Students' flexible use of ontologies and the value of tentative reasoning: Examples of conceptual understanding in three canonical topics of quantum mechanics

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As part of a research study on student reasoning in quantum mechanics, we examine students' use of ontologies, or the way students' categorically organize entities they are reasoning about. In analyzing three episodes of focus group discussions with modern physics students, we present evidence of the dynamic nature of ontologies, and refine prior theoretical frameworks for thinking about dynamic ontologies. We find that in a given reasoning episode ontologies can be dynamic in *construction* (referring to when the reasoner constructs the ontologies) or *application* (referring to which ontologies are applied in a given reasoning episode). In our data, we see instances of students flexibly switching back and forth between parallel stable structures as well as constructing and negotiating new ontologies in the moment. Methodologically, we use a collective conceptual blending framework as an analytic tool for capturing student reasoning in groups. In this research, we value the messiness of student reasoning and argue that reasoning in a tentative manner can be productive for students learning quantum mechanics. As such, we shift away from a binary view of student learning which sees students as either having the correct answer or not.

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I. INTRODUCTION

The fact that learning and teaching quantum mechanics (QM) is difficult has been known for a long time. Rather than just accept this or focus on why QM is difficult, we examine how students negotiate meaning of difficult QM topics in productive ways. That is, rather than focus on student difficulties we ask (in line with Refs. [2,3]) what kinds of sophisticated reasoning do the students engage in? We investigate students' knowledge structures in QM, and focus on a fine-grained analysis of student reasoning. In improving quantum physics education, our goal is not only to promote and document pre to post shifts on conceptual surveys (e.g., QMCS [4] or QMCA [5]), but to value and encourage students' engagement in scientific discourse and reasoning about complex interpretive phenomena. Developing this capacity for meaning making and discussions of interpretation are part of our explicit goals of instruction. In understanding what our students are capable of within the current system, we can value what they are doing and ultimately build on curricula supportive of such ends. We seek to understand student capacity to further advance student capability.

While prior work has examined student conceptual mastery and interpretive skills by documenting pre-post shifts [6,7], we seek to extend such outcomes and to better understand how students organize and understand ideas in QM. Consider the following prompt for students: *When not being observed, an electron in an atom still exists at a definite (but unknown) position at each moment in time*. Students from a sophomore-level modern physics course are asked to respond to this statement on a five-point Likert scale ranging from strongly disagree to strongly agree. This particular question gives us an idea of how students are (or are not) thinking about indeterminacy in the context of atoms. If a student agrees with the statement, they might say "the electron does exist at a certain location but we do not know where it is until we measure it," which we would classify as a hidden variable interpretation [8]. If a student disagrees with this statement, they might say "the electron is not in a definite position until measured" [11]. These responses provide valuable information about students' reasoning and interpretation of quantum phenomena. Ultimately, we would like our students to shift toward disagree after a semester of modern physics. We value asking these questions of our students, have incorporated these questions of interpretation into our learning objectives, and see these pre-post results as meaningful for our course and our research [6,7]. However, we must consider the limitations of this pre-post multiple choice survey

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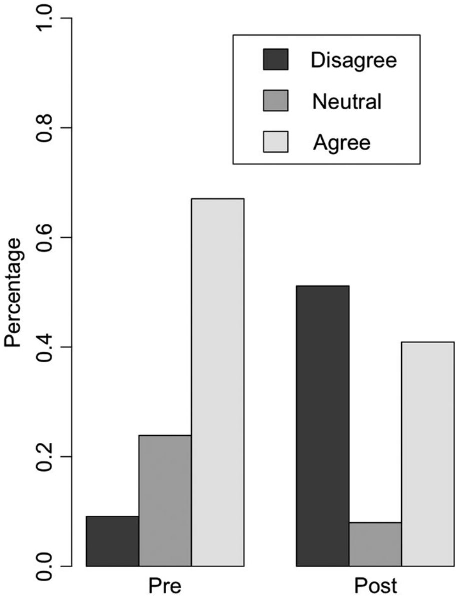

FIG. 1. Sophomore level modern physics class ($N = 88$) responses to *When not being observed, an electron in an atom still exists at a definite (but unknown) position at each moment in time.* (Following the work of Baily [6,7]) Highly statistically significant ($p \ll 0.001$) shifts from pre- to post-survey with the Bhapkar test. These data were collected in the same course and semester as the focus group data presented in Sec. V.

approach. Figure 1 shows the results from this question during one semester of a modern physics course. Although we see a statistically significant shift from the beginning to end of the semester, in a direction that we define as "more sophisticated," these coarse-grained, aggregate survey data do not tell us much about students' actual reasoning. This approach to understanding student reasoning, while a good tool for triangulation, is limited and when used alone reinforces a binary view of student learning, characterizing student responses as either correct or incorrect, with no nuance in between. One interpretation of the famous Feynman quote:

> *"I think I can safely say that nobody understands quantum mechanics."—Richard P. Feynman [1].*

is that there is a monolithic understanding of quantum physics, aligning with a binary "get it or don't" view. We find value in taking a broader look at students' learning experiences and conceptual development [12,13], and in emphasizing the productive nature of students' responses even when their ideas may be faulty or scientifically incorrect in some ways [2]. We find that student reasoning is nuanced and it may be the case that, on a given multiple choice question, students are answering favorably for the wrong reasons or that they may not exhibit consistent views across conceptual, epistemological, or social contexts [14,15]. Focused on the goal of obtaining a more nuanced description of student reasoning, we shift away from this binary view and seek a finer-grained analysis of students' reasoning about these types of questions.

As a part of the present research study, we are particularly focused on the nature of ontologies. Ontologies describe our categorization of the kinds of entities in the world—grouping them categorically by fundamental properties or characteristics. Ontological categorization is important for understanding physics concepts [16,17], and understanding students' ontologies is an essential component of understanding student reasoning. In physics, for example, energy can be conceptualized (by both experts and novices) as a quasimaterial substance, a stimulus to action, or a vertical location [18,19]. These can be considered as three different ontologies for energy.

Historically there has been a debate about the nature of ontological structures and the ways in which a learner is able to move between ontologies. One line of thought suggests a stable view of ontological structure including a notion of ontological correctness [16,20–22], and another framework posits a more flexible ontological structure, describing dynamic movement between ontologies on the part of both students and experts [23,24]. There is mounting evidence [6,19,25] for the latter (dynamic) perspective. We provide further evidence, arguing from our data that underlying ontological structures and movements between them can be flexible. Pre-post results like the those in Fig. 1 can imply that there are unambiguous [26] before and after states of the students' ontological commitment, and they are either correct or incorrect. Using this measure alone does not acknowledge the nuance of student reasoning, and thus can reinforce a static or fixed view of ontologies.

This paper makes two main theoretical contributions. First, we provide evidence of the dynamic nature of ontological structures, and go further to identify *different types* of dynamic ontologies. In our data, we see students flexibly switching back and forth between parallel stable structures as well as constructing and negotiating new ontologies in the moment. Our new framework of dynamic ontologies thus includes some stable or robust notions that can be moved between flexibly, in addition to the ability for reasoners to construct and refine ontologies on the fly.

Next, we argue that it can be productive for students to reason in a tentative or messy way when learning and grappling with quantum ideas. In making this argument, we shift away from a binary view of student learning [2]. There are many within the physics education research field who study and value the messiness of student reasoning (see Refs. [3,23,27–30]) and who argue for moving beyond reductionist metrics [2] because we know that nuance supports efforts for things like understanding student reasoning and building inclusive environments [31]. We contribute to this framing and philosophical commitment that values the complexities of student reasoning, and we bring it to a different area of research—modern physics and quantum mechanics. In this paper, we refer to the "messiness" of student reasoning as having two parts: (i) students' flexible use of ontologies (whether that means moving back and forth between parallel structures trying to test when each is appropriate to use, or constructing and negotiating ontologies in the moment, or a little bit of both, or something different all together), and (ii) students' tentativeness around ideas they contribute to group conversations (this comes along with negotiating in the moment and flexibly playing with ontological structures). We argue that this messiness can be productive for students. Some researchers have addressed the notion of productivity even when students' ideas are not canonically correct [28,32,33]; Scherr and Robertson define an idea as productive if it "supports, initiates, or sustains progress" Ref. [28], (p. 3). Our notion of productivity is consistent with this definition, but also includes the existence of discourse and practices similar to those of professional physicists (e.g., reasoning across ontological categories, or flexibly playing with ideas they are unsure about).

Methodologically, we develop a system of studying the nuances of collective reasoning, thereby contributing to the research community that values the messiness and complexities of student reasoning. We employ conceptual blending [34] as a tool for describing student reasoning and elucidating the dynamics of ontological negotiation. Although conceptual blending is a theory of cognition, traditionally referring to what goes on in an individual's mind, we demonstrate how the framework can be used to model and analyze *collective discourse*, thus treating conceptual blends as conceptions that are socially constructed and distributed. We do not claim that blending is the cognitive mechanism by which students reason, but use the framework as a descriptive tool for understanding student reasoning.

In summary, this paper addresses two research questions:

1. In learning and reasoning conceptually in QM, do students use dynamic ontological structures, and what does it mean for ontologies to be dynamic?
2. In what ways (if any) can reasoning in a messy and tentative way be productive for students learning QM?

II. THEORETICAL FRAMING

There are three theoretical perspectives which together explain why we ask the questions that we ask in our research study: (i) We view learning as a social process and understand that learning environments are situated within larger social environments, thus positioning both individual reasoning and collective discourse as cognitive tools; (ii) we draw on the resources framework (e.g., Ref. [14]) of cognitive structure; (iii) we study the nature of ontologies to inform a theoretical view of cognition, and we understand ontological categorization and movement between ontologies to be flexible. We discuss each of these briefly as they relate to and inform our research, and then we discuss how these perspectives combine with the theory of conceptual blending [34] to form a theoretical foundation for our method of analysis, which uses conceptual blending as a tool to analyze group discourse.

A. Sociocultural perspectives

We take the perspective that both the objects and processes of learning are social [35]. The objects of learning are social in the sense that the domain of quantum mechanics (and physics more broadly) continues to evolve, and it is the community of physicists (including physics educators) that defines the consensus knowledge [36]. Furthermore, it is increasingly recognized that physics education is not simply about the transfer of a body of knowledge but also about engaging people and developing their capacities in (and in the long run helping define) the discourse, practices, and community norms of physics (Refs. [37–41]).

The process of learning is similarly social; sociocultural perspectives consider learning as the act of internalizing social norms and practices [42,43]. To such ends, valuing the social practice of physics and learning physics means valuing the interactions of individuals in collective engagement in the processes of physics—talking about, reasoning about, and solving of physics problems. Indeed, it is the doing of physics that supports (or in many cases can be considered) its learning [44,45].

Such a perspective shapes not only our pedagogical approaches, but also frames our research questions and methods. Because we consider the process of learning to be a social act, we look at students' individual as well as their collective reasoning. In doing so, we value collective discourse as a cognitive tool, and use it to help us understand the reasoning structures being used by students as they learn QM. We value both individual ideas and associated inferred reasoning structures as well as the negotiated collective meanings (inferred from collective discourse) that students develop as they solve physics problems. In the present piece, we do not always seek to distinguish between individual and collectively developed reasoning. Not only do socially and collectively developed tools (like language) mediate thought [43], but other people and their use of tools also mediate cognitive processes, including those involved in doing physics and solving physics problems [46,47]. Aligned with our pedagogical approaches grounded in sociocultural framing, we look at collective student discourse, document the complex and dynamic nature of reasoning structures, demonstrate the utility of such reasoning, and broaden the application of a tool that has historically only been used for analysis of individual reasoning.

