0.14	0.26 ± 0.34	0.841	13	0.416
Level 3–Level 5	-0.00 ± 0.14	0.63 ± 0.23	2.297	13	0.039^{*}
Level 4–Level 5	0.26 ± 0.34	0.63 ± 0.23	1.148	13	0.272

*significant at 0.05 level.
**significant at 0.01 level.

To further investigate this issue, we performed a *post hoc* honestly significant difference (HSD) Tukey test to identify subsets of levels, whose corresponding items have similar mean difficulty. Results show that three partially overlapping groups of levels can be identified: subgroup 1, which includes levels 1 and 2 ($p = 0.251$, average difficulty $= -0.86$); subgroup 2, which includes levels 2 up to 4 ($p = 0.271$, average difficulty $= -0.07$); subgroup 3, which includes levels 3 up to 5 ($p = 0.396$, average difficulty $= +0.30$). Then, we performed three new contrasts: level 1 of the HCM against subgroup 2 ($t = 3.402$; $df = 13$; $p = 0.005$); level 1 against level 5 ($t = 5.054$; $df = 13$; $p < 10^{-4}$); subgroup 2 (level 2 up to 4) against level 5 ($t = 2.942$, $df = 13$; $p = 0.011$). All new contrasts resulted significant at least at the $p = 0.01$ level. On such a basis, we finally opted for a three-level construct map, characterized by a lower level, which corresponds to level 1 of the HMC; an intermediate level, which corresponds to levels 2–4, and an upper level, which corresponds to level 5 of the HCM. The three revised levels are briefly summarized in Table IV.

C. TLS and textbook group performances

The mean ability of the TLS group is 0.27 logit ± 0.99 (st. dev.), while for the textbook group it is −1.13 logit ± 1.01 (st. dev). The difference is statistically significant ($t = 14.082$, $df = 406$, $p < 10^{-4}$). Table V reports the distribution of TLS and textbook group students across the five levels of the hypothesized construct map, using as cut values for abilities the average difficulty of the levels. A chi-square analysis shows that the number of TLS group students at the first level is significantly smaller than that of the textbook students. Similarly, TLS students at the fourth and fifth level of the construct map is significantly greater than that of textbook group students ($\chi^2 = 165.921$, $df = 5$, $p < 10^{-4}$; Cramer's $V = 0.64$, $p < 10^{-4}$).

TABLE IV. Revision of the initial HCM.

Description of the revised HCM levels
Upper level
Students grasp the concepts of chemical bond and of molecular orbital. They distinguish between conductors and insulators in terms of energy bands. They know that only the conduction-band electrons contribute to the electrical current in metals and that the number of charge carriers can be changed in semiconductors. They can qualitatively discuss how LED and solar cells work.
Intermediate level
Students know that the Heisenberg principle sets an intrinsic limit to the possibility to determine the particle law of motion and trajectory and that it also constrains measurement errors in the quantum limit. They are also able to qualitatively explain the atom stability by using the uncertainty principle. They know the electronic structure of atoms in terms of energy levels and can compute the energy of emitted or absorbed photons in terms of levels difference. They are acquainted with the probabilistic interpretation of atomic orbitals.
Lower level
Students know that classical physics cannot fully explain the interaction between matter and radiation. They can use Planck's constant h to compute photon energy. They qualitatively know that matter atoms exchange energy with radiation by emitting and absorbing photons.

TABLE V. Distribution of TLS and textbook group students across the levels of the initial HCM (Table I).

		% of students at the ith level		
Level	Mean difficulty (logit)	Whole sample ($N = 408$)	TLS group ($N = 200$)	Textbook group ($N = 208$)
0	···	26.0[a]	9.5[a]	41.8[a]
1	−1.23	27.5[b]	12.5[b]	41.8[b]
2	−0.48	13.0[b]	17.5[b]	8.7[b]
3	0.00	5.9[b]	9.0[b]	2.9[b]
4	+0.26	7.4[b]	14.0[b]	1.0[b]
5	+0.63	20.3[c]	37.5[c]	3.8[c]

[a] Students with ability lower than first level difficulty.
[b] students with ability between ith and $(i+1)$th level difficulty.
[c] students with ability greater than level 5 difficulty.