B. Resources framework

We approach our analysis from a resources perspective of cognition [14], understanding students to have multiple resources available to them in a given moment and that certain resources will be activated depending on the conceptual and/or social context. In short, in many instances the notion of a "concept" has been demonstrated to be too coarse a grain size for capturing the nuanced and rich nature of student reasoning [48,49]. Furthermore, building on the knowledge-in-pieces perspective, it has been demonstrated that student epistemological stances can be better explained by examining finer-grained commitments than static, robust beliefs [14]. We draw from this work to consider the ontological reasoning that students use [23]. The resources perspective lends to a view of student reasoning that is flexible and context dependent. Although we consider organizational structures to be flexible, we also find utility in the notion of fixed or robust ideas. We echo Niedderer and Schecker's description of cognition as something that includes stable elements as well as those constructed in the moment [50]. We use the resources perspective to document and catalog fine-grained elements of students' ontological reasoning, but we do not go as far as to argue for a mechanism of ontology development, noting that our approach is consistent with the resources framework, and that investigating or describing a mechanism will be a subject of future study.

C. Ontologies

The goal of our research project is to study how students reason when learning QM, for both theoretical interest and to ultimately further curricula and student capabilities in the subject areas of modern physics and quantum mechanics. One way to understand patterns and processes of student reasoning is to investigate students' use of ontologies—what properties and behaviors are they assigning to the entity they are reasoning about, and how does their ontological reasoning help them make sense of the concept or phenomenon at hand? Ontologies describe mental categorizations of entities [51] (e.g., objects, processes, concepts), grouped according to fundamental characteristics.

Historically, there has been some debate about the nature of ontologies, which can be broken up into two aspects: (i) the nature of the organization of ontological structures and (ii) the nature of the movement between categories. One framework for characterizing ontologies describes discrete ontological categories and assumes a single correct ontology for every entity [16,20–22,52]. According to this view, for two categories to be ontologically distinct, there can be no overlapping ontological attributes [53]. In this line of thought, students come to physics instruction with a preexisting treelike structure of ontological categories (e.g., matter, processes) and then the physics concepts they learn are assimilated into the existing structure. The assumption of ontological correctness requires movement between categories, described by Chi's *Incompatibility Hypothesis* [16], a key feature of this framework. When a student assigns a concept to the wrong ontological category (e.g., thinking of electrical current as a material substance instead of a process), their conception must be reassigned through "radical conceptual change." This mismatch between a student's ontology and the correct (or scientifically accepted) ontology presents a conceptual barrier that is difficult to cross, or can be "resistant to instruction" [22]. Another line of research challenges the "attribution of stable, constraining ontologies" [23] (p. 286), and posits an ontological structure which is more flexible [6,24]. This framework does not include the assumption of ontological correctness, but argues that learners' commitment to a substance-based ontology for a concept that scientists typically conceptualize as a process or interaction can be productive [54–56]. In this view, not only is the underlying organization of ontological categories flexible, but the movement between ontologies is described as a dynamic process. Gupta, Hammer, and Redish argue that both novices and experts reason across ontological categories, and that these dynamic processes are ubiquitous and productive [23]. They suggest that a person's ontological categorization of a given entity is context dependent and can vary moment to moment. In recent work on the historical development of metaphors for energy, Harrer demonstrates that expert physicists *necessarily* use multiple ontological metaphors [57]. Additionally, more recent work argues that ontologies can be blended to form new categories [19].

Some of our prior work has provided evidence for the dynamic nature of ontologies [6,25]. Coming from a resources perspective, we treat ontologies as flexible structures that can be cued in certain contexts and that a reasoner can move between or construct in the moment. This view of both underlying ontological structures and movement between ontologies influences our research questions and our methodologies. However, we avoid an either-or mentality in terms of the theoretical debate (static vs dynamic), because while we adhere to the flexible nature of ontological structure and investigate how students construct ontologies in the moment, we also find utility in the part of Chi's approach which suggests stable and robust notions. When students arrive at a modern physics classroom to begin learning about QM, we expect that they will have conceptions of a classical particle and classical wave that could be considered stable or robust (perhaps more so for particles than for waves), and these ontological structures will influence the way they learn and reason about QM (especially if we teach wave-particle duality as particle *or* wave rather than a different category altogether). Ontologies of quantum entities, like electrons and photons, often include properties or behaviors which resemble those of classical particles and classical waves in specific contexts. The resemblance remains piecemeal in the sense that attribution of one property does not always imply attribution of others (e.g., a photon in a double slit experiment is detected at a single point on the screen like a classical particle would be, but a photon is not always a localized entity like the classical particle). In our data, we see students constructing ontological structures for quantum entities in specific contexts by borrowing from and combining canonical particle and wave language. We see the students flexibly using these already stable conceptions as well as constructing new ontologies in the moment. In this sense, the ontological framework we use here sharply departs from that of Chi [16,22] (or its precursors from linguistics [58,59]), where the assumption is that new knowledge is assimilated into the existing structures and a "mix and match" model based on context specific needs is not suitable.

A slightly different scenario of how classical particle and wave ontologies are used in QM is one that involves switching back and forth between the two ontologies depending on the specific context. This reflects the way we often teach students to think about wave-particle duality (i.e., we model an electron as a particle in some contexts and as a wave in others). Following the work of Baily, we refer to this type of structure as *parallel ontologies* [6]. Figure 2 depicts this quantum ontological structure for an electron compared to a classical ontological structure which would list "electron" as one example of an entity in the "particle" category. In a quantum context though, the electron might be the umbrella category and within that one would switch back and forth between particle and wave. Moving between parallel ontologies aligns with the context dependence (that can happen moment to moment or across broader contexts) of the Gupta *et al.* framework of dynamic ontologies [23], but also aligns with the aspect of Chi's framework that describes stable notions [22]. In our data, we see examples of students switching back and forth between parallel categories for a given entity in this manner, as well as of students constructing blended ontologies in the moment. These examples help us to describe the ontological flexibility we see students engaging in as they learn QM, and to refine our framework for dynamic ontologies.

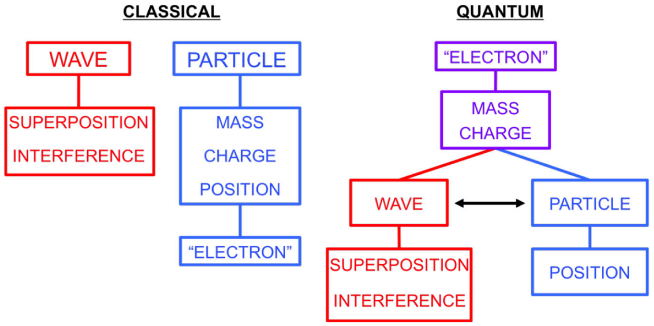

FIG. 2. Baily's [6] representation of classical versus quantum ontologies of an electron. The double headed arrow represents movement back and forth between parallel ontological structures of particle and wave, which are both subsumed within a broader ontology of "electron."

D. Conceptual blending

We use a conceptual blending framework as an analytic tool to capture the dynamics and nuances of students' reasoning. Conceptual blending [34] is a theory of cognition developed by Fauconnier and Turner (FT) that describes the dynamic process of creating *mental spaces*. FT define mental spaces as "small conceptual packets constructed as we think and talk, for purposes of local understanding and action" [34] (p. 102). Through conceptual blending, two (or more) input spaces merge in some way to create a blend space. Select elements from each input space are projected to the blend space where new elements that do not occur in either input space also emerge. Using blending as a tool to get at the nuances of students' ontological reasoning, our present work responds, in part, to a call from Brookes and Etkina for conceptual blending analysis that "may better account for "local" or "personal" ways of expression observed among individual professors and students. The dynamics of blending may also be useful for answering questions about how we can make students more aware of the myriad of models encoded by the metaphors in physicists' language" [60] (p. 15).

FT describe blending as a ubiquitous and unconscious activity involving dynamic processes that result in emergence of new meaning. To illustrate the process of conceptual blending, we present an example from Podolefsky's work on analogical scaffolding [61], which describes the conceptual blend behind the Rutherford model of the atom (Fig. 3). Podolefsky's description assumes students' prior knowledge of atoms as consisting of a nucleus and electrons, which could be arranged in a variety of ways. This knowledge forms one of the input spaces. The other input space is the solar system, which consists of the Sun, and planets in concentric orbits around the Sun. The Sun is counterpart to the nucleus, and the planets are counterpart to the electrons. In the blend space that emerges, electrons orbit the nucleus at fixed radii. Just as the Sun attracts the planets in the input space, the nucleus in the blend space attracts the electrons.

As the structure of a blend space emerges, FT identify three processes: composition, completion, and elaboration [34]. The *composition* of elements from both input spaces creates the possibility for new relations between said elements that would not have occurred had the elements remained in distinct spaces. *Completion* refers to bringing a familiar structure to the blend, or assigning properties based on prior knowledge of one or more of the inputs. *Elaboration* is a dynamic process of mental simulation, and is also referred to as the "running of the blend." As the blended scenario plays out, new ideas come up, along with new meaning of the elements in the blend space. This

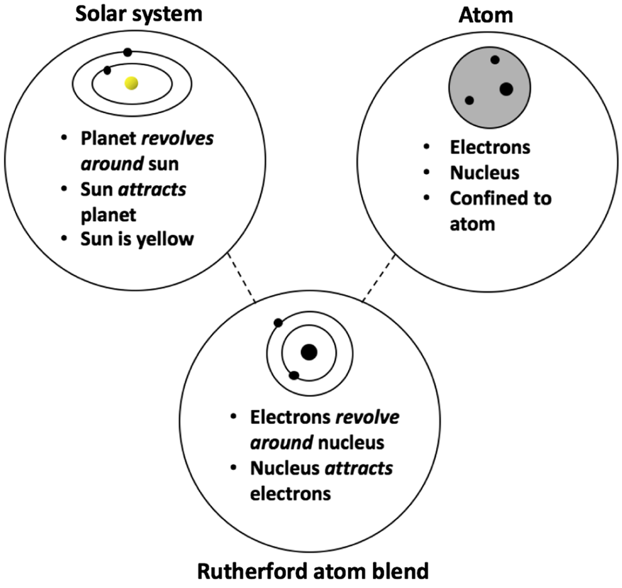

FIG. 3. Adapted from Ref. [61]. Example of a common conceptual blend in physics. Solar system and atom form the input spaces for the Rutherford model of the atom in which electrons revolve around the nucleus and the nucleus attracts the electrons.

emergent meaning is a cornerstone of the conceptual blending theory. FT emphasize that the standard blend diagrams (e.g., Rutherford atom blend diagram in Fig. 3) are static representations of a *dynamic* and imaginative process, which leads to emergent meaning.