TABLE VI. Distribution of TLS and textbook group students across the levels of the revised HCM (Table IV).

		% of students at ith level		
Level	Mean difficulty (logit)	Whole sample ($N = 408$)	TLS group ($N = 200$)	Textbook group ($N = 208$)
0	···	26.0[a]	9.5[a]	41.8[a]
Lower	−1.23	35.5[b]	22.5[b]	48.1[b]
Intermediate	−0.07	18.1[c]	30.5[c]	6.3[c]
Upper	+0.63	20.3[d]	37.5[d]	3.8[d]

[a] Students with ability lower than 1st level difficulty.
[b] students with ability between 1st level and intermediate level difficulty.
[c] students with ability between intermediate level and upper level difficulty.
[d] students with ability greater than upper level difficulty.

As shown in Table VI, also using the condensed three-level construct map the association between the levels and the experimental groups is significant ($\chi^2 = 149.605$, $df = 3$, $p < 10^{-4}$; Cramer's $V = 0.60$, $p < 10^{-4}$).

VI. DISCUSSION

The main aim of this paper was to contribute toward the organization of a core QM curriculum of introductory topics—the structure of matter and the behavior of atoms and photons [6]—in a construct map form. We started from a hypothetical construct map (HCM) closely linked to curriculum that describes students' increasing understanding of such introductory QM topics, and then operationalized the envisioned progression into a teaching sequence that develops along a short-medium timescale in the classroom (about 14 h). We adopted a two-experimental groups research design with 23 classes in order to collect evidence for the validity of the HCM and for the effectiveness of the proposed activities in comparison with the usual Italian textbook-based teaching. In the following, we discuss in more detail to what extent the collected evidence supports the validity of the hypothetical construct map reported in Table I (RQ1) and the effectiveness of the activities informed by the construct map levels (RQ2).

A. To what extent does the hypothesized construct map describe students' reasoning about introductory QM topics?

Analysis of data substantially confirms the ordinal sequence of the initial HMC levels, i.e., that the lower levels are easier to achieve than upper levels, but does not support the hypothesized five level fine structure. We found that the revised HCM, organized in three levels, better describes students' progression from the knowledge of simple quantization models (difficulty $= -1.23$ logit) to the qualitative use of more complex models as energy bands of solids (average difficulty of the upper level $= +0.63$ logit).

The new, intermediate level of the revised HCM condenses levels 2, 3, and 4 of the initial HCM (average difficulty $= -0.07$ logit) and it is characterized by the knowledge of the Heisenberg principle and its applications, the atomic structure, and orbitals (average difficulty of the second, third, and fourth level $= -0.07$ logit).

As a consequence of the collapse of levels 2–4, the grain size of the revised HCM has increased. Now, the shift in ability needed to move from the lower level to the intermediate level (about 1.1 logit) is almost twice as much than that required to move from the intermediate level to the upper one (about 0.6 logit).

In our view, the ability shift to reach the intermediate level is greater than the one needed to reach the upper level for at least three reasons.

First, it is at the intermediate level that students develop a more complex conception of QM: they become acquainted with the intrinsic limitations to the description of particle motion; they can interpret the processes of emission and absorption in terms of energy conservation; they are able to describe orbitals in terms of probability distributions.

Second, the students have to reinterpret key concepts of classical physics (e.g., energy, wave, particle) in terms of the new QM framework that entails the discretization of energy and the dual nature of matter and radiation [8,9]. They also revisit the concept of measurement and of measurement errors, by introducing a description that is far from everyday life.

Third, the students should also connect these newly acquired concepts to what they have previously learned in chemistry, e.g., orbitals, energy levels, electronic configuration. Building the latter connection can be often very difficult given the current instructional practice in Italy, because physics and chemistry teaching at the secondary school level differs from two main viewpoints: they are taught by different teachers with different backgrounds (physics teachers are often graduated in math, while chemistry teachers graduated in biology); physics and chemistry vertical curricula are not aligned (e.g., the atomic structure is addressed in chemistry classes at least two years before it is taught in the physics course). Indirectly, our results prove that a harmonization and integration of physics and chemistry teaching, a process that is rarely achieved even at the university level, would be greatly beneficial.