Coming from the sociocultural perspective that considers collective discourse as a cognitive tool, we take conceptual blending beyond the analysis of an individual's cognition and describe collectively constructed blends [25]. That is, we model group conversations as conceptual blends, paying attention to shared meaning around ontologies for quantum entities as constructed by the group. Individual students' ideas or contributions to the conversation are modeled as elements in the input or blend spaces. The blending framework then allows us to map out processes within the group that lead to emergent meaning. In using conceptual blending as a tool for analyzing group discourse, we are not claiming that the blends and mental spaces exist within one individual's mind, but rather that they are socially distributed conceptions constructed and utilized by the group to make sense of the phenomenon at hand.

III. METHODOLOGY

A. Course context

Our data come from one semester of a modern physics course at University of Colorado Boulder. This course is the third semester of the introductory physics sequence and it comes in two versions: one primarily for physics (and engineering physics) majors, and one primarily for engineering majors, although students can choose to take the other course if they wish. Prior to modern physics, most students have completed two semesters of introductory calculus-based physics (Physics 1 and 2) or occasionally received credit from AP Physics in high school. This study focuses on the modern physics course for engineers and took place in a transformed version of the course. The curriculum is a result of several years of ongoing course transformation; it focuses on the conceptual foundations and real world applications of QM [62], and explicitly addresses physical interpretation of quantum phenomena [7]. The course [63] in which this study took place was a large lecture-style course enrolling approximately 130 students. Physical interpretation of QM was explicitly addressed in lectures and on homework assignments, and many times students were told that there was not necessarily a "right" answer to these interpretation questions but that they were expected to back up their answers with evidence. This curricular approach was informed in part by prior research in this course which found that when instructors do not explicitly attend to interpretation when teaching QM, students will arrive at their own interpretations anyway, which most often rely on intuitive classical views [7].

B. Methods of data collection

We use qualitative methods to elicit and analyze student reasoning. Expanding on the historical debate about the nature of ontologies [16,23], we dive into what it looks like for students to reason in the moment and flexibly negotiate ontologies around quantum phenomena. We recruited students from the modern physics course, and ran biweekly focus groups throughout one semester. The students volunteered to be a part of the focus group study and were paid for their time each week. The recruitment process involved making announcements in class and sending out class-wide emails; the study was presented to the students as a paid opportunity that would likely benefit the participants' learning in the class, but was in no way connected to course grades, and participation in the group would be kept anonymous for the instructor until after the course was over. From the volunteers we formed two focus groups—referred to here as group A and group B—organized by scheduling constraints. Each group met once every other week for a total of six one-hour-long sessions. We also conducted brief individual interviews with each participant at the middle and end of the semester to gauge how the social dynamics and learning of the groups were progressing, and to ask about their overall experiences in learning QM. Within each weekly session, the students were presented with prompts directly designed to investigate student reasoning on topics drawn from the class. Some prompts involved multiple choice questions or structured problems (with correct answers) that we asked them to discuss, while others were more open ended and interpretive. Most prompts were designed to facilitate conceptual understanding of quantum phenomena. The students were encouraged to think aloud and discuss with their peers, and two interviewers were in the room to facilitate and probe the conversation with questions. The focus group sessions were video and audio recorded, and after all sessions and interviews were completed we collected background information from the participants (e.g., race or ethnicity, gender, prior math experience, prior exposure to QM).

C. Participants

Group *A* comprised three students, all freshmen mechanical engineering majors: Eric (white male), Tara (female; chose not to identify race or ethnicity), and Bryan (white male). Group *B* comprised four students, three of whom were present for the data clip we will present here: Fernando (junior astrophysics major; Hispanic and white male), Zach (junior geophysics major; Japanese and white male), and Jacob (sophomore mechanical engineering major; white male). These six students were top-performing students in the class, receiving either A's or B's for their final grade. At the first focus group session, these students did not know each other (other than having seen one another in class). By the end of the semester, some of the students reported talking with the other focus group participants about coursework outside of class and the focus group sessions.

D. Methods of analysis

Upon collecting the twelve hours of video data between the two focus groups, we identified specific clips to analyze. Selection of these initial clips was based on existence of rich conversations around ontologies, including but not limited to (a) particlelike or wavelike language, (b) a phrase or sentence combining characteristics from multiple ontological categories (e.g., a hybrid phrase like "blob of electromagnetic wave"), or (c) an analogy relating quantum and classical ideas. We then looked at the surrounding discussion in an attempt to determine how this language, phrase, or analogy came into existence. For the present analysis, we chose video clips in which students were thinking about photons and/or electrons, because ontological conceptions of these entities are fundamental for students in building a strong conceptual understanding of QM, and thus the topic areas in the selected episodes could be widely recognizable across many different types of QM and modern physics instruction. Because we selected episodes of conversation rich in ontological negotiation, the examples of reasoning we present here are mostly conceptual. We acknowledge that mathematical reasoning is essential to understanding QM, but choose to foreground the conceptual understanding, which we believe to be an important aspect of learning QM (and is aligned with the emphasis of the course). We also note that a robust understanding of QM includes *both* mathematical and conceptual understanding, and our overall approach seeks to link these (such linkages are the subject of future work). In this paper, we present three episodes chosen from the video data collected between the two focus groups. The three episodes were selected with the above-mentioned criteria, but there are many such discussions between the two groups; the type of reasoning we see in the selected clips is not unique to just a small subset of the data.

Our analysis of the selected episodes is what we consider to be a coarse-grained discourse analysis. Attending to language used by the students, but also paying attention to how the conversation is constructed and owned by the group as a whole, we go through the transcript line by line and map out the ontological structures we see the students using. We draw from the grammatical analysis of Brookes and Etkina [60,65] as we use linguistic cues to guide our analysis. Paying attention to the words students use helps us determine what ontologies they might be using or how individual students are contributing to the collectively constructed ontologies.

In the transcriptions, ellipses (…) represent pauses longer than those natural in speech; gestures or nonverbal actions are indicated in [square brackets]; square brackets also contain information added to the transcript by the researchers for clarity; interruptions in conversation are indicated by an em dash (—).

Paying attention to how a particular entity (photon or electron) is being treated or characterized within a group conversation, we map the conversation (or a chunk of it) onto the conceptual blending framework. Individuals' ideas or turns of talk become elements in mental spaces. As such, the mental spaces constructed in the analysis do not belong to any one student. Rather, the ontologies or conceptions modeled by the blends are socially constructed and distributed. Often in the analysis, we model the blend space first and then work backwards to define the input spaces. This happens in practice by attending to language used by the students that gives us a sense of the ontological properties the individual students and/or the group as a whole are assigning to a given entity. In the specific case of electrons and photons, we look for particlelike and wavelike language and ask how that language is being contrasted or combined to form an understanding—whether shared or not—of the given entity in the group. The conceptual blending framework is not always useful for modeling the group discourse. Sometimes, as we will see in episode 3, one or more students (or the group as a whole) hold parallel ontologies and move between them in a form of ontological negotiation different from the in the moment construction of blended ontologies. This is an example of where some of the data lend themselves to a conceptual blending analysis and others require a different structure to map out the ontological reasoning.

We use the conceptual blending framework as a tool for discourse analysis. This tool is well suited to our research questions for the following three reasons: First, it was designed (originally as a theory of cognition) to investigate the connection of distinct ideas and thus is a model suited for meaning making through analogy or comparison. In our case we sought a tool that would allow us to identify *multiple* ontological resources that students were using and describe how they were being connected together. Second, the framework is inherently dynamic. It allows us to examine the dynamics of students' ontological negotiation in the moment, and describes a mechanistic process for emergent meaning. Finally, conceptual blending also allows us to move fluidly between facets of the "static" and "dynamic" views of ontologies, and in fact helped us to develop the refined typology of ontologies we present below. These unique aspects of the conceptual blending framework as a methodological tool make it well suited for our research purposes. We could have conducted similar analyses using other tools, but none were as well suited to our research questions. For example, many forms of discourse analysis do not highlight the conceptual spaces (and instead tend to favor microgenetic reasoning strategies of individuals). Concept mapping [66] and metaphor theory [67–69] tend to emphasize static concepts and metaphors, and not the dynamic development and reorganization of ideas. Our use of conceptual blending could be considered as one approach to either concept mapping or metaphor theory.

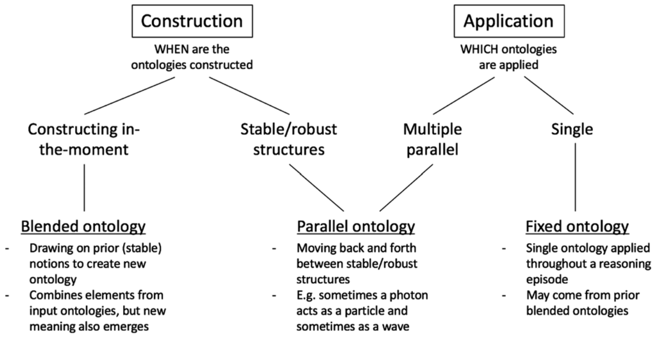

FIG. 4. Framework for the nature of ontologies. *Within a given reasoning episode*, ontologies can be dynamic in their construction or application. Blended ontologies are new structures constructed in the moment, as opposed to stable structures that a reasoner brings with them to a reasoning episode. Parallel ontologies occur when multiple stable structures are applied.

Each of the episodes in this paper exist as an example of student reasoning about a quantum phenomenon. This work provides evidence of dynamic ontologies, while working towards refinement of an ontological framework. We demonstrate the potential for tentative reasoning on the part of the students to be productive, and argue that is important to value the messiness of students' reasoning. Additionally, these analyses demonstrate the utility of conceptual blending as an analytic tool for understanding the nuances and complexities of group discourse. The arguments laid out here are based on three examples of student reasoning, including a total of six students. We do not attempt to generalize the findings outside of our population of students in this specific course context, or to make broad claims about patterns of student reasoning or impacts of a curriculum. These episodes—each a conversation among three students—provide evidence for the types of reasoning that can be used by, and are valuable for, students in learning QM.

IV. TOWARDS A COMMON NOMENCLATURE

Building on the theoretical and methodological tools above and putting them into practice to understand and characterize student reasoning requires the development of a refined nomenclature. The definitions that we operationalize here emerge from and are tested in the episodes of student reasoning in our data. These definitions serve as a refinement of previous dynamic ontologies frameworks [23], and allow us to distinguish among different forms of dynamics in developing and applying ontological categories. The ultimate framework we rely upon (captured in Fig. 4) comes from both these prior frameworks and adapting them to the data and analysis in Sec. V.

In the episodes below, we use the terms *blended ontology* and *parallel ontology*. A blended ontology is a new ontological structure that is constructed or emerges in the moment and draws on prior (usually stable) ontologies. The new ontological structure cannot be fully mapped onto the structures it draws from, as new meaning emerges in the blending of the prior structures. In our data, these blended ontologies are locally sustained. They can be temporary structures, used by a student in a given moment to make sense of a quantum entity or phenomenon, or perhaps they can become compiled into robust and stable structures to be used again in other contexts. In our analyses, we do not make claims about the ontological structures used by students *beyond* the local moments in the given episodes.

As described in Sec. II C, our notion of parallel ontologies stems from Baily's work [6]. A student who holds a parallel ontology moves back and forth between two (or possibly more) stable structures. This is often seen in the context of wave-particle duality, when students (or expert physicists) may think about an electron as a particle in some situations and a wave in others. We see this in episode 3 below. This particular episode includes both parallel and blended ontologies, and we refer to the parallel stable structures as the input spaces of a blend. When there is no blended ontology, and only parallel, we do not use the language of input spaces.