Similar conclusions can be drawn when observing that the mean difficulties of the items related to levels 4 and 5 of the initial HCM do not significantly differ. This means that once students have acquired a sound knowledge of the probability-based model of an atom, which connects the notion of orbitals from chemistry and the qualitative knowledge of the electron behavior from physics, they can reach a satisfactory knowledge of the physical mechanisms underlying materials' behavior with a relatively small ability shift. For instance, to correctly answer item 2 (the most difficult question of the questionnaire with difficulty $= +1.26$ logit), the students should reject the classical model of conduction (answer choices A and B) in favor of the most advanced model of the energy band in solids. The capability to distinguish between the classical and QM model of conduction can be fostered by the above connection between chemistry and physics. This evidence suggests that a stronger connection between chemistry and physics may help students not only overcome the idea that QM is "too theoretical" [62] and confined to the microscopic world, but, also, acquire awareness that QM allows interpreting the macroscopic behavior of everyday devices [63,64].

B. To what extent are the instructional activities informed by the hypothesized construct map effective in improving students' knowledge about introductory QM topics?

We found that students involved in the TLS activities outperformed the students that were taught through textbook instruction. From Table V, we note that about 40% of the textbook group students have less than 50% probability to reach the first level of the construct map, while this percentage is less than 10% for the TLS group. Conversely, more than one third of the TLS group students have more than 50% of probability to reach the upper level, while this is the case for only 4% of the textbook group students. While we acknowledge that such results may also be related to different teaching skills across the 23 classes—despite our efforts to minimize possible effects external to the TLS through the professional development course—we believe that our evidence indicates the potential of the proposed activities, which were informed by the construct map levels definition.

In particular, TLS group students seem to have connected more effectively the concepts addressed at the first level with those addressed at the intermediate level of the revised construct map. On the contrary, the textbook group students had more difficulties in connecting the concepts underlying the levels of the construct map. For instance, looking at Table V, textbook students had difficulty in shifting from quantization to Heisenberg principle (only 9% of these students has an ability equal to the second level difficulty) and in connecting both ideas to justify the atom's stability (level 3).

In our view, this result may be due to the typical textbook presentation of phenomena and concepts. Such presentation is sequential (for instance, photoelectric effect, discretization of energy, dual nature of matter) without a clear connecting conceptual path. Differently, in our approach, the proposed conceptual chain likely helped the students improve their explanations of the behavior of quantum systems both at the microscopic level (hydrogen atom) and at the macroscopic level (everyday materials). This result is similar to that obtained in Ref. [33], where students were able to quantitatively describe emission and absorption of radiation in terms of energy diagrams. To be able to build such connections is at the core of modeling skills, which are deemed as very relevant for scientific literacy [1,2].

Another possible advantage of our TLS in comparison to the traditional sequence is the low intensity of mathematical formalism, except for basic relationships such as $E = h\nu$ or $\Delta x \Delta p \sim h$, which may have further reduced the cognitive load required of students. This benefit is especially evident when dealing with concepts such as orbitals: while controlled instruction defined the orbitals focusing on a simplified expression of the wave function, our activities first guided the students to the need to introduce the orbitals (as a new representation of the atom that takes into account energy levels) and only later the wave function with no mathematical details.

Similarly, from the analysis of the percentage of the answer choices of the 18 retained items, we can also infer that the emphasis in the textbook-based instruction on manipulations of formulas have likely lead teachers of the textbook group to not address well-known misconceptions related to the target topics.

The TLS proposed in this study adds to previous approaches aimed at innovating the physics curriculum in the Italian educational context [48,65]. For instance, in Ref. [48] the authors exploit the Feynman approach to address wave particle duality and the role of probability in QM. In Ref. [65] the authors propose an approach focused on the axiomatic definition of probability and Dirac formalism. To make sense of QM, both TLSs promote conceptual viewpoints that are completely new to students, i.e., very different from those previously adopted. Differently, in our TLS the students are helped to make sense of the microscopic world by deepening the previous knowledge from chemistry classes.