We describe both blended and parallel ontologies as dynamic in nature, but they are dynamic in different ways. Figure 4 depicts a framework that further describes the nature of blended and parallel ontologies. In a given reasoning moment, ontologies can be dynamic in their *construction* or their *application*. By construction, we refer to *when* the ontologies are developed for the reasoner. Ontological structures can be constructed in the moment, which would be a dynamic and perhaps messy process, or they could be already compiled as stable structures. That is, sometimes we see students bring robust ontological conceptions (i.e., of a classical particle) with them into a reasoning episode, and other times we see new, blended ontologies being constructed in the moment. When we think about the application of ontologies, we are thinking about which ontologies are applied in a given reasoning episode. A student could apply a single ontology (static application), or they could apply multiple ontologies in a juxtaposing or complementary manner (dynamic application). We see parallel ontologies when the ontological structures are stable in construction (i.e., students bring stable notions with them into the reasoning episode), and multiple ontologies are applied within a single reasoning episode. We note that this parallel application could occur on short or long time scales (switching back and forth within one or a few sentences, or a broader context dependence between different conceptual situations). We choose to focus on single reasoning episodes, thus honing in on ontological flexibility at the shorter time scales.

One goal of our educational environments is for the stable construction and single application of an ontology in a reasoning episode, noting that experts exhibit context dependence of ontologies, but have the ability to apply a single stable quantum ontology for an electron in a given moment. At the same time, such ontological flexibility may arise from the construction and application of blended and parallel ontologies, which while messy can be valuable for students learning and making sense of QM.

V. DATA AND ANALYSIS

We present three episodes from focus group sessions in which students reason about a canonical topic area of modern physics:

Episode 1: double slit experiment with a single photon,
Episode 2: Mach-Zehnder interferometer with a single photon,
Episode 3: tunneling of an electron in a wire.

In episode 1, we demonstrate the utility of the collective conceptual blending method of analysis with a simple example of three students collectively negotiating an ontology of a photon. In episode 2, we illustrate how students use a common blended ontology of a photon to come to different interpretations of superposition. In episode 3, we see different types of dynamic ontologies within the group: moving back and forth between parallel ontologies and negotiation of a new blended ontology. All three examples provide evidence of the dynamic nature and students' flexible use of ontologies, and help us to refine our framework for the nature of ontologies. Each episode also speaks to the value of students' tentative reasoning, especially as they learn about and grapple with quantum phenomena. We note that different problem statements, representation and simulation use, and conceptual contexts likely cue different types of reasoning [70] and potentially invoke different mechanisms of ontology development. Investigating these impacts is not the purpose of this paper, but will be a subject of future study. Rather, the purpose of the present study is to provide evidence of different types of dynamic ontological reasoning, and investigate the value of students' reasoning in a messy and tentative manner.

A. Episode 1: "Blob of EM wave"

The first episode occurred in the second week of the focus group sessions in group *A* with Eric, Tara, and Bryan. The students were given a screenshot from the PhET Quantum Wave Interference simulation [71] which shows a double slit experiment with a single point on the detection screen (Fig. 5). Accompanying the picture was a multiple-choice question: *A single photon is shot towards the slits and detected at the point shown on the screen. What is the most reasonable interpretation of where the photon was just before it was detected? (a) it was located just in front of where it was detected, (b) it was spread out evenly in the space in front of the screen, (c) it was spread out in a non-even pattern in the space in front of the screen, or (d) it was spread out evenly through all space.* This question highlights both the particlelike and wavelike natures of a photon (energy quanta of light). When a single photon is fired at the apparatus, it is detected at a single point on the screen. When this is repeated many times, the individual photons form an interference pattern on the screen—there are some places with many dots (measurement of a photon), and others with very few. The correct answer to this question is (c) because there are some points on the screen that are more likely to detect a photon than others. The photon is not localized until it is detected. The three students agreed that (c) was the answer and then proceeded to set up the PhET simulation on a laptop. A reductionist framing of student learning that focuses on whether or not students got the answer "correct" would stop here, but we believe there is so much more to learn about student thinking.

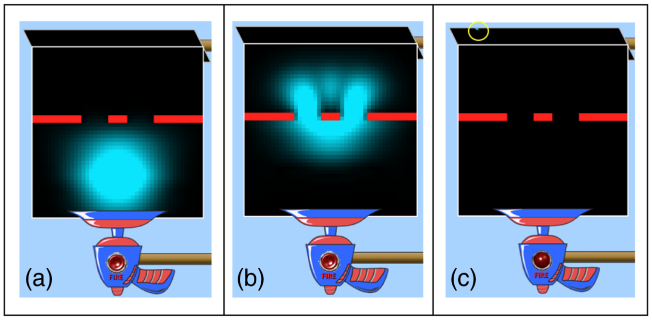

FIG. 5. In episode 1, students were given the screenshot in (c) which shows a single photon as a point on the detection screen of a double slit experiment. (a), and (b) The PhET simulation [71] shown as a single photon is fired and travels toward the screen.

Looking at the simulation, Eric initiates the conversation by saying,

1 *Eric: It's different from how I would otherwise*
2 *think about it because it's a big blob*
3 *of…electromagnetic…stuff, instead of like a single*
4 *point [gestures a small point with his fingers] that's*
5 *like flying through space…And because of that it can*
6 *interfere with itself and make the interference pattern.*

We believe it is reasonable to assume that the simulation, which represents the single photon as a circular bloblike entity traveling toward the slits, cues the word "blob" in Eric's explanation. The students were then prompted by the interviewer to consider how the simulation helped them think about where the photon was just before detection.

7 *Tara: I mean you can see it like as the photon, like*
8 *it's just this big blob of light, and it hits the screen,*
9 *and there's—it kind of spreads out into the interference*
10 *pattern sort of [gestures fingers spreading out].*
11 *And so right before a dot appears, you can see, there's*
12 *like the [gestures horizontal lines]…spread out photon*
13 *[mutters] makes sense.*
14 *Eric: When you think of the photon as–like this blob*
15 *of electromagnetic wave then, I think it becomes more*
16 *complicated to talk about where it is, because, like, it's*
17 *an electromagnetic field…now, instead of like…a*
18 *particle.*
19 *Bryan: Yeah it's weird to think that it's like in*
20 *more…places than one at a single time. Like, I don't*
21 *know—*

In the dialogue, we see the students collectively negotiating the ontology of the photon. In this moment, the photon is described by Eric as a "blob of electromagnetic wave" which draws from both particle and wave characteristics. The pauses and hesitations in the conversation suggest in the moment construction of these ideas. Additionally, in lines 19–21 Bryan tentatively offers his thoughts, explicitly flagging "weirdness" of the idea that the photon could be unlocalized. We model the above dialogue with the conceptual blending framework, and describe the group's conversation as a collective construction of a conceptual blend with classical particle and classical wave input spaces. As evidenced by Eric's first statement, particle characteristics include localization ("single point") and particle path ("flying through space"), while wave characteristics include creation of an interference pattern and interfering with itself. The blend space is the blob of electromagnetic (EM) wave. This new entity inherits some of the localization property from the particle (the word blob suggests something contained in a finite space) as well as a nonlocalized wave property (it is "complicated to talk about where it is," line 16). A particle interacting with the screen would leave a dot, but a wave would create an interference pattern. The blob of EM wave inherits both of these properties—Tara explains that it "spreads out into the interference pattern…right before a dot appears" (lines 9–11).

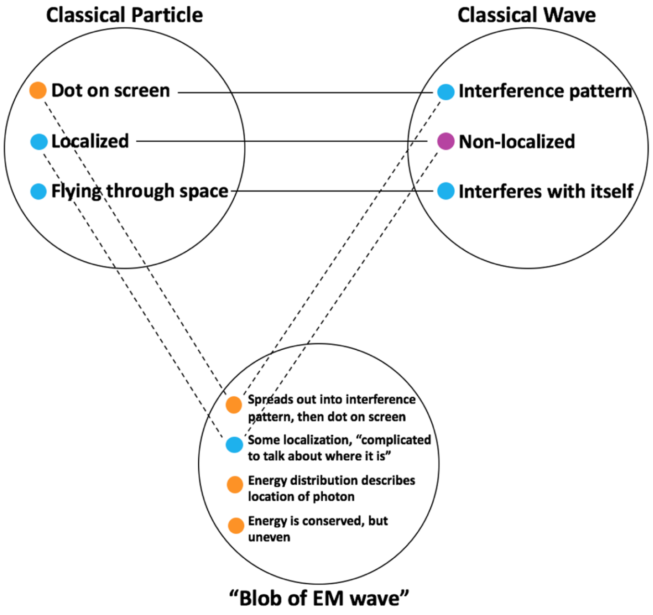

FIG. 6. Conceptual blending diagram for episode 1—a discussion between Eric (blue), Tara (orange), and Bryan (pink) about the behavior of a single photon in a double slit experiment. Classical particle and classical wave form the input spaces for the Blob of EM wave blend space.

The diagram in Fig. 6 illustrates this blend. The particle input space is on the left, the wave input space is on the right, and the blob of EM wave blend space forms the third vertex of the triangle. The horizontal lines on the diagram connect an element in one space to its counterpart in another space. The dashed lines represent projections of elements from the input spaces to the blend space [72]. The *composition* of localization and nonlocalization gives the blob of EM wave the property that it is somewhat localized, but it does not have an easily defined position. The elements "dot on the screen" and "interference pattern" form the blend space element described by Tara: the blob spreads out into an interference pattern and then appears as a dot on the screen.

As the conversation continues, we see the students elaborating on the ideas set out in the construction of the blend space, blob of EM wave, which we model as an *elaboration*, or running of the blend process. Initiated by Eric's statement in line 17, the students are thinking about an electromagnetic field, and a question arises:

22 *Eric: So then we're saying that if the…if the amplitude*
23 *of the wave is zero it's like because of destructive interference*
24 *…bec—if it's uh staying at zero, does that mean*
25 *the photon isn't there, because there's no …field there?*
26 *Tara: Right, 'cause it's interfering with itself, but can*
27 *a photon cancel itself out?*

Prior to the construction of the blend space, this question of whether a photon can cancel itself out would have held no meaning for the group. However, in the blend space, the photon can simultaneously take on both wave and particle characteristics (e.g., localization and interference). The wave property of interference leads to this new question among the group; there is now a particlelike entity that exhibits interference behavior. Upon constructing the blend space, the students begin to try on new ideas. They are negotiating the structure of this blend space—*What are the rules? How does the photon behave? What are its fundamental properties?* They do this in a tentative manner, posing their ideas as questions (lines 22–27) as if to explicitly mark them as exploratory ideas that they are playing around with in order to make sense of the quantum phenomenon at hand. Eric responds to Tara's question about whether or not a photon could cancel itself out:

28 *Eric: Completely?…Like if you bounced a photon off of*
29 *a wall, and then like halfway through the bounce it was*
30 *like halfway over itself and it just canceled out to*
31 *nothing—*
32 *Bryan: Is that possible?*
33 *Eric: But then you can't do that because of conservation*
34 *of energy. Also, I'm not sure that photons bounce*
35 *off walls.*
36 *Tara: Well, I mean if you're thinking of a photon as a*
37 *wave that's sort of spread out, it's not like—it's like,*
38 *it's spread out so if it doesn't exist at a certain point it*
39 *doesn't mean it doesn't exist at other points.*
40 *Eric: Can you say that again?*
41 *Tara: Like, you can have points where there is no*
42 *energy from the photon, but you can have points where*
43 *there is a lot of energy from the photon so overall,*
44 *energy is conserved, it's just uneven.*

The concepts of energy and energy conservation were not present in either input space initially—they emerge as a result of elaboration of the blob of EM wave blend. The students bring in their prior knowledge of energy conservation that has not been explicitly addressed in this prompt (this is an example of what FT call *completion*). As they reason through their new description of a photon, the idea arises that a photon could bounce off of a wall and cancel itself out. The students reject this idea immediately, holding on to the idea that a particlelike entity cannot simply cancel itself out and disappear. After running into the insufficient description of a photon bouncing off of a wall, Tara presents a different explanation. Throughout the discussion, the students are negotiating collectively: *Does it make sense to describe a photon like this?*

The students negotiate the emergent meaning of energy conservation as it applies to the blob of EM wave. Through the running of the blend, students return to the input spaces in order to construct a reasonable description of energy and energy conservation in the blend space. It is this dynamic interplay between all elements of the network that characterizes the student reasoning about photons in this episode.