A second difference between our TLS and previous efforts in the Italian educational context [7] relies on the connection with technological applications of QM. Our prior work suggests that an approach based on the integration between science and technology may be beneficial for students to build such relationship [66]. Accordingly, we chose in our TLS to guide the students to make sense of the behavior of everyday devices (e.g., an LED), using the quantum viewpoint. To this aim, as suggested by general results in cognitive sciences [67,68], the TLS activities were driven by questions that challenged students with competing explanations that linked the microscopic behavior of matter to the macroscopic phenomena.

VII. IMPLICATIONS AND FUTURE DIRECTIONS

Overall, this study contributes to research in the field by increasing our understanding of how to describe and support students in developing sound knowledge about introductory quantum physics concepts in high school physics classes. This study has also three main implications.

First, the revised three-level construct map reliably describes high school students progression when learning introductory QM topics, starting from energy discretization (lower level), and landing, through increasingly complex atomic models (intermediate level) to the energy band model of solids (upper level). As such, the revised HCM can be included in the wider learning progression we developed in a previous study [35] to provide a more detailed description of how students progress in their understanding of QM from high school to upper-level physics university courses. As an example, research is warranted to investigate whether students at the lower level of the revised HCM experience more difficulties about advanced QM concepts (e.g., the Schrödinger equation, the concept of measurement or superposition of states) than students at intermediate and upper level, as indirectly suggested by prior work [69–71]. Similarly, research would be much needed to investigate how a strict calculation-based discourse in QM [72] impacts on students initially at different levels of the HCM. Finally, research is also needed to explore ways to connect our construct map with existing learning progressions about the structure of matter [17,37,73].

The second implication concerns the used assessment tool. In the present study, we validated the HCM through an assessment tool whose items are strictly linked to the proposed levels. To improve its reliability, the Rasch 1D model allowed us not only to detect malfunctioning items that showed potential DIF, but also to safely compare the average difficulty of items corresponding to different levels of the HCM. However, as any construct map, ours is not a static outcome, despite a substantial confirmation of the hypothesized levels, but it should be subjected to a necessary revision of the levels and of the corresponding items according to the collected evidence [74–76]. For instance, while some of the initially designed items have been already removed due to scarce statistical robustness, further revisions of the items may be in order. We plan to revise the questionnaire and administer it to a different sample so to improve the validity and reliability of the instrument.

Third, the proposed TLS seems to be more effective than traditional textbook-based instruction in helping students progress through the levels of the construct map. Clearly, our TLS is strictly dependent on the Italian educational context. However, in view of the chosen contents, which can also be found in other curricula [6,10] and of the validation method carried out, we expect a wider applicability beyond the Italian case. Similarly, more research is needed to investigate whether the emphasis of the TLS activities on the relationship between the scientific concepts and the students' everyday experience about metals and semiconductor behavior may also improve the students' attitude toward scientific research and raise their awareness of the relevance of QM in their lives.

Finally, it would be worth to investigating whether students' ability to assess their own knowledge of QM concepts—namely, their confidence in QM—is affected by a transformative didactic intervention like the proposed TLS. We address this issue in a companion paper [77].

VIII. LIMITATIONS

A number of limitations must be acknowledged when interpreting the results.

First, we were not able to administer a baseline test to independently verify that all the involved classes were at the same proficiency level concerning reading capability and general physics and chemistry knowledge. Second, an additional reasoning tier to our questionnaire could have provided more accurate information about students' performance. Third, our results are strictly dependent on the chosen mapping of the items of the questionnaire onto the levels of the construct map. As pointed out above, there were some discrepancies between the proposed categorizations, which were resolved by discussions among the authors. Hence, while a further refinement of the items' wording could result in a slightly different items' mapping, the number of analyzed items for each level is large enough to ensure a reliable measure of the difficulty of each level of the HCM. Finally, it is worth noting that the collapse of levels 2, 3, and 4 of the initial HCM into the intermediate one stemmed from the statistical analysis of the data collected with the sample involved in this study. Therefore, it is possible that further work on new data may lead to a finer structure of the HCM more aligned to the initial one.