After Tara brings in explanations that use energy and energy conservation, Eric tries to clarify them, attempting to solidify some common understanding within the group about how the photon behaves:

45 *Eric: So, it would not be possible for a photon to*
46 *completely cancel itself.*
47 *Tara: Right, it's like at a point it would be possible for*
48 *none of the energy from the photon to be there, but over*
49 *the area that the photon is spread out on, it would still*
50 *have the total energy*
51 *Eric: So then, a photon can either have all of its energy*
52 *concentrated in one spot or spread out over a large*
53 *area?*
54 *Tara: I didn't say that.*
55 *Eric: Oh, ok. [Chuckles]. That was just something I*
56 *was throwing out there.*
57 *Tara: I don't know if you can have…um, I don't know*
58 *if it's either-or, I don't think anything is either-or.*

In this last exchange, Tara maintains a tentative position about the nature of the photon. In this moment, she is unwilling to assign deterministic properties to the photon and ends the conversation by saying "I don't think anything is either or." In addition to describing the indeterminacy of the photon, this statement could be a reference to the nature of knowledge itself. This last statement from Tara marks the end of the episode, because at this moment there is a long pause and then the group moves on to a different topic. The silence among the group members following Tara's statement can be taken as a signal of agreement that this type of incertitude is appropriate here. Because they end with this note of hesitation and do not have anything further to add to the sense making process at this moment, we infer that in this local moment, the tentative stance is conceptually satisfactory for the group. This is aligned with instructional goals where the course instructor explicitly argued for the value of tentative knowledge and claims in QM. An alternative interpretation could be that the conversation ended on this note because of an awkward social interaction (lines 54–57), or that the other group members used this tentative stance as an escape hatch [73] to get out of a socially uncomfortable or conceptually confusing situation and move on to the next problem. We take the tentative stance to be productive for them in this moment, whether it be conceptual or social productivity, or both.

We characterize episode 1 as a conversation of dynamic ontological construction—the group collectively develops an understanding and negotiates meaning of the blob of EM wave ontology of the photon, which we label as a blended ontology. This structure is constructed on the fly by the group (i.e., the blob of EM wave was not a conception of the photon that the students brought with them into the reasoning episode). While blob and EM wave may be existing ontological structures (the word blob is likely brought in by the representation on the PhET simulation), the blob of EM wave is a new object for them to discuss and takes on new meaning during their conversation. The students elaborate on the blob of EM wave blend by questioning the possibility of a photon canceling itself out. Through this in the moment and messy reasoning, energy conservation takes on a new meaning for the students in the context of the single photon in a double slit experiment. The emergence of new ideas about energy and the ultimate rejection of the idea that the photon is able to cancel itself out is a dynamic process to which each group member contributes. While the episode certainly takes on the flavor of a blended ontology, we also see some hints of parallel ontological structure in the blend space element (coming from Tara's statement, lines 9–12) that the photon spreads out into an interference pattern and *then* forms a dot on the screen (the temporal order is the key thing here that suggests a parallel structure). This suggests not only that students can apply multiple ontologies in a given reasoning episode or construct new ontologies on the fly, but that they can use multiple types of dynamic ontologies (i.e., parallel and blended ontologies are not mutually exclusive). This flexible use and in the moment collective negotiation contributes to what we refer to as the messiness of student reasoning in this particular episode.

Also contributing to this messiness is the students' tentativeness around ascribing characteristics to the photon (namely, Tara's bid in line 58 that nothing is "either or" in regards to the energy of the photon). In addition to the bid for uncertainty that concluded the episode, throughout the conversation, each of the three students flagged their contributions with hesitation and incertitude—posing their ideas as questions, and including the caveats "I'm not sure" or "I don't know." We take these tentative stances to be productive for the students. They are engaging in patterns of scientific discourse with their peers—playing around with ideas to make sense of the quantum phenomenon at hand, pushing one another to articulate their ideas, and entertaining questions of interpretation of QM—activities that we would define as goals of this course. Additionally, these students are putting forth ideas they are unsure about, which is something we hope our physics students feel comfortable doing in our educational environments and is one way that novices and experts alike learn and make sense of physics concepts [12,13].

Through modeling the group's conception of the photon as a collective blend of classical particle and classical wave input spaces, we are able to "see" the dynamics of the group's ontological negotiation. This conceptual blending analysis demonstrates the productivity of tentative and messy reasoning.

B. Episode 2: "Superimposed photon"

Episode 2 also involves group *A* and occurred two weeks after episode 1, during the first ten minutes of the third focus group session. In this episode, Eric, Tara, and Bryan are collectively negotiating the ontology of a single photon in the context of a Mach-Zehnder interferometer. Along with the schematic shown in Fig. 7, the students were presented with the following questions: *(1) What is going on when photons are sent through the experiment? (2) How would this experiment be different with classical particles or EM waves? (3) How do you think about the energy of the photon in this situation?* When this experiment is performed with a beam of light, half of the wave is reflected and the other half transmitted at the beam splitter (BS1). The reflected beam then follows the path to mirror A (M_A) and into detector A (PMA), while the transmitted beam goes to mirror B (M_B) and into detector B (PMB). When a single photon is sent through the interferometer, its state can be described as a superposition of the two paths. The photon will then be detected by *either* PMA or PMB. The students also had access to the IOP *Interferometer experiments with photons, particles and waves simulation* [74], which they set up on a laptop before they began to answer the questions. Looking at the schematic of the experiment, Eric begins the conversation:

59 *Eric: Umm…well when photons go through the beam*
60 *splitter, and it splits the photon, and then, when it hits*
61 *the detectors it goes into one of them randomly.*
62 *Bryan: Yeah, it's a 50-50 shot, which one it's detected*
63 *by.*
64 *Tara: But, I–I think you have to be careful about your*
65 *language there, because the photon itself I don't think*
66 *is split, it has an equal probability of going through*
67 *either of these paths, but the photon is still like…a photon*
68 *[gestures a container or ball with her hands].*
69 *Eric: So it's not physically split, but like, it's in the*
70 *ambiguous state of being—*
71 *Tara:—It's superimposed—*
72 *Eric:—on both paths. [nods]*

The students construct a collective conception of the photon as something that has equal probability of being detected by either of the two detectors, and does not physically split, but is "superimposed"—a term that at this point in the discussion refers to an "ambiguous state of being on both paths" (lines 70–72) and will later be referred to by the students as a "state of superposition." This characteristic of being superimposed arises in the conversation as a response to Tara's correction of Eric's language (lines 64–65) and assertion that the photon is not physically split. This is one of several times in this focus group session when the idea that the photon may physically

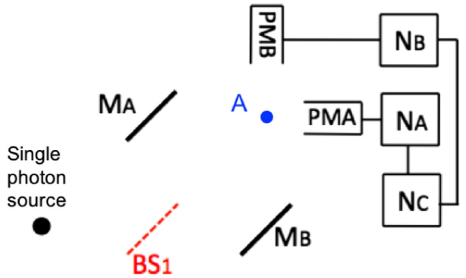

FIG. 7. Schematic of a Mach-Zehnder interferometer with one beam splitter presented to students in episode 2. Point A marks the point that Tara references in line 74 (this was not present in the diagram given to students). PMA, PMB are detectors, N_A, N_B, N_C are counters, M_A, M_B are mirrors, and BS_1 is a beam splitter.

split is rejected. The group as a whole is playing around with ideas, rejecting some elements that are not productive in this specific context. Elaborating on the superposition aspect, Eric and Tara seem to reach some shared understanding that the photon, after being superimposed, decides which detector to go into:

Tara: Right…And it's still superimposed after it crosses like this little point right here [draws on paper and identifies point A in Fig. 7].

Eric: Right

Tara: So once it goes this way it's still superimposed and somehow it decides one of these based on probability.

Eric: Yeah…Um, and there's no chance that they—that it would go into both…detectors at the same time because you're only shooting a single photon.

Tara: Right.

Eric: So it just chooses one and goes entirely into that one.

In grappling with the Mach-Zehnder experimental results and with what it means for a photon to be in a state of superposition, Eric and Tara have anthropomorphized the photon by assigning it the ability to choose which detector it hits. This may suggest that the students are actually thinking of the photon as an entity with decision-making abilities. It could also be interpreted as language that the students do not take literally and that, as they negotiate in the moment, they are searching for the words with which they can talk about this new, "weird" phenomenon in a coherent physical way. In order to continue to make sense of this superimposed photon, the students move on to the second question and compare the properties of the photon with those of a classical particle or electromagnetic wave. Referring to a classical particle, Tara begins the discussion:

Tara: Like, if-if you think of this as like a, like a literal ball, it can either go like that, or like that [tracing out two paths on the paper]. And so can the photon, but not as physically as this would. Like this [particle] would exist at this path the whole time—

Eric: Yeah, you would be able to like see which path it was on as it traveled.

Tara: Right, but the photon's like, I'm gonna do one of these things…we'll find out. [shrugs]

Eric: [Laughs] Um, versus with EM waves, it would actually go down both paths.

Bryan: Yeah, it's actually split.

Eric: So like, it's the two extremes, like with the particle it would go down one path only and with the wave it would go down both paths, and with like the photon it neither—it does like something in between where it just superpositions itself along both paths and then goes into one of them.

Tara: Right, like with the EM wave you have half the wave going on one part and half the…wave on the other.

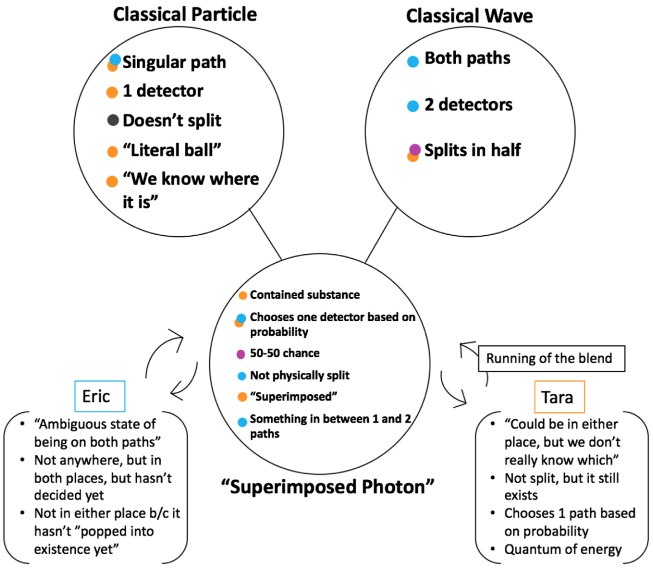

FIG. 8. We model the collective conversation in episode 2 between Eric (blue), Tara (orange), and Bryan (pink) as a conceptual blend. Superimposed photon is formed by input spaces of classical particle and classical wave. Eric and Tara run the shared blend to arrive at different interpretations of superposition. The elements indicated with gray were not spoken by any one individual, but rather implied by statements from various group members.