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APPENDIX A: COMPARISON BETWEEN TOPICS COVERED IN THE TLS AND TEXTBOOK INSTRUCTION

In Appendix A we report the list of topics addressed in the proposed TLS and in a typical Italian textbook [34]. In Appendix B we report the questionnaire used in the study.

Proposed TLS	Textbook
1) Applications of QM (e.g., LED)	1) Historical presentation of the blackbody radiation
2) The concept of photon	2) The photoelectric effect and the concept of photon
3) The interaction between matter and radiation	3) The Bohr model and the discrete energy levels of hydrogen
4) The concept of mechanical action	4) Absorption or emission of photons
5) The uncertainty principle	5) Wave-particle behavior of matter
6) The atom stability	6) Electron diffraction by a double slit
7) The electronic structure of atoms (energy levels)	7) The uncertainty principle
8) Atomic orbitals and probability distributions	8) The wave function and the probabilistic interpretation of the concept of orbital
9) Molecular orbitals	9) Many-electron atoms and electronic behavior of materials
10) Energy band model of solids: metals, insulators, and semiconductors	10) Applications of QM (lasers and diodes)

APPENDIX B: INSTRUMENT USED IN THIS STUDY

Administration instructions

Time allotted to complete the questionnaire: 30 min

Tick only the answer choice you think it is correct

For each item, please indicate how confident you feel in your answer

Note: correct answer choices in **bold** face. Number of respondents is reported for each question. Percentages for each answer choice are reported in round brackets

Level 1—Discrete energy levels and photons

Item 1. The Planck's constant ($N = 383$):

A) should be added to the other universal constants when we use the Newton's laws for electrons (13%)

B) **allows to explain how matter emits radiation (71%)**

C) is a phenomenological constant (11%)

D) has the same physical dimensions of electrical power (5%)

Item 13. The energy of a photon ($N = 384$):

A) **depends on the color of the light (67%)**

B) at a given wavelength, depends on the type of source (9%)

C) depends on the intensity of the source (17%)

D) is very small (7%)

Level 2—Heisenberg's principle

Item 4. According to the Heisenberg uncertainty principle ($N = 400$):

A) **if the position of a quantum particle is known with high precision, its velocity is completely undefined (47%)**

B) it is impossible to measure with infinite precision any physical quantity (7%)

C) classical mechanics does not provide an experimental apparatus that measures at the same time position and velocity of an electron with sufficient precision (28%)

D) the measurement errors on a particle position and velocity are inversely proportional (18%)

Item 6. According to the Heisenberg uncertainty principle ($N = 394$):

A) the measurement errors do not depend on the experimental apparatus, but on a fundamental law of nature (14%)

B) **the better we know the velocity of an electron, the worse we know its position (62%)**

C) the measurement errors only depend on the experimental apparatus (5%)

D) it is impossible to measure both the position and the velocity of an electron (19%)

Item 10. The product $\Delta x\ \Delta p$, where x and p are, respectively, the electron position and momentum ($N = 358$):

A) **is of the same order of the Planck constant h in all microscopic processes (53%)**

B) is of the same order of the Planck constant h only in the emission or absorption of one photon (23%)
C) is a useful tool to determine the momentum as a function of position (17%)
D) is constant in the presence of conservative forces only (7%)

Level 3—Atom stability and emission of radiation

Item 3. How would you describe light emission from a white-hot bulb? In this process…($N = 384$)
A) the emitted photons change their wavelength due to the high temperature of the filament (32%)
B) photons of different wavelength are emitted (46%)
C) photons are emitted by the current flowing through the bulb filament (11%)
D) electromagnetic waves are emitted by oscillating charges in the hot filament (11%)

Item 7. Two light pulses of the same color carry the same energy E_{tot}. Hence ($N = 383$):
A) the energy E_{tot} only depends on the light frequency (43%)
B) the two pulses have the same number of photons (38%)
C) the energy of the photons of each pulse depends on the intensity of the electromagnetic radiation (12%)
D) the longest pulse has more photons (7%)