Here, scaffolded by the prompts, the conversation turns to explicitly identifying particle and wave characteristics between which the photon characteristics are situated. We model this conversation as a collective conceptual blend where the superimposed photon is the blend space, which draws from classical particle and classical wave inputs (Fig. 8). First, the group constructs an ontology of the photon and begins to negotiate the properties of the superimposed photon. Then they return to the input space elements in order to further make sense of the blend. Eric and Tara continue by identifying how the photon is different from either of these entities. They agree that the photon does not act like a wave "because it doesn't get detected by both detectors," and that it does not act like a particle because they "don't know where [the photon] is." Once the students have reached some shared understanding around the properties of the photon (elements of the blend space in the diagram) and how they do and do not draw from classical particle and classical wave properties (elements of the input spaces), new meaning arises for individual students in what we label as elaboration or running of the blend processes. Addressing the third prompt, the students begin talking about the energy of the photon and elaborate further on the superposition aspect of the photon:

Bryan: [Looking at the paper] Um. How do you think about the energy? I don't really know. Um…I don't know. Like when I think of it, I think of it as it's like split, just because it's easier to think about that way. But, I know that's not…true.

112 *[Eric and Tara laugh]*
113 *Eric: Yeah, it's weird to think about where the photon*
114 *is when it's in this like superposition state of not being*
115 *anywhere. But also being in both places, but not really,*
116 *because it hasn't decided yet.*
117 *Bryan: Yeah.*
118 *Eric: It's like it goes back and changes history when it*
119 *hits. But, I don't think that's a good way to think about*
120 *it. [laughs] I don't think that's accurate.*
121 *Tara: Well, I mean, I always think of the energy of the*
122 *photon always being [gestures ball with her hands],*
123 *like together. Like it can't really be split 'cause it's*
124 *a quantum of energy.*

Through pauses, "ums," saying I don't know or labeling something as weird, the three students explicitly flag their ideas with incertitude. Bryan (line 111) acknowledges that he knows the way he thinks about the photon is not the "right" way to think about it, making the distinction between his notion of the energy splitting in half versus the unarticulated "correct" way to think about the energy (an idea that the group is seeking to articulate and make sense of). Signaling the difference between these two ontological spaces, Bryan voices his thinking with the caveat that he does not think it is correct. Similarly, Eric brings in a new idea that the photon changes history when it hits one of the detectors (line 118). He then immediately distances himself from this idea with laughter, perhaps suggesting that a photon behaving this way would be too weird. His subsequent statement, "I don't think that's accurate" shows his lack of confidence in and noncommittal to the idea he just put forth, suggesting that Eric is making sense of the photon in the moment, and putting forth ideas he is unsure about. Tara also brings in a new idea when she invokes quantization. Following her description of the photon as a "quantum of energy" (line 124), Eric makes another bid for the photon physically splitting in half at the beam splitter. Similar to the exchange at the beginning of the episode (lines 60–68), the idea of splitting is rejected from the shared understanding of the photon. As the conversation continues, the students negotiate the meaning of the superposition state of the photon. In doing so, Eric and Tara draw different interpretations. Bryan takes much fewer turns of talk [75] and thus we cannot discern how he is thinking about the superposition element in running the blend. However, we do note that despite his minimal contributions to the conversation, Bryan *is* a part of the collective construction of the blended ontology of the superimposed photon. At the end of the ten-minute episode, Eric and Tara each articulate their ideas about what the state of superposition means:

125 *Eric: [laughs]…Um, yeah I mean with the particle,*
126 *obviously it's that same thing where all the energy is*
127 *traveling along one path and with the wave the energy*
128 *is split between two paths…So the ener–the energy like*
129 *follows its position, but because we don't really know*
130 *the position of the photon, I don't really know where*
131 *the energy is. I assume it's wherever the photon is…So*
132 *I guess the energy is also in, a sup—state of*
133 *superposition.*
134 *Tara: I think that's a good way of thinking about it.*
135 *Like it could be in either place, all of it could be in*
136 *either place, but we don't really know which, and it's*
137 *not split, but it still exists.*
138 *Eric: And it's not actually in either of those places*
139 *yet, because it hasn't been detected yet, so it doesn't–it*
140 *hasn't like popped into existence yet. If that's the*
141 *correct way of thinking about that…*

In this exchange, Eric elaborates on his original statement of the "ambiguous state of being on both paths" (lines 70–72) by describing the photon as "not being anywhere," but also "being in both places," while having "not decided yet." Despite the apparent agreement in their language towards one another, this contrasts Tara's descriptions of the photon as a "quantum of energy" that could be on one path or the other, we just "don't really know which". Tara is more confident in her interpretation while Eric approaches his ideas tentatively as if to suggest he is still trying them on and making sense of the quantum phenomenon at hand. We note that these roles are reversed from those in episode 1 where Eric was attempting to be more concrete and Tara was hesitant to do so as she made a bid for nothing being "either or." Two weeks later and in a different experimental, although conceptually similar, context Tara's statements are now more consistent with a deterministic interpretation (this language from Tara shows up repeatedly in episode 2, leading us to believe she is, in this moment, utilizing an interpretation similar to what we would characterize as a deterministic hidden variable interpretation of QM), while Eric tentatively develops a notion of indeterminacy. We believe this longer time scale change in stances around indeterminacy suggests an underlying flexible nature of ontological structure, although we focus just on the local reasoning episodes for this analysis. Future work may explore this more, and follow the changes in ontological reasoning of one or more individuals within one group over the course of a semester.

In this episode we see evidence of blended ontologies. In order to explore the dynamic processes that help shape the negotiation of the ontology of the photon, we describe the collective conversation using the conceptual blending framework. At the beginning of the conversation, the group's conception of the photon begins to emerge when they decide that the photon is not physically split, goes into one of the two detectors with equal probability, and is superimposed. These properties can be described as elements of a quantum photon blend space (Fig. 8). From this initial description the students explore the superposition element in order to refine their ideas about how the photon behaves. Eric and Tara agree that the photon "decides one of [the detectors] based on probability" to coordinate the idea that the photon is not able to physically split and the fact that it has equal probability of hitting either detector. This can be described as a running of the blend process, which contributes to the emergence of meaning for the quantum photon. Following the collective construction of the conception of the superimposed photon, the students identify classical particle and wave properties that the photon draws from. While a classical particle, or a "literal ball" (line 87) takes one path and does not physically split, an electromagnetic wave takes both paths simultaneously and is "actually split" (line 97). The students explicitly peg the classical particle and wave ontologies as the two extremes between which the photon lies; the photon exhibits some properties of each, but is itself a different kind of entity as it does not fully map onto either particle or wave. As shown in Fig. 8, in mapping the group's conversation onto the blending framework, classical particle and classical wave form the input spaces which merge to form the quantum photon blend space. We remind the reader here that our use of the terms *input space* and *blend space* do not refer to conceptual packets held in one person's mind as originally intended by FT, but rather they are socially distributed conceptions constructed and utilized by the group to make sense of the phenomenon in question. The input and blend space in Fig. 8 reflect the shared meaning reached in the group.

The conceptual blending framework helps to elucidate the dynamic nature of the conversation: first the students come to shared understanding about the superimposed photon, then they articulate the particle and wave inputs in order to make sense of the blend space photon, and then elaboration of the constructed ontology brings in new ideas about energy, all scaffolded by the prompts. In the latter stage of the conversation, the students begin to use energy as a proxy with which to think about the photon. For example, now instead of saying that the photon does not split, Tara says that the *energy* does not split and thus describes the photon as a quantum of energy. Through this running of the blend, Eric and Tara reach different interpretations of the state of superposition. Tara is more aligned with a hidden variable interpretation: the photon takes one path or the other, but we cannot know which one until it is detected. Although we ultimately want the students to reject the hidden variable interpretation, this may be an important stepping stone for Tara in her learning of QM. While she seems sure of her conception of superposition in this context, Tara still uses some unsure language and gestures throughout the conversation ("I don't even know," tone of voice, confused facial expressions, shrugging). In this episode, Tara's hidden variable interpretation of the photon in a Mach-Zehnder interferometer is elaborated upon and locally sustained. We make no claims about her cognition or interpretation beyond this situated moment, and we are not concerned with any of the individual students having "right" or "wrong" answers in this particular moment. In shifting away from the binary view of student learning, we focus on the value of having students engage in messy and flexible discourse with their peers. Contrary to Tara's apparent assuredness in her interpretation of superposition, Eric is less confident in his ontological stance, stating that the photon is not anywhere because it does not exist yet, but that it could also be on both paths. We emphasize here the value and productivity of tentative knowledge structures as we note that this is one example (of many) in QM where certainty is not always appropriate. We distinguish students' tentativeness (being unsure about their ideas) from uncertainty in QM (the lack of ability to make predictions with certainty, inherent in quantum mechanical systems), noting that tentative reasoning may help students to develop a sophisticated understanding of inherent uncertainty. Additionally, we see the students flexibly playing with ideas they have put forth, and negotiating in the moment with one another in order to make sense of the quantum phenomenon at hand. These are skills we find important as they mirror activities of professional physicists, and demonstrate a depth of understanding and learning. The conceptual blending analysis of this episode provides a way to look beyond the checkbox of "they used the physics language, so they get it." That is, within the dynamic construction of a collective ontology, the quantum terminology of superposition takes on certain meaning for individual students. This episode provides evidence of collective dynamic construction and negotiation of ontologies and the conceptual blending framework allows us to see how different interpretations of shared meaning arise through running of the blend processes.

C. Episode 3: "Fuzzy ball of probability"

Episode 3 occurred in the first few minutes of the fifth focus group session in group *B* (week 13 of the semester), with Zach, Fernando, and Jacob. The students were presented with an image of a copper wire that ends in space about half way across the page, along with the following question: *We have an electron in a copper wire. Can you draw the potential energy, and the wave function of the electron?* The second part of the prompt (that the students attend to later in the session, after this particular episode) brings a second copper wire near the first with a small insulating gap in between, and asks the students to again draw the potential energy and wave function of the electron. In this second part of the question we would expect tunneling, a quantum phenomenon where the electron can be found in the second wire despite not having enough energy to "get over" the potential barrier. In the first part of the prompt that comprises episode 3, the potential is zero inside the wire and nonzero outside. The wave function of the electron is sinusoidal inside the wire, and exponentially decays outside of the wire (the "classically forbidden" region). After working independently for a brief time, the students compare the wave functions that they have drawn. All three students have indicated that the wave function extends outside of the wire in an exponential decay. Zach says, "the wave function extends on either side of the well—of the wire, into the classically forbidden zone, which has something to do with quantum tunneling." To that, Fernando responds with a description of the electron:

142 *Fernando: Yeah I guess it's like that, the fuzzy nature*
143 *of, of electrons kinda like you don't know. I think that—*
144 *that's always been kind of confusing, whether it goes*
145 *out because it like, it's like a fuzzy ball of probability*
146 *that extends out—into that zone? Physically? Or…I*
147 *don't know that's the way it seems to me.*

This language of an electron being "fuzzy" was never used in class, and had not come up in prior focus group discussions. Fernando may have been thinking about the electron in this way before (or a prior simulation may have cued the word fuzzy for him much like the blob in episode 1), or he may have constructed this description in the moment. Either way, he proposes the idea of the fuzzy ball of probability in this moment as a tool for linking the wave function representation in front of him on the paper to a physical picture of what the electron is doing. Fernando's initial statement about the fuzzy nature of electrons hints at some notion of probability or indeterminism when he says "kinda like you don't know" (line 143). He then labels the electron as a fuzzy ball of probability and questions the physicality of this description. Is the electron something that physically extends outside of the wire, or is it something else? Fernando explicitly flags his idea with reluctance—he labels it as "confusing" (line 144), presents the fuzzy ball description as a question (line 146), and makes the reflective statement "I don't know that's the way it seems to me" (line 147). Fernando throws out the fuzzy ball of probability as a catch-all phrase, and then the students begin to unpack what it might mean in the specific context of an electron in a wire:

148 *Jacob: Huh I guess I never really thought about…that.*
149 *If it just extends out 'cause it's like a fuzzy ball of stuff,*
150 *it's not really anywhere. [laughs] But that's kind of—*
151 *yeah, I never really thought about why it extends out.*
152 *Fernando: Yeah I feel like a lot of times we think about*
153 *like only the math, and then like—I, I don't know, I*
154 *guess physically thinking about a lot of this quantum*
155 *stuff is not very intuitive, and kind of hard to do, but…*
156 *Jacob: I just kind of took it for granted 'cause I was*
157 *like oh negative kinetic energy—pshh [puts hands up]*
158 *don't tell me.*
159 *Fernando: [laughs] Yeah, exactly. Doesn't really*
160 *mean anything I guess, but—*
161 *Zach: I kinda imagine like a, an electron like going*
162 *back and forth and like getting to the end and like—*
163 *like going out, and then almost like magnetically like—*
164 *like going outside a little bit and then shooting back in.*
165 *[gesturing with his finger a particle moving back and*
166 *forth]*
167 *Fernando: …Yeah, I like the analogy in class where we*
168 *had, where like the water was with the rubber [brings*
169 *right hand to meet left]*
170 *Jacob: Yeah that's what I was just thinking. How it*
171 *would like hit the rubber wall and like, can stretch out*
172 *[gesturing stretching rubber wall] and then just shoot*
173 *it back.*

Responding to Fernando's proposition for the electron as a fuzzy ball of probability, Jacob engages in metacognition as he recognizes that he has never thought about the physical meaning of the decay of the wave function. He attempts to take up Fernando's fuzzy ball description, but refers to a fuzzy ball of stuff, which we take to be ontologically distinct [76] from a fuzzy ball of probability. In lines 149–151, the ambiguity of pronouns in Jacob's statement is a signal of in the moment reasoning. We infer that Jacob means that the *wave function* extends out because the *electron* is a fuzzy ball of stuff. In lines 152–155, Fernando sets the tone for the conversation by revealing his expectations about the roles of math and thinking about physical interpretation in QM. He says that thinking physically about quantum phenomena is difficult and not intuitive, yet the group continues attempting to do just that. For the remainder of the episode (and much of the entire hour-long session), they work collectively to make sense and construct a physical description of quantum tunneling. We note that in this focus group session the students have not been explicitly prompted to engage in this interpretive discussion (although by this time in the semester, there are expectations established in the class that attending to interpretive aspects is part of learning QM). We take the word "but" in lines 155 and 160 to represent an epistemic bid from Fernando that although thinking about the physical interpretation of QM is difficult, it is important and valuable for him and his peers to continue to engage in this discussion in the focus group setting.

The group puts forth different ideas for how to think physically about tunneling, beginning with Zach imagining the electron as a pointlike particle (evidenced by his gesture) moving in and out of the wire "almost magnetically" (lines 162–166). The water and rubber analogy the students reference in lines 167–173 is an analogy for tunneling that was presented in class, where a rubber barrier is analogous to a potential barrier and the water wave is analogous to a wave function. As water sloshes up against a rubber wall, some of the wave will "leak" to the other side of the wall, going through the barrier and not over it (nor breaking through it). Although the analogy as presented in class was meant to draw on wave characteristics, the students are attending to the material aspects of the description in their conversation. This is evidenced by their particlelike language: [the water] *hits* the rubber wall and then the rubber *shoots* it back. In this episode, the water and rubber analogy is brought up as a continuation of the description of the electron as a particle that moves out of and back into the wire magnetically.

Thus far in the conversation the students have been shopping around for ideas to try to link the mathematically derived wave function to a physical picture. In a metacognitive statement about his ontologies, Fernando highlights the difference between these spaces and then identifies what he sees as a conflict between them:

174 *Fernando: Yeah, that was pretty good. And then…I don't know, I*
175 *think it's just that this, this concept*
176 *still…escapes me a little bit. Yeah because I feel like*
177 *the same thing [gestures to Zach], I feel like, I, I*
178 *always like when I look at this I see an electron kinda*
179 *bouncing back and forth, but in class he keeps saying*
180 *it's like distributed throughout this at all times. And*
181 *then like at the nodes, it can never like bounce through*
182 *that. [Shrugs.]*
183 *Zach: Oh yeah.*
184 *[Fernando laughs]*

Fernando identifies that there is a difference between his intuition of a particlelike electron and the "right answer" from class that involves the electron being nonlocalized, and being represented as a wave function (which is sinusoidal inside the wire), such as the one drawn on his paper. In trying to reconcile these two ontologies, he encounters a conflict because the wave function representation suggests there are points (nodes) where the electron will never be found. With a shrug, Fernando marks this as something that needs to be reconciled. Zach agrees that this poses a problem, and returns to the fuzzy ball of probability idea:

185 *Zach: Yeah that's right…I like the fuzzy ball thing. It's*
186 *like, in that um…what was it, the matter wave demo sim*
187 *thing, where it like dims out when it splits [separates*
188 *his fingers on the table]…*
189 *Fernando: Oh yeah when it hits a wall and it kind of*
190 *dampens.*
191 *Zach: I kinda think about that. Yeah it's*
192 *like not an electron, it's just like a dim bigger—*
193 *electron…[gesturing a blob with his hands]*
194 *Fernando: Thing?*
195 *[Zach and Fernando laugh]*
196 *Jacob: It'd probably almost be useful to start thinking*
197 *of electrons like we would think of photons or whatever,*
198 *we would have to think about them like a particle*
199 *in some situations but like a wave in others 'cause like*
200 *right here [pointing to the wave function] we're thinking*
201 *about it like a wave for sure, but when we're talking*
202 *about it bouncing back and forth that's like a particle*
203 *kinda deal to me.*

Here, Zach and Fernando have used the fuzzy ball to help reconcile their perceived contradiction between particlelike and wavelike characteristics of the electron. Zach references a simulation that he relates to the fuzzy ball description of the electron; from his gestures and the overall course context, we believe he is referencing the PhET Quantum Wave Interference simulation (the simulation used in episode 1 to run a double slit experiment with a single photon [71]). In line 187, the "it" that dims out is ambiguous. We infer that Zach is suggesting that the electron is a fuzzy ball that acts similar to an entity in the sim which "dims out when it splits." In the class, the students have most often used this particular simulation to talk and think about photons, but the simulation gives the option of choosing a photon, electron, neutron, or helium atom. This recall of the simulation helps Zach and Fernando bring meaning to the fuzzy ball. That is, the picture from the sim helps them link the wave characteristics that they know are part of the right answer with their intuitive physical picture of material entities. The simulation representation can be thought of as a material anchor [77] for the electron as a fuzzy ball of probability. Zach and Fernando share some common understanding around the blended entity, fuzzy ball of probability, but Jacob on the other hand does not take up this language. Instead he advocates for holding the particle and wave ontologies in parallel: sometimes we should think about electrons as waves and other times we should think about them as particles.

In this episode, we see the group utilizing two different types of ontological structures—blended ontologies and parallel ontologies—both of which can be described as dynamic, although the nature of dynamics in these two cases are distinct. For the bulk of the conversation, the students are shopping around for ideas, trying out different analogies and descriptions. In doing so, they are attempting to reconcile two competing pictures: electron as a wavelike entity (invoked by mathematically derived representations on paper, and things the professor has said in class) and the more intuitive material or particlelike picture of the electron. Fernando begins the conversation by throwing out the phrase fuzzy ball of probability as a possible way to link the wave function representation with a physical picture of the electron, explicitly acknowledging his hesitancy around this idea as he does so. Although attempting to link the particlelike and wavelike ontologies, in practice the students hold them in parallel to one another. This is evidenced by the word "but" in Fernando's statement, "when I look at this I see an electron kind of bouncing back and forth, *but* in class he keeps saying it's like distributed throughout this at all times [emphasis added]" (lines 178–180).

The material or particlelike view of the electron includes thinking of it "bouncing back and forth," gestures of a small localized entity that has a definite location, and using language such as "it hit the wall" and "shooting back in." The wavelike ontology includes the mathematically derived wave function representation which has nodes and a notion of probability, and thinking of the electron as an unlocalized entity which is "distributed throughout." The students attempt to draw meaning from intuition as they try to connect the two parallel ontologies and form a cohesive description of the quantum tunneling phenomenon. The lack of reconciliation comes in line 181 when Fernando says "And then like at the nodes, it can never bounce through that." This marks a turn in the conversation where

Fernando and Zach no longer find it constructive to hold the two ontologies in parallel. Using recall of a simulation representation as a material anchor, they turn to the fuzzy ball of probability and collectively negotiate meaning around this idea. The fuzzy ball is an entity which draws from both particle and wave characteristics. The name "ball" suggests a material property; it includes a notion of probability or indeterminism, a uniquely quantum characteristic that the students do not discuss in depth but is mentioned in the name of the fuzzy ball of probability and is buried in their wave function representations; the fuzzy ball is "not really anywhere," suggesting a wavelike property of unlocalization; it hits a wall and then dampens, drawing on both particlelike language ("hits") and wavelike characteristics of propagation and diffuseness; it dims out when it splits, eluding to wavelike interference; and it is like a "dim bigger electron thing" suggesting a material entity with a wave-like characteristic of intensity. Figure 9 illustrates the fuzzy ball of probability as a conceptual blend, drawing from input spaces of particle and wave, and including elements (in italics) which come from or are informed by the recalled simulation.

While Fernando and Zach come to some common understanding of the fuzzy ball as a blend of two competing ontological spaces, Jacob does not take up this same language. The episode ends with Jacob making a bid for continuing to hold these ontologies in parallel by thinking about the electron as a wave sometimes, but a particle at other times. We note here that in this analysis there is no judgment as to whether the blended or parallel ontologies are "better." Rather, this episode provides evidence of *different types* of dynamic ontologies.