Item 9. Why can't classical physics explain the stability of the hydrogen atom? ($N = 388$)
A) While rotating around the positively charged nucleus, the electron is attracted by the electrostatic force and dissipates energy (22%)
B) In classical physics there is not the uncertainty principle and, hence, it is not possible to describe the trajectory of the electron (24%)
C) Classical physics concerns only massive particles, much heavier than the electron and the nucleus (15%)
D) While rotating around the positively charged nucleus, the electron emits radiation and dissipates energy (39%)

Item 19. When an atom emits a photon… ($N = 378$)
A) the wavelength of the emitted photon is always similar to the dimension of the atom (22%)
B) it is in an excited state (47%)
C) the photon carries the energy of an electron (18%)
D) it has an electron that accelerates, because a moving charge emits e.m. radiation (13%)

Item 21. When we calculate the energy of an electron in an atom, we can neglect the gravitational force, since ($N = 392$):
A) the gravitational force is balanced by the centrifugal force due to the electron rotation around the nucleus (20%)
B) the electron has a much smaller mass than the nucleus (15%)
C) the gravitational force is many orders of magnitude smaller than the Coulomb force (52%)
D) the nuclear energy is large enough to attract the electrons and consequently the gravitational force is negligible (13%)

Level 4—Orbitals, wave function and probability in QM

Item 12. Concerning orbitals, we can safely claim that ($N = 375$)
A) orbitals $1s$, $2s$, $2p$ have equal dimensions for all atoms (29%)
B) have microscopic dimensions (34%)
C) can be described by classical mechanics only if they have macroscopic dimensions (9%)
D) in the presence of a chemical bond, orbitals can extend over different atoms (28%)

Item 15. In any atom ($N = 381$):
A) the orbitals are the orbits of electrons that move with constant energy around the nucleus (20%)
B) each orbital corresponds to a constant value of the electron mechanical energy (49%)
C) each orbital corresponds to a constant value of the electron kinetic energy (17%)
D) an orbital is a region of the atom where the electrons rotate around the nucleus (14%)

Item 20. The wave function of an electron allows to calculate ($N = 377$):
A) the measurement error on the electron's momentum (13%)
B) the most likely position of the electron, but it gives no information on the momentum (31%)
C) the momentum of the electron vs. time (13%)
D) the probability of measuring a given value of the momentum (43%)

Level 5—Molecules and solid state matter

Item 2. Consider a metal as Cu. Which of the following statements regarding its properties is the most correct one ($N = 386$)?
A) the electrical resistivity is due to the collisions of the conduction electrons against the ions (33%)
B) between two consecutive hits, a conduction electron has constant acceleration (24%)
C) the orbitals of conduction electrons have macroscopic dimensions (22%)
D) the electrical current flowing in a wire is due to the motion of all the electrons (21%)

Item 8. The average atomic radius ($N = 383$):
A) is determined by the orbitals of valence electrons (32%)
B) is the same for both atoms and ions of each element (10%)
C) is proportional to the atomic number (36%)
D) is the radius of the orbit of valence electrons (22%)

Item 11. When an external negative charge is placed close to a metal ($N = 389$):

A) all the electrons are displaced to the opposite side of the metal (25%)
B) **the innermost orbitals of the metal remain unchanged (27%)**
C) some of the electrons of each orbital are displaced to the opposite side of the metal (21%)
D) the positive charges of the metal move in the direction of the external charge (27%)

Item 14. An insulating material ($N = 377$):
A) **when exposed to photons of suitable energy, it may carry an electrical current (36%)**
B) it has no conduction band (40%)
C) when exposed to an external electric field, its charges move following the lines of the field (16%)
D) its electrons cannot move (8%)

Item 17. In a semiconductor… ($N = 373$)
A) the particles with positive charge and the particles with negative charge move in opposite directions (10%)
B) **the number of charges that move freely within the material can be modified (48%)**
C) is partially a conductor and partially an insulator (23%)
D) the electric current can flow in only one direction (19%)

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