The conversation is full of hedging and tentativeness, signaling in the moment ontological negotiation. Even further, Fernando puts forth an idea that he questions and delivers with hesitation. We take this type of tentative reasoning to be inherently productive. As educators, we want our students to be able to put forth ideas they are unsure about and play with them in a flexible way to make meaning and develop understanding. Fernando does this here with the help of his peers, which we take to be valuable for the group because they continue to search for and make sense [28,32] of a satisfactory physical description of the tunneling phenomenon they see represented in the form of a wave function drawn on their papers. Fernando and Zach use the fuzzy ball idea to reconcile a contradiction they see between two ontologies, and in doing so they begin to develop a physical description of the wave function in this specific context, a goal that Fernando identified for the group at the beginning of the episode when he eluded to the value of thinking physically about quantum phenomena despite the unintuitive nature of doing so. For Jacob, the tentative reasoning and collective negotiation resulted in a different ontological structure which was productive for him in the moment. In this episode, we see two students (Fernando and Zach) shifting locally from a material ontology to something that can be defined as a hybrid between matter and wave ontologies (the fuzzy ball). We make no claims of stability beyond these observed moments. The blended ontology is locally sustained for Fernando and Zach, and the parallel ontologies are productive in the moment for Jacob. This episode provides examples of how ontologies can be dynamic in either construction (blended) or application (parallel).

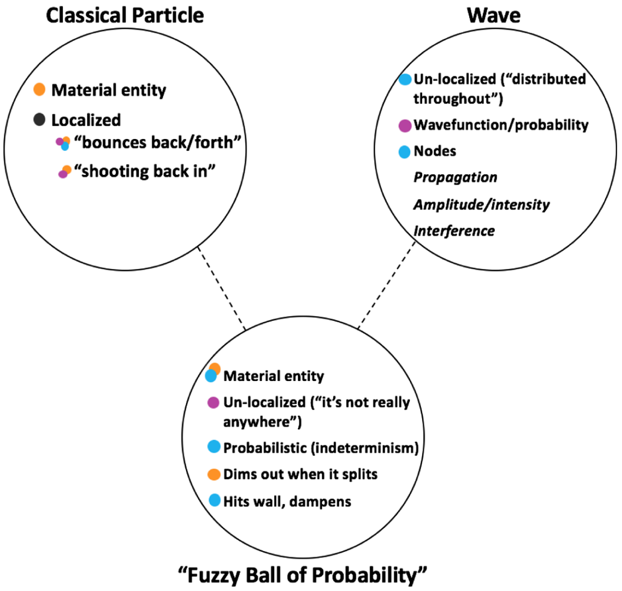

FIG. 9. We model the conversation between Fernando (blue), Zach (orange), and Jacob (pink) in episode 3 as a collective conceptual blend, where classical particle and (semiclassical or quantum) wave form the input spaces for the fuzzy ball of probability blend. Part of the wave input includes the material anchor [77] of a simulation that the students recall (elements indicated in italics). The elements indicated with a gray circle were not spoken by any one individual, but rather implied by statements from various group members.

VI. SYNTHESIS AND DISCUSSION

The three focus group episodes provide examples of students reasoning collectively about the nature of a quantum entity. In each example, we see nuanced and complex reasoning processes and flexible use of ontologies. In episode 1 we see Eric, Tara, and Bryan collectively negotiating the ontology of a photon, which they come to describe as a blob of EM wave, an ontological conception that draws from both classical particle and wave but that cannot be fully mapped onto either. We see the students construct emergent meaning around the energy of the photon, and we also see hints of parallel ontological structure within the blended ontology (lines 11–12), suggesting that the different types of dynamic ontologies may not be mutually exclusive. Episode 2 illustrates how two students can come to different interpretations of a superposition state from a shared understanding of the properties and behaviors of a photon. In episode 3 we see evidence of different types of dynamic ontologies: parallel (switching back and forth between stable notions of particle and wave) and blended (constructing a new ontology on the fly, which is a blend of particle and wave properties). All three episodes provide evidence of the dynamic nature and students' flexible use of ontologies, and demonstrate what it looks like for students to engage in messy and tentative reasoning when negotiating conceptual understanding of quantum phenomena. The large scale methods of selecting and analyzing the data are the same across all episodes, but the results are determined by which form of analysis the data lend themselves to. For example, in episodes 1 and 2 we map the ontological reasoning using the conceptual blending framework, but in episode 3 only part of the conversation can be mapped using the conceptual blending tool while the rest is best described with a parallel ontological structure.

Our analyses lead us to identify a preliminary model of four different approaches to understanding quantum entities (Fig. 10). The first is a purely classical ontology (e.g., electrons are classical particles). Chi's framework would say that if a student holds this (wrong) ontology, they must undergo radical conceptual change in order to reassign the electron to the correct category [20]. We would say that this ontology is bad in some circumstances (i.e., in a quantum context it is wrong to think of electrons only as classical particles). A dynamic ontologies framework [23] would suggest that students have the ability to reason across ontological boundaries and to use ontologies flexibly. So, a student holding a classical particle ontology of an electron in one moment could be cued into activating different ontological resources and flexibly utilize a different ontology in another context or moment. The second type of quantum ontological structure is when one applies robust classical structures to quantum entities (unidirectional arrow). This could be the Bohr model of the atom (which is productive in many situations), or a hidden variable interpretation of quantum mechanics (which may or may not be useful as a stepping stone to reason through); these can be destructive if they are the *only* models used. Similarly, the third type is a combination of classical and quantum structures, with a bidirectional arrow in between. That is, instead of simply applying the structure and properties of classical entities to a quantum entity, the student uses both classical and quantum properties flexibly. An example of this could be the analogical mapping of a rubber barrier and water wave to potential barrier and wave function in quantum tunneling. If students applied this analogy as a one-way mapping of one domain to another [78], we would classify it as the second type of approach. If it were applied as more of a blend analogy [79], blending the hybridity of quantum understanding with classical mechanics concepts, we would consider this to be the third type of approach. Another example could be particle-wave duality. The second and third approaches describe different applications of classical concepts that create tools for QM understanding. The fourth ontological approach is one that uses purely quantum descriptions of a quantum entity, identifying mathematical and physical entities which come together to describe QM. In Fig. 10 these four approaches to reasoning about quantum entities are listed in increasing degree of sophistication from our perspective. We are not suggesting that a student learning quantum mechanics must go in order 1–4 (but maybe sometimes they do), or that types 2 and 3 are bad. In fact, most of our data live in these two middle types, and even expert physicists often make use of combined classical and quantum descriptions of quantum entities. We do not wish to make judgments about the relative utility or sophistication of types 2 and 3 (as shown in Fig. 10, we consider them to be equivalent in this sense). A deterministic hidden variable interpretation (such as we see reflected in Tara's statements in episode 2) is ultimately not correct and not useful for physicists, but may be a stepping stone to understanding QM. In comparison, the Bohr atomic model, also an example of applying classical structures to quantum entities, is productive for students [80] and often used by expert physicists. We suspect that each of the four models could be applied productively or unproductively. This is a preliminary model based on the data and analysis presented in this paper as well as our experience teaching and learning QM. Future work will continue to investigate and refine these approaches.

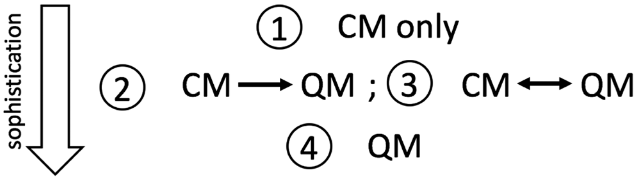

FIG. 10. Preliminary model of four different ontological approaches to QM sense making. Classical mechanics (CM).

VII. LIMITATIONS

A key limitation of this work is the small sample size. We do not seek to map the small group conversations between a total of six modern physics students onto a broader modern physics or QM student population (here at the University of Colorado, or even broader to other universities or contexts). These six students were some of the top-performing students in the class, and self-selected into the focus group study. Nonetheless, the three episodes presented here demonstrate that this type of dynamic ontological reasoning is plausible. The snippets of student reasoning we have selected and analyzed here are not representative of all students, but they *are* recognizable kinds of discourse. That is, not all students (or groups) go through these exact kinds of reasoning practices, but the methods we provide and the resulting refinement of a dynamic ontologies framework are particularly useful for thinking about the nature of student reasoning, and further, how to leverage these student practices and capabilities. While qualitative research of this kind may not be externally generalizable in the probabilistic sense of large scale quantitative work [81], we do believe the results are transferable and applicable to other individuals who matter in the enterprise of physics education.

Coming from a sociocultural perspective, we chose to focus on collective conversations, attending to both individual and collective reasoning. There are some limitations to this approach. First, analyzing group discourse makes it hard to know how individual students are reasoning. We often cannot tell if statements voiced by students are personally held ideas. Ultimately, knowing each student's personal relationship to the ideas put forth in the groups was not the purpose of this study, and thus this limitation did not present an issue for our analysis. It is, however, important to keep in mind when reading and interpreting these results. Second, there are social dynamics of the groups which intersect strongly with the collective reasoning. In this paper we have chosen to foreground the reasoning structures of the group, but note that this is certainly influenced by the social dynamics unique to each group of students. We leave further investigation of the social dynamics for future work.

As noted in Sec. V, it is likely that in addition to the prompts and content areas, the simulations influence the type of ontological reasoning we see the students engaging in. In episodes 1 and 2, the simulation is part of the task itself and so we cannot separate it from the prompt. In episode 3, the students are not asked to use a simulation, but in discussing the phenomenon of tunneling they reference a prior simulation. We can think about (the recall of) the simulation in this case as playing the role of a material anchor [46] for the blended ontology. We are not able to account for the influence of the simulations on students' reasoning in our analysis. The nature of the contexts (whether social or pedagogical) in which students are answering and discussing these questions is a subject of future study.

VIII. CONCLUSIONS

Using evidence from three episodes of student reasoning, we have outlined a framework of dynamic ontologies. This framework includes an underlying organizational structure which is flexible in nature, and in which reasoners can make flexible use of existing stable structures as well as develop and negotiate new blended structures. We argue that the messiness of student reasoning should be valued and that the tentative nature of students' contributions to collective discourse are valuable for learning QM. The quote from Richard Feynman in the introduction (Sec. I) can have many interpretations. One is that there is a monolithic understanding of QM—you either get it or you don't. We choose an alternative interpretation which aligns with our argument for valuing the messiness of student reasoning: Everybody understands QM, incompletely, in their own messy ways. Lastly, we have demonstrated the utility of collective conceptual blending as an analytic tool to unpack the nuances of students' dynamic and tentative reasoning structures in QM, contributing to a broader research community that values complexities of student reasoning.

The theoretical contributions of this paper are not (yet) designed to focus on instructional implications, though the work here does suggest that instructors of modern physics and QM courses (and probably physics in general) should allow students space to engage in messy reasoning. One specific instructional implication is a challenge to us; namely, how we should go about teaching wave-particle duality. Do we allow students the space to engage in messy and tentative reasoning or to construct their own nuanced ontological conceptions of photons and electrons? Do we encourage students to use blended or parallel ontologies with the words we use and the questions we ask? These questions are worth investigating. Additionally, when designing curricula for modern physics or QM researchers and teachers should attend to the dynamic nature and students' flexible use of ontologies.

